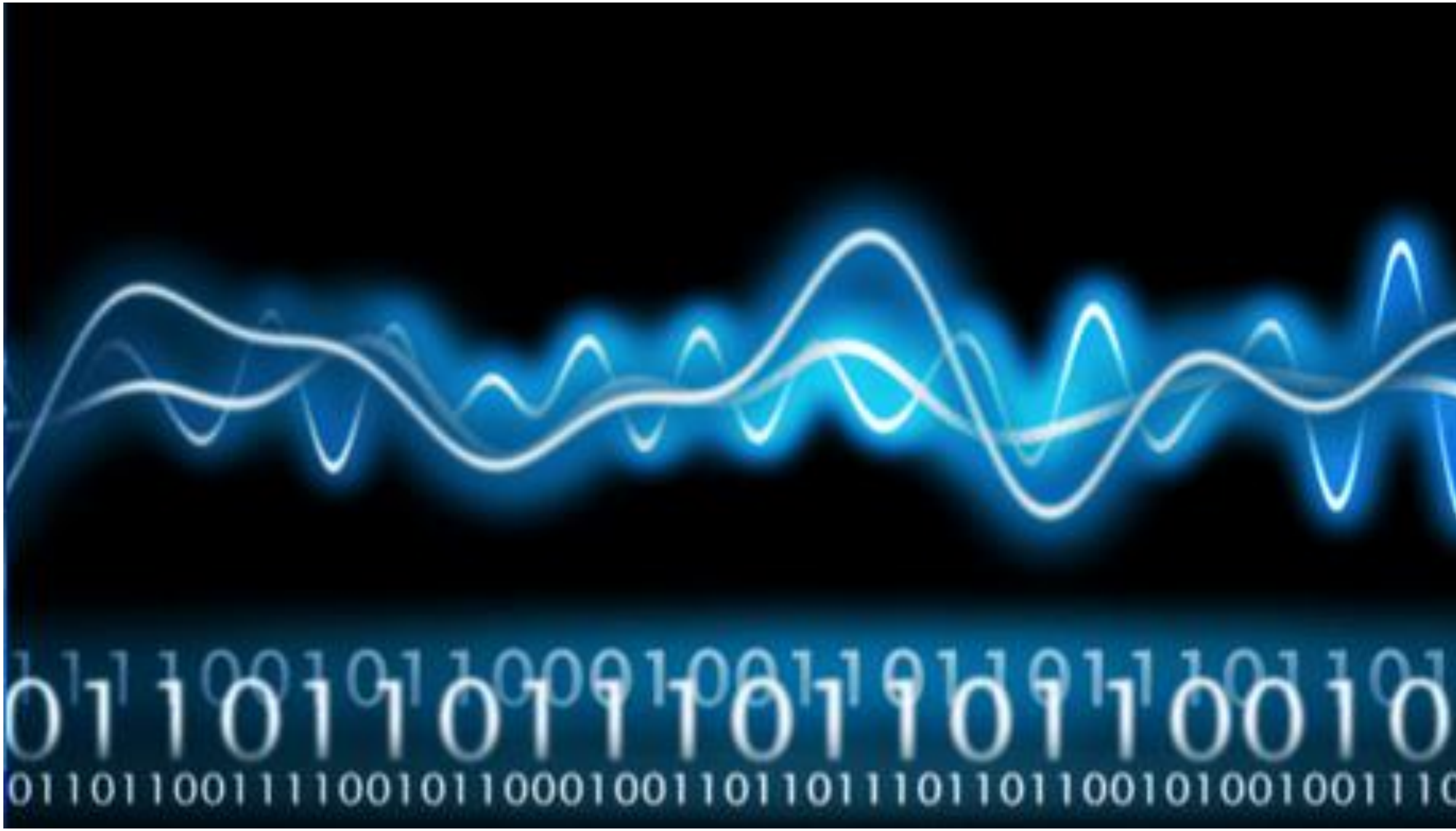


Digital Signal Processing (DSP)

LECTURE 01

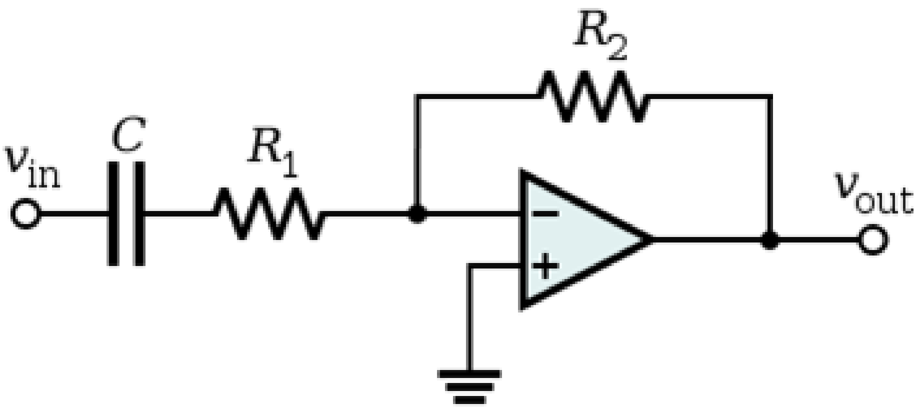
Introduction to Digital Signal Processing

- Introduction Digital Signal Processing
- Application of Digital Signal Processing
- Discrete-Time and Continuous-Time Signals
- Fundamental Sequence
- Basic Sequence and Operations
- Periodic and Aperiodic Sequences
- Even and Odd symmetry

Introduction to DSP

- Digital signal processing (DSP) refers to anything that can be done to a signal using code on a computer or DSP chip".

- Originally signal processing was done only on analog or continuous time signals using analog signal processing (ASP) e.g. Electrical filters using Resistors, capacitors and inductors, amplifiers etc.



Analog Signal Processing Higher pass filter

Application of the DSP

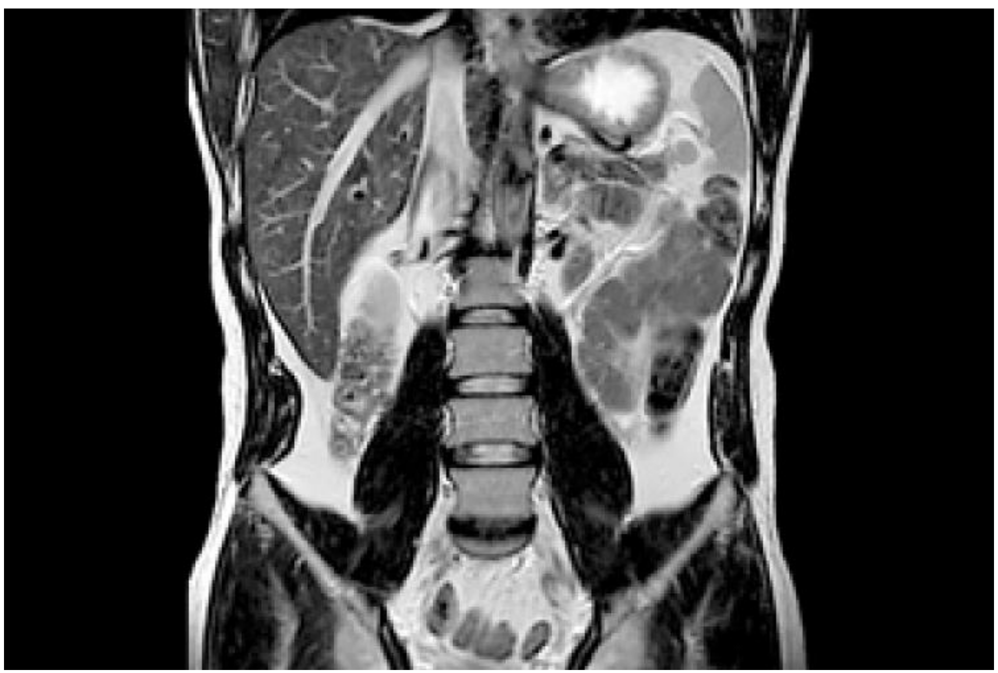
1. Space:

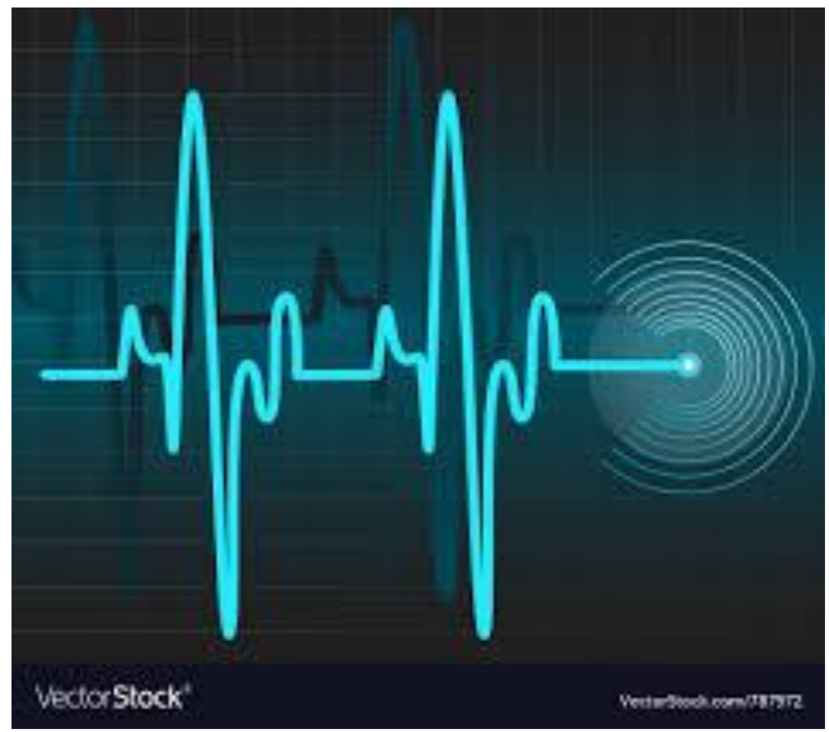
- Space photograph enhancement
- Remote space probes
- Data compression


Application of the DSP

2. Medicine:

- Diagnostic imaging (CT, MRI, ultrasound,)
- Electrocardiogram analysis
- Medical image storage/retrieval

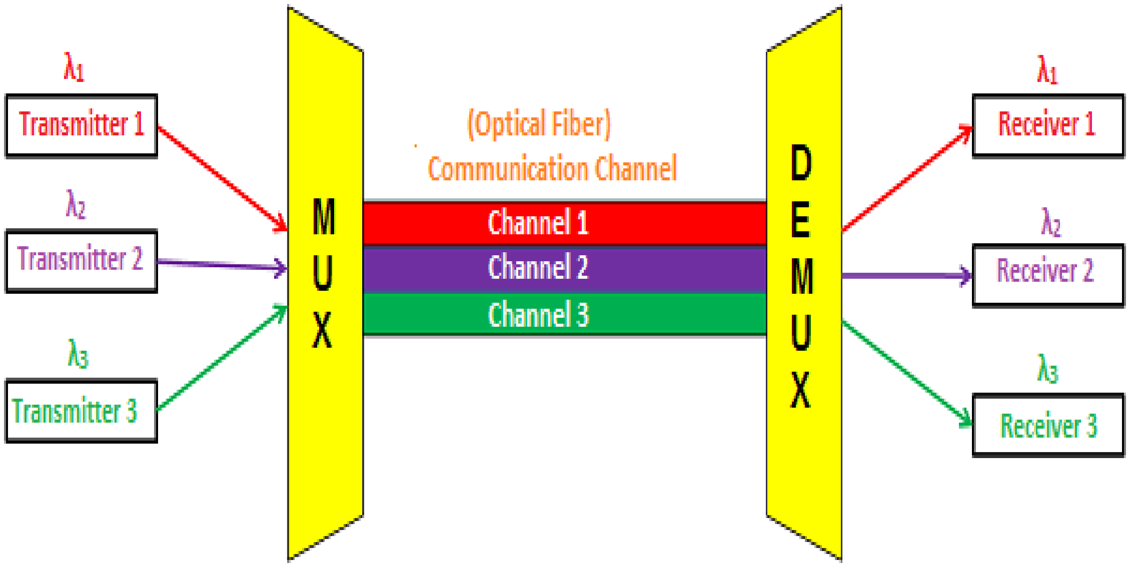




Application of the DSP

3. Telephone:

- Voice and data compression
- Signal multiplexing
- Filtering



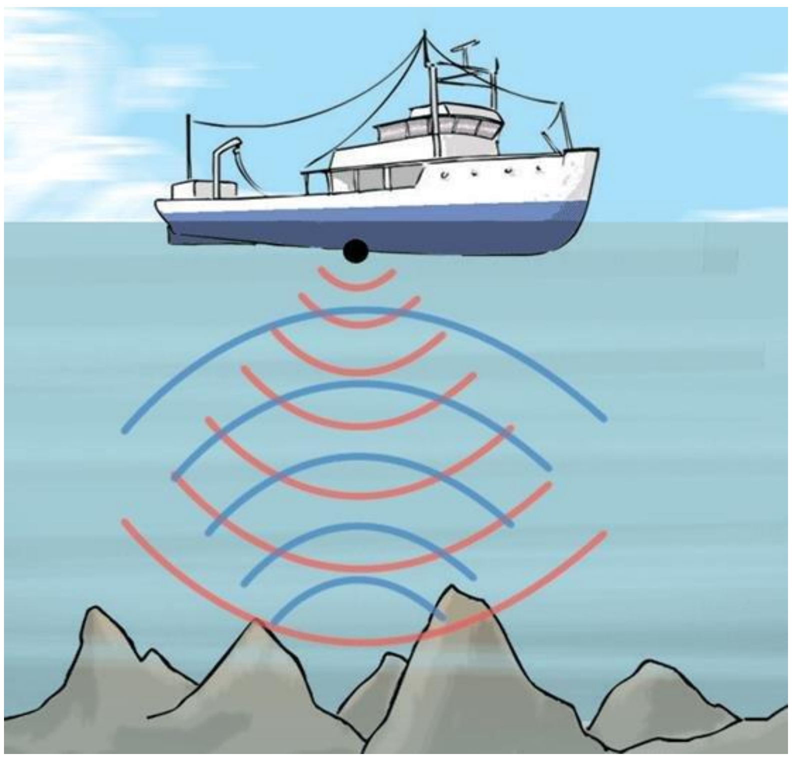
Wavelength Division Multiplexing

Physics and Radio-Electronics

Application of the DSP

4. Military:

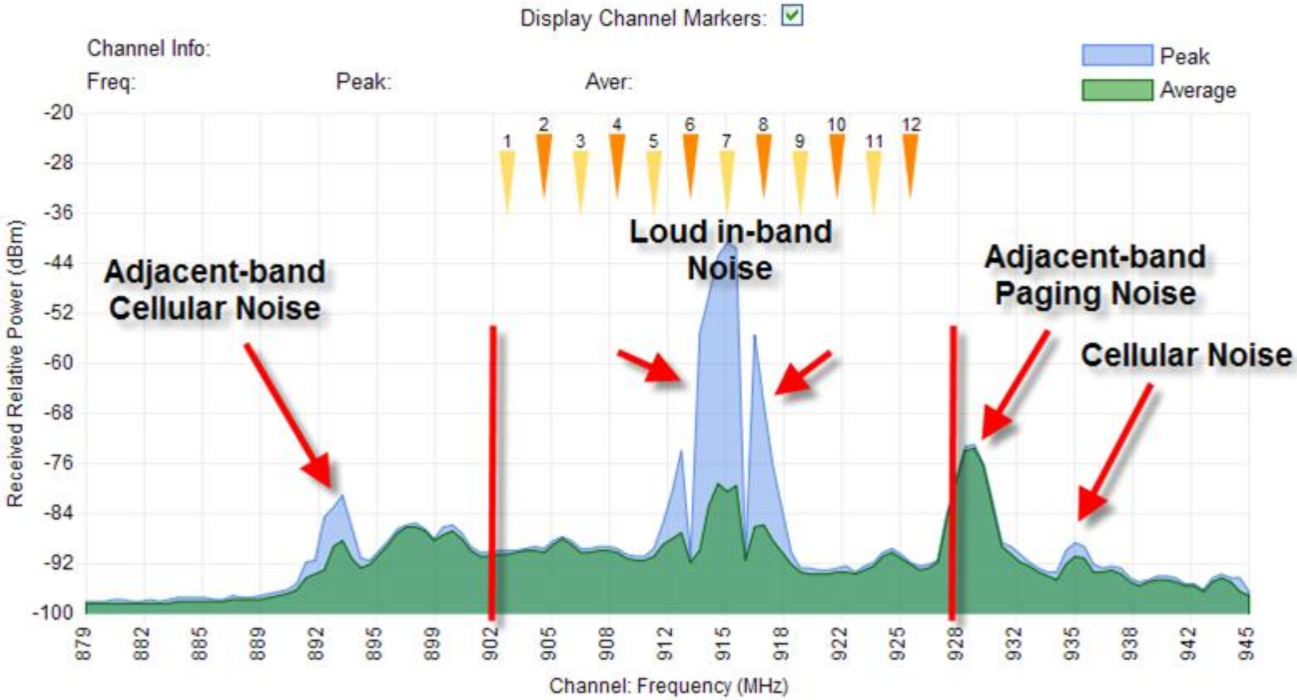
- Radar (Radio Detection and Ranging)
- Sonar (Sound Navigation and Ranging)
- Secure communication



Application of the DSP

5. Scientific:

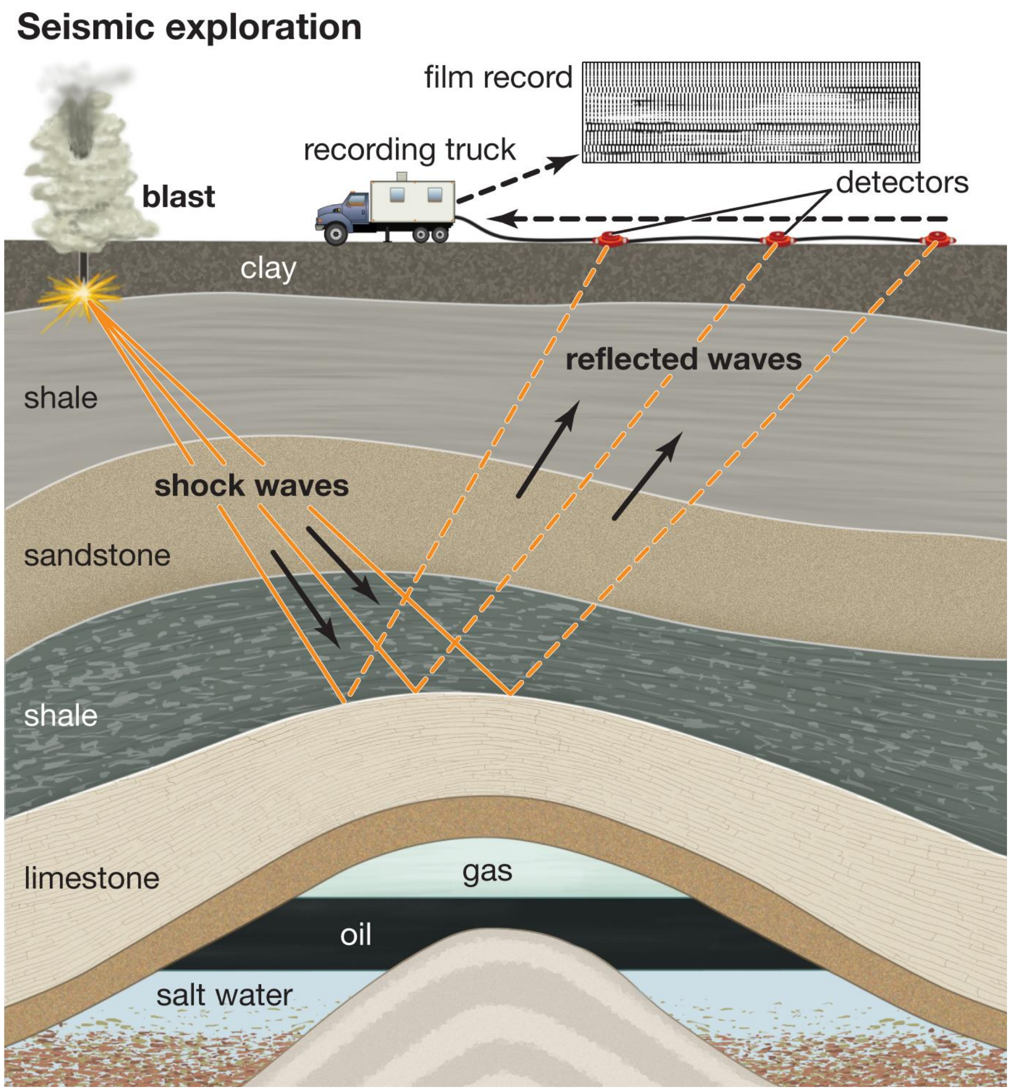
- Earthquake recording & analysis
- Data acquisition
- Spectral analysis
- Simulation and modeling



Application of the DSP

6. Industrial:

- Oil and mineral prospecting
- Process monitoring & control
- CAD and design tools



Application of the DSP

7. Commercial:

- Movie special effects
- Video conference calling

Overview of the DSP System

- In real life most of the signals are analog in nature, to implement DSP on computer some fundamental steps are followed”.

1. An analog signal is sampled at a regularly spaced time interval to form a sequence of the signal amplitude.
2. The sampled sequence of the analog magnitude is converted into a binary number.
3. The sampling and conversion are done with an analog to digital converter **(ADC).**
4. Perform some operations (Processing) on the digitized analog signal to get an output value.
5. Convert the processed output value into an analog signal using analog to digital Converter **(ADC)**

Overview of the DSP System

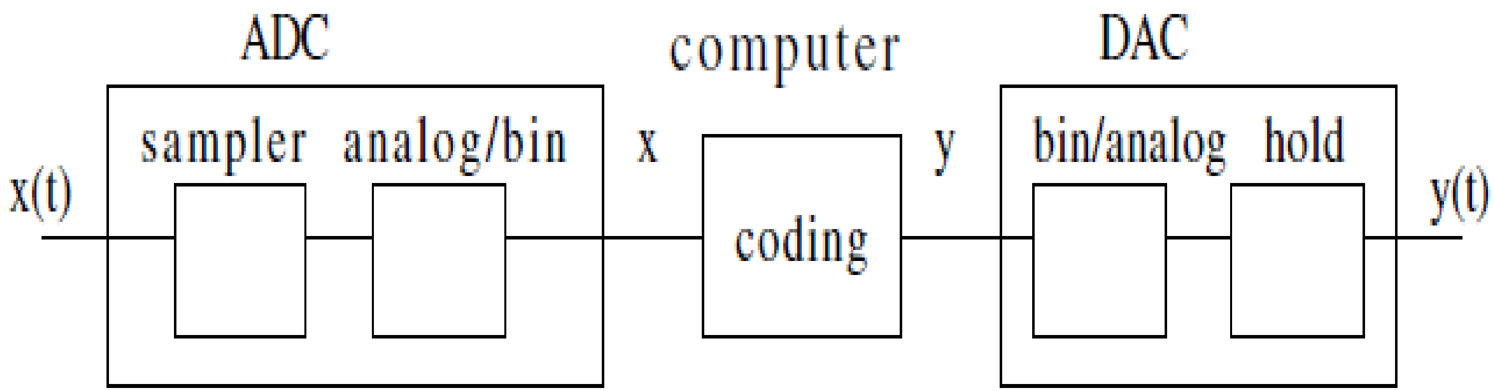


Figure 1.1 Basic DSP Systems

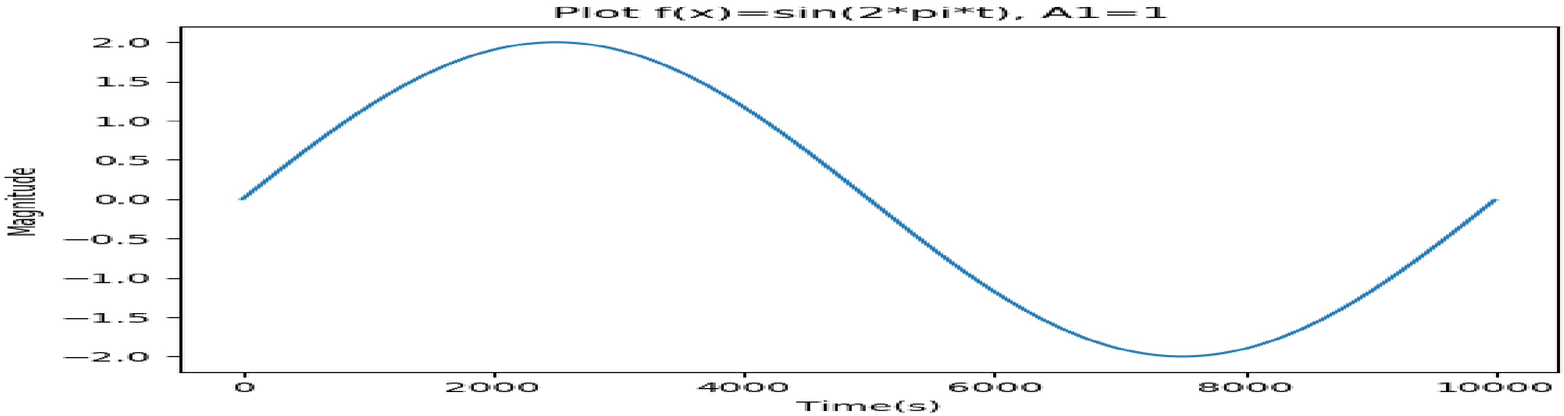
$DSP\ Signal\ conversion\text{:}\ x(t) \rightarrow x(n) \rightarrow y(n) \rightarrow y(t)$

$x(t), y(t), \rightarrow Continous - Time\ Signals, or\ analog\ signals$

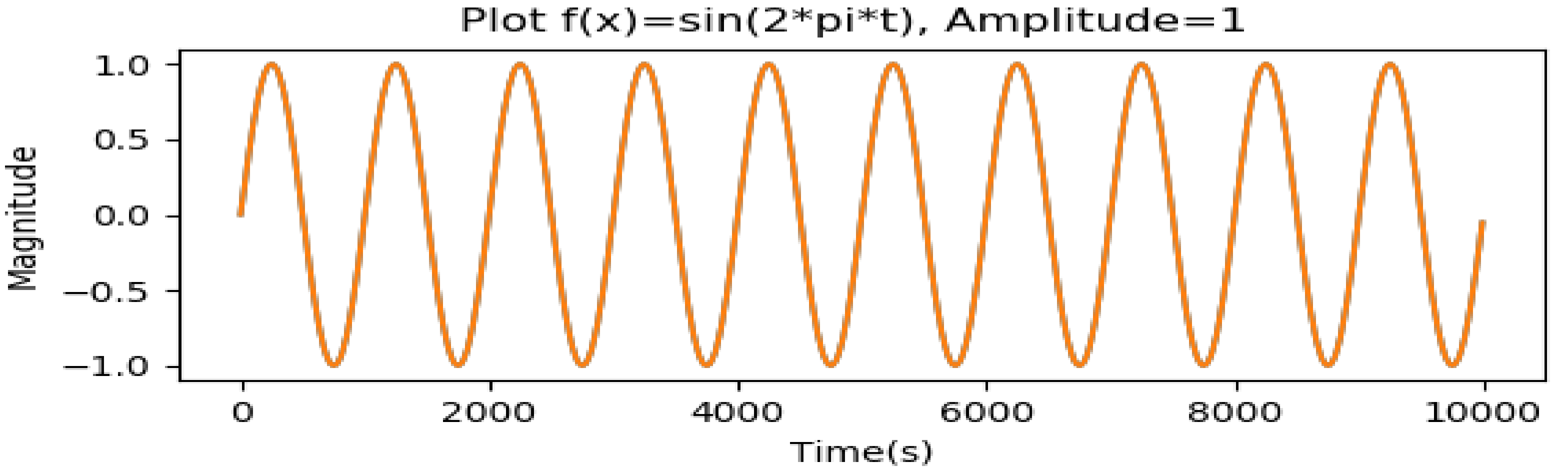
$x(n), y(n), \rightarrow Discrete - Time\ Signals\ or\ Digital\ Signals$

Continuous Time Signals

$$F_1(t) = a_1 \sin(2\pi f_1 t)$$

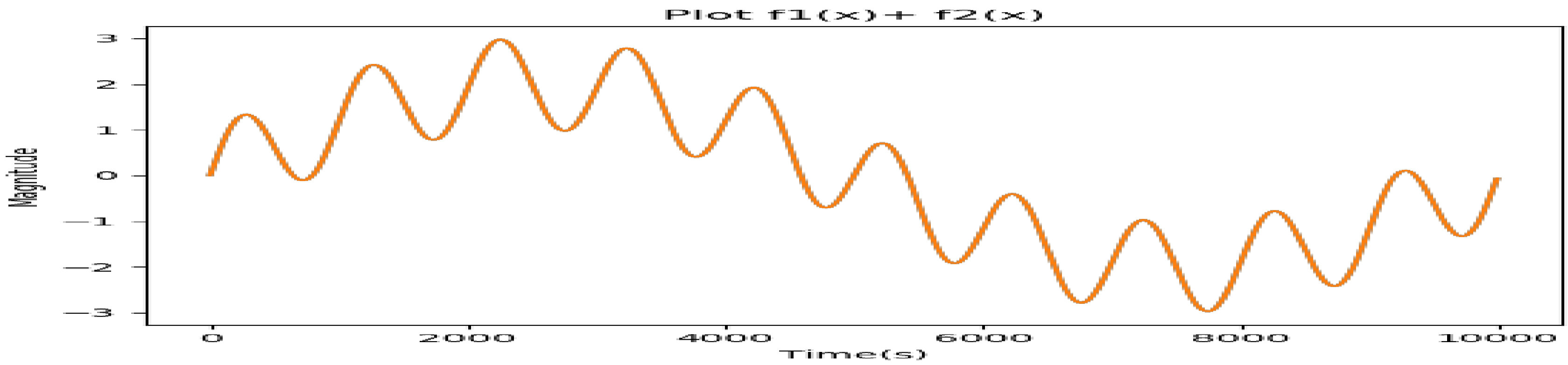


$$F_2(t) = a_2 \sin(2\pi f_2 t)$$

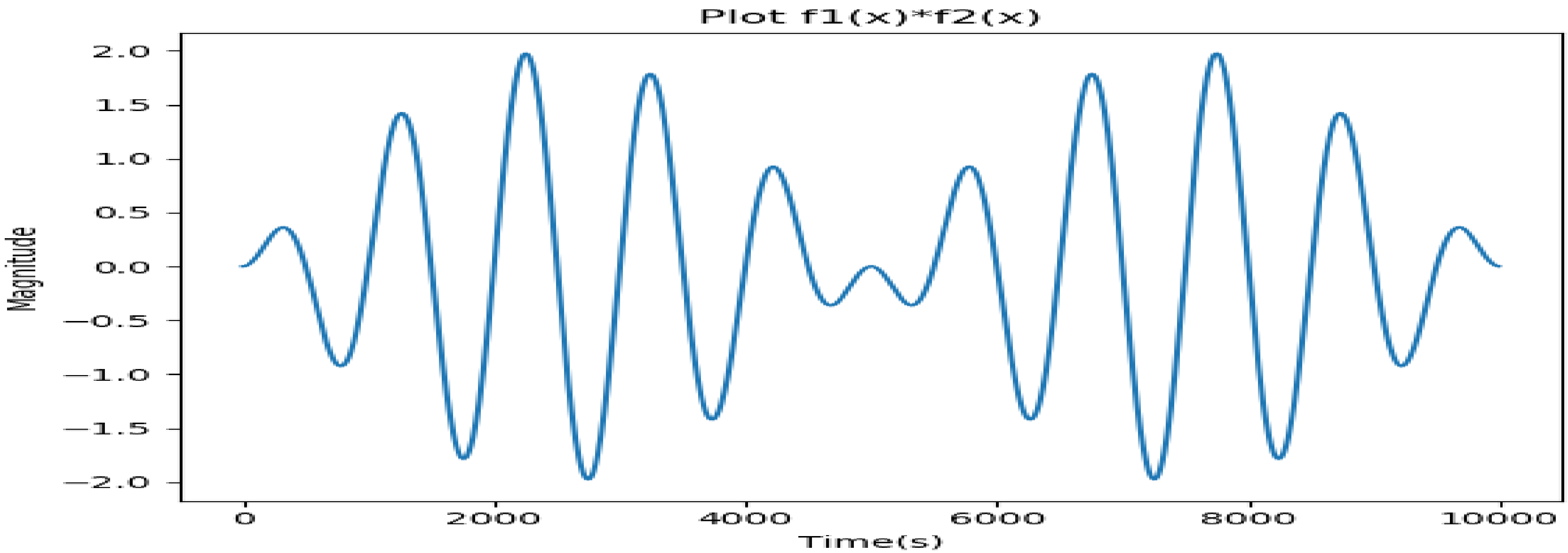


Continuous Time Signals

$$addtion\ of\ two\ CT\ signals\ F_3(t) = a_1 \sin(2\pi f_1 t) + a_2 \sin(2\pi f_2 t)$$



$$Product\ of\ two\ CT\ signals, F_4(t) = a_1 \sin(2\pi f_1 t) * a_2 \sin(2\pi f_2 t)$$



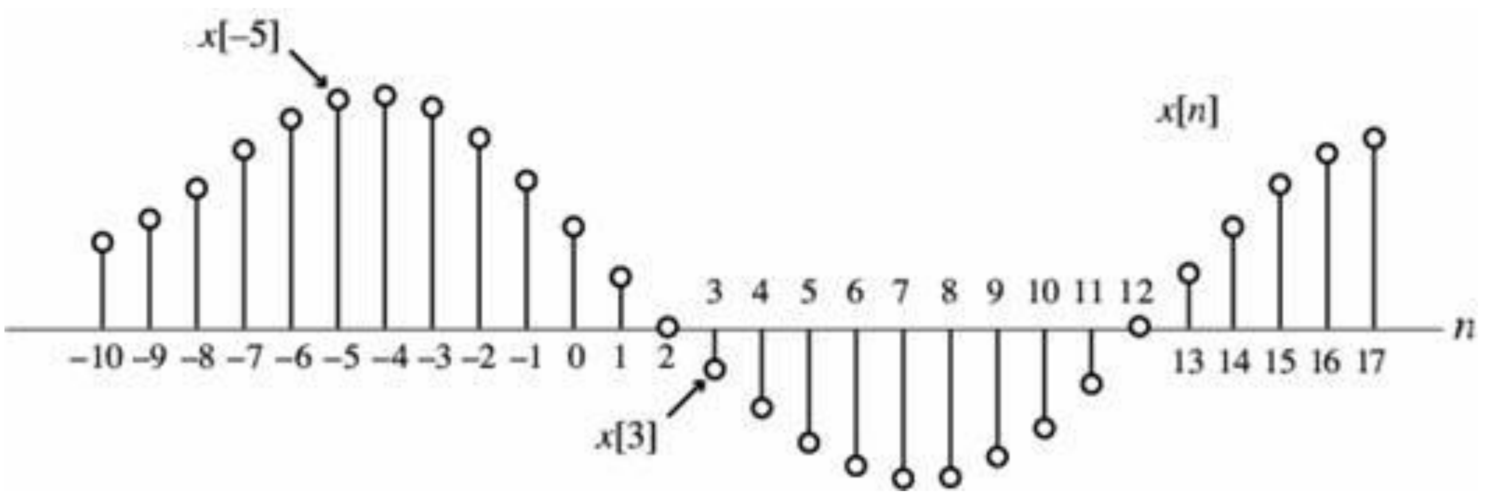
Discrete Time Signals

Discrete Time (DT): are signals obtained by Sampling CT signals at regularly time interval T. They are mathematically represented as sequences of numbers.

- The nth sequence of numbers x, is denoted as $x[n]$

$$x = \{x[\mathrm{n}]\} \qquad\qquad -\infty < \mathrm{n} < +\infty$$

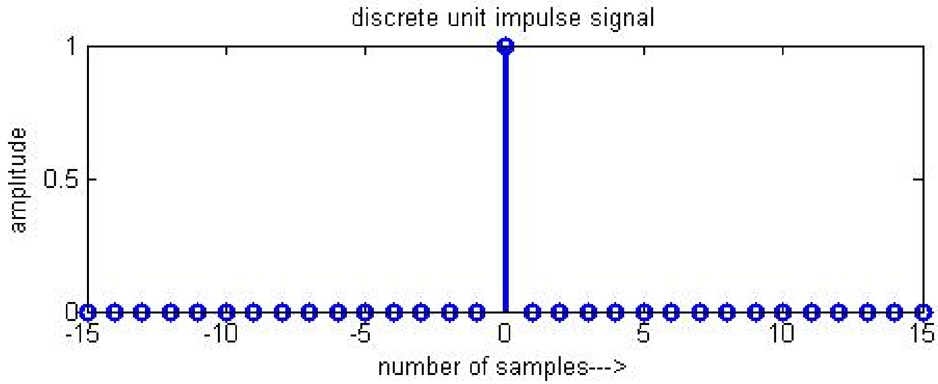
$$x[n] = f(t) = f(nT) \qquad t = nT$$

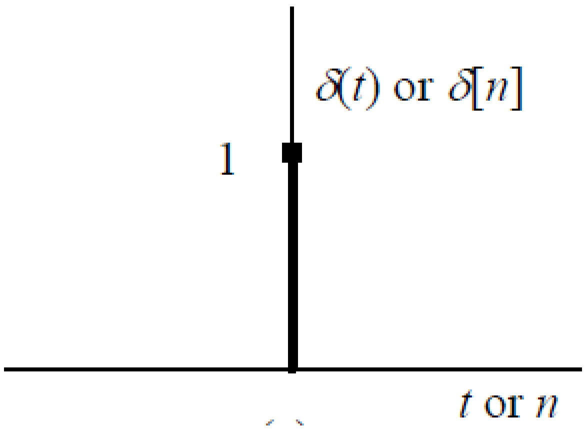


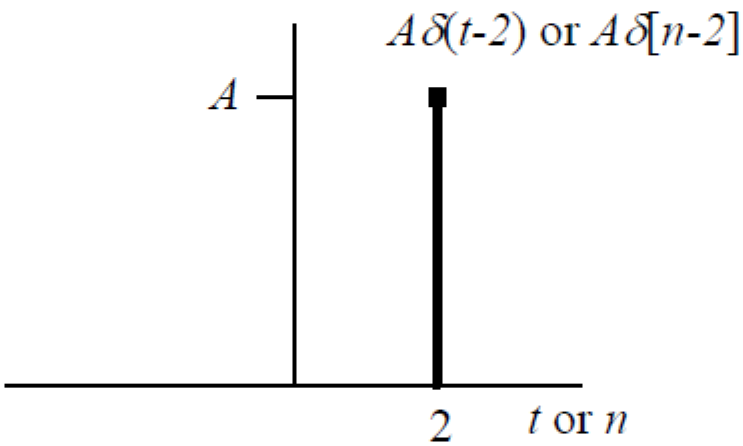
Fundamental Sequence

Unit impulse : The unit function is one of the most important function and is defined as $\delta(n)$ (delta)

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$$



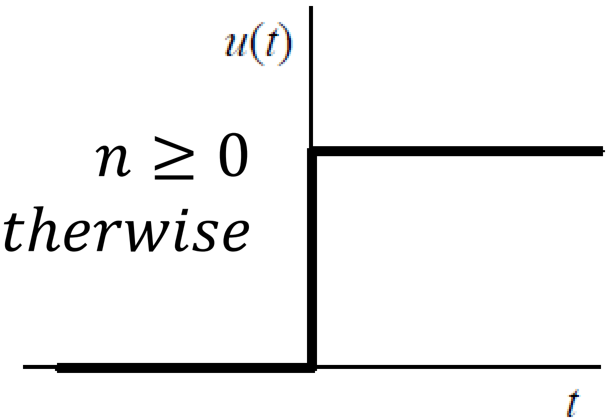


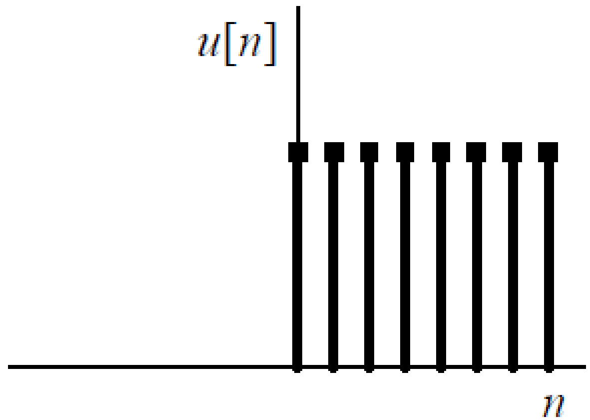


Fundamental Sequence

Unit step: The unit step function $u(n)$ is defined as:

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & otherwise \end{cases}$$





- Relation ship between $\delta(n)$ & $u(n)$

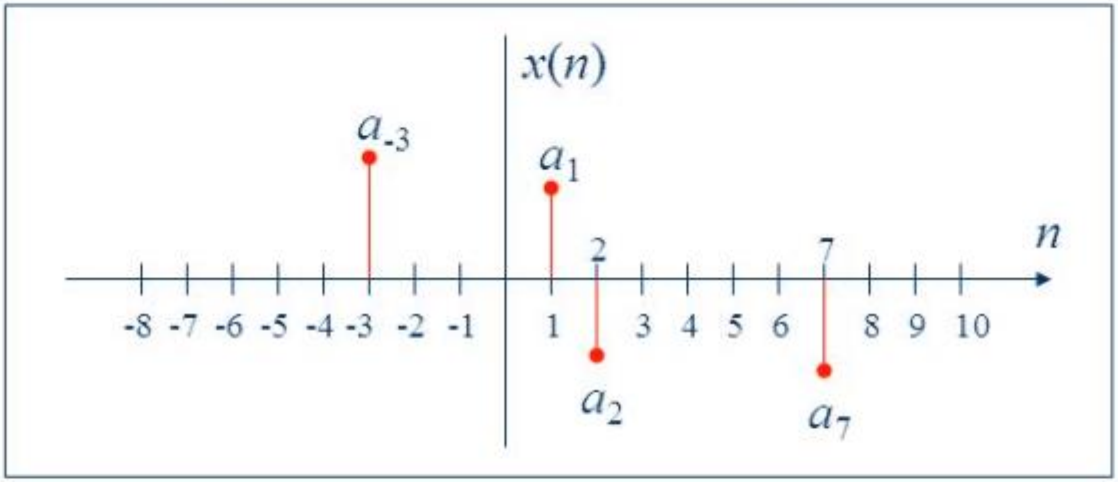
$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

$$\delta(n) = u(n) - u(n-1)$$

Fundamental Sequence

Unit Impulse sequence Representation

$$x(n)=\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$



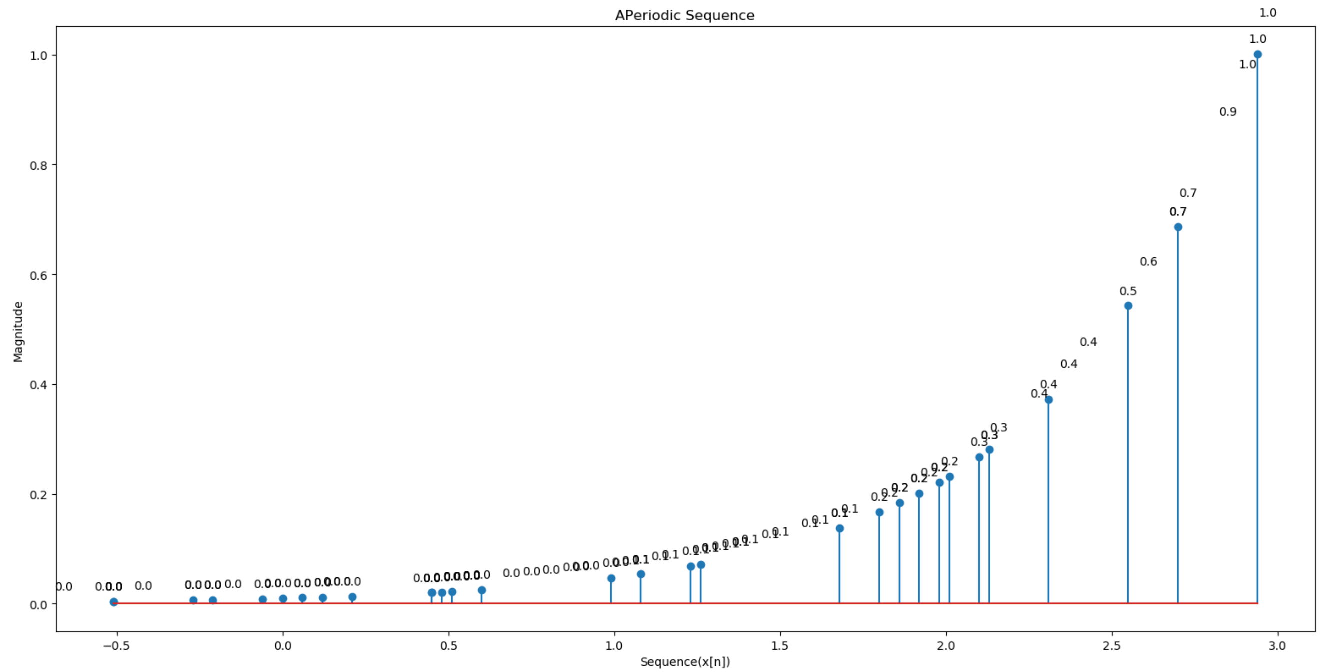
$$x(n)=a_{-3}\delta(n+3)+a_1\delta(n-1)+a_2\delta(n-3)+a_7\delta(n-7)$$

$$u(n)=\sum_{k=-\infty}^{n}\delta(k)$$

Fundamental Sequence

Exponential: an exponential sequence is defined such that it takes the form

$$x(n) = a^n$$



- Complex exponential can be expressed in the form $e^{-i\omega}$

$$x(n) = e^{-i\omega n} \quad or \;\; x(n) = \cos(\omega n) - i\sin(\omega n)$$

Basic Sequence and Operations:

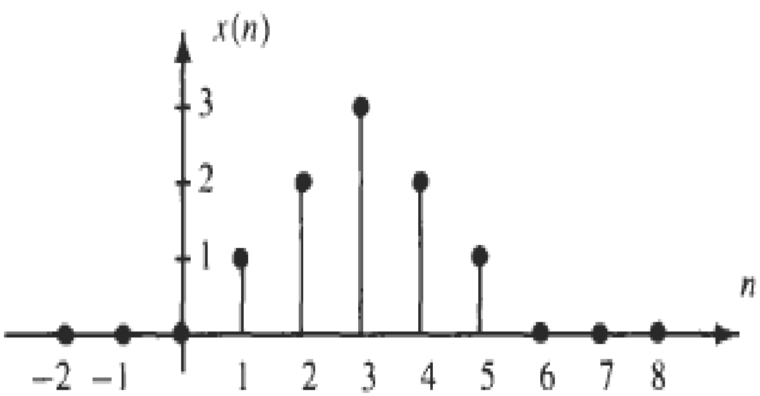
Shifting: is the transformation defined by $f(n)$

$$f(n) = n \pm n_0.$$

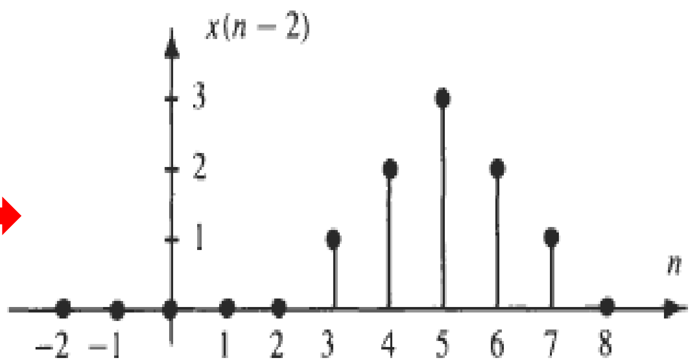
- For $y(n) = x(n - n_0)$, the sequence $x(n)$ is shifted to the right by n_0 samples and is referred to as a **Delay**.

Example: $x(n) = \{0,1,2,3,2,1,0,0,0\}$ $\quad n = [0, 1, 2,3,4,5,6,7,8]$

$Delay$: $x(n-2) = \{0,0,0, 1,2,3,2,1,0\}$ $\quad n = [0,1,2, 3, 4,5,6,7,8]$



(a) A discrete-time signal.



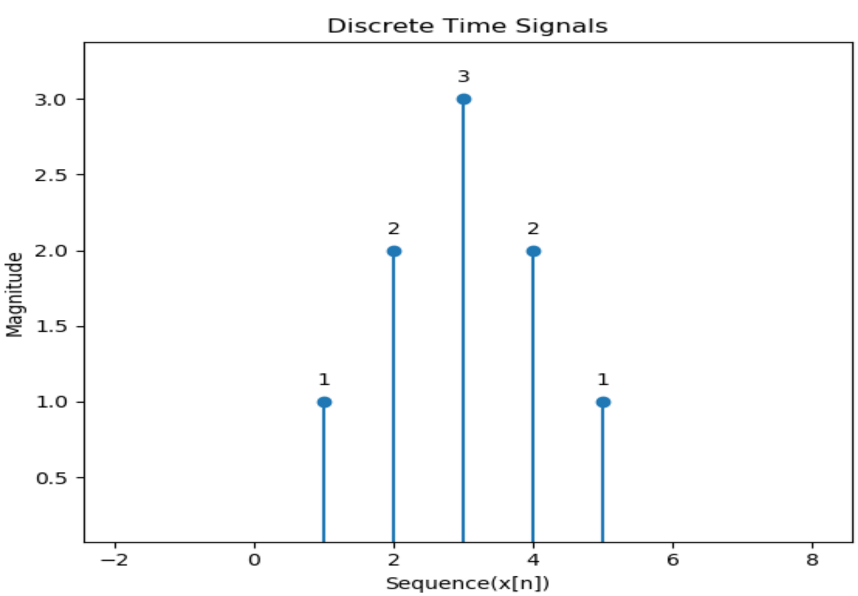
(b) A delay by $n_0 = 2$.

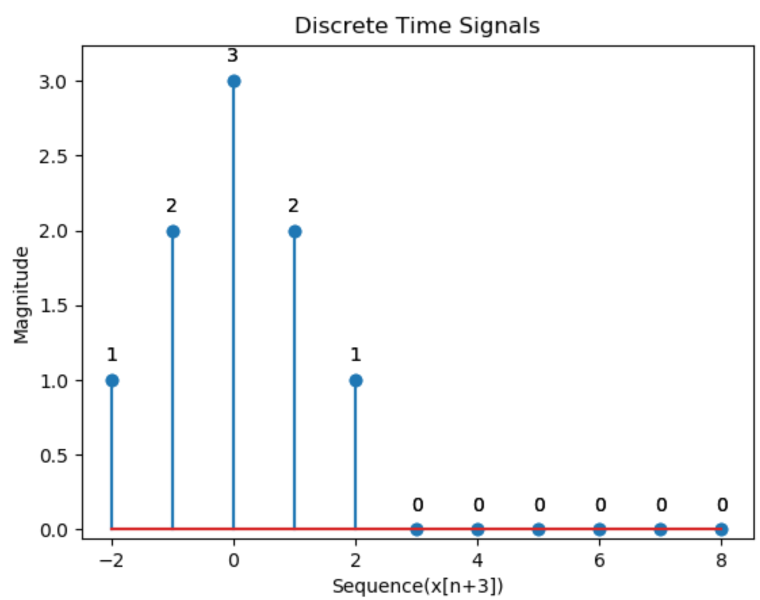
Basic Sequence and Operations:

- For $y(n) = x(n + n_0)$ the sequence $x(n)$ is shifted to the left by n_0 samples and is referred to as a **Advance**.
- Example:

$$x(n) = \{0,1,2,3,2,1,0,0,0\} \qquad n = [0, 1, 2,3,4,5,6,7,8]$$

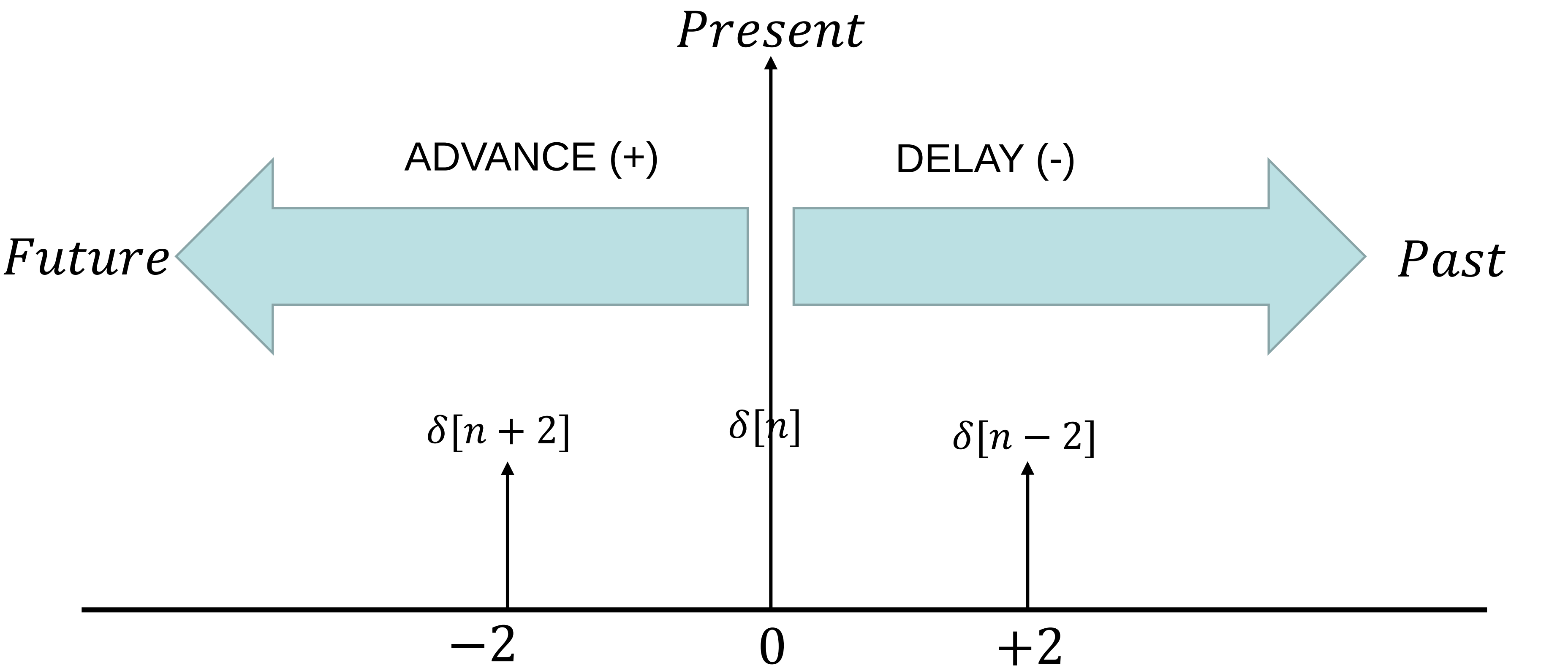
$$Advance: \; x(n + 3) = \{0,1,2,3,2,1,0,0,0\} \;\; n = [-3, -2, -1,0,1,2,3,4,5]$$





Basic Sequence and Operations:

Shifting Rule



Basic Sequence and Operations:

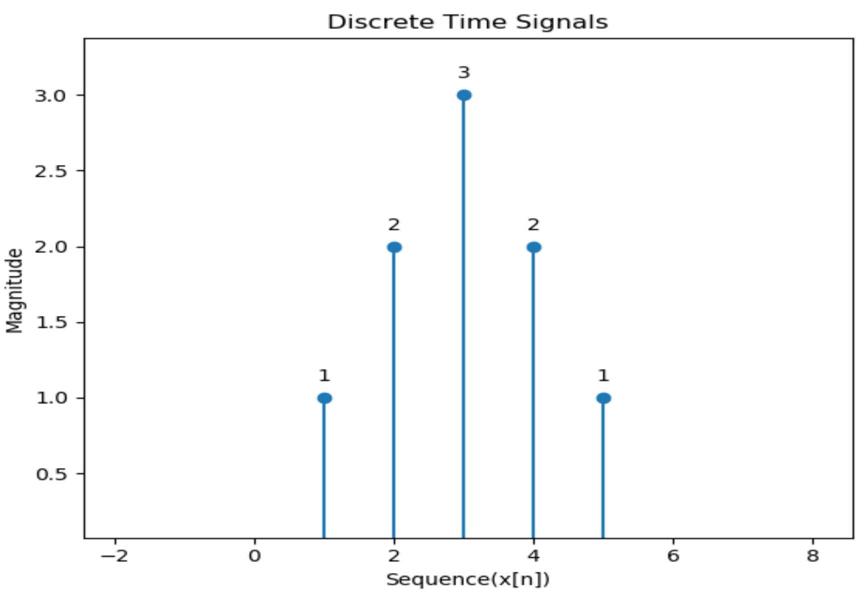
Scaling: a sequence $x(n)$ is said to be scaled by a constant real number A, if defined according to the relation below.

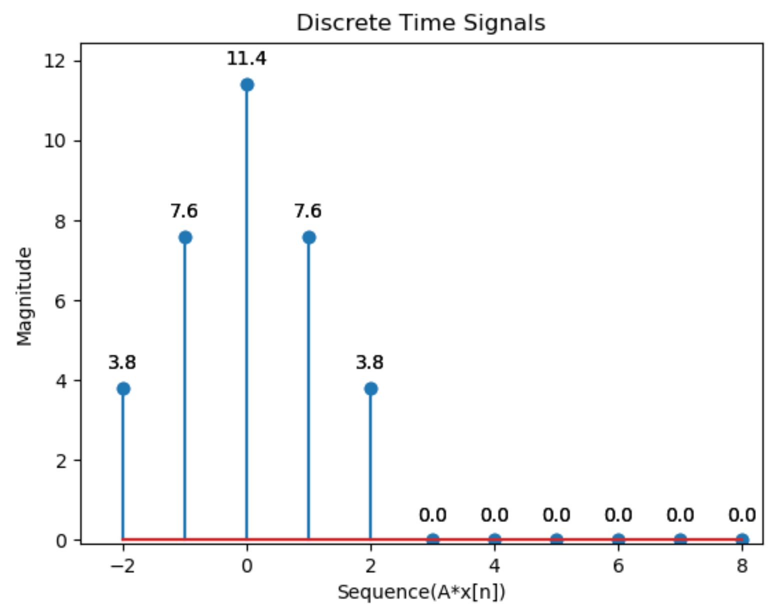
$$y(n) = A * x(n)$$

Example:

Let $x(n) = \{1,2,3,2,1,0,0,0,0,0\}$ $and\ A = 3.8$

Scaling: $A * x(n) = \{3.8, 7.6, 11.4, 7.6, 3.8,0,0,0,0, 0,0, \}$





Basic Sequence and Operations:

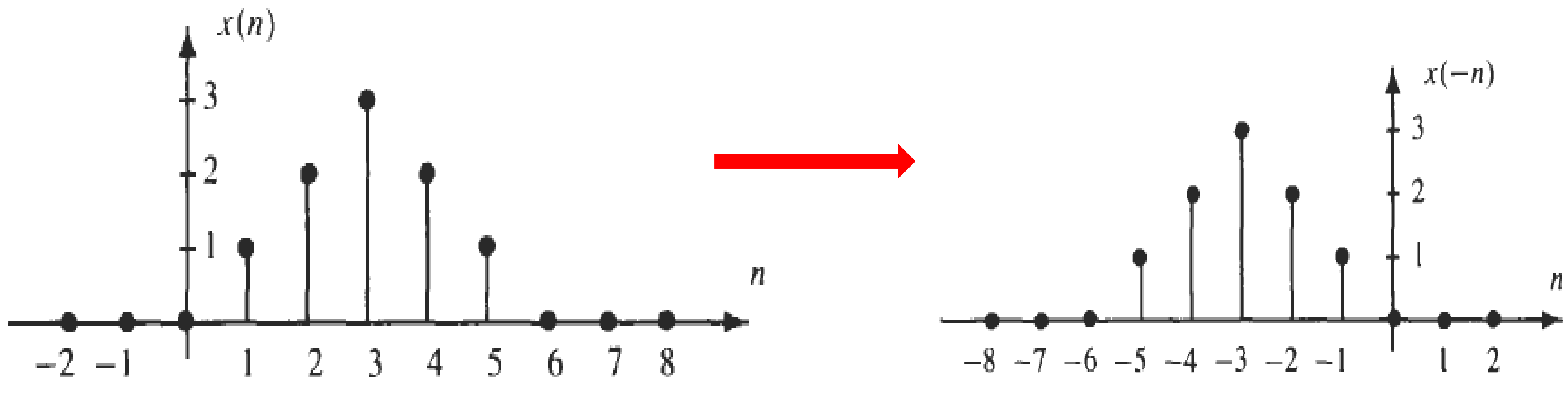
Time Reversal: the sequence $x(n)$ is said to be time-reversed or flipped if

$$\text{y}(n) = x(-n)$$

Example:

Let $x(n) = \{0,1,2,3,2,1,0,0,0\}$ $\quad n = [0, 1, 2,3,4,5,6,7,8]$

$\boldsymbol{Flipped}: x(-n) = \{0,0,0,1,2,3,2,1,0, \}$ $n = [-8, -7, -6, -5, -4, -3, -2, -1,0]$



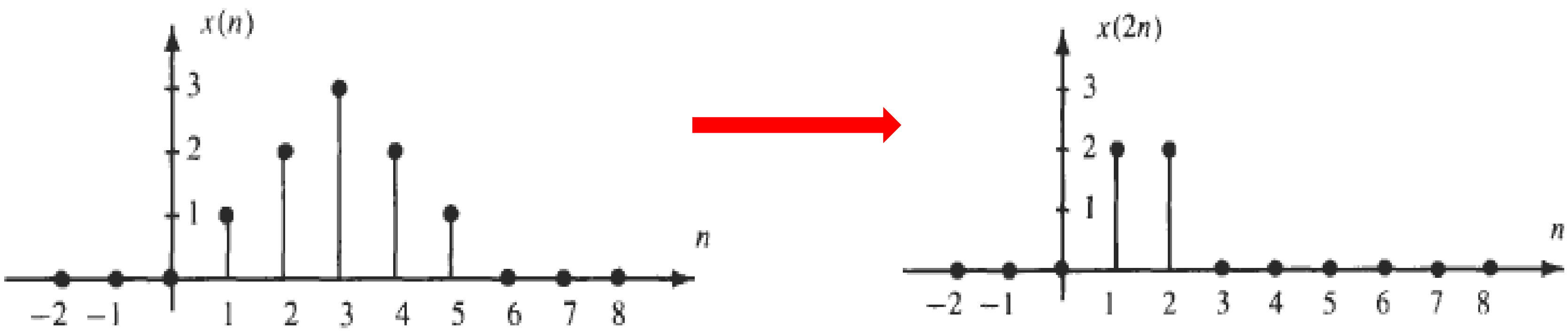
Basic Sequence and Operations:

Down Sampling: the sequence $x(n)$ is said to be down sampled by a factor of integer A defined as

$$y(n) = x(An)$$

- Down sampling reduces the size of a sequence $x(n)$

Example: down sample a sequence $x(n) = \{0,1,2,3,2,1,0,0,0\}$ by a factor of 2 (i.e. A=2)



(a) A discrete-time signal.

- Perform same operation with *A=3 i.e.* $y(n) = x(3n)$

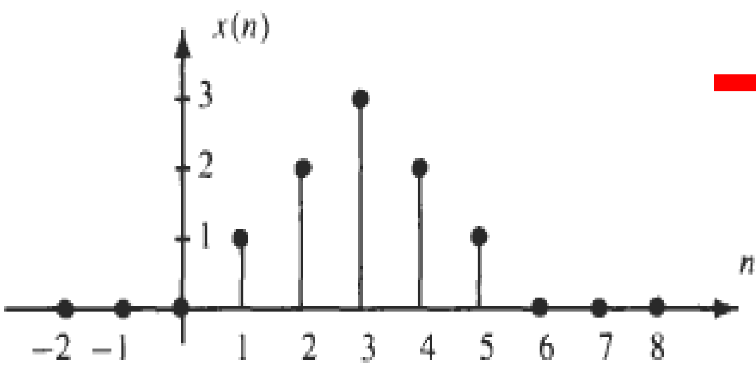
Basic Sequence and Operations:

Up Sampling: the sequence $x(n)$ is said to be up-sampled by an integer factor A defined the relation,

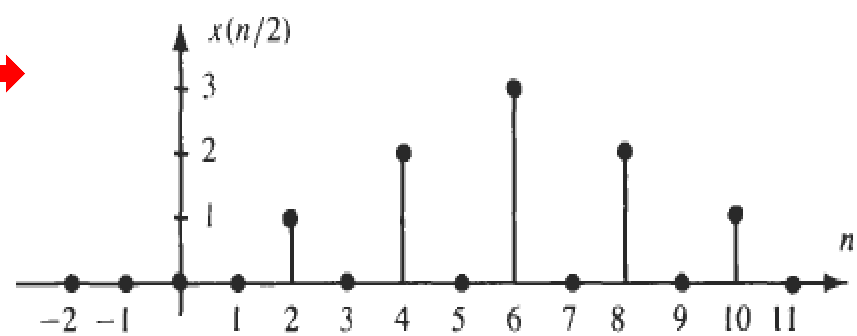
$$y(n) = x(n/A)$$

- Up sampling increases the size of a sequence $x(n)$.

Example: up sample a sequence $x(n) = \{0,1,2,3,2,1,0,0,0\}$ by a factor of 2 (i.e. A=2)



(a) A discrete-time signal.

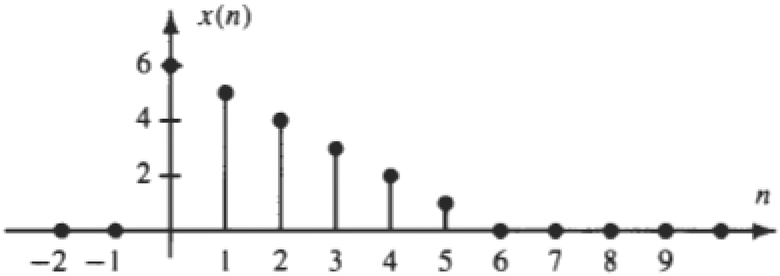


- Perform same operation with $A=3$ *i.e.* $y(n) = x(n/3)$

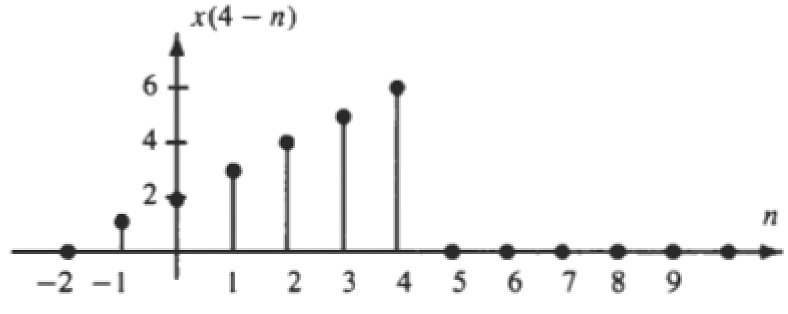
Basic Sequence and Operations:

Signal Manipulation

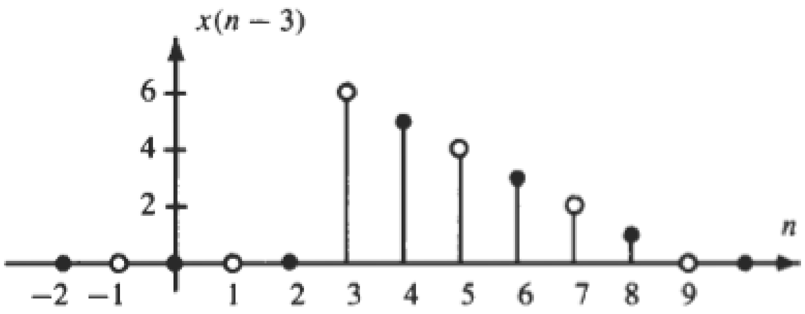
- consider a sequence begins at index n = 0 and ends at n=5



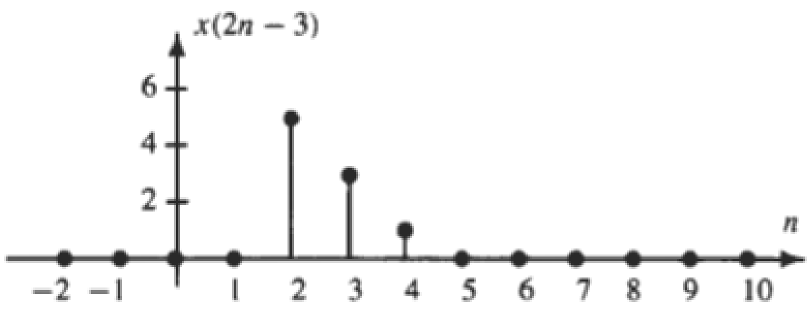
Original signal



Shifting followed by time reversal



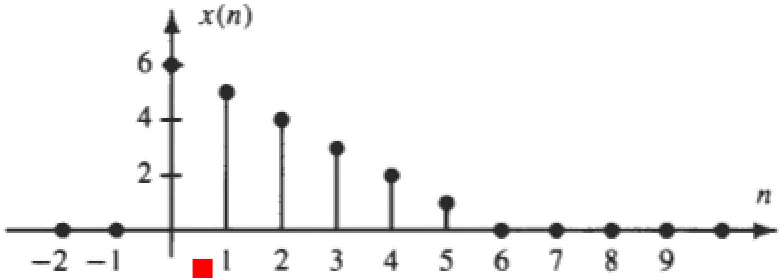
Shifting



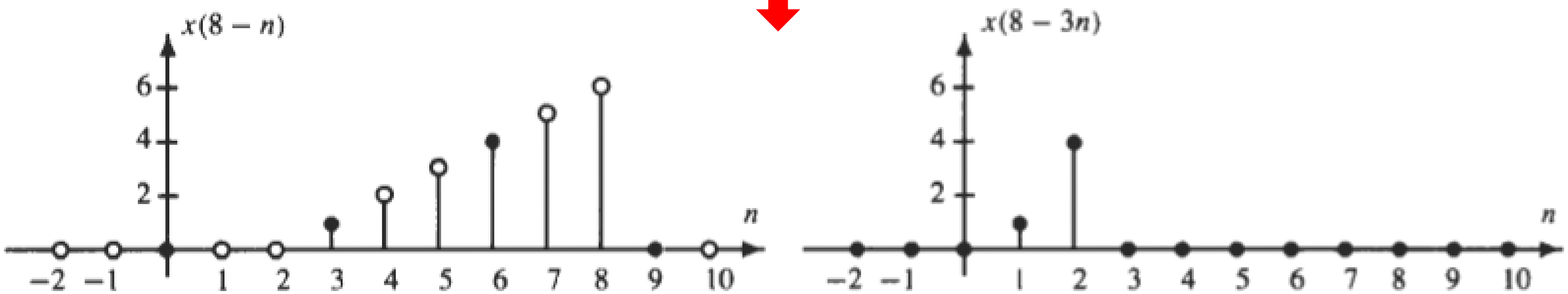
Shifting followed by down sampling

Basic Sequence and Operations:

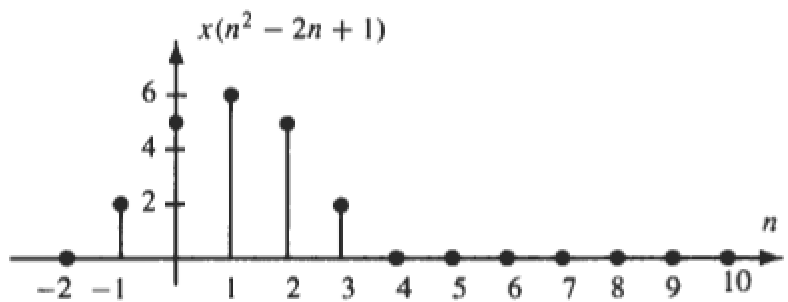
Signal manipulation



1. Time reversal
2. Shifting
3. Down/Up sample



(e) (f)



(g)

Periodic and Aperiodic Signals:

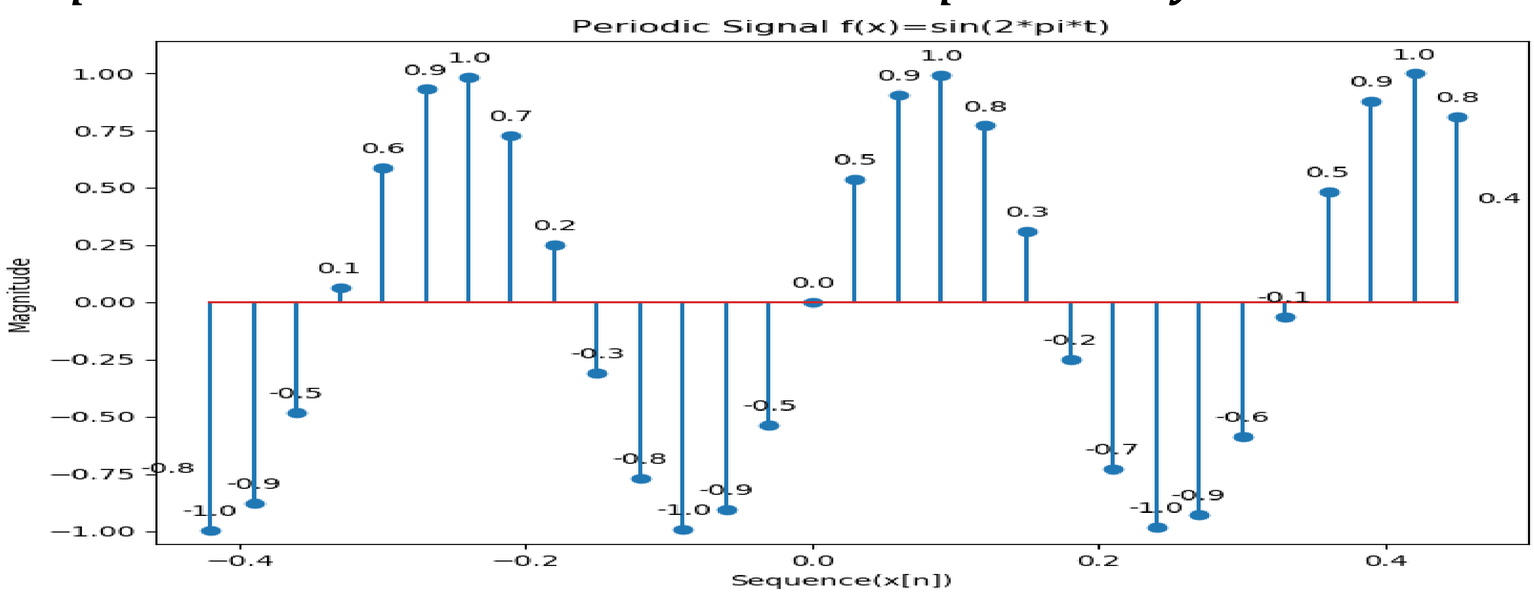
Periodic Signal: **a** discrete signal $x(n)$ is said to be periodic if, for some positive real integer N, such that

$$x(n) = x(n+N) \qquad\qquad for\ all\ values\ of\ n$$

- The sequence repeats itself after every N samples period. N is called the fundamental period.

$$Examples:\ x_1(n) = \sin\left(\frac{\pi n}{7}\right)\ and\ \ x_2(n) = e^{-\pi n/8}$$

$$with\ period\ N = 14\ and\ N = 16\ \ respectively$$



Periodic and Aperiodic Signals:

Periodicity Check

- Discrete periodic signal must meet the following condition: The ratio $\frac{\omega_0}{2\pi}$ must be a rational fraction.

$$\frac{\omega_0}{2\pi} = \left(\frac{M}{N}\right)$$

- Where M is the number of full cycles and **N** is the number of period.
- Sum of any two periodic signal results in another periodic signal with combined signal Period **N** given by:

$$N = \frac{N_1 N_2}{\gcd(N_1, N_2)}$$

Where **gcd** is the greatest common divisor between N_1 *and* N_2

Periodic and Aperiodic Signals:

Example 1: Determine if the following discrete signals are periodic or not. If is periodic, find the sample period N.

a) $x_1(n) = e^{-i\pi n/6}$

By comparison, $e^{i\omega n} = e^{-i\pi n/6}$

$$Standard\ Discrete\ exponential\ signal\ is\ \ e^{i\omega n}$$

$$i\omega n = -\frac{i\pi n}{6}, \qquad \omega = -\frac{\pi}{6}$$

$$\frac{\omega_0}{2\pi} = \frac{-\pi/6}{2\pi} = -\frac{1}{12}$$

$$is\ a\ ratonal\ fraction.\ hence\ the\ signal\ is\ periodic$$

Periodic and Aperiodic Signals:

$-\frac{1}{12} = \left(\frac{M}{N}\right)$, hence by comparison the sample fundamental Period

N=12

a) $x_2(n) = \cos(n^2)$

Standard Discrete sinosuidal signal is $\mathbf{cos}(\boldsymbol{\omega n} + \boldsymbol{\phi})$

By comparison

$$n^2 = \omega n, \qquad \omega = n$$

$$\frac{\omega_0}{2\pi} = \frac{n}{2\pi} = \frac{7n}{44}$$

- Since for value of n>6 the fraction $\frac{7n}{44}$ becomes irrational hence the discrete signal $x_2(n) = \cos(n^2)$ is non-periodic.

Periodic and Aperiodic Signals:

a) Is the sequence x(n) = $\boldsymbol{sin}(\frac{\boldsymbol{3\pi n}}{\boldsymbol{7}} - \frac{\boldsymbol{\pi}}{\boldsymbol{3}})$ periodic or aperiodic? If period what is the samples period N ?.

- If $x_1(n)$ is a sequence that is periodic with a period N_1, and $x_2(n)$ is another sequence that is periodic with a period N_2, the sum is always periodic

$$x(n) = x_2(n) + x_2(n) \qquad\qquad is\ \ periodic$$

- Product of periodic sequence is also results in a periodic sequence

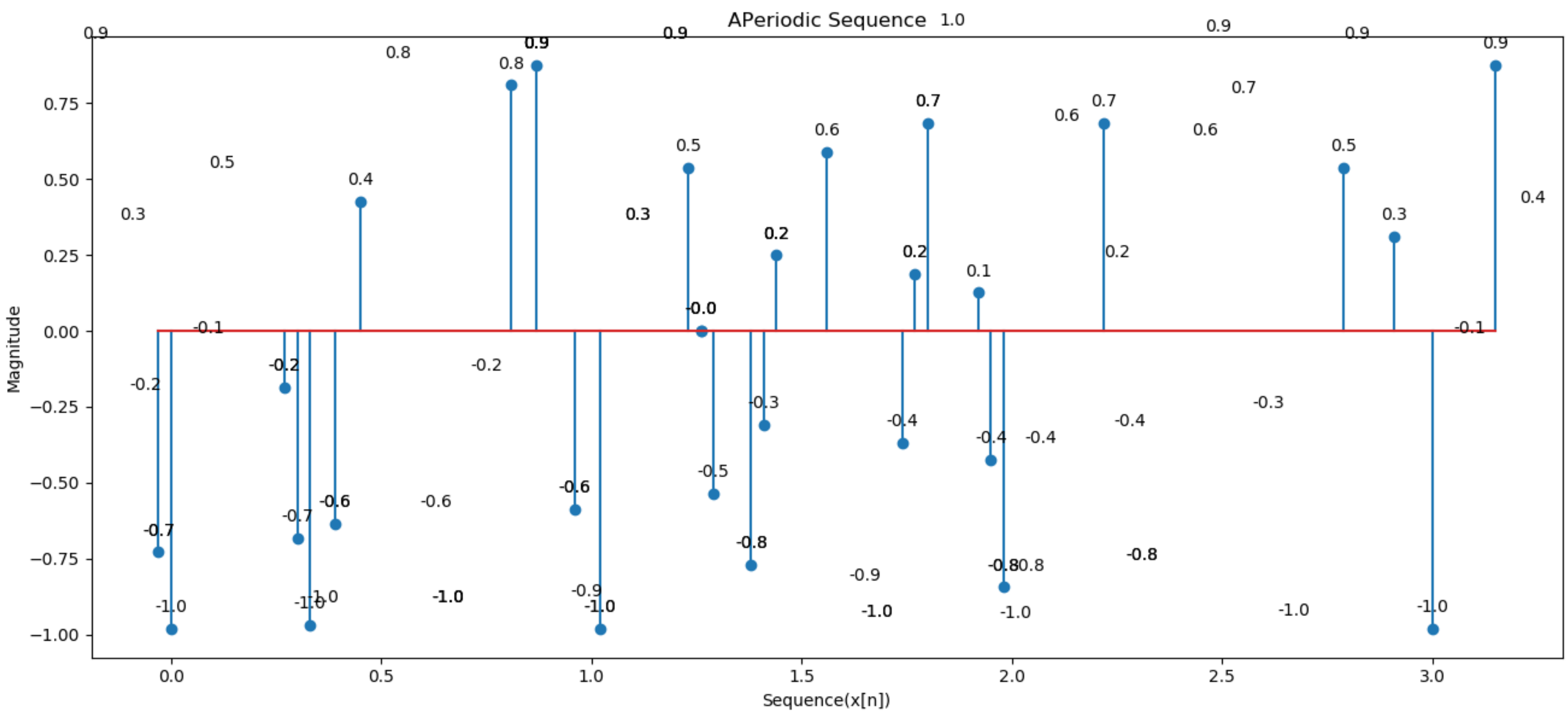
$$x(n) = x_2(n) * x_2(n) \qquad\qquad is\ \ periodic$$

Periodic and Aperiodic Signals:

Aperiodic Signal: a discrete signal $x(n)$ is said to be aperiodic or non-periodic for any positive real integer N, such that

$$x(n) \neq x(n+N) \qquad\qquad for\ all\ values\ of\ n$$

- The sequence does not repeats itself periodically



Symmetric Sequence:

Symmetric sequence

- A discrete-time signal will often possess some form of symmetry that may be exploited in solving problems. Two symmetries of interest are as follows:

Even Sequence: a discrete time signal is said to have be **even symmetry** if for all n in the sequence:

$$x(n) = x(-n)$$

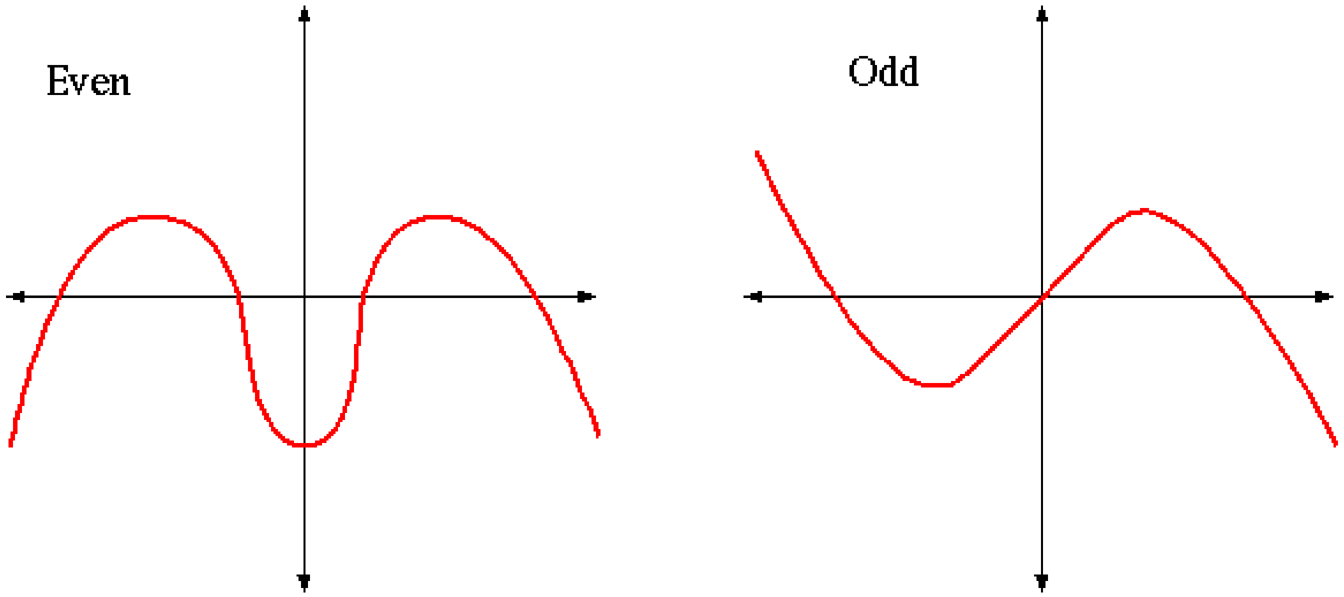
- That is a sequence on one symmetry is equivalent to its time reversal on the other symmetry

Symmetric Sequence:

Odd Sequence: a discrete time signal is said to be an **odd sequence** if for all n in the sequence

$$x(n) = -x(-n)$$

- That is a sequence on one symmetry is equivalent to the negative of its time reversal on the other symmetry



Symmetric Sequence:

- Any symmetrical signal $x(n)$ may be decomposed into a sum of its even part, $x_e(n)$ and its odd part, $x_o(n)$ as follows:

$$x(n) = x_e(n) + x_o(n)$$

- **Even parts:**

$$x_e(n) = \frac{1}{2}\{x(n) + x(-n)\}$$

- **Odd parts:**

$$x_o(n) = \frac{1}{2}\{x(n) - x(-n)\}$$

Symmetric Sequence:

Example 2: Decompose the following sequence into its odd and even symmetry and show that the decomposed signal is equals to the original sequence

$x[n]$	1	2	4	0	3
n	0	1	2	3	4

- **Solution:**

Time reversal of $x[n]$

$x[-n]$	3	0	4	2	1
$-n$	-4	-3	-2	-1	0

Even part:

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$$

$$x_e[n] = \frac{1}{2}\{[1, 2, 4, 0, 3] + [3, 0, 4, 2,1]\} = [2, 1, 4, 1, 2]$$

Symmetric Sequence:

Odd part:

$$x_e[n] = \frac{1}{2}\{x[n] - x[-n]\}$$

$$x_o[n] = \frac{1}{2}\{[1, 2, 4, 0, 3] - [3, 0, 4, 2,1]\} = [-1, 1, 0, -1, 1]$$

$$x_e[n] = \frac{1}{2}\{x[n] - x[-n]\}$$

$$\begin{aligned} x_o[n] + x_e[n] &= [-1, 1, 0, -1, 1] + [2, 1, 4, 1, 2] \\ &= [1, 2, 4, 0, 3] \end{aligned}$$