Digital Logic Design

Combinational Logic Analysis

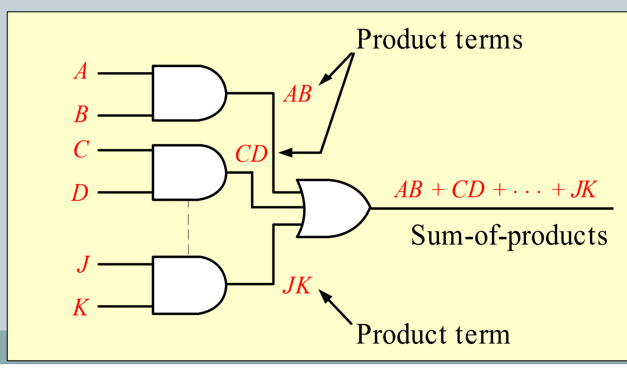
CHAPTER 5

Content

- Basic Combinational Logic Circuit
- Implementing Combinational Logic
- The Universal Property of NAND and NOR Gates
- Combinational Logic Using NAND and NOR Gates
- Logic Circuit Operation with Pulse Waveform Inputs

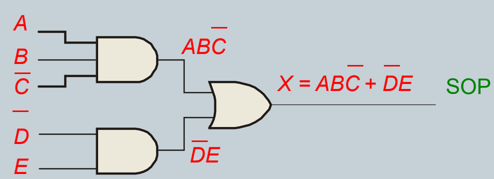
Combinational Logic Circuits

- In Sum-of-Products (SOP) form, basic combinational circuits can be directly implemented with AND-OR.



AND OR Logic

- An example of an SOP implementation is shown. The SOP expression is an AND-OR combination of the input variables and the appropriate complements.



Example 5-1

In a certain chemical-processing plant, a liquid chemical is used in a manufacturing process. The chemical is stored in three different tanks. A level sensor in each tank produces a HIGH voltage when the level of chemical in the tank drops below a specified point.

Design a circuit that monitors the chemical level in each tank and indicates when the level in any two of the tanks drops below the specified point.

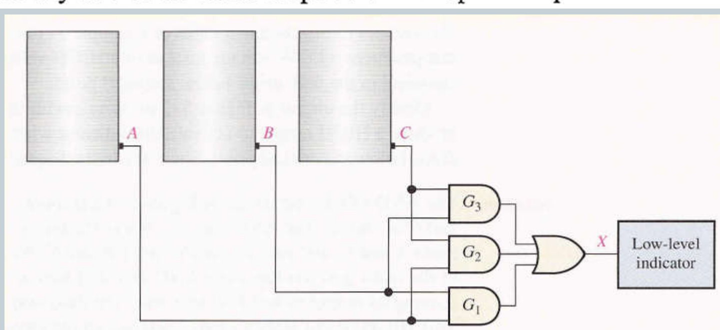
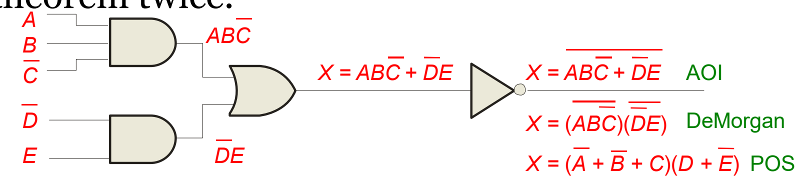


Figure 5-2

AND-OR-Invert Logic

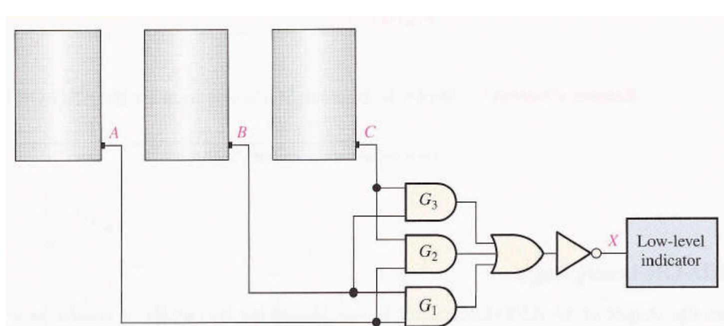
When the output of a SOP form is inverted, the circuit is called an AND-OR-Invert (AOI) circuit. The AOI configuration lends itself to product-of-sums (POS) implementation.
An example of an AOI implementation is shown. The output expression can be changed to a POS expression by applying DeMorgan's theorem twice.



Example 5-2

The sensors in the chemical tanks of Example 5–1 are being replaced by a new model that produces a LOW voltage instead of a HIGH voltage when the level of the chemical in the tank drops below a critical point.

Modify the circuit in Figure 5–2 to operate with the different input levels and still produce a HIGH output to activate the indicator when the level in any two of the tanks drops below the critical point. Show the logic diagram.



Exclusive-OR Logic

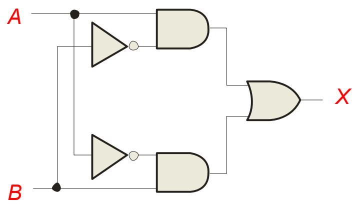
The truth table for an exclusive-OR gate is: ---------------------------------→

Inputs A	Inputs B	Output X
0	0	0
0	1	1
1	0	1
1	1	0

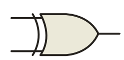
Notice that the output is HIGH whenever *A* and *B* disagree.

The Boolean expression is $X = \overline{A}B + A\overline{B}$

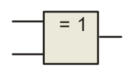
The circuit can be drawn as



Symbols:



Distinctive shape outline



Rectangular

Exclusive-NOR Logic

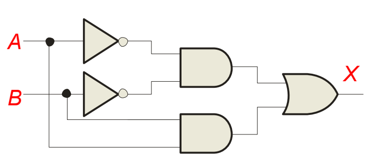
The truth table for an exclusive-NOR gate is

Inputs A	Inputs B	Output X
0	0	1
0	1	0
1	0	0
1	1	1

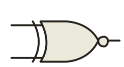
Notice that the output is HIGH whenever *A* and *B* agree.

The Boolean expression is $X = \overline{A}\overline{B} + AB$

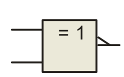
The circuit can be drawn as



Symbols:



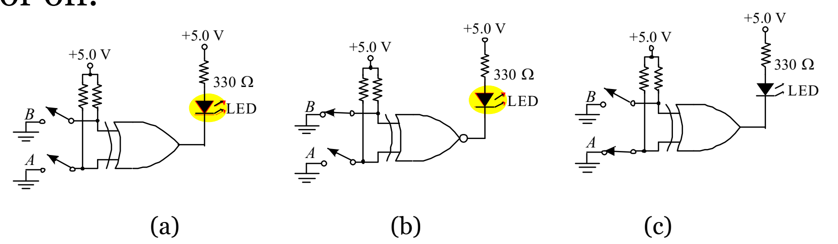
Distinctive shape outline



Rectangular

Example

For each circuit, determine if the LED should be on or off.

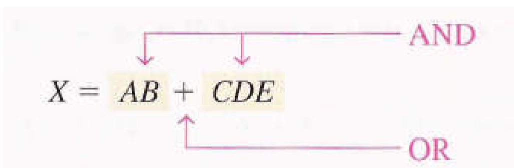


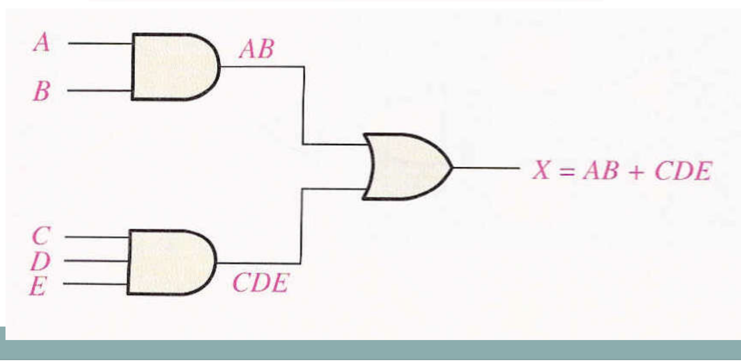
Circuit (a): XOR, inputs agree, output is LOW, LED is ON.

Circuit (b): XNOR, inputs disagree, output is LOW, LED is ON.

Circuit (c): XOR, inputs disagree, output is HIGH, LED is OFF.

From Boolean Expression to Logic Circuit



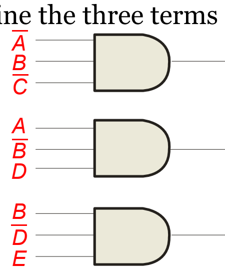


Implementing Combinational Logic

Implementing a SOP expression is done by first forming the AND terms; then the terms are ORed together.

Example Show the circuit that will implement the Boolean expression $X = \overline{A}B\overline{C} + A\overline{B}D + B\overline{D}E$. (Assume that the variables and their complements are available.)

Start by forming the terms using three 3-input AND gates.
Then combine the three terms using a 3-input OR gate.



$X = \overline{A}B\overline{C} + A\overline{B}D + B\overline{D}E$

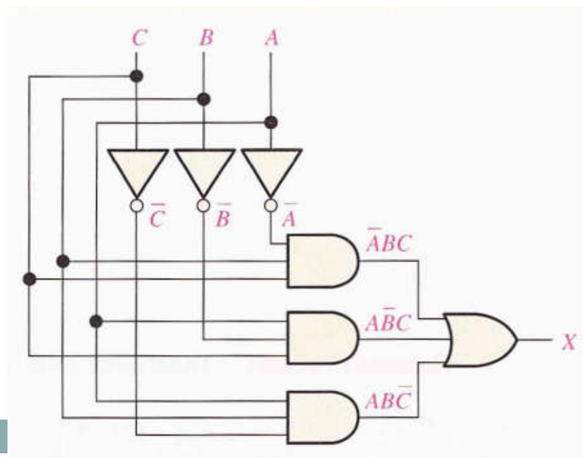
From a Truth table to a Logic Circuit

INPUTS A	B	C	OUTPUT X	PRODUCT TERM
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\overline{A}BC$
1	0	0	0	
1	0	1	1	$A\overline{B}C$
1	1	0	1	$AB\overline{C}$
1	1	1	0	

$X = 1$ for only three of the input conditions.

$$X = \overline{A}BC + A\overline{B}C + AB\overline{C}$$

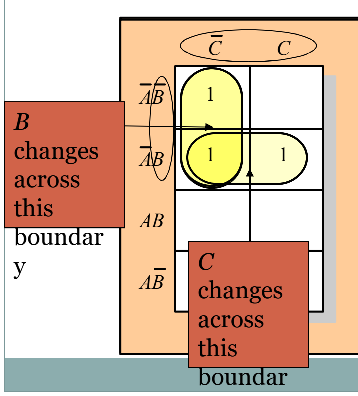
$$X = \overline{A}BC + A\overline{B}C + AB\overline{C}$$



Karnaugh Map Implementation

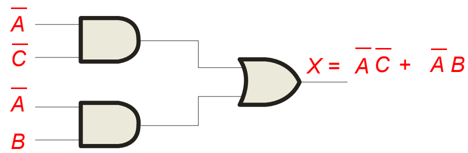
For basic combinational logic circuits, the Karnaugh map can be read and the circuit drawn as a minimum SOP.

A Karnaugh map is drawn from a truth table. Read the minimum SOP expression and draw the circuit.



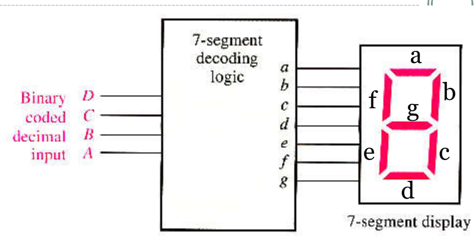
1. Group the 1's into two overlapping groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The vertical group is read $\overline{A}\,\overline{C}$.
4. The horizontal group is read $\overline{A}B$.

Circuit:



The result is shown as a sum of products.

Application Example



DIGIT	SEGMENTS ACTIVATED
0	*a, b, c, d, e, f*
1	*b, c*
2	*a, b, d, e, g*
3	*a, b, c, d, g*
4	*b, c, f, g*
5	*a, c, d, f, g*
6	*a, c, d, e, f, g*
7	*a, b, c*
8	*a, b, c, d, e, f, g*
9	*a, b, c, d, f, g*

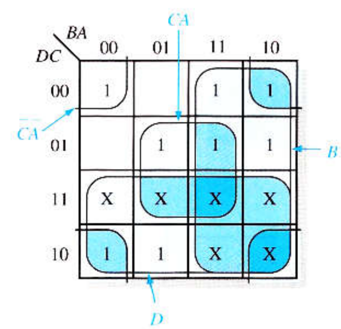
Case of numbers only

DECIMAL DIGIT	INPUTS *D*	*C*	*B*	*A*	SEGMENT OUTPUTS *a*	*b*	*c*	*d*	*e*	*f*	*g*
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
10	1	0	1	0	X	X	X	X	X	X	X
11	1	0	1	1	X	X	X	X	X	X	X
12	1	1	0	0	X	X	X	X	X	X	X
13	1	1	0	1	X	X	X	X	X	X	X
14	1	1	1	0	X	X	X	X	X	X	X
15	1	1	1	1	X	X	X	X	X	X	X

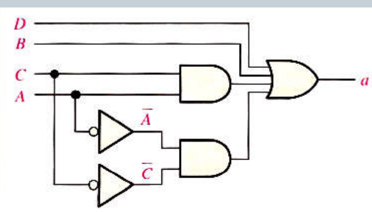
Out put a for example

Standard SOP expression:

$\overline{D}\,\overline{C}\,\overline{B}\,\overline{A} + \overline{D}\,\overline{C}B\overline{A} + \overline{D}\,\overline{C}BA + \overline{D}C\overline{B}A + \overline{D}CB\overline{A} + \overline{D}CBA + D\overline{C}\,\overline{B}\,\overline{A} + D\overline{C}\,\overline{B}A$



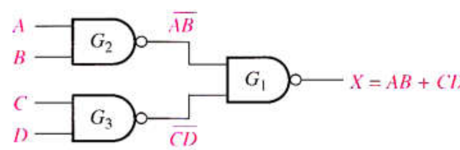
Minimum SOP expression: $D + B + CA + \overline{C}\,\overline{A}$

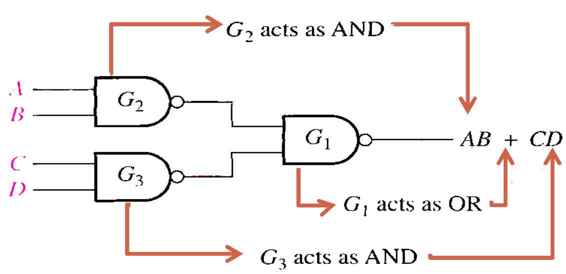


NAND Logic

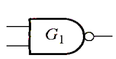
$$\overline{AB} = \overline{A} + \overline{B}$$

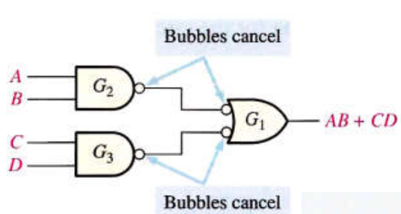
$$\begin{aligned} X &= \overline{(\overline{AB})(\overline{CD})} \\ &= \overline{(\overline{A} + \overline{B})(\overline{C} + \overline{D})} \\ &= \overline{(\overline{A} + \overline{B})} + \overline{(\overline{C} + \overline{D})} \\ &= \overline{\overline{A}}\,\overline{\overline{B}} + \overline{\overline{C}}\,\overline{\overline{D}} \\ &= AB + CD \end{aligned}$$

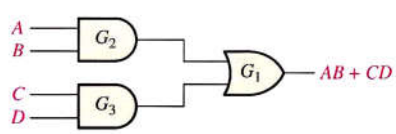




(a) Original NAND logic diagram showing effective gate operation relative to the output expression

$\overline{AB} = \overline{A} + \overline{B}$ 

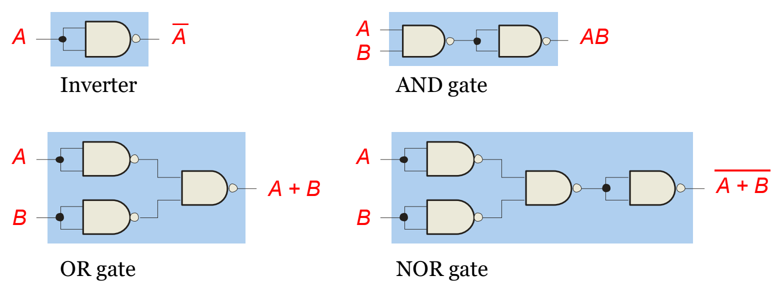




Dual Symbols

Universal Gates

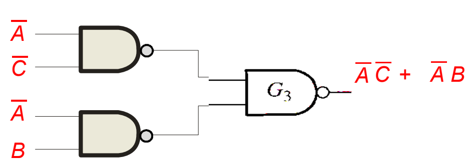
NAND gates are sometimes called **universal** gates because they can be used to produce the other basic Boolean functions.



NAND Logic

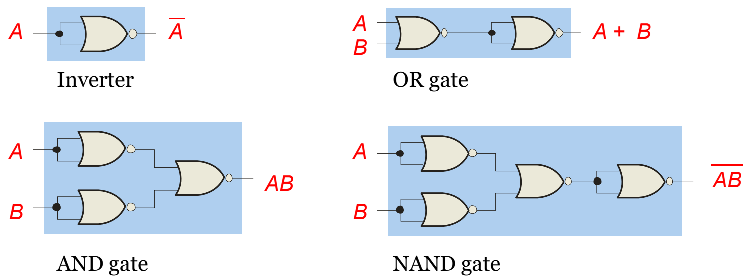
Convert the circuit in the previous example to one that uses only NAND gates.

Recall from Boolean algebra that double inversion cancels. By adding inverting bubbles to above circuit, it is easily converted to NAND gates:



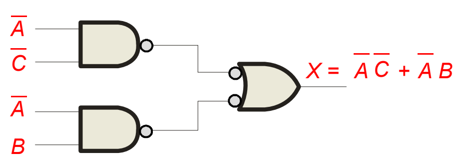
Universal Gates

NOR gates are also **universal** gates and can form all of the basic gates.



NAND Logic

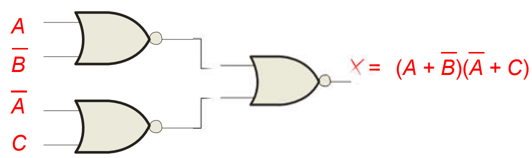
Recall from DeMorgan's theorem that $\overline{AB} = \overline{A} + \overline{B}$. By using equivalent symbols, it is simpler to read the logic of SOP forms. The earlier example shows the idea:



The logic is easy to read if you (mentally) cancel the two connected bubbles on a line.

NOR Logic

Alternatively, DeMorgan's theorem can be written as $\overline{A + B} = \bar{A}\bar{B}$. By using equivalent symbols, it is simpler to read the logic of POS forms. For example,

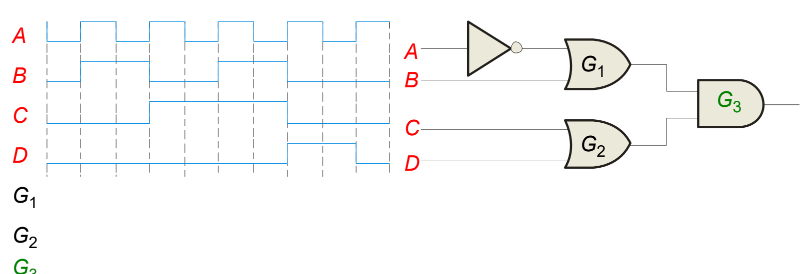


Again, the logic is easy to read if you cancel the two connected bubbles on a line.

Pulsed Waveforms

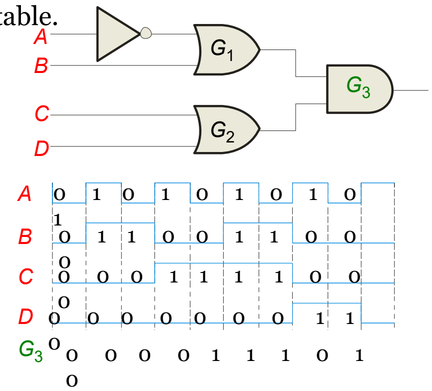
For combinational circuits with pulsed inputs, the output can be predicted by developing intermediate outputs and combining the result.

For example, the circuit shown can be analyzed at the outputs of the OR gates:



Pulsed Waveforms

Alternatively, you can develop the truth table for the circuit and enter 0's and 1's on the waveforms. Then read the output from the table.



Inputs				Output
A	*B*	*C*	*D*	*X*
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

References

1. T. Floyd, "Digital Fundamental", 10th Ed., USA: PrenticeHall, 2008
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3. W. Kleitz, "Digital Electronics: A Practical Approach", 8th Ed., USA: Prentice-Hall, 2007
4. Begnell and Donovan, "Digital Electronics", 5th Ed., USA: Delmar Thomson Learning, 2006