Chapter three

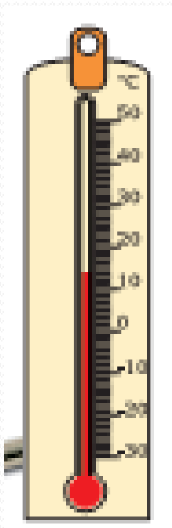
Scalar and Vector Quantities

You will learn:

1. The principles of scalar and vector quantities
2. Mathematical combinations of vector quantities
3. Unit vectors

Scalar Quantities:

- A **SCALAR** is a quantity of physics that has MAGNITUDE only, however, direction is not associated with it.
- Magnitude – A numerical value with units.
- Some examples of scalar quantities



Scalar Example	Magnitude
Speed	20 m/s
Distance	10 m
Age	15 years
Heat	1000 calories

Vector Quantities:

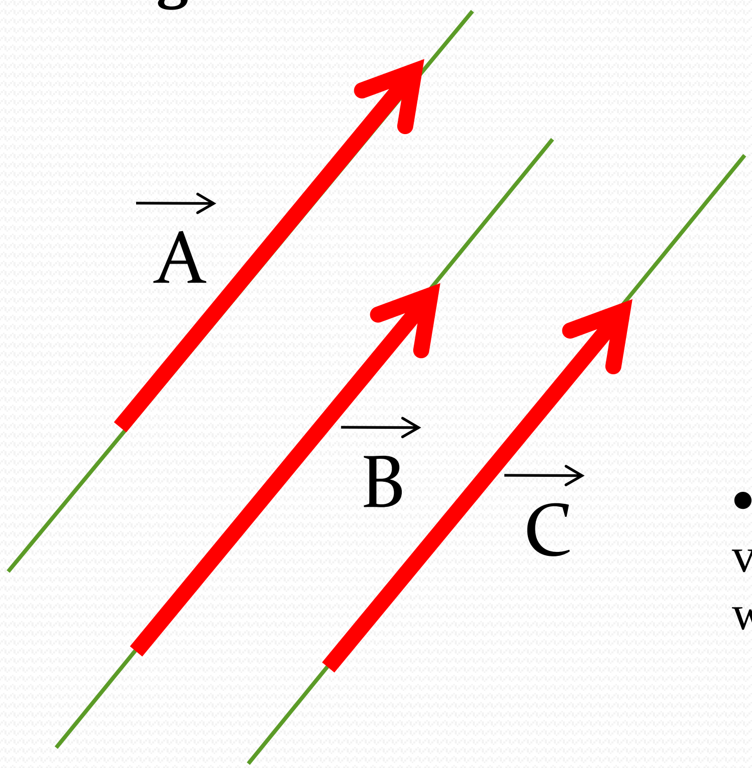
- A **VECTOR** is a quantity which has both MAGNITUDE and DIRECTION.
- Examples: force, displacement, velocity….

Vector	Magnitude & Direction
Velocity	20 m/s, North
Acceleration	10 m/s/s, East
Force	5 N, West

Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.

Properties of Vectors

- **Equality of Two Vectors**
  - Two vectors are **equal** if they have **the same magnitude and the same direction.**

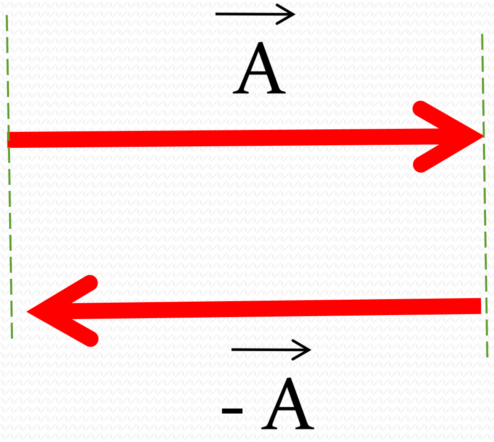


$$\vec{A} = \vec{B} = \vec{C}$$

•This property allows us to translate a vector parallel to itself in a diagram without affecting the vector.

Negative Vectors

- Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)



Vector Applications:

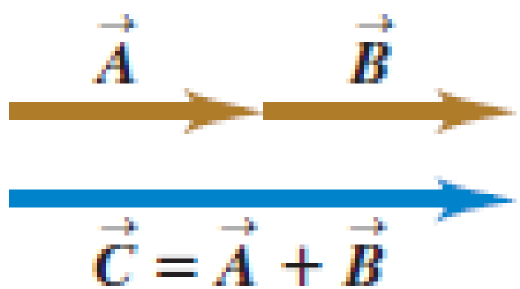
- Adding vectors
- Subtracting vectors
- Vector multiplication

- There are two approaches of vector applications:

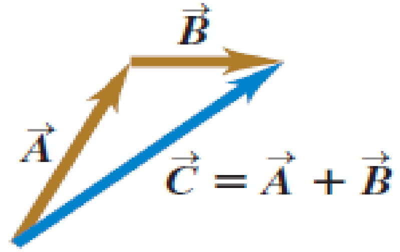
1. Graphical Methods: the vectors are plotted.
   - Use scale drawings.
2. Algebraic Methods: mathematics is used.
   - More convenient

Adding Vectors

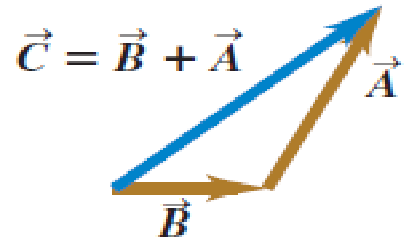
- When adding vectors, their **directions must be taken into consideration.**
- Units must be the same.



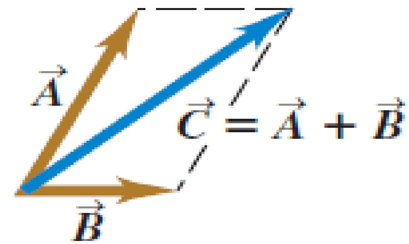
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result.



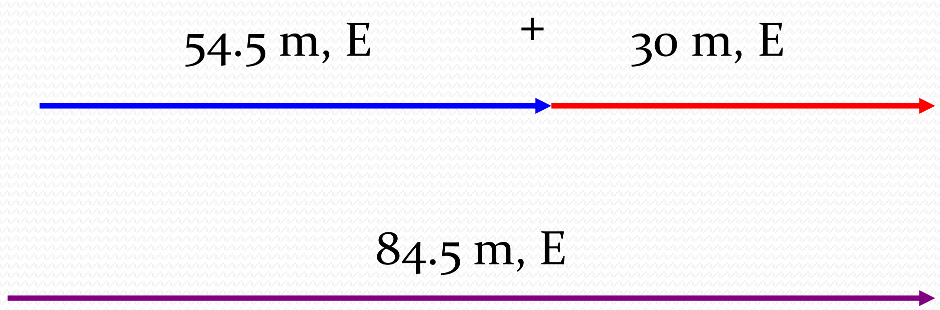
(c) We can also add them by constructing a parallelogram.



Adding Vectors Graphically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector (A) with the appropriate length and in the direction specified, with respect to a coordinate system.
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector **A** and parallel to the coordinate system used for **A.**

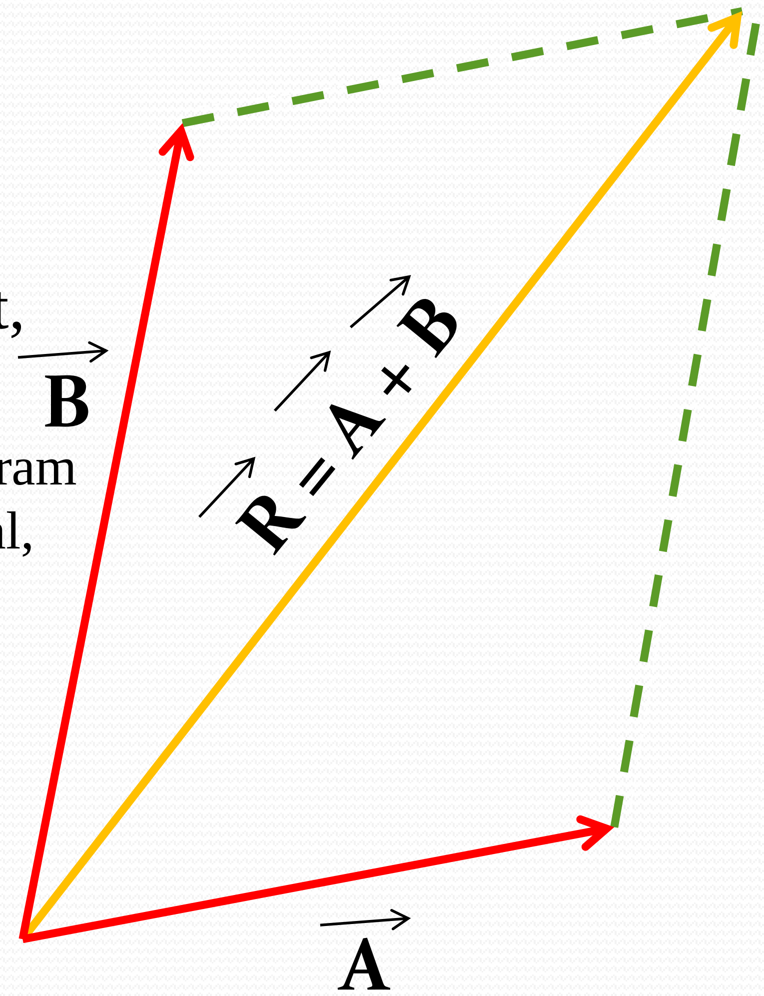
- **Example: A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?**

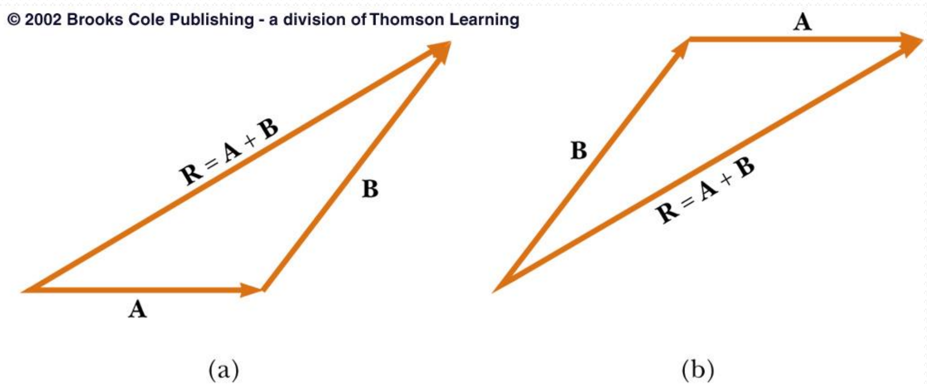


Notice that the SIZE of the arrow conveys MAGNITUDE and the way it was drawn conveys DIRECTION.

Alternative Graphical Method

- When you have only two vectors, you may use the **Parallelogram Method.**
- All vectors, including the resultant, are drawn from a common origin.
  - The remaining sides of the parallelogram are sketched to determine the diagonal, **R**

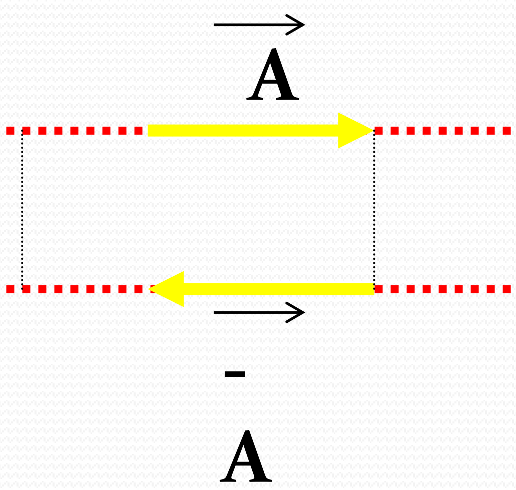


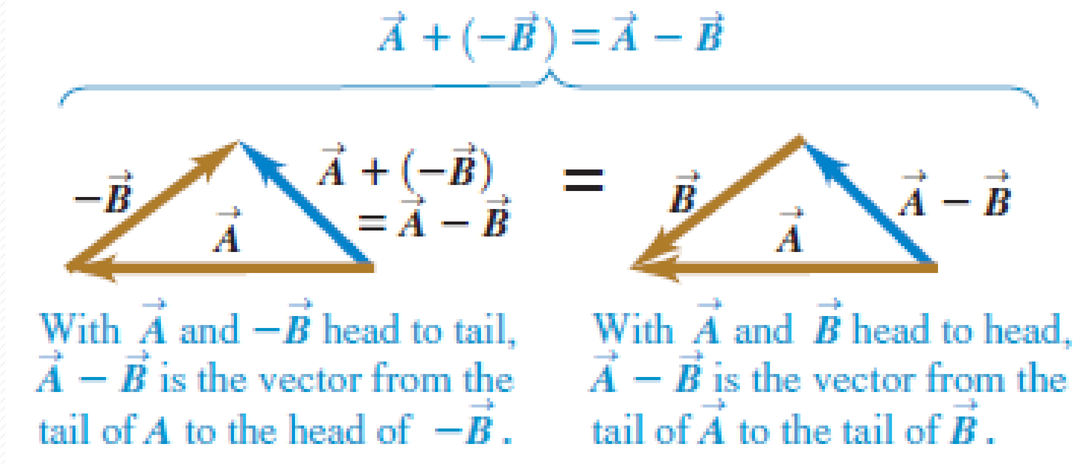


Subtracting vectors:

- The vectors **A** & –**A** have the same magnitude but point in opposite directions. Therefore when we add negative vectors we get zero.

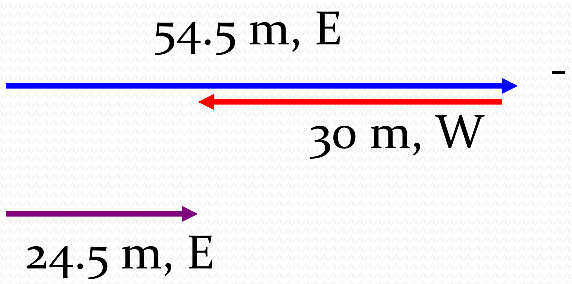
- $\vec{\mathbf{A}} + \overrightarrow{(\mathbf{-A})} = \mathbf{0}$





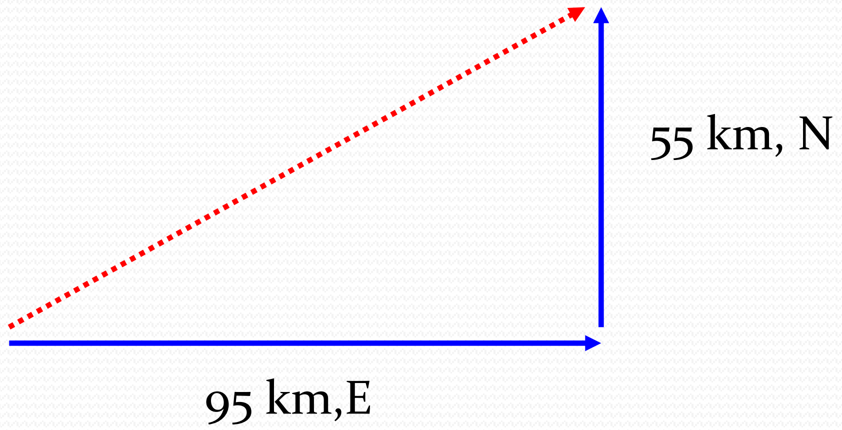
- **Example: A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started?**

Solution



When two vectors are perpendicular, you must use the Pythagorean theorem.

Example: A man walks 95 km East then 55 km north. Calculate his RESULTANT DISPLACEMENT.



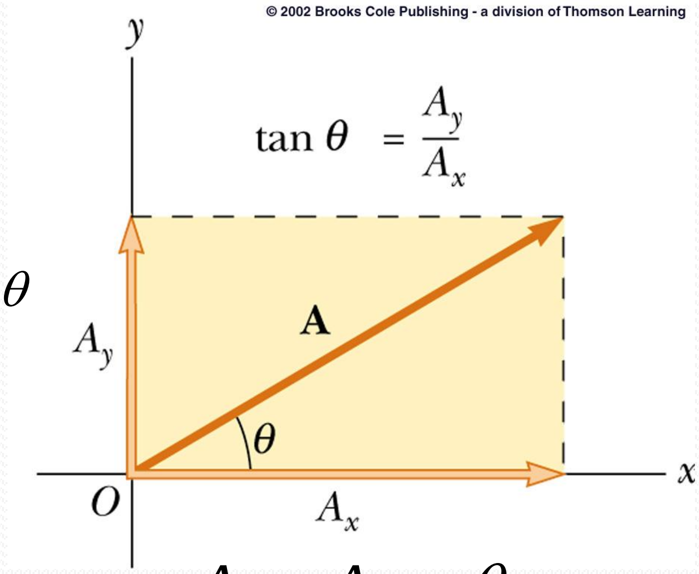
$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$

$$c = \sqrt{12050} = 109.8\ km$$

Components of a Vector

- The projections of a vector on the x and y axis are called the components of the vector.



$$A_y = A\sin\theta$$

$$A_x = A\cos\theta$$

- *The y-component of a vector is the projection along the y-axis.*
- *The x-component of a vector is the projection along the x-axis.*

$$A = \sqrt{A_x^2 + A_y^2} \quad \textit{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

HW: from the diagram below:

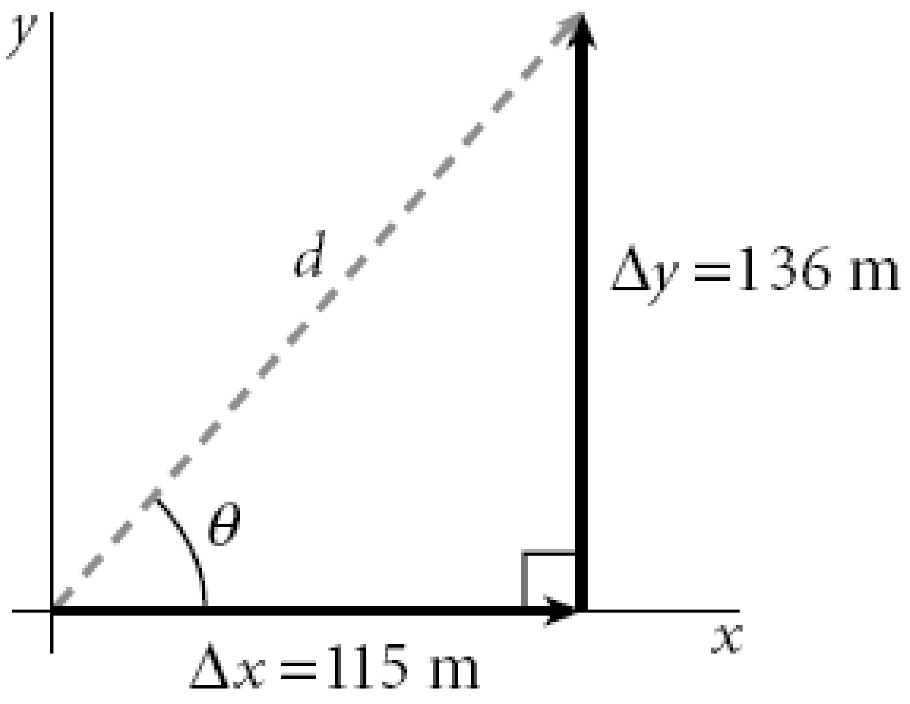
Given:

Δy = 136 m

Δx = 115 m

Find:

d = ? θ = ?

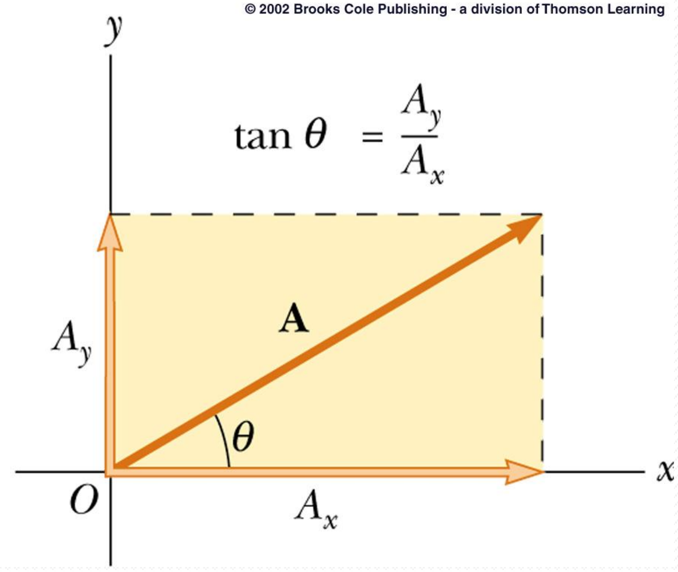


Vectors with two and three components

- Vectors with two components

$$\vec{\mathbf{A}} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}$$

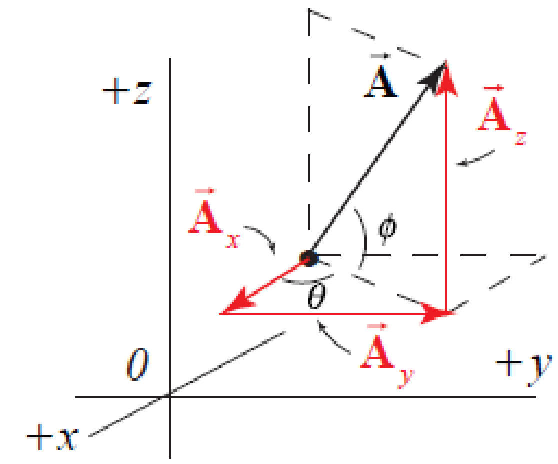
$$A = \sqrt{A_x^2 + A_y^2}$$



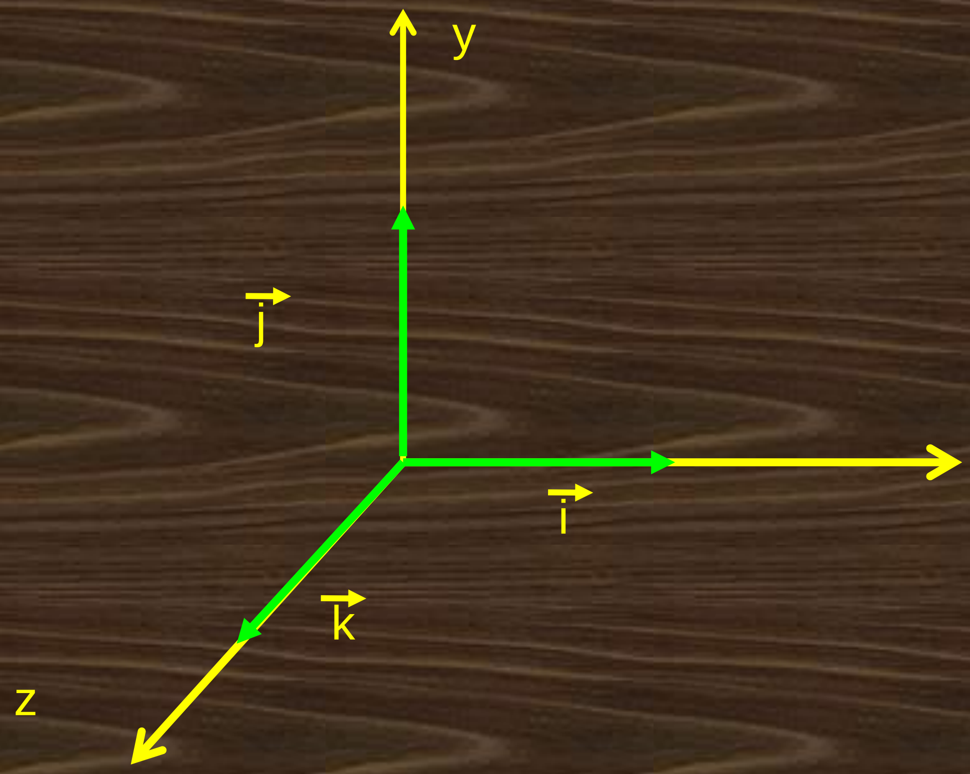
- Vectors with three components

$$\vec{\mathbf{A}} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}}\,.$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Unit Vectors



Unit Vectors

- Vector quantities are often expressed in terms of unit vector.
- A unit vector is a dimensionless vector having a magnitude of exactly one.
- Unit vectors are used to specify a given direction and have no other physical significance.
- Symbols **i, j and k** are used for unit vectors and pointing in the positive x, y and z direction respectively.

Vector multiplication

- Vector multiplications can be:

1. Dot product
2. Cross product

- Notes:
  - Dot product (**A.B**) multiplication results a scalar quantity.
  - Cross product (**A** x **B**) multiplication results a vector quantity.
  - When vectors are added, subtracted or multiplied, the unit vectors have to be considered.
  - As two vectors added or subtracted, the similar components (i with i, j with j and k with k) are combined or subtracted together, only.
  - When two vectors multiplied with dot (.), non-similar components multiplications are equal to zero.
  - We can also measure the angle between the two vectors using the below formular: **A.B**=$|\boldsymbol{A}|\ |\boldsymbol{B}|\boldsymbol{cos\theta}$ ($\boldsymbol{check\ question\ bank\ three}$)

3. The "Dot" Product (Vector Multiplication)

- Multiplying two vectors (for example A and B) sometimes gives you a SCALAR quantity which we call it the **SCALAR DOT PRODUCT**.

$$\mathbf{A} = \mathbf{A_x i} + \mathbf{A_y j} + \mathbf{A_z k}$$

$$\mathbf{B} = \mathbf{B_x i} + \mathbf{B_y j} + \mathbf{B_z k}$$

$$A \bullet B = A_x B_x + A_y B_y + A_z B_z$$

Example:

Let A = (3i - 4j - 5k)

Let B = (2i + 7j + 3k)

Therefore, A "dot" B = **(3)(2) + (-4)(7) +(-5)(3) = -37**

- HW: From the vector below, determine: A+B, B+A, A-B, B-A, A.B and B.A.

A= 2i+3j-2k, **B**= i-4k

Multiplying or Dividing a Vector by a Scalar quantity

- A vector can be multiplied or divided by a quantity (such as a number).
- If the scalar is positive, the direction of the resultant is the same as of the original vector.

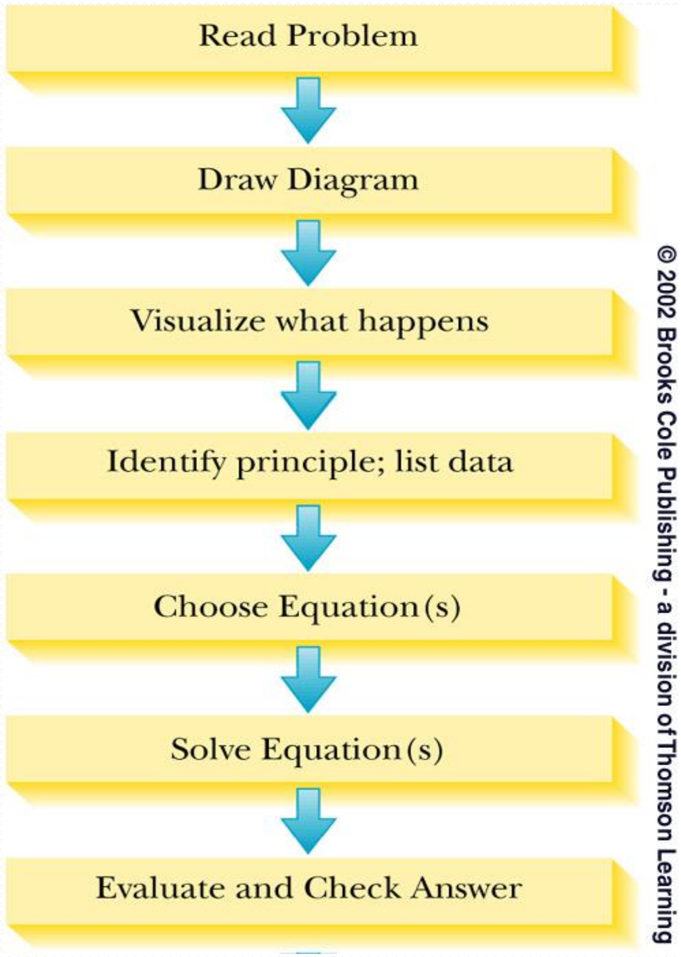
$\vec{A}$ $\quad$ $5 \text{ x } \vec{A} = 5\vec{A}$ $\quad$ $\frac{1}{2} \vec{A}$

- If the scalar is negative, the direction of the resultant is opposite that of the original vector.

$\vec{A}$ $\quad$ $-2 \text{ x } \vec{A} = -2\vec{A}$

Problem Solving Strategy

- Read the problem
  - identify type of problem, principle involved
- Draw a diagram
  - include appropriate values and coordinate system
  - some types of problems require very specific types of diagrams



- Visualize the problem
- Identify information
  - identify the principle involved
  - list the data (given information)
  - indicate the unknown (what you are looking for)

End of chapter three