Chapter 2: <u>Flexural Analysis and Flexural Strength</u>

<u>Flexural Analysis</u>

In developing elastic equations for flexural stress, the effects of <u>(a) prestress force</u>, <u>(b) self-weight moment</u>, and <u>(c) dead and live load moments</u> are calculated separately, and the separate stresses are superimposed.

P_i = initial prestress force

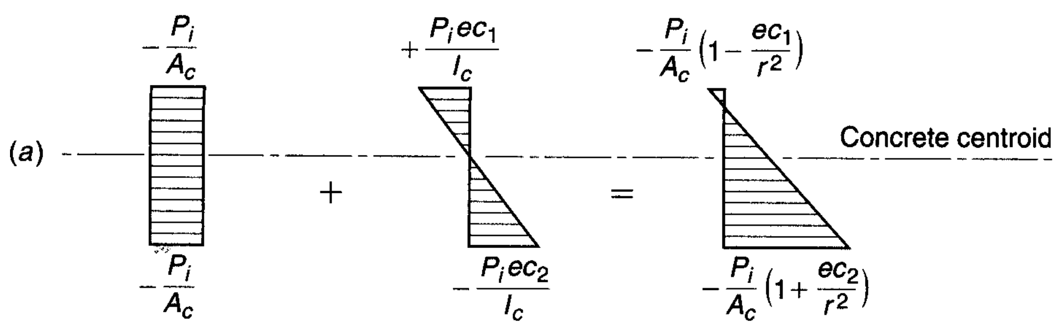
e = eccentricity below the centroid of the cross section.

A_c = area of the cross section

c_1 and ***c_2*** = top and bot fiber distances, respectively.

f_1 and ***f_2*** = top and bot fiber stresses, respectively.

a. **<u>initial prestress alone</u>** *<u>Sign convention:</u>* ***Tension* (+)** ***Compression* (−)**



$$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c} = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) \qquad f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c} = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right)$$

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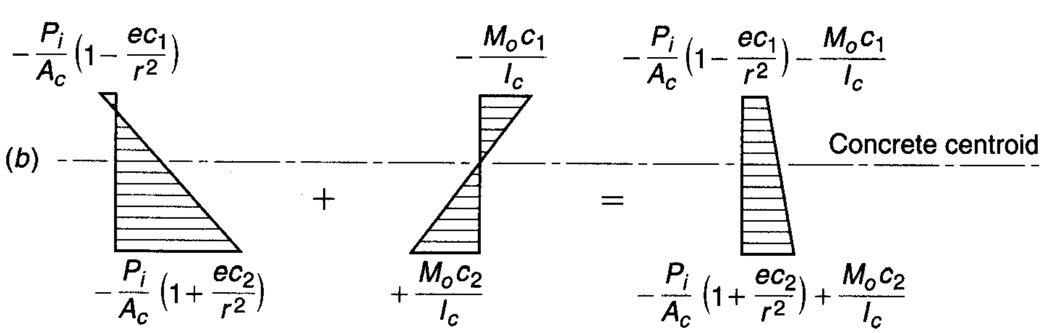
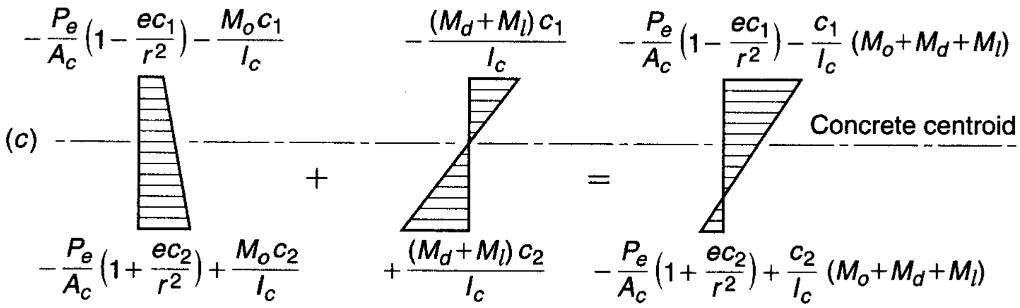
b. **<u>initial prestress + self-weight</u>**



$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_o c_1}{I_c} \qquad f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_o c_2}{I_c}$$

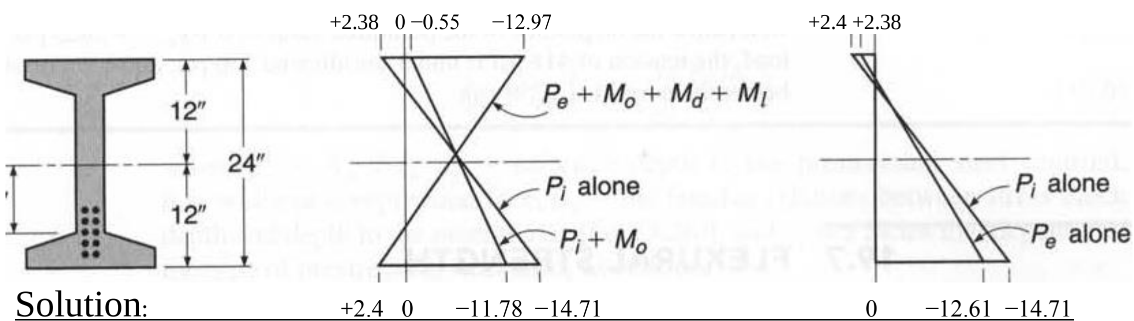
c. **final prestress + full service load**



$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{(M_o + M_d + M_l)c_1}{I_c} \qquad f_2 = -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{(M_o + M_d + M_l)c_2}{I_c}$$

Example 1: A simply supported symmetrical I beam of depth = **610 mm** (24″) pre-tensioned with multiple 7-wire strands with $\boldsymbol{e}$ **= 200 mm**, will be used on a **12 m** simple span. Given: $\boldsymbol{I_c = 5\times10^9}$ **mm**^{4}, $\boldsymbol{A_c}$ **= 113548 mm**^{2}, $\boldsymbol{r^2}$ **= 44000mm**^{2}, $\boldsymbol{S}$ **= 1.639×10**^{7} **mm**^{3}, self-weight $\boldsymbol{w_o}$**= 2.67 kN/m**.
Superimposed $\boldsymbol{D}$ + $\boldsymbol{L}$ 'sustained' **= 11 kN/m.**
$\boldsymbol{P_i}$ **= 700 kN,** $\boldsymbol{P_e}$ **= 600 kN,** $\boldsymbol{f'_{ci}}$ **= 26 MPa,** $\boldsymbol{f'_c}$ **= 35 MPa.** Calculate:

1. Concrete flexural stresses at midspan section at time of transfer.
2. Concrete flexural stresses at midspan section after all losses.
3. Compare with ACI allowable stresses for Class U



Solution:

Stresses at midspan(EX.1) *Stresses at ends (EX.2)*

a) Initial P_i

$$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c} = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right)$$

= (− 700,000/113,548)(1 − 200×305/44,000) = + 2.38 MPa (+ tension)

$$f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c} = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right)$$

= = (− 700,000/113,548)(1 + 200×305/44,000) = − 14.71 MPa (− compression)

b) Due to P_i + self-weight;

M_o = 2.67 $(12)^2$/ 8 = 48.06 kNm

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_o c_1}{I_c}$$

= +2.38 − 48.06×10^6×305 / 5×10^9 = +2.38 – 2.93 = − 0.55 MPa (comp, OK)

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_o c_2}{I_c}$$

= − 14.71 + 2.93 = − 11.78 MPa (11.78 < 15.6 OK)

Check ACI permissible limits: Tension at transfer =**0.25**√***f'***$_{ci}$ = 0.25 √26 = 1.27 MPa
Check ACI permissible limits: Compr at transfer = **0.60** ***f'***$_{ci}$ = 0.60 × 26 = 15.6 MPa

c) Due to P_e + full service load;

$M_d + M_l$ = 11 $(12)^2$/ 8 = 198 kNm

$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{(M_o + M_d + M_l)c_1}{I_c}$$

= + 2.38(600/700) – 2.93 −198×10^6×305 / 5×10^9 = 2.04 – 2.93 – 12.08 = – 12.97 MPa
(12.97 < 15.75 OK)

$$f_2 = -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{(M_o + M_d + M_l)c_2}{I_c}$$

= − 14.71 (600/700) + 2.93 + 198×10^6×305 / 5×10^9 = −12.61 + 2.93 + 12.08 = + 2.4 MPa
(2.4 < 3.55 OK)

Check ACI permissible limits: Tension at service load =**0.6**√***f'***$_c$ = 0.6 √35 = 3.55 MPa
Check ACI permissible limits: Compr at service load = **0.45** ***f'***$_c$ = 0.45 × 35 = 15.75 MPa

Example 2: For example 1, calculate the same requirement at ends of beam and compare with ACI limits.

Solution

a) Due to P_i

f_1 = + 2.38 MPa < **0.50**√***f'***$_{ci}$ = 0.50 √26 = 2.55 MPa OK
f_2 = − 14.71 MPa < **0.70** ***f'***$_{ci}$ = 0.70 × 26 = 18.2 MPa OK

b) Due to P_e

f_1 = + 2.4 MPa < **0.60** √***f'***$_c$ = 0.60 ×√ 35 = 3.55 MPa OK
f_2 = − 12.61 MPa < **0.45** ***f'***$_{ci}$ = 0.45 × 35 = 15.75 MPa OK

Flexural Strength

After cracking, the prestressed beam behaves essentially as an ordinary RC beam. The strength of PC beam can be predicted by the same methods developed for RC beams, with modifications to account for

a. The different shape of the stress-strain diagram for prestressing steel
b. The tensile strain already presented in prestressing steel before the beam is loaded.

ACI 18.7 includes approximate equations for flexural strength:

Stress in prestressing steel at failure f_{ps}

f_{ps} = Stress in prestressing steel at failure, MPa
f_{pe} = effective prestress = P_e / A_{ps} , MPa
f_{pu} = tensile strength of prestressing steel, MPa
If $f_{ps} \geq 0.50 f_{pu}$, ACI Code 18.7.2 permits use of certain approximate equations for f_{ps}

1. For members with ***bonded*** tendons

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\frac{\rho_p f_{pu}}{f'_c}\right)$$

Where $\rho_p = A_{sp} / b\, d_p$, d_p =effective depth to the prestressing steel centroid.
$\beta 1 = a / c$
$\beta 1 = 0.85$ …………………. for $f'c \leq 28$ **MPa**
$\beta 1 = 0.85 - 0.05\,(f'c - 28) / 7$ ……….. for $f'c > 28$ **MPa**
$0.65 \leq \beta 1 \leq 0.85$
γ_p = factor depending on the type of prestressing steel

$$\gamma_p = \begin{cases} 0.55 & \text{for } f_{py}/f_{pu} \geq 0.80 \text{ (typical high-strength bars)} \\ 0.40 & \text{for } f_{py}/f_{pu} \geq 0.85 \text{ (typical ordinary strand)} \\ 0.28 & \text{for } f_{py}/f_{pu} \geq 0.90 \text{ (typical low-relaxation strand)} \end{cases}$$

2. For members with ***unbonded*** tendons , with $l / h \leq 35$

$$f_{ps} = f_{pe} + 70 + f'_c / 100\rho_p \quad \leq f_{py}$$
$$\leq f_{pe} + 420$$

3. For members with ***unbonded*** tendons , with $l / h > 35$

$$f_{ps} = f_{pe} + 70 + f'_c / 300\rho_p \quad \leq f_{py}$$
$$\leq f_{pe} + 210$$

<u>Nominal Flexural Strength and Design Strength</u>

<u>a.</u> For ***rectangular*** sections, or ***I*** sections, or ***T*** sections in which the stress block depth is ≤ average flange thickness ($a \le h_f$), the <u>*nominal moment*</u>:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) \qquad a = \frac{A_{ps} f_{ps}}{0.85 f'_c b}$$

These equations can be combined to:

$$M_n = \rho_p f_{ps} b d_p^2\left(1 - 0.588\,\frac{\rho_p f_{ps}}{f'_c}\right)$$

<u>*Design moment*</u>: $M_u = \varphi M_n$ φ = strength reduction factor for flexure

<u>b.</u> If ($a > h_f$), the <u>*nominal moment*</u> is analogous to that used in ordinary RC T-sections:

For overhanging part: for the remaining part:

$$A_{pf} = 0.85\,\frac{f'_c}{f_{ps}}(b - b_w)h_f \qquad A_{pw} = A_{ps} - A_{pf}$$

The sum of the both parts contributions:

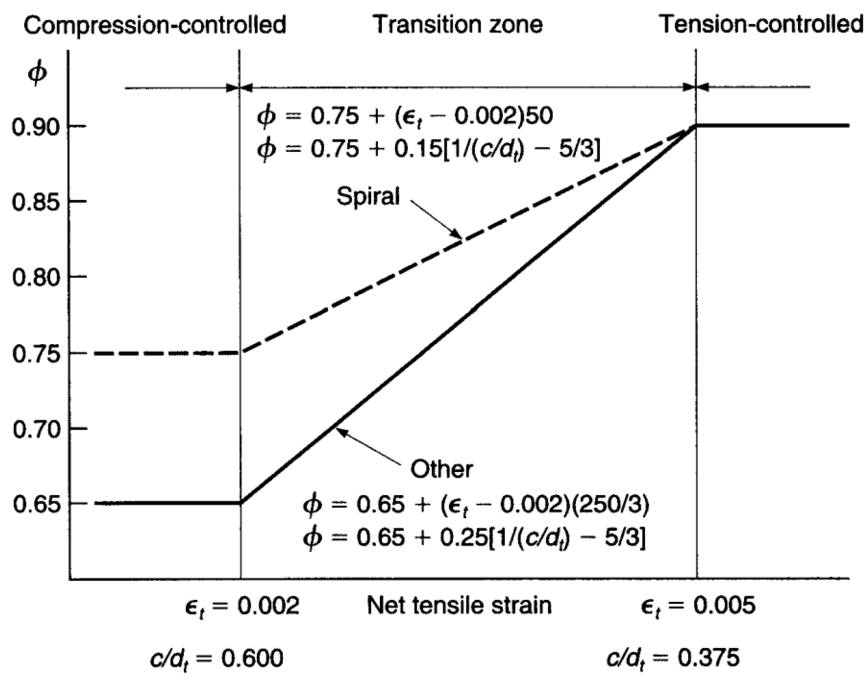
$$M_n = A_{pw} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f\left(d_p - \frac{h_f}{2}\right)$$

$$a = \frac{A_{pw} f_{ps}}{0.85 f'_c b_w}$$

<u>*Design moment*</u>: $M_u = \varphi M_n$ $\varphi = 0.9$

Limits of Reinforcement

To maintain φ = 0.9 and ensure a ductile failure: $\varepsilon_{t,\,net} \geq \mathbf{0.005}$ is required. This is ensured if $c/d_t \leq \mathbf{0.375}$. *'Tension-controlled beam'*
In PC beams, $c/d_p \leq \mathbf{0.375}$
If $c/d_t > \mathbf{0.375}$, the beam is *compression-controlled* and φ must be determined



Minimum tension reinforcement

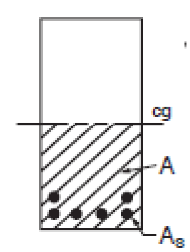
ACI 18.8 requires that, *for bonded prestressed beam*, the total tension reinforcement be adequate to support a **factored load ≥ 1.2 × cracking load** of the beam, calculated on the basis of $f_r = \mathbf{0.62}\sqrt{f'_c}$

Minimum bonded reinforcement

ACI 18.9 requires that, *for unbonded prestressed beam*, some bonded reinforcement must be added in the form of ordinary steel bars to control cracking:

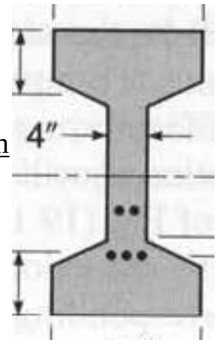
$$A_s = 0.004\,A$$

A = area of part of cross section between tension face and the centroid of the section



<u>Example 3</u>: The prestressed **I** beam shown is pretensioned using **5** low relaxation stress-relieved 7- wire, **Grade 1860 MPa**, **12.7mm** dia strand, carrying effective prestress $\boldsymbol{f_{pe}}$ **= 1100 MPa**. $\boldsymbol{f'_c}$ **= 28 MPa**. Calculate the design strength of the beam.

Section properties:
Ave. h_f = 115 mm
Total depth = 610 mm
e = 130 mm
d_p = 435 mm
d_t = 500 mm



<u>b_w = 100 mm</u>
(4″)
<u>b = 300 mm</u>

Solution:

A_{ps} = 5×98.7 = 493 mm^2
Fpe = 1100 MPa > 0.5 fpu = 0.5(1860) = 930 MPa OK ACI eqs are applicable.
$\boldsymbol{\rho_p = A_{sp} / b\, d_p}$ = 493 / 300×435 = 0.0038

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\frac{\rho_p f_{pu}}{f'_c}\right)$$ = 1860(1 – 0.28×0.0038×1860 / 0.85×28) = 1705 MPa

$$a = \frac{A_p f_{ps}}{0.85 f'_c b}$$

a = 493×1705 / 0.85×28×300 = 117.7 mm > *Ave.* h_f = 115 mm
Thus, equations of T-section should be used

$$A_{pf} = 0.85\frac{f'_c}{f_{ps}}(b - b_w)h_f \qquad A_{pw} = A_{ps} - A_{pf}$$

= 0.85×28(300 – 100)×115 / 1705 = 321 mm^2, A_{pw} = 172 mm^2
Actual a =172×1705 / 0.85×28×100 = 123.2 mm
$c = a / \beta_1$ = 145 mm
Check c / d_t = 145 / 500 = 0.290 < 0.375 OK, tens-controlled, φ = 0.9

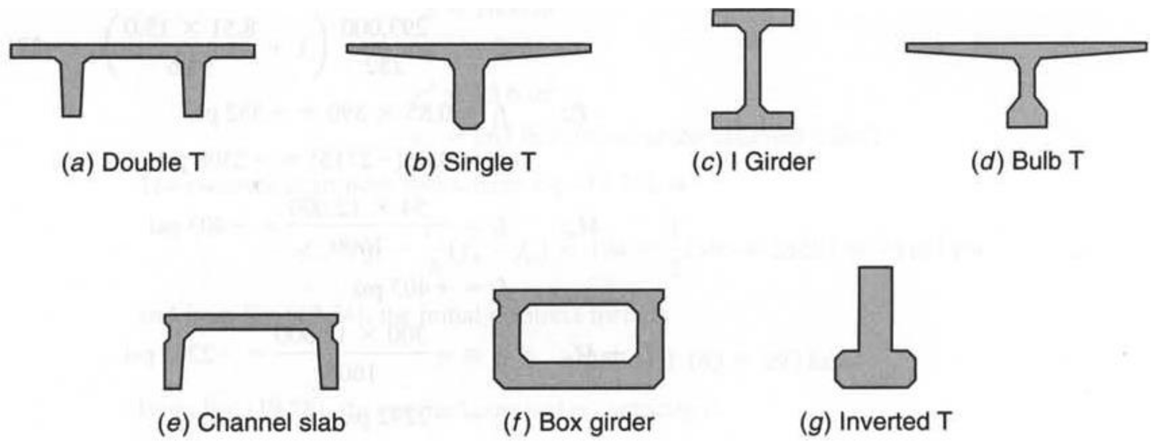
$$M_n = A_{pw} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(d_p - \frac{h_f}{2}\right)$$

= 109.50×106 + 206.64×106 = 316.14 kNm

$\boldsymbol{M_u}$ = 0.9×316.14 = <u>284.53 kNm</u>

Shape Selection

In PC design there is a freedom to select cross-section proportions and dimensions to suit special requirements. The member depth, the web thickness, and the flange widths and thicknesses can be changed and modified to produce a nearly ideal beam for a given case.
Several common precast shapes are shown below



The *double T* is the most widely used, ***b*** ~ 1.25 – 3.5 m, spans up to 18 m.
The *single T* spans up to 36 m and carries heavier loads.
The *I* and *bulb T* are used for bridges and roof girders up to 42m.
The *channel slab* is suitable for floors of intermediate spans.
The *box girder* is used for bridges of intermediate to major spans.
The *inverted T* provides a bearing ledge to carry precast deck members in the perpendicular direction.