Control Systems

Block Diagram Algebra

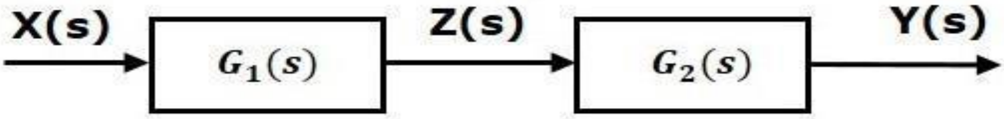
Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

Basic Connections for Blocks

There are three basic types of connections between two blocks.

Series Connection

Series connection is also called cascade connection. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in series.



For this combination, we will get the output $Y(s)$ as:

$Y(s)=G_2(s)Z(s)$

Where,

$Z(s)=G_1(s)X(s)$

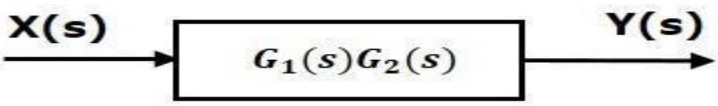
$\Rightarrow Y(s)=G_2(s)[G_1(s)X(s)]=G_1(s)G_2(s)X(s)$

$\Rightarrow Y(s)=\{G_1(s)G_2(s)\}X(s)$

Compare this equation with the standard form of the output equation, $Y(s)=G(s)X(s)$.

Where, $G(s)=G_1(s)G_2(s)$.

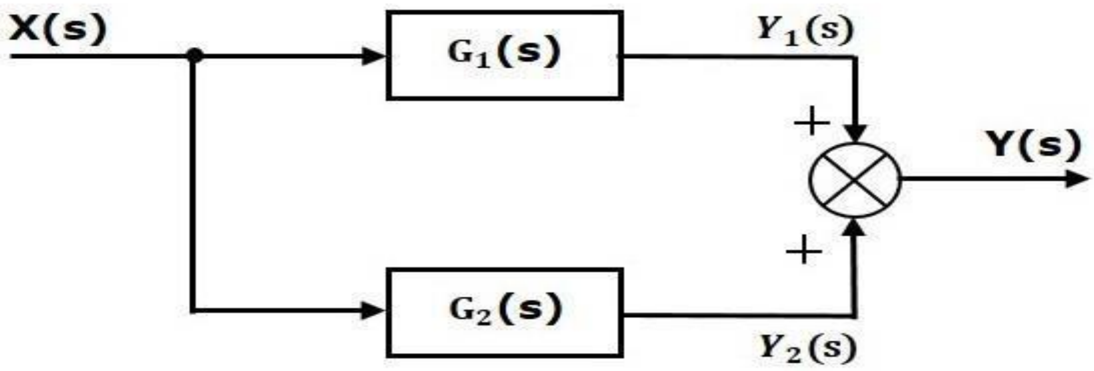
That means we can represent the series connection of two blocks with a single block. The transfer function of this single block is the product of the transfer functions of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Parallel Connection

The blocks which are connected in parallel will have the same input. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output $Y(s)$ as

$Y(s)=Y_1(s)+Y_2(s)$

Where,

$Y_1(s)=G_1(s)X(s)$ and $Y_2(s)=G_2(s)X(s)$

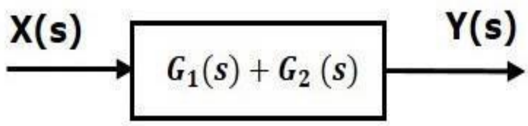
$\Rightarrow Y(s)=G_1(s)X(s)+G_2(s)X(s)=\{G_1(s)+G_2(s)\}X(s)$

Compare this equation with the standard form of the output equation,

$Y(s)=G(s)X(s)$.

Where, $G(s)=G_1(s)+G_2(s)$.

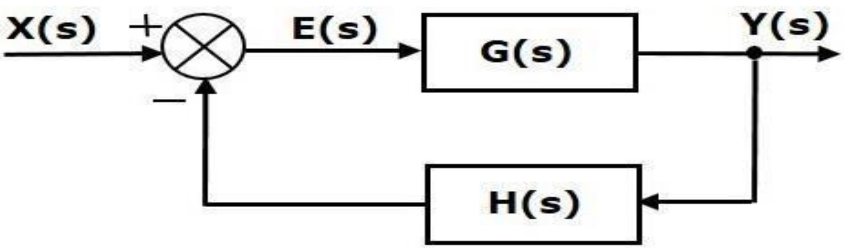
That means we can represent the parallel connection of two blocks with a single block. The transfer function of this single block is the sum of the transfer functions of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.

Feedback Connection

As we discussed in previous lectures, there are two types of feedback , **positive feedback** and **negative feedback**. The following figure shows negative feedback control system. Here, two blocks having transfer functions $G(s)$ and $H(s)$ form a closed loop.



The output of the summing point is -

$E(s)=X(s)-H(s)Y(s)$

The output $Y(s)$ is -

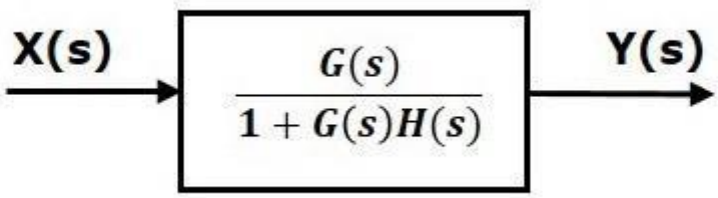
$Y(s)=E(s)G(s)$

Substitute $E(s)$ value in the above equation.

$Y(s)=\{X(s)-H(s)Y(s)\}G(s)\}$

$Y(s)\{1+G(s)H(s)\}=X(s)G(s)\}$
$\Rightarrow Y(s)/X(s)=G(s)/1+G(s)H(s)$

Therefore, the negative feedback closed loop transfer function is
$G(s)/1+G(s)H(s)$
This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown below.

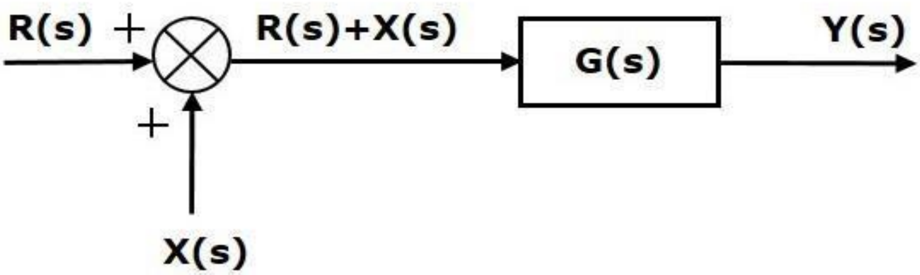


Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $G(s)/1-G(s)H(s)$.

Block Diagram Algebra for Summing Points
There are two possibilities of shifting summing points with respect to blocks −

Shifting summing point after the block ,and Shifting summing point before the block

<u>Shifting Summing Point After the Block</u>
Consider the block diagram shown in the following figure. Here, the summing point is presented before the block.

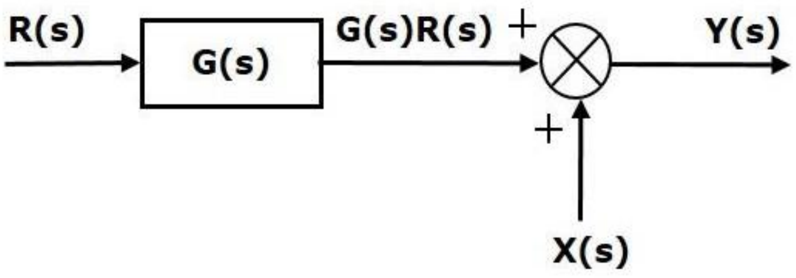


Summing point has two inputs $R(s)$ and $X(s)$. The output of summing point is $\{R(s)+X(s)\}$. So, the input to the block $G(s)$ is $\{R(s)+X(s)\}$ and the output from it is –

$$Y(s)=G(s)\{R(s)+X(s)\}$$

$$\Rightarrow Y(s)=G(s)R(s)+G(s)X(s) \quad \ldots\ldots\ldots\ldots\ldots\text{(Equation 1)}$$

Now, shift the summing point after the block. This block diagram is shown in the following figure:

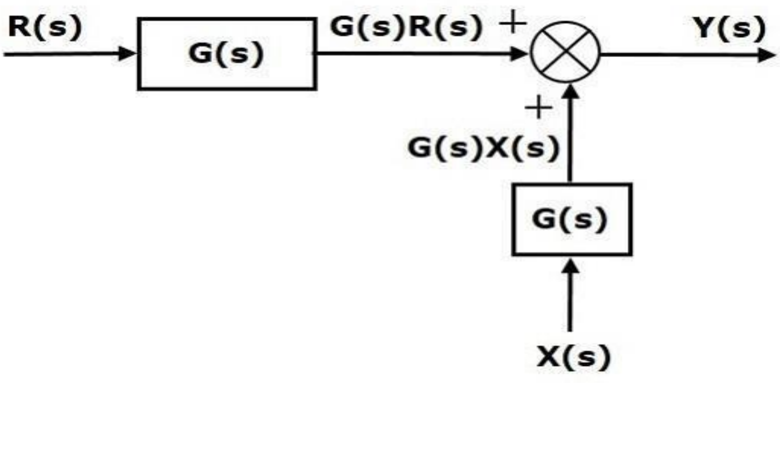


Output of the block $G(s)$ is $G(s)R(s)$. The output of the summing point is

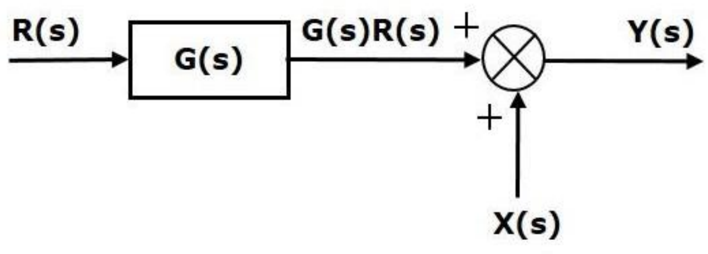
$$Y(s)=G(s)R(s)+X(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(Equation 2)}$$

Compare Equation 1 and Equation 2.

The first term '$G(s)R(s)$' is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $G(s)$. It is having the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. This block diagram is shown in the following figure.



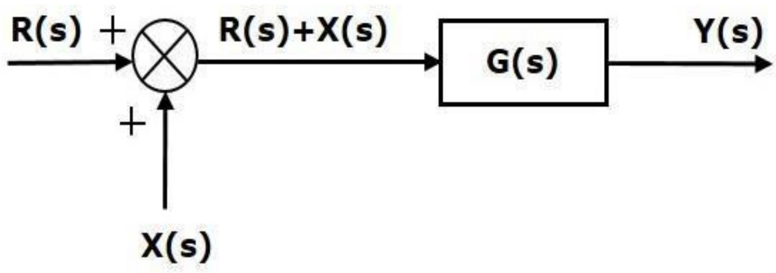
Consider the block diagram shown in the following figure. Here, the summing point is presented after the block.



Output of the previous block diagram is -

$Y(s)=G(s)R(s)+X(s)$ ………………..(Equation 3)

Now, shift the summing point before the block will produce a block diagram shown in the following figure :

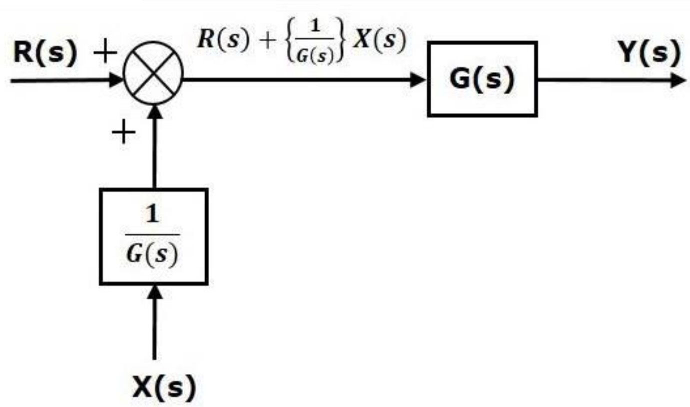


Output of this block diagram is -

$Y(S)=G(s)R(s)+G(s)X(s)$ ……………….. (Equation 4)

Compare Equation 3 and Equation 4

The first term '$G(s)R(s)$' is same in both equations. But, there is difference in the second term. In order to get the second term also the same, it requires one more block $1/G(s)$. It has the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. the new block diagram is shown in the following figure.



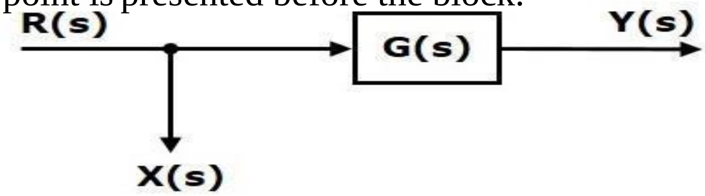
Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to the blocks, Shifting take-off point after the block and Shifting take-off point before the block.

Let us now see what kind of arrangements are to be done in the above two cases, one by one.

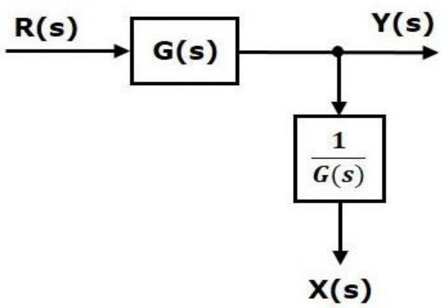
Shifting Take-off Point After the Block

Consider the block diagram shown in the following figure. In this case, the take-off point is presented before the block.



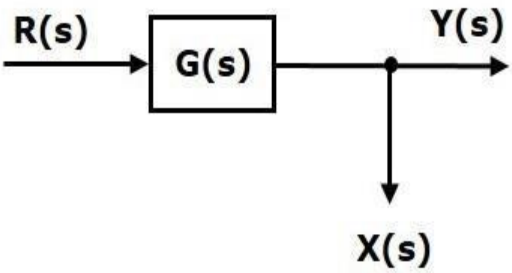
Here, $X(s)=R(s)$ and $Y(s)=G(s)R(s)$

When you shift the take-off point after the block , the output $Y(s)$ will be the same. But, there is difference in $X(s)$ value. So, in order to get the same $X(s)$ value, we require one more block $1/G(s)$. It has the input $Y(s)$ and the output is $X(s)$. This block diagram is shown in the following figure:



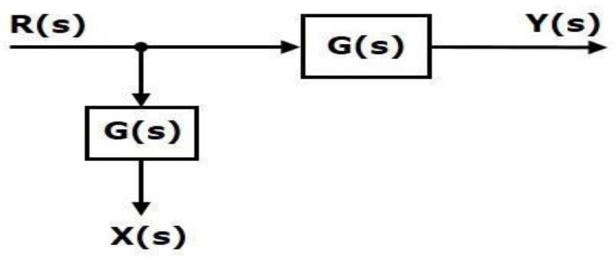
Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



Here, $X(s)=Y(s)=G(s)R(s)$

When you shift the take-off point before the block, the output $Y(s)$ will be the same. But, there is a difference in $X(s)$ value. So, in order to get same $X(s)$ value, we require one more block $G(s)$. It has the input $R(s)$ and the output is $X(s)$. This block diagram is shown in the following figure.



Control Systems - Block Diagram Reduction

Follow these rules for simplifying (reducing) the block diagram, which it has many blocks, summing points and take-off points.

Rule 1 − Check for the blocks connected in series and simplify.

Rule 2 − Check for the blocks connected in parallel and simplify.

Rule 3 − Check for the blocks connected in feedback loop and simplify.

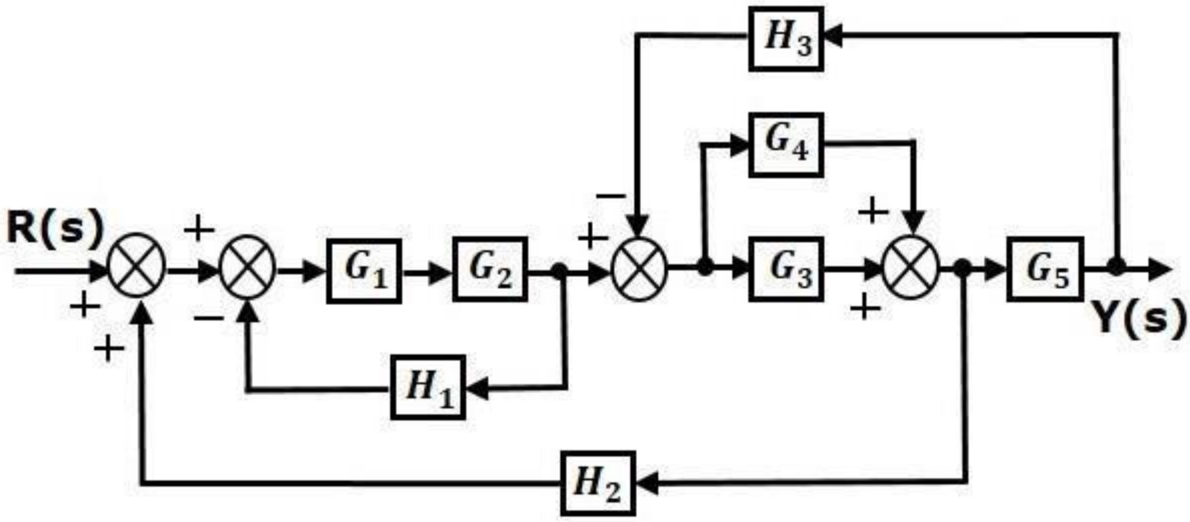
Rule 4 − If there is difficulty with take-off point while simplifying, shift it towards right.

Rule 5 − If there is difficulty with summing point while simplifying, shift it towards left.

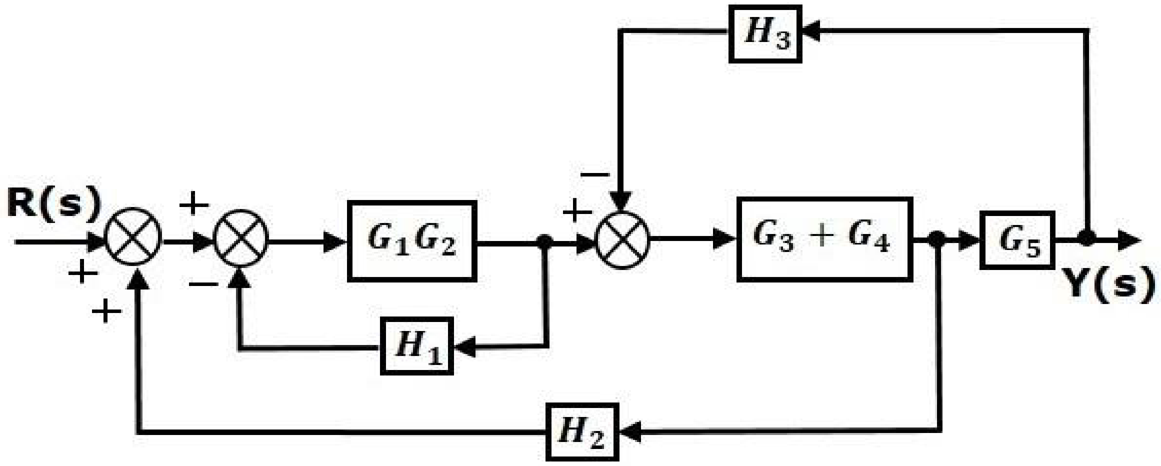
Rule 6 − Repeat the above steps till you get the simplified form, i.e., single block.

Note − The transfer function present in this single block is the transfer function of the overall block diagram.

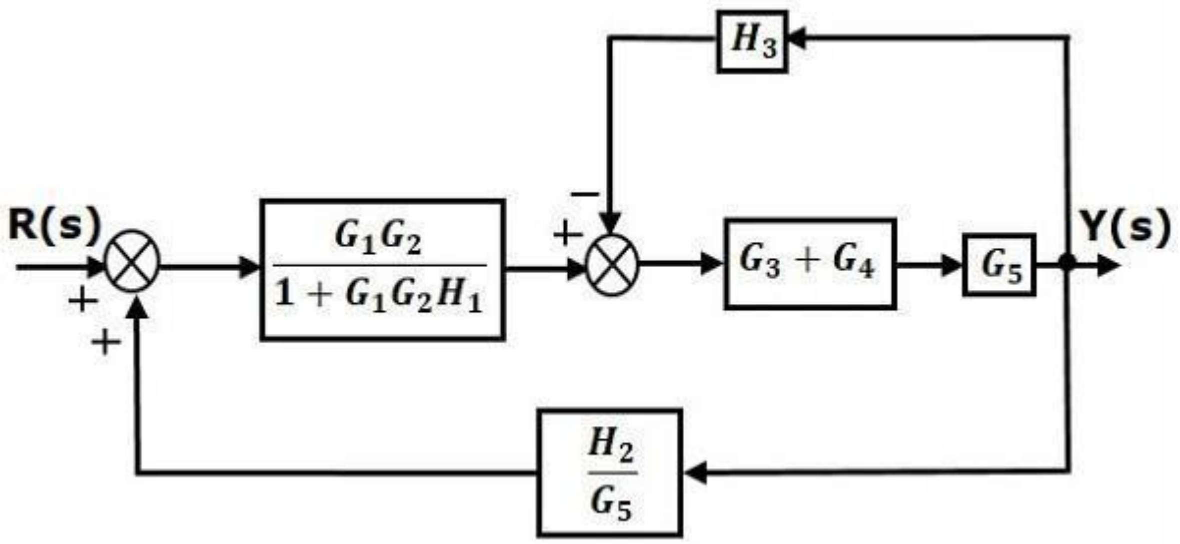
Example: Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



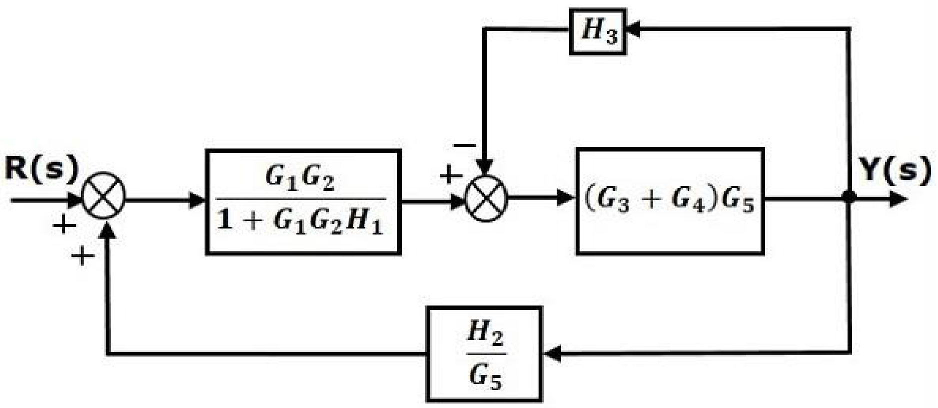
Step 1 − Use Rule **1** for blocks ***G*1 and *G*2**. Use Rule **2** for blocks ***G*3 and *G*4**. The modified block diagram is shown in the following figure.



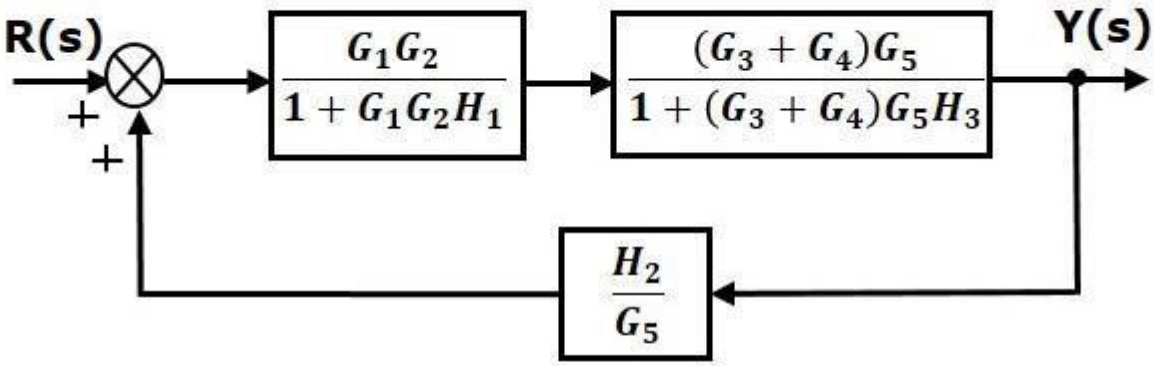
Step 2 − Use Rule 3 for blocks *G*1 *G*2 and *H*1. Use Rule 4 for shifting take -off point after the block *G*5. The modified block diagram is shown in the following figure.



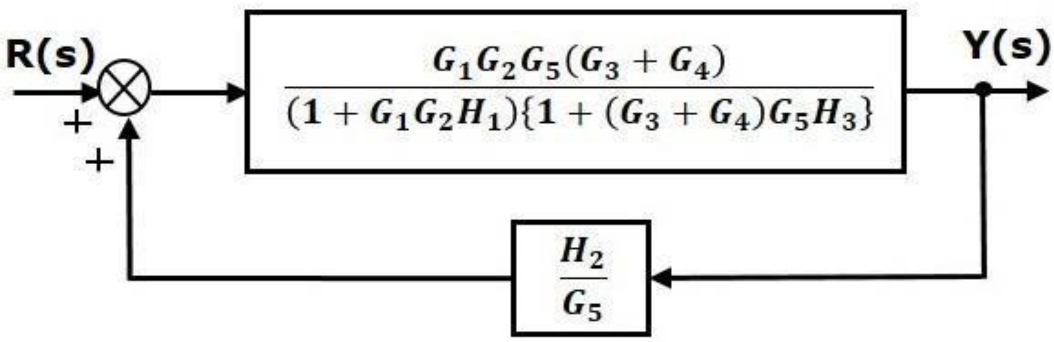
Step 3 − Use Rule 1 for blocks (*G*3+*G*4) and *G*5. The modified block diagram is shown in the following figure.



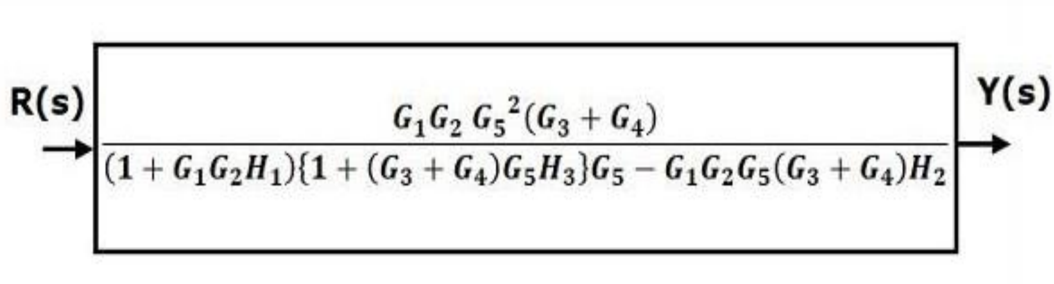
Step 4 − Use Rule 3 for blocks $(G3+G4)G5$ and $H3$. The modified block diagram is shown in the following figure.



Step 5 − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



$\boldsymbol{Y(s)/R(s)}=G1G2G_5^2\ (G3+G4)$ **/**

$[(1+G1G2H1)\ \{1+(G3+G4)G5H3\}G5-G1G2G5(G3+G4)H2]$