CHAPTER TWO

AC-DC CONVERSION:
UNCONTROLED RECTIFICATIONS

2.1 INTRODUCTION TO BASIC RECTIFIER CIRCUITS

Several types of rectifier circuits are available: Single-phase and three-phase, half-wave and full-wave, controlled and uncontrolled, etc. For a given application, the type used is determined by the requirements of that application. In general the types of rectifiers are:

1. Uncontrolled Rectifiers : Provide a fixed d.c. output voltage for a given a.c. supply where diodes are used only.

2. Controlled Rectifiers : Provide an adjustable d.c. output voltage by controlling the phase at which the devices are turned on, where thyristors and diodes are used. These are of two types:

    (a) Half-controlled : Allows electrical power flow from a.c. to d.c. (i.e. rectification only) .
    (b) Fully-controlled : Allows power flow in both directions (i.e. rectification and inversion).

There are many applications for rectifiers. Some of them are:

» Variable speed d.c. drives,
» Battery chargers,
» DC power supplies for electric railways and,
» Power supply for a specific application like electroplating.

The power rating of a single-phase rectifier tends to be lower than 10 kW, whereas the three-phase bridge rectifiers are used for delivering higher power output, up to 500 kW at 500 V d.c. or even more.

In study of the rectifier circuits; there are certain electrical properties of interest that should be taken into consideration. These are the properties on the supply side and the properties on the load side of the rectifier respectively. The supply-side will be assumed to have zero impedance such that the sinusoidal supply voltages remains undistorted even when the rectifier current pulses drawn from the supply are nonsinusoidal.

Basically the study of rectifier circuit is concentrated on study of waveforms as well as circuit analysis with the assumptions that:

(1) No energy is stored within a rectifier.
(2) The forward vo1tge drop, and reverse and forward leakage currents in diodes and thyristors are neglected.
(3)The pulse is of sufficient amplitude to switch on the appropriate thyristor.

Although the controlled rectification is the most important type that is mainly used in many industrial applications, it is of interesting to demonstrate first the uncontrolled diode rectifiers to clarify the presentation. Hence, the single-phase and poly-phase, half-wave and full -wave uncontrolled rectifiers loaded with different types of load will be discussed in the following sections.

2.2 UNCONTROLLED RECTIFICATIONS

In this type of rectifier, the produced d.c. output power is fixed with the converter used. They usually employ diodes as their power switches. The following subsections deal with the basic operation of some examples of uncontrolled rectifiers single-phase half-wave rectifier loaded with resistive and series resistive inductive loads.

2.2.1 Single-Phase Half-Wave Uncontrolled Rectifier with Resistive Load

Fig.2.1 shows the basic circuit for a single-phase, half-wave uncontrolled rectifier supplying a resistive load. The circuit is supplied by a single-phase transformer whose secondary represents the rectifier's circuit a.c. source (v_s) that is represented by a sinusoidal wave given by,

$$v_s(\omega t) = V_m sin\omega t \tag{2.1}$$

where v_s is the instantaneous supply voltage, V_m is the peak value of the supply voltage, $\omega = 2\pi f$ is the angular frequency, and t and is the time.

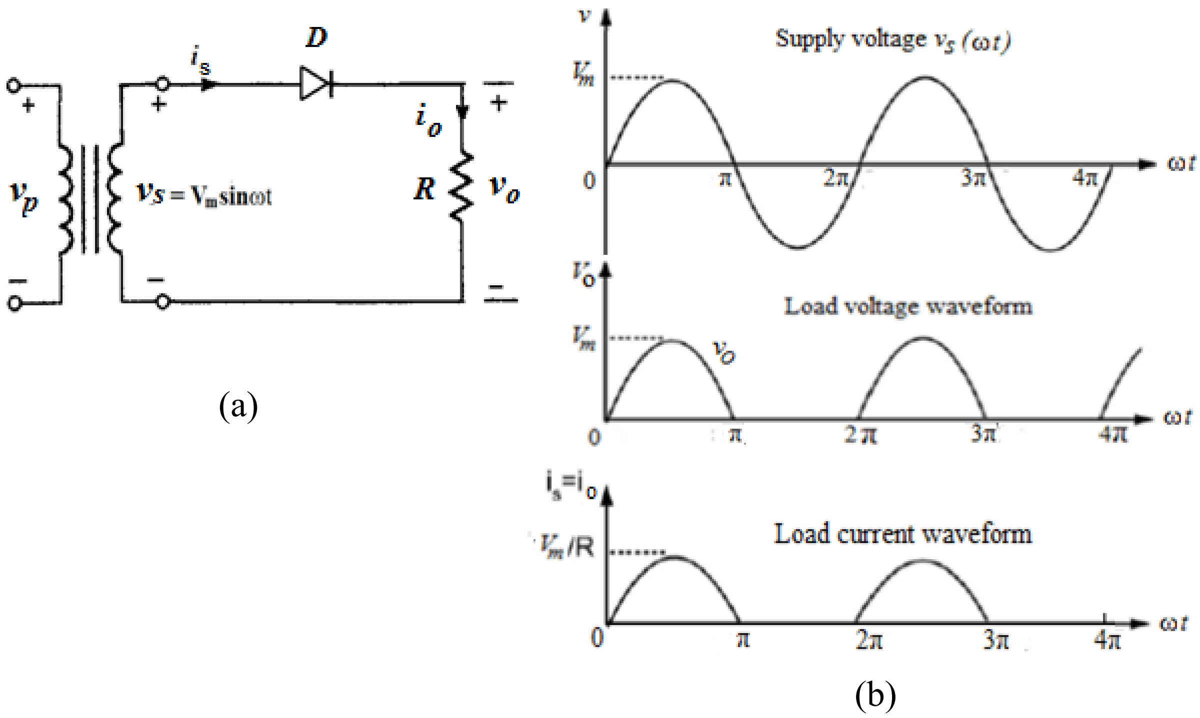


Fig.2.1 Single-phase half-wave rectifier: (a) Circuit and, (b) Waveforms.

For this configuration, the diode will conducts (becomes forward biased) whenever the supply voltage v_S is positive. This means that, during the positive half cycle, ($0 < \omega t < \pi$), The diode conducts and behaves like a closed switch connecting the supply to the load. Current i_o will flow through the load with value $i_o = v_o / R$ and since the load is resistive, the load current waveform will be replica of the voltage waveform.

The average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{2\pi}\int_0^{\pi} v_s\,(\omega t) d\omega t \qquad (2.2)$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m sin\omega t\, d\omega t = \frac{V_m}{2\pi}\,(-cos\pi + cos0^o) = \frac{V_m}{\pi} \qquad (2.3)$$

Since the load is resistive, therefore the load voltage and current are in-phase and they are related by $i_o = v_o / R$. Consequently, the average value of the load current I_{dc} is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R} \qquad (2.4)$$

The output d.c. power is given by:

$$P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} \qquad (2.5)$$

The *rms* value of the load voltage V_{orms} can be calculated over one cycle as follows:

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} v_s^2\,(\omega t) d\omega t} \qquad (2.6)$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_m sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{2\pi}\int_0^{\pi} \frac{1}{2}(1 - cos2\omega t) d\omega t} = \frac{V_m}{2} \qquad (2.7)$$

Since: $(sin\omega t)^2 = \frac{1}{2}(1 - cos2\omega t)$

Therefore, the *rms* value of the load current I_{orms} is :

$$I_{orms} = \frac{V_{orms}}{R} = \frac{V_m}{2R} = \frac{I_m}{2} \qquad (2.8)$$

The *rms* value $I_{orms} = I_m/2$ for half-wave operation compared with the corresponding value for pure sinusoidal operation which is $I_{orms} = I_m/\sqrt{2}$.

The output a.c. power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.5V_m)^2}{R} \qquad (2.9)$$

Performance parameters

From the above discussion, the performance of a rectifier can be, therefore, evaluated in terms of the following parameters:

1.The output d.c. power is given by

$$P_{dc} = V_{dc} I_{dc} \qquad (2.10)$$

2.The output a.c. power is given by

$$P_{ac} = V_{orms} I_{orms} \qquad (2.11)$$

3. The efficiency of the converter is given by

$$\eta = \frac{P_{dc}}{P_{ac}} \tag{2.12}$$

4. Output a.c. component

The output voltage can be considered to have two components: including (i) d.c. value and (ii) the a.c. components or ripple. The *rms* value of the a.c. component of the output voltage is

$$V_{ac} = \sqrt{V_{orms}^2 - V_{dc}^2} \tag{2.13}$$

5. Form Factor, *FF* : It is a measure of the shape of the output voltage.

$$FF = \frac{V_{orms}}{V_{ac}} \tag{2.14}$$

6. Ripple factor, *RF* : It is a measure of the ripple content or the degree of distortion in a rectified voltage waveform which can be calculated as

$$RF = \frac{\text{rms value of the ac components}}{\text{average value}} = \frac{V_{ac}}{V_{dc}}$$

$$= \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} \tag{2.15}$$

7. Transformer utilization factor, *TUF* : This is defined as

$$TUF = \frac{P_{dc}}{V_s I_s} \tag{2.16}$$

where

V_s : *rms* voltage of the transformer secondary
I_s : *rms* current of the transformer secondary

8. Displacement Factor, *DF*

$$DF = cos\emptyset \tag{2.17}$$

"$\emptyset$" is the phase angle between the fundamental component of the input current and voltage.

9. Harmonic Factor, *HF*

$$HF = \frac{\sqrt{I_s^2 - I_1^2}}{I_1} \qquad (2.18)$$

where
I_1 : the fundamental *rms* component of the input current.

10. Power factor *PF* : For rectifier , this is defined as

$$PF = \frac{\text{a.c. output power}}{\text{input volt ampere}} = \frac{P_{ac}}{S_{in}} \qquad (2.19)$$

Example 2.1

An ideal single-phase source, 240 V, 50 Hz, supplies power to a load resistor $R = 100\ \Omega$ via a single ideal diode.

(a) Calculate the average and *rms* values of the load current and the power dissipation.
(b) Calculate the circuit power factor and the ripple factor.
(c) What must be the rating of the diode?

Solution

$$I_{dc} = \frac{V_m}{\pi R} = \frac{240\sqrt{2}}{\pi \times 100} = 1.08\text{A}$$

$$I_{orms} = \frac{V_m}{2R} = \frac{240\sqrt{2}}{2 \times 100} = 1.7\text{A}$$

The average power dissipation in the load resistor *R* is given by the average a.c. power:

$$P_{ac} = I_{orms}^2 \times R = \left(\frac{V_m}{2R}\right)^2 = \frac{{V_m}^2}{4R^2} = (1.7)^2 \times 100 = 289\text{ W}$$

$$S_{in} = V_s \times I_{orms} = 240 \times 1.7 = 408\text{ W}$$

$$PF = \frac{P_{ac}}{S_{in}} = \frac{289}{408} = 0.708$$

$$RF = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = \frac{\sqrt{I_{orms}^2 - I_{dc}^2}}{I_{dc}} = \sqrt{(\frac{1.7}{1.08})^2 - 1} = 1.21$$

The value 1.21 for half-wave rectification is large, since the ideal value of the ripple factor should be zero for the output d.c. voltage. The diode must be rated in terms of a peak reverse voltage and a mean forward current.

Diode $PRV = V_m = \sqrt{2} \times 240 = 339.4V \longrightarrow$ Choose 400V diode.

Either the *rms* or the mean (average) current could be use as the basis of current rating: Since $I_{orms} = 1.7$A choose 2A diode rating.

2.2.2 Single-Phase Half-Wave Uncontrolled Rectifier with *R-L* Load

If the load consists of a series resistor and inductor, the current will flow through the negative cycle as well; Fig.2.2 shows the circuit diagram and Fig.2.3 shows the load voltage and current waveform for this case.

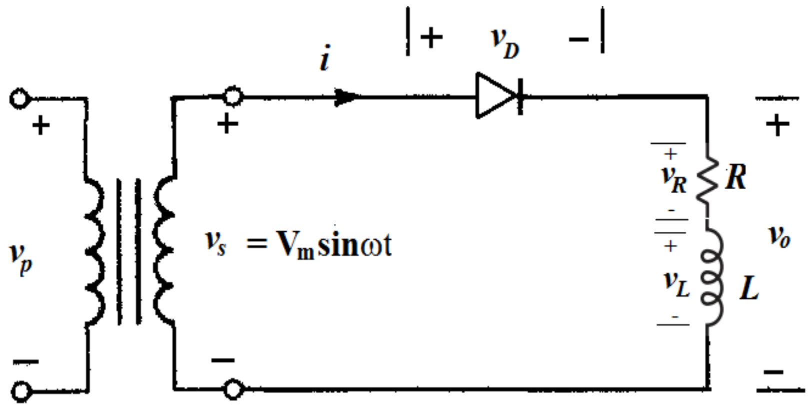


Fig .2.2 Single-phase half-wave rectifier with *R -L* load.

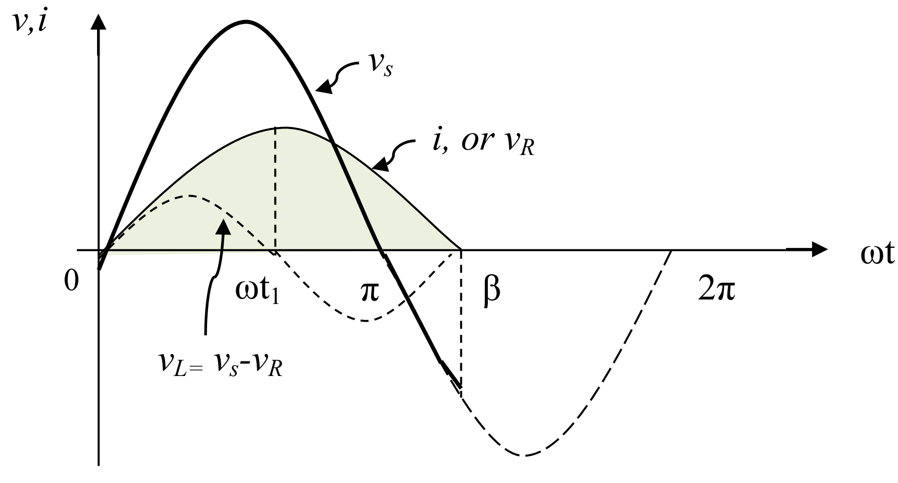


Fig.2.3 Waveforms for the circuit of Fig.2.2.

During conduction period, we have by KVL;

$$v_s = v_o = v_R + v_L$$

$$V_m sin\omega t = i\,R + L\frac{di}{dt} \tag{2.20}$$

Each supply period (cycle) in Fig.2.3 can be divided into 4-distinct regions:-

- From 0 – ωt_1: The current rises from zero to peak, which lags the voltage peak due to circuit inductance; v_L is positive and the inductor store energy.
- From $\omega t_1 - \pi$: The current decays, and hence v_L is negative. Both source and inductor supply energy to *R*.
- From $\pi - \beta$: The current continues to decay until it reaches zero, v_L remains negative, and hence energy is supplied by the inductor to both source and resistor.
- From $\beta - 2\pi$: At β current reaches zero and the diode cut-out. Current remains zero until the beginning of the next positive half cycle. The average output voltage is

$$V_{dc} = \frac{1}{2\pi}\int_0^\beta v_s\,(\omega t)d(\omega t) = \frac{1}{2\pi}\int_0^\beta V_m sin\omega t\,d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi}\left[-\cos\omega t \begin{Bmatrix}\beta\\0\end{Bmatrix}\right] = \frac{V_m}{2\pi}[-\cos\beta + \cos 0]$$

$$= \frac{V_m}{2\pi}[1-\cos\beta] \tag{2.21}$$

The average output current is

$$I_{dc} = \frac{V_m}{2\pi R}[1-\cos\beta] \tag{2.22}$$

Equation of the current:

The equation for the current through *R-L* load can be found from the solution of the differential equation (2.20) which can be re-written as:

$$L\frac{di}{dt} + i\,R = V_m sin\omega t \tag{2.23}$$

This is a first order differential equation. The solution of this equation has two parts:

1- Steady state solution : $i_{ss}(\omega t) = \frac{V_m}{|Z|}\sin(\omega t - \theta)$

2- Transient solution : $i_{tr}(\omega t) = Ae^{-\frac{t}{\tau}}$

where

$$|Z| = \sqrt{R^2 + (\omega L)^2} \qquad , \qquad \tan\theta = \frac{\omega L}{R}$$

and

$$\tau = \frac{L}{R} = \text{time constant}\,, \qquad A = \text{constant}$$

The complete solution is : $i(\omega t) = i_{ss}(\omega t) + i_{tr}(\omega t)$

$$i(\omega t) = \frac{V_m}{|Z|}\sin(\omega t - \theta) + Ae^{-\frac{t}{\tau}} \qquad (2.24a)$$

The constant A can be found from initial conditions: when t =0 , $i(\omega t) = 0$:

$$0 = \frac{V_m}{|Z|}\sin(-\theta) + Ae^{-0}$$

$$\therefore A = \frac{V_m}{|Z|}\sin(\theta) \qquad (2.24b)$$

Substitute Eq. (2.24b) into Eq. (2.24a) yields

$$i(\omega t) = \frac{V_m}{|Z|}\sin(\omega t - \theta) + \frac{V_m}{|Z|}\sin(\theta)\,e^{-\frac{t}{\tau}}$$

The final equation of the current is

$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\omega t - \theta) + sin\theta e^{-cot\theta\,.\omega t}] \qquad (2.25)$$

The current extinction angle β at which the current becomes zero can be determined for a given load impedance angle θ as:

From the final condition of the current, when ωt = β , i = 0 , hence from the above equation (2.25) :

$$0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$0 = [\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$\sin(\beta - \theta) = -\,sin\theta e^{-cot\theta\,.\beta} \qquad (2.26)$$

This is a transcendental equation which cannot be solved analytically. It can only be solved numerically by iteration technique. The initial guess of β is : $\beta_o = 180^o + \theta + \Delta$, where Δ is few degrees.

Example 2.2

For the half-wave uncontrolled rectifier circuit supplying a series resistive-inductive load shown in Fig.2.2. The supply voltage is $v_s(\omega t) = \sqrt{2} \times 220 \sin\omega t$ and the supply frequency is 50 Hz and the load parameter values are: $R = 5\ \Omega$, and $L = 16$ mH.

(a) Sketch the load voltage and current waveforms for two cycles.
(b) Calculate the current extinction angle β.
(c) Calculate the average value of the d.c. load voltage (V_{dc}) .
(d) Calculate the angle of maximum current (ωt_1) .

Solution

(a) The output voltage and current waveforms are as shown in Fig.2.4.

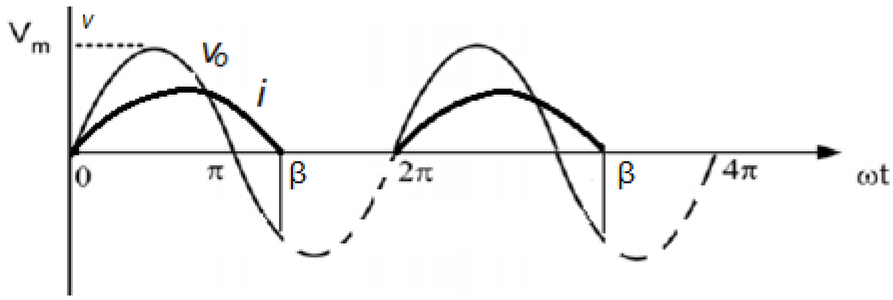


Fig.2.4 Waveforms.

(b) From the final condition of the current, when $= \beta$, $i(\beta) = 0$, hence substitute these values in Eq.(2.25) to find β ,

$$i(\beta) = 0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$0 = [\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$\sin(\beta - \theta) = -\,sin\theta e^{-cot\theta\,.\beta}$$

The solution of this transcendental equation numerically by iteration technique is as follows:
The initial guess of β is : $\beta_o = 180^o + \theta + \Delta$, where Δ is few degrees.

$\tan\theta = \omega L/R = 2\pi fL/R = (100\pi \times 16 \times 10^{-3})/5 = 1$

hence $\theta = 45^{o}$. and therefore : $\beta_o = 180^{o} + \theta + \Delta = 180^{o} + 45^{o} + 0 = 225^{o}$ as a starting value for iteration (here choose $\Delta = 0^{o}$):

Table 2.1.

Estimated β			
β (degrees)	**β (rad)**	**$\sin(\beta - \theta)$**	**$-\sin\theta\, e^{-\beta}$**
210°	**3.663**	**0.2588**	**- 0.0181**
220°	**3.837**	**0.0870**	**-0.0152**
225°	**3.925**	**0.0000**	**-0.0139**
226°	**3.942**	**-0.013**	**-0.0136**

By intuition : $\beta = 225.8^{o}$.

(c)The average DC output voltage is :

$$V_{dc} = \frac{1}{2\pi}\int_0^{\beta} v_s(\omega t)d(\omega t) = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\omega t\, d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi}\left[-\cos\omega t \begin{Bmatrix}\beta\\0\end{Bmatrix}\right] = \frac{V_m}{2\pi}[-\cos\beta + \cos 0°]$$

$$= \frac{V_m}{2\pi}[1 - \cos\beta]$$

$$V_{dc} = \frac{\sqrt{2}\times 220}{2\pi}[1 - \cos 225.8] = 84\text{ V}$$

(d) Angle of maximum current can be found by differentiating Eq.(2.25) and equating to zero , hence

$$\frac{di(\omega t)}{dt} = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\omega\cos(\omega t - 45°) - \omega \sin 45° e^{-(\cot 45).\omega t}] = 0$$

$$\frac{di(\omega t)}{dt} = [\cos(\omega t - 45°) - \sin 45° e^{-(\cot 45°).\omega t}] = 0$$

Maximum current occurs at $\omega t = \omega t_1$ as shown in Fig. 2.5, thus

$$\cos(\omega t_1 - 45°) = \sin 45° e^{-(\cot 45°)\,\omega t_1}$$

Solve the above transcendental equation numerically by starting at $\omega t_1 = 110^\circ$, we get the final solution $\omega t_1 = 146^\circ$.

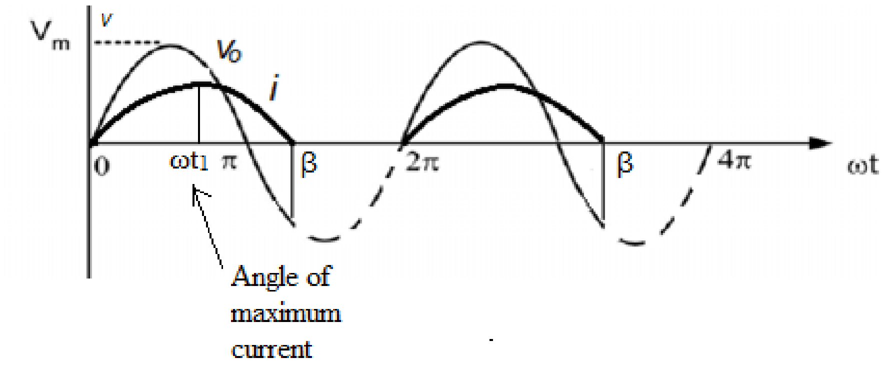


Fig.2.5 Angle of maximum current.

Example 2.3

A circuit is connected as shown in Fig.2.6 to a 240 V, 50 Hz supply. Neglect the diode voltage drop; determine the current waveform, the average load voltage and the average load current of (a) A pure resistor of 10 Ω, (b) An inductance of 0.1 H in series with a 10 Ω resistor. Assume the extinction angle $\beta = 265\,^\circ$.

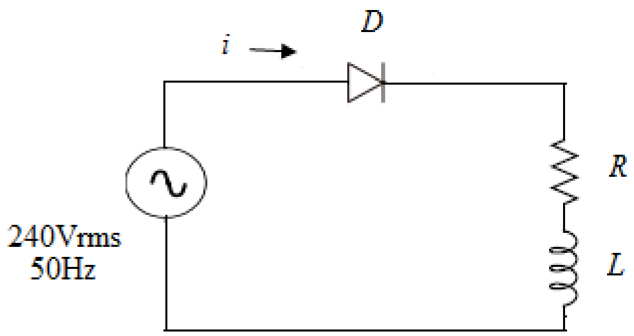


Fig.2.6.

Solution

(a) For resistive load

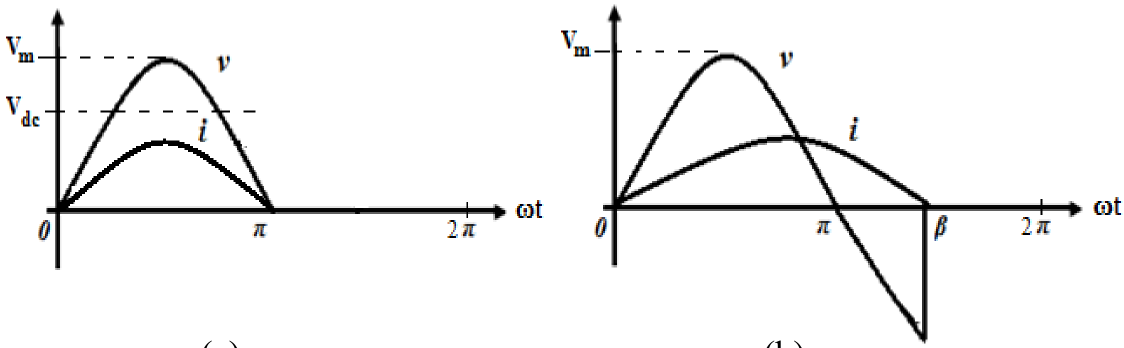
$$v_s = V_m \sin \omega t = \sqrt{2} \times 240 \sin 2\pi \times 50t = 339.4 \sin 314t$$

For single-phase half-wave uncontrolled rectifier:

$$V_{dc} = \frac{V_m}{\pi} = \frac{339.4}{\pi} = 108 \text{ V}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{108}{10} = 10.8\text{ A}$$

The voltage and current waveforms are shown in Fig.2.7 (a).



(a) (b)

Fig.2.7 Current and voltage waveforms : (a) Case of *R*-load, (b) Case of *R-L* load.

(b) For *R-L* load :

$$i(\omega t) = \frac{V_m}{|Z|}\sin(\omega t - \theta) + \frac{V_m}{|Z|}\sin\theta\, e^{-\frac{R}{L}t}$$

$$|Z| = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \times 50 \times 0.1)^2} = 32.97\ \Omega$$

$$\theta = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{31.4}{10} = 72.3^\circ = 1.262\text{ rad}$$

$$i(\omega t) = \frac{339.4}{32.47}[\sin(\omega t - 72.3^\circ) + \sin 72.3^\circ e^{-\frac{10}{0.1}t}]$$

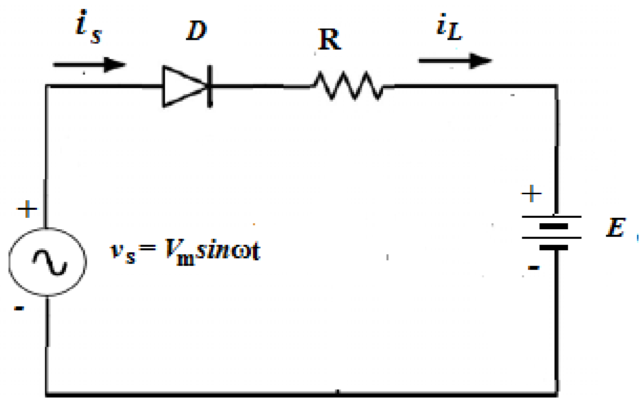
$$= 10.29\sin(100\pi t - 72.3^\circ) + 9.1\, e^{-100t}$$

$$V_{dc} = \frac{1}{2\pi}\int_0^{4.62=265^\circ} 339.4\ \sin\omega t\ d\omega t = 58.8\text{ V}$$

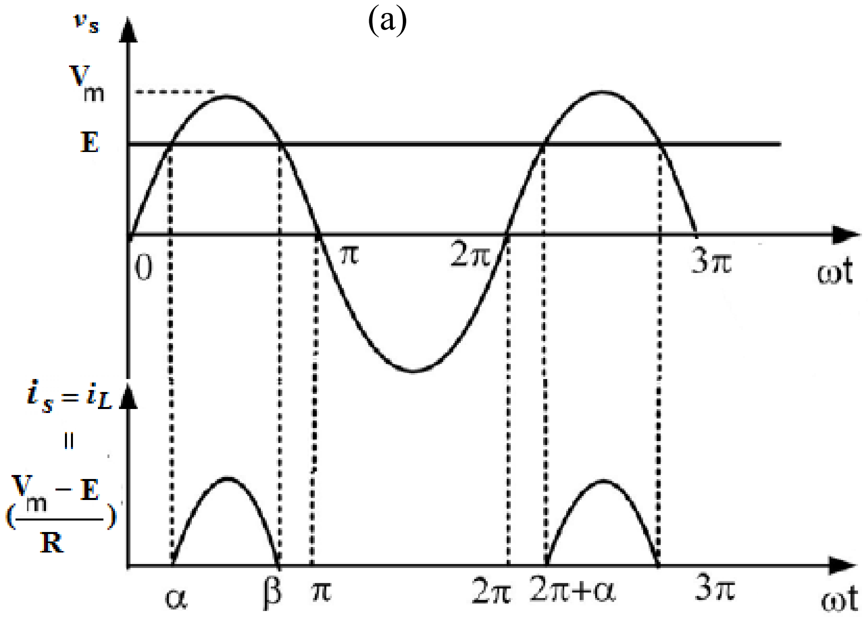
$$I_{dc} = \frac{V_{dc}}{R} = \frac{58.8}{10} = 5.88\text{ A}$$

2.2.3 Single-Phase Half-Wave Uncontrolled Rectifier Circuit for Battery Charging

The single-phase half-wave controlled rectifier circuit containing simple diode and a current limiting resistance *R* can be used to charge a battery of *emf E* from a single-phase supply (Fig.2.8). The battery opposes the unidirectional flow of current so that the net driving voltage is v_s-*E*.



(a)



(b)

Fig.2.8 Single-phase half-wave controlled rectifier circuit for battery charging: (a) Circuit, (b) Waveforms.

Neglecting any voltage drop on the diode (which is of the order 1–2 V) the current flows in the circuit is therefore,

$$i_L = \frac{v_s - E}{R} = \frac{1}{R}(V_m \sin \omega t - E) \qquad \alpha \leq \omega t \leq \beta \qquad (2.27)$$

where α and β define the current pulse in Fig. 2.8(b). These angles are defined as

$$v_s - E = V_m \sin\alpha - E = 0$$

Therefore

$$\alpha = \sin^{-1}\frac{E}{V_m} \tag{2.28}$$

and $$\beta = \pi - \alpha \tag{2.29}$$

The average value I_{av} of the battery charging current is defined by

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i_L\,(\omega t) d\omega t$$

Substituting Eq. (2.27) into the above defining integral expression gives

$$I_{av} = \frac{1}{2\pi}\int_\alpha^\beta \frac{1}{R}(\,V_m \sin\omega t - E)\,d\omega t$$

$$= \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\beta) + E(\alpha - \beta)]$$

$$\therefore\ I_{av} = \frac{1}{2\pi R}[2V_m\cos\alpha + E(2\alpha - \pi)] \tag{2.30}$$

Example 2.4

The battery charger circuit of Fig. 2.8 is used to charge a 270 V battery of electric vehicle from the main supply voltage of 230 V, 50 Hz through a limiting resistor R =5 Ω. Calculate the average charging current of the battery.

Solution

From equations (2.28) and (2.29)

$$V_m = 230 \times \sqrt{2} = 325.2\,V$$

$$\alpha = \sin^{-1}\frac{E}{V_m} = \frac{270}{325.2} = 0.830$$

$$\alpha = 56^\circ \qquad \text{and} \qquad \beta = \pi - \alpha = 124^\circ$$

Now from Eq.(2.30) :

$$I_{av} = \frac{1}{2\pi R}[2V_m \cos\alpha + E(2\alpha - \pi)]$$

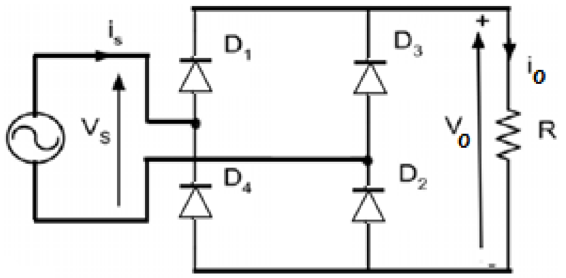
$$= \frac{1}{2\pi \times 5}[2 \times 325.2 \cos 56^\circ + 270(2 \times 0.976 - \pi)] = 1.38 \text{ A}$$

2.3 SINGLE-PHASE FULL-WAVE UNCONTROLLED RECTIFIERS

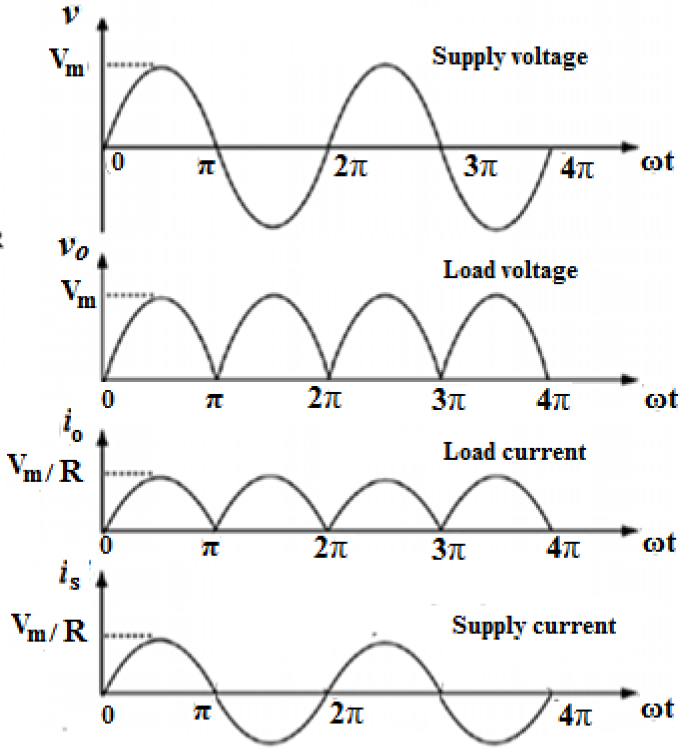
2.3.1 Case of Resistive Load

Fig.2.9 (a) presents the circuit connection for a single-phase full-wave, bridge rectifier loaded with a resistive load. It is sometimes referred to as the full-wave bridge rectifier. For this configuration, two diodes always conducting during the same interval to provide a closed loop for the current. D_1 and D_2 conduct whenever the supply voltage (v_S) is positive while D_3 and D_4 conduct whenever the supply voltage (v_S) is negative as illustrated by Fig .2.9 (b).

Since the load is a resistive load. Then, the load current will have the same waveform as the load voltage , $i_o = \frac{v_o}{R}$.



(a) Circuit.



(b) Waveforms

Fig.2.9 Single-phase full-wave rectifier with resistive load.

The average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m sin\omega t \, d\omega t = \frac{V_m}{\pi}\,(-cos\pi + cos0^o) = \frac{2V_m}{\pi} \qquad (2.31)$$

The value of Eq. (2.31) is seen to be twice the corresponding value of the half-wave rectifier given in Eq. (2.3).

The average value of the load current I_{dc} is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} \qquad (2.32)$$

$$P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} \qquad (2.33)$$

The *rms* value of the load voltage V_{orms} can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v_s^2\,(\omega t) d\omega t} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m sin\omega t)^2 \, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{\pi}\int_0^{\pi} \frac{1}{2}(1 - cos2\omega t) d\omega t} = \frac{V_m}{\sqrt{2}} \qquad (2.34)$$

The value of V_{orms} is seen to be √2 times the corresponding value of the half-wave rectifier given in Eq. (2.34). It should be noted that V_{orms} has the same *rms* value as a sinusoidal voltage of same maximum value . This means that the *rms* value is not affected by the polarity of the waveform.

The *rms* value of the load current I_{orms} is ,

$$I_{orms} = \frac{V_{orms}}{R} = \frac{V_m}{\sqrt{2}R} \qquad (2.35)$$

The output a.c. power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.707V_m)^2}{R} \qquad (2.36)$$

The *PRV* for any diode in this configuration is (V_m) ; and The load voltage ripple factor is

$$RF = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = 0.48$$

which is smaller than the corresponding *RF* value of the half-wave rectifier (see Example 2.1).

The supply current i_s is a pure sinusoidal as shown in Fig.2.9 (b). This is because the application of sinusoidal voltage to the resistive load causes sinusoidal supply current which is in-phase with the voltage. The average and *rms* values of the supply current are,

$$I_{s(av)} = \frac{1}{2\pi}\int_0^{2\pi} V_m sin\omega t\, d\omega t = 0 \qquad (2.37)$$

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (\frac{V_m}{R} sin\omega t)^2\, d\omega t} = \frac{V_m}{\sqrt{2}R} \qquad (2.38)$$

If we neglect the voltage drops across the diodes, the input power will be equal to the output power and therefore the power factor of the circuit will be unity.

2.3.2 Single-Phase Full-Wave Bi-Phase (Center-Tapped) Uncontrolled Rectifier with *R*-Load

This rectifier is shown in Fig.2.10. The waveforms and analysis are same as that for the bridge rectifier of Fig.2.9. Hence, it will not be repeated here.

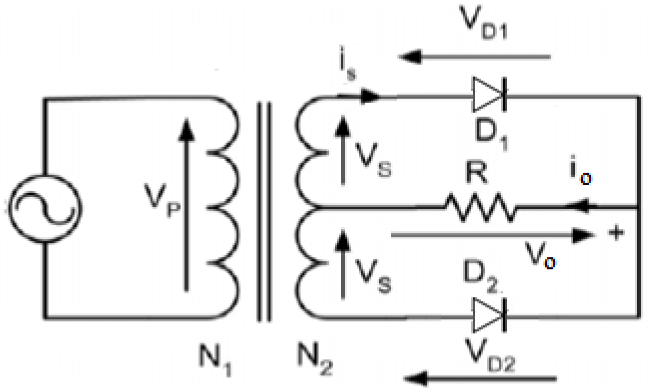


Fig.2.10 Bi-phase (center- tapped) full wave uncontrolled rectifier.

Example 2.5

For the single-phase, full-wave, uncontrolled rectifier shown in Fig.2.9, the supply voltage is 110 V, 50 Hz. The load resistor is 25 Ω, calculate:

(a)The average values of the output voltage and current.

(b)The *rms* values of the output voltage and current.

(c) The d.c. power consumed by the load (P_{dc}) and the average value of the power delivered to the load (P_{ac}).

Sketch the appropriate load voltage and diode voltage waveforms.

Solution

(a) The average value of the load voltage V_{dc}

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi} = \frac{2\sqrt{2}\times 110}{\pi} = 99.035 \text{ V}$$

The average value of the load current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{99.035}{25} = 3.961 \text{ A}$$

(b) the *rms* value of the load voltage V_{orms} is calculated from Eq.(2.34) :

$$V_{orms} = \frac{V_m}{\sqrt{2}} = \frac{\sqrt{2}\times 110}{\sqrt{2}} = 110 \text{ V}$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{110}{25} = 4.4 \text{ A}$$

(c) The d.c. and a.c. power,

$$P_{dc} = \frac{V_{dc}^2}{R} = \frac{(99.035)^2}{25} = 392.31 \text{ W}$$

$$P_{ac} = \frac{V_{orms}^2}{R} = \frac{(110)^2}{25} = 484 \text{ W}$$

The load voltage and diode voltage waveforms are shown in Fig.2.11.

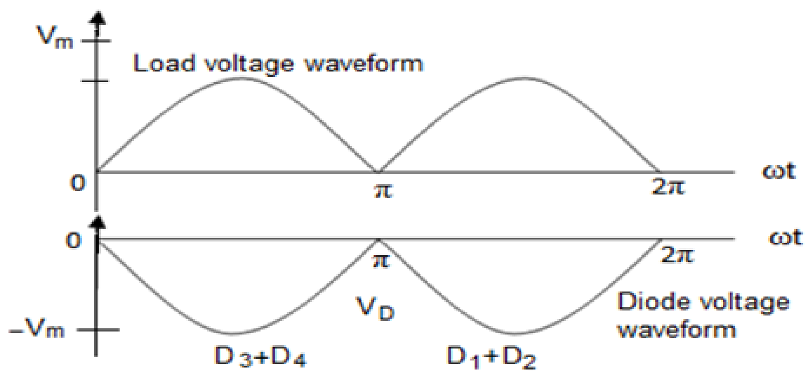


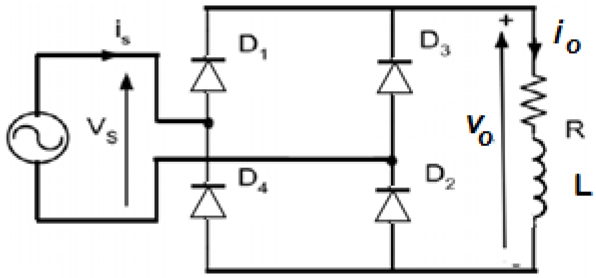
Fig.2.11 Voltage and current waveforms for Example 2.5.

2.3.3 Single-Phase Full-Wave Uncontrolled Rectifier Loaded with Highly Inductive Load

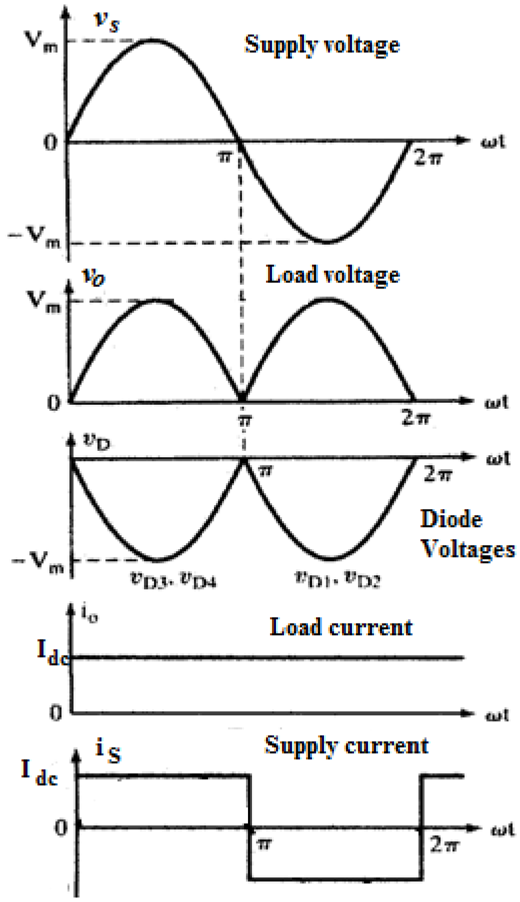
Fig. 2.12 (a) presents the circuit connection for a single-phase, full-wave, rectifier loaded with a highly inductive load. Highly inductive loads are basically *R-L* loads where $L >>> R$. Therefore, the load time constant $\tau = L / R$ is very high and can be considered infinity. Consequently, the load current is assumed constant. For one total period of operation of this circuit, the corresponding waveforms are shown in Fig. 2.12 (b).

During the conduction of D_1 and D_2 simultaneously the supply voltage appears directly across the load so that the load voltage $v_o(\omega t)$ remains the same form as shown in Fig.2.9 (b) (same as the case of resistive load). Hence, the average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m sin\omega t\, d\omega t = \frac{V_m}{\pi}\,(-cos\pi + cos0^o) = \frac{2V_m}{\pi} \qquad (2.39)$$



(a) Circuit.



(b) Waveforms.

Fig.2.12 Single-phase full-wave rectifier loaded with highly inductive load.

Since the load is a highly inductive load. Then, the load current is considered constant (ripple free current) and its average value is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} \tag{2.40}$$

and the d.c. power is

$$P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} \tag{2.41}$$

The *rms* value of the load voltage V_{orms} can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m sin\omega t)^2 \, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{\pi}\int_0^{\pi} \frac{1}{2}(1 - cos2\omega t) d\omega t} = \frac{V_m}{\sqrt{2}} \tag{2.42}$$

Therefore the *rms* value of the load current I_{orms} is

$$I_{orms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2}R} \tag{2.43}$$

Since the load current is constant over the studied period, therefore, the rms value of the load current I_{orms} is

$$I_{orms} = I_{dc} = I_s \tag{2.44}$$

The output a.c. power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.707V_m)^2}{R} \tag{2.45}$$

2.4 HARMONIC CONSIDERATIONS OF THE OUTPUT VOLTAGE AND CURRENT WAVEFORMS OF THE SINGLE-PHASE FULL - WAVE RECTIFIER

2.4.1 Voltage Waveform Harmonics

The output voltage waveform of the single-phase full-wave rectifier is shown in Fig.2.13. It is re-drawn with the supply voltage is taken as cosine wave instead of sine wave for sake of simplifying analysis.

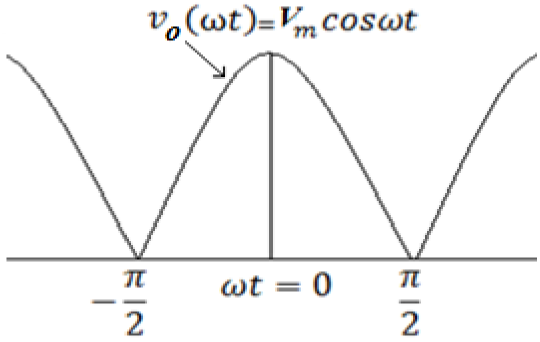


Fig.2.13. Voltage waveform of full-wave rectifier.

The output voltage waveform can be expressed in a Fourier series as,

$$v_o(\omega t) = a_0 + \sum_{n=1,2,3,..}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \tag{2.46}$$

where $n =$ is the order of harmonic.

The Fourier coefficients are defined as (See Appendix-A)

$$\frac{a_o}{2} = \frac{1}{2\pi} \int_0^{2\pi} v_o(\omega t) d\omega t \tag{2.47}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) \cos n\omega t \, d\omega t \tag{2.48}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} V_o(\omega t) \sin n\omega t \, d\omega t \tag{2.49}$$

where

a_o is the average value or the d.c. component of the output voltage waveform.
a_n is the cosine-term of the nth order harmonic.
b_n is the sine-term of the nth order harmonic.

In this case, the Fourier coefficients are:

$$a_0 = V_{dc} = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_m \cos \omega t \, d\omega t = \frac{2V_m}{\pi} \tag{2.50}$$

$$a_n = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_m \cos \omega t \cos n\omega t \; d\omega t$$

$$a_n = \frac{4V_m}{\pi} \sum_{n=2,4,6,..}^{\infty} \frac{1}{(n+1)(n-1)} \tag{2.51}$$

$$b_n = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_m \cos \omega t \sin n\omega t \; d\omega t = 0 \tag{2.52}$$

The amplitude of the nth order harmonic c_n can be found as,

$$C_n = \sqrt{a_n^2 + b_n^2} \tag{2.53}$$

and the phase-angle ψ_n of the n^{th} harmonic component is given as,

$$\psi_n = \tan^{-1} \frac{a_n}{b_n} \tag{2.54}$$

Hence, the output voltage waveform expressed in Fourier series as,

$$v_o(\omega t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos 2\omega t + \frac{4V_m}{15\pi} \cos 4\omega t - \frac{4V_m}{35\pi} \cos 6\omega t + \cdots + \cdots \tag{2.55}$$

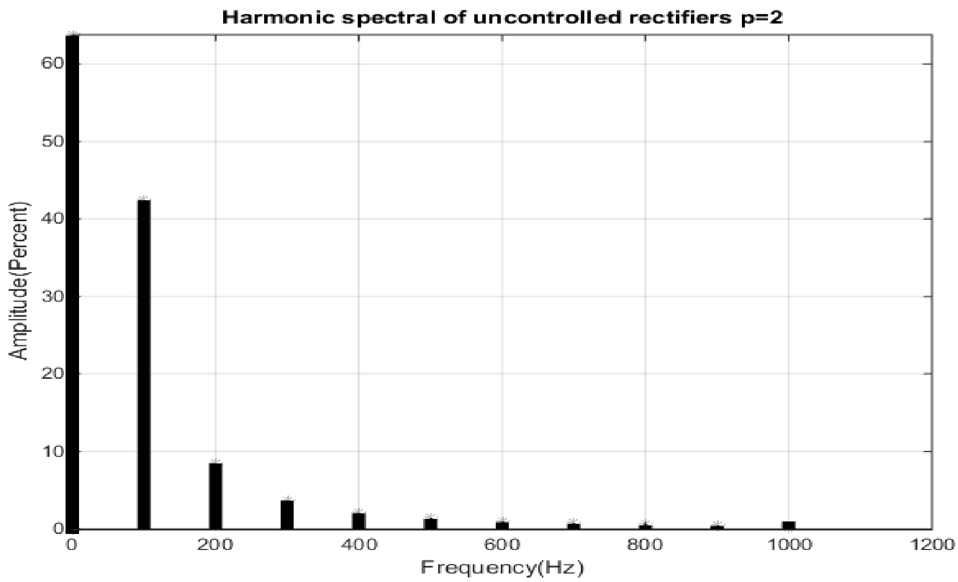
The first term on the RHS is the d.c. value. The second term is the dominant output ripple, which in this case is at twice the supply frequency; the third other terms are the higher order ripples. The amplitude of this ripples reduce as the harmonics number increases. All ripple components are unwanted.

For the half-wave rectifier, it is found that the Fourier series for the voltage waveform of Fig.2.1 is

$$v_o(\omega t) = \frac{V_m}{\pi} + \frac{V_m}{2\pi} \sin \omega t - \frac{2V_m}{3\pi} \cos 2\omega t - \frac{2V_m}{15\pi} \cos 4\omega t + \cdots + \cdots \tag{2.56}$$

Harmonic spectra of the output voltage waveform of the full-wave rectifier are shown in Fig.2.14 for the case when the load is purely resistive.

From Fig.2.14, it is seen that the zero frequency component n = 0 or the d.c. component is the dominant one in the spectrum, and that the second harmonic, corresponding to n=2 is the most significant harmonic amplitudes. Only even harmonics are existing and any odd harmonic number are all of zero value in this case. The harmonic spectrum of Fig.2.14 has an envelope identical to a Fourier integral of the mathematical form *sincx*.



n	0	2	4	6	8	10	12	16	20
Hz	0	100	200	300	400	500	600	800	1000
c(n)	63.63635	42.42423	8.484847	3.636363	2.020202	1.285583	0.890019	0.499109	0.318979

Fig.2.14 Harmonic spectra for the output voltage of the single-phase full-wave rectifier, with normalised V_m (=100%).

2.4.2 Input Current Harmonics

If ripple-free load current in steady-state is assumed, the input current i_s waveform of the single-phase full-wave rectifier may then be depicted as shown in Fig.2.15 for an ideal input transformer.

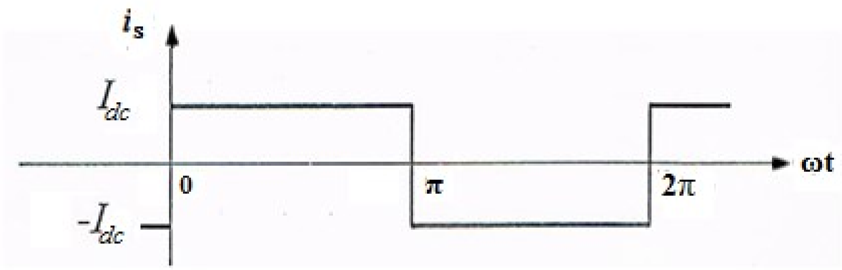


Fig.2.15 Input current waveform for single-phase full-wave rectifier.

Diodes D_1 and D_2 carry each half cycle of the load current and input current i_s is a square wave a.c. waveform as indicated. The actual input current waveform includes the transient behavior of the load in each half cycle and its harmonics are quite difficult to obtain analytically. With the assumption that the supply is ideal and perfectly smooth and ripple free load current, which implies an infinitely large load inductance, it is quit straightforward to obtain an analytical expression for the input current harmonics.

The rectangular wave of Fig.2.15 is defined as,

$$f(\omega t) = i_s(\omega t) = I_{dc} \qquad 0 \leq \omega t \leq \pi$$

$$= -I_{dc} \qquad \pi \leq \omega t \leq 2\pi$$

Hence, the Fourier coefficients of the rectangular wave are,

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} i_s(\omega t)\ d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} i_s(\omega t)\ d\omega t$$

$$= \frac{1}{2\pi}\int_0^{\pi} (I_{dc})\ d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} (-I_{dc})\ d\omega t\ = 0 \qquad (2.57)$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} (I_{dc})\, cosn\omega t\ d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} (-I_{dc}) cosn\omega t\ d\omega t\ = 0 \qquad (2.58)$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} (I_{dc})\, sinn\omega t\ d\omega t + \frac{1}{\pi}\int_{\pi}^{2\pi} (-I_{dc}) sinn\omega t\ d\omega t$$

$$= \frac{I_{dc}}{n\pi}\left[- cosn\omega t\ \Big|_0^{\pi} + cosn\omega t\ \Big|_{\pi}^{2\pi}\ \right]$$

$$= \frac{I_{dc}}{n\pi}[(coso - cosn\pi) + (1 - cosn\pi\ )]$$

$$= \frac{I_{dc}}{n\pi}[1 - cosn\pi + 1 - cosn\pi] = \frac{I_{dc}}{n\pi}(1 - cosn\pi)$$

- When n is even (2, 4, 6…) cosnπ = 1, $\Longrightarrow$ $b_n = 0$.
- When n is odd (3, 5, 7…) cosnπ = -1

$$\therefore b_n = \frac{4I_{dc}}{n\pi} \qquad (2.59)$$

The Fourier series is given by:

$$i_s(\omega t) = a_o + \sum_{n=1}^{\infty} (a_n \, cosn\omega t + \, b_n sinn\omega t)$$

Hence the input current can be represented by Fourier series as,

$$i_s(\omega t) = 0 + \sum_{n=1}^{\infty} (0 + \frac{4I_{dc}}{n\pi} sinn\omega t)$$

$$or \qquad i_s(\omega t) = \sum_{n=1}^{\infty} \frac{4I_{dc}}{n\pi} sinn\omega t$$

$$= \frac{4I_{dc}}{\pi} \sin \omega t + \frac{4I_{dc}}{3\pi} \sin 3\omega t + \frac{4I_{dc}}{5\pi} \sin 5\omega t + \cdots \qquad (2.60)$$

In more general form (2.60) may be written as

$$i_s(\omega t) = \frac{4I_{dc}/k}{\pi} \sin \omega t + \frac{4I_{dc}/k}{3\pi} \sin 3\omega t + \frac{4I_{dc}/k}{5\pi} \sin 5\omega t + \cdots \qquad (2.61)$$

where k is the transformer turns ratio between primary and secondary windings. The transformer magnetizing current has been neglected in this analysis .

Note that in the above circuit, the input current waveform i_s has no d.c. value. The first term for n = 1, is called the fundamental and the higher order terms are the harmonics which are unwanted. The harmonic spectrum of the input current wave is as shown in Fig.2.16.

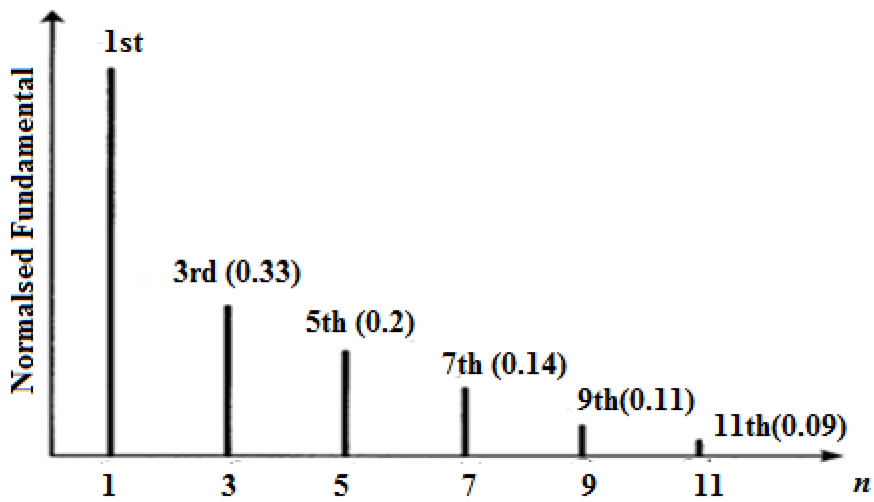


Fig.2.16 Harmonic spectrum of the input current wave.

It is clear from Fig.2.16 that:

- The harmonic amplitudes reduce as the harmonic number increases.
- No even harmonics.
- Nearest harmonic is the third, if the fundamental is 50 Hz, then the third harmonic is 150Hz and the fifth harmonic 250 Hz as shown in Fig.2.16.

2.5 POLY-PHASE UNCONTROLLED RECTIFICATION

For most industrial applications poly-phase rectifier circuits are used. The circuit employed may give either half-wave or full-wave, controlled or uncontrolled rectifier circuits. In these rectifier circuits, the commutation processes involved are purely natural by the cycling of the supply-side voltages. Hence there is no reason in using gate turn-off devices and such rectifiers can employ thyristors as switches. When poly-phase a.c. is rectified, phase-shifted pulses overlap each other to produce d.c. output that is much "smoother", i.e. has less a.c. content than that produced by single-phase rectification. This is considered as an advantage in high-power rectifier circuits, where sheer physical size of filtering components would be prohibitive but low-noise d.c. power must be obtained.

The following assumption are made during the studying and analysing the polly-phase rectifier circuits :

(a) All phases of the supply are identical, balanced, pure sinusoidal and displaced exactly by 360°/p from each other, where p = number of phases (p = 3 for three-phase system and P = 6 for six-phase system …etc).

(b) The leakage reactances of the supply are neglected for all the phases.

(c) The switching functions are symmetrical and are phase shifted by 360°/ p with respect to each other.

2.5.1 Three-Phase Half-Wave Uncontrolled Rectifier

Fig.2.17 shows a three-phase half-wave uncontrolled rectifier with resistive load. The rectifier is fed from an ideal three-phase supply through delta-star three-phase transformer.

The principle of operation of this convertor can be explained as follows:

- Diode 1 which has a more positive voltage at its anode conducts for the period from $\pi/6$ to $5\pi/6$. In this period D_2 and D_3 are off. The neutral wire provides a return path to the load current.
- Similarly, diode 2, and 3, whichever has more positive voltage at its cathode conducts.
- The conduction pattern is: D_1 ,D_2, D_3.

The output voltage and current waveforms are shown in Fig.2.18.

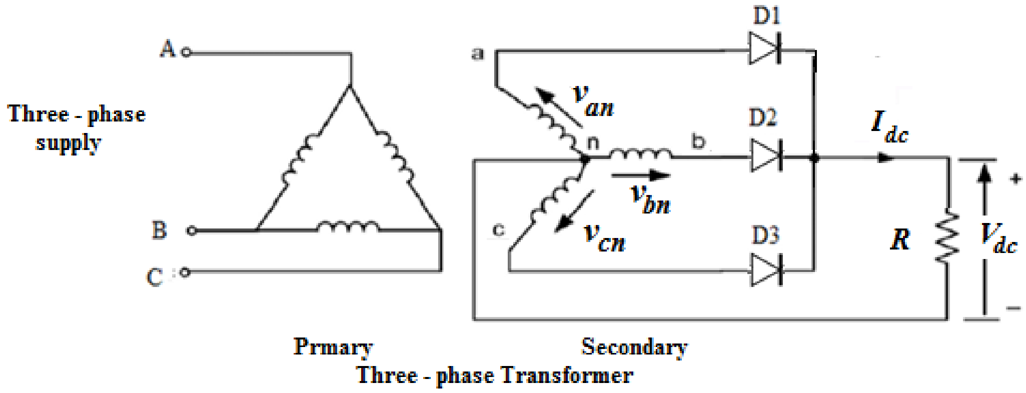


Fig.2.17 Three-phase half-wave uncontrolled rectifier with *R*-Load.

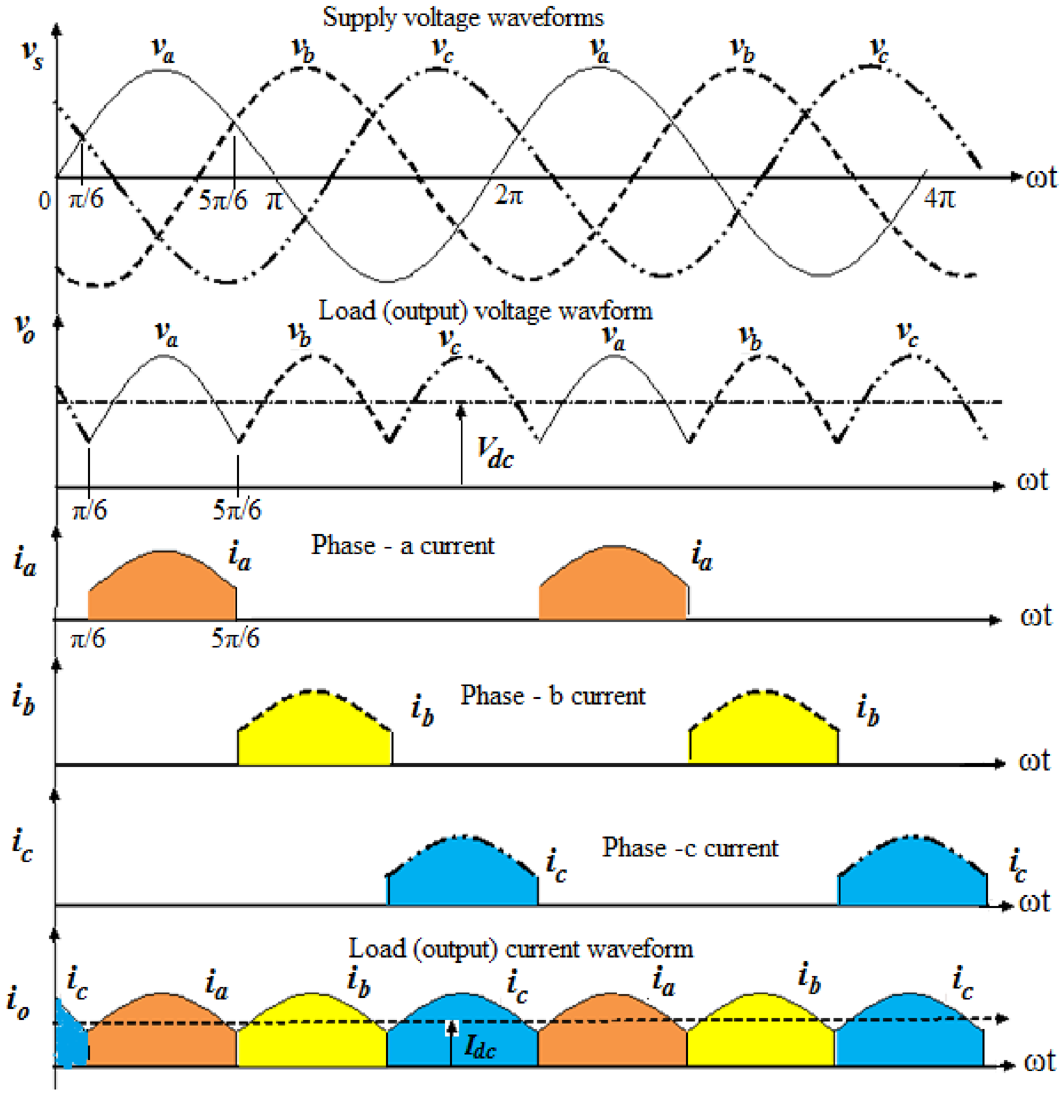


Fig .2.18 Load voltage and current waveforms for the three-phase, half-wave uncontrolled rectifier.

Analytical properties of the output voltage waveform

Let $v_{an} = V_m \sin \omega t$
$v_{bn} = V_m \sin (\omega t - 2\pi / 3)$
$v_{cn} = V_m \sin (\omega t - 4\pi / 3)$

The average value V_{dc} of the output (load) voltage waveform $v_o(\omega t)$ shown in Fig.2.17 can be found as

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m \ sin\omega t \ d\omega t = \frac{3V_m}{2\pi} [-cos\omega t] \begin{matrix} 5\pi/6 \\ \pi/6 \end{matrix}$$
$$= \frac{3V_m}{2\pi} \left[-(\cos\frac{5\pi}{6} - cos\frac{\pi}{6}) \right] = \frac{3V_m}{2\pi} \left[-(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}) \right]$$
$$= \frac{3\sqrt{3}V_m}{2\pi} \qquad (2.62)$$

The load current I_{dc} is:

$$I_{dc} = \frac{3\sqrt{3}V_m}{2\pi R} \qquad (2.63)$$

Note that the secondary windings of the supply transformer carry unidirectional currents, which leads d.c. magnetization of the transformer core. This implies that the transformer cores have d.c. flux, so that for the same a.c. voltage and hence flux swing, it must have larger core size than is necessary. This problem of d.c. magnetization is avoided using bridge rectifier circuit.

Example 2.6

Power is supplied to a load resistor R from a three-phase zero-impedance supply of balanced sinusoidal voltages using half-wave bridge circuit of Fig.2.19 . The three diodes may be considered as ideal switches.

(a) Sketch the phase current waveform i_a (ωt) in phase-a with respect to the phase voltage v_{an} (ωt) = V_m sin ωt .

(b) Show that the rms value of the phase current current is:

$$I_a = 0.485\frac{V_m}{R}$$

Hence, calculate I_a of the phase current, compare with the value for sinusoidal operation.

(c) Sketch the load current waveform and specify, by observation, the lowest ripple frequency.

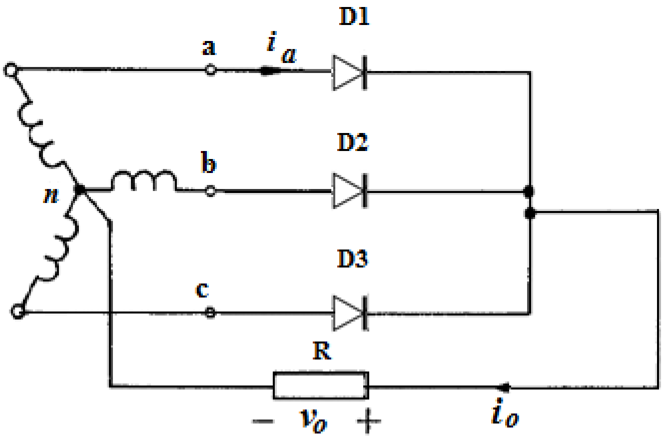


Fig.2.19 Three-phase Half-wave rectifier.

Solution

(a) The waveform of the current in phase - a is depicted in Fig.2.20.

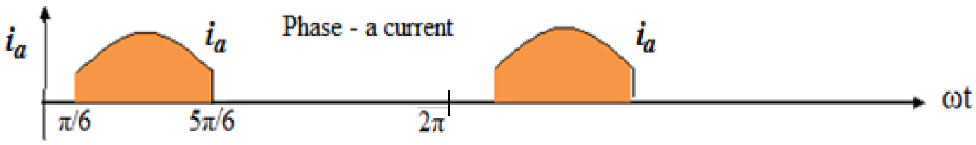


Fig.2.20 Current waveform in phase - a.

(b) Phase current (i_a) :
Let the phase current i_a (ωt) in phase - a with resistive load be ,

$$i_a(\omega t) = I_m \sin\omega t$$

where : $I_m = V_m / R$, hence the *rms* value of i_a is:

$$I_a = \sqrt{\frac{1}{T}\int_0^T i_a^2\,(\omega t)dt} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_m^2 sin^2(\omega t)d(\omega t)}$$

$$\therefore I_a = \frac{I_m}{\pi\sqrt{2}}\sqrt{\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(1 - cos\omega t)d(\omega t)} = 0.485 I_m$$

For sinusoidal operation

$$I_{arms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

(c) The ripple period is given by $\frac{2\pi}{3}$ which represents the lowest ripple frequency = 3 times the supply frequency as depicted in Fig.2.21.

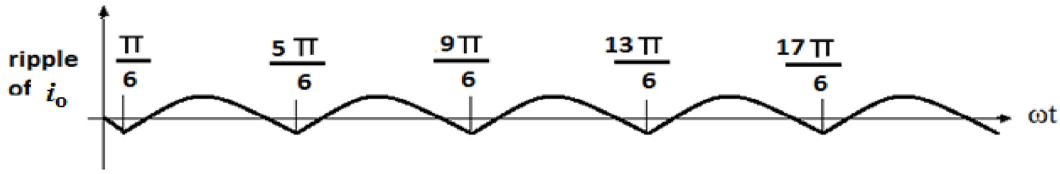


Fig.2.21 The ripple frequency.

2.5.2 Three-Phase Full-Wave Uncontrolled Bridge Rectifier

Fig.2.22 shows a three-phase full-wave uncontrolled bridge rectifier with resistive load. The rectifier is fed from an ideal three-phase supply through delta-star three-phase transformer. The principle of operation of this convertor can be explained as follows:

Each three-phase line connects between pair of diodes, one to route power to positive (+) side of load, and other to route power to negative (-) side of load.

- Diode 1, 3 and 5, whichever has a more positive voltage at its anode conducts.
- Similarly, diode 2, 4 and 6, whichever has more negative voltage at its cathode return the load current.
- The conduction pattern is: **16-36-34-54-52-12.**
- Each diode conducts for 120° in each supply cycle as shown in Fig.2.23.

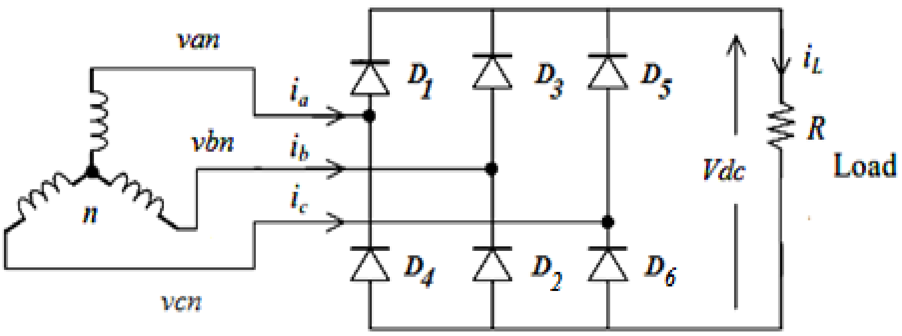


Fig.2.22 The three-phase full-wave, uncontrolled rectifier.

The output voltage is the instantaneous difference between two appropriate phases at each instant as depicted in Fig.2.23, and the resultant d.c. output voltage wave is shown in Fig.2.24.

To find the average voltage V_{dc} on the load, assume that the line to line voltages are represented by the following equations,

$$v_{ab} = v_{an} - v_{bn} = V_m \sin(\omega t) - V_m \sin(\omega t + 2\pi/3),$$

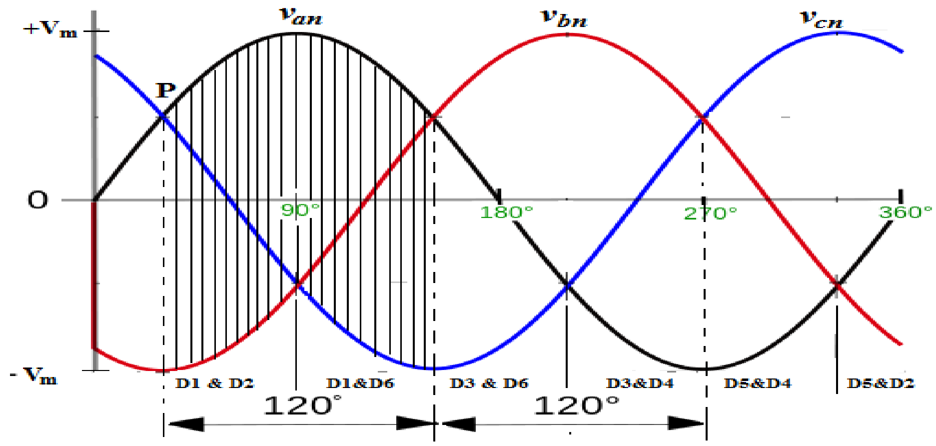


Fig.2.23 Each diode conducts for 120˚.

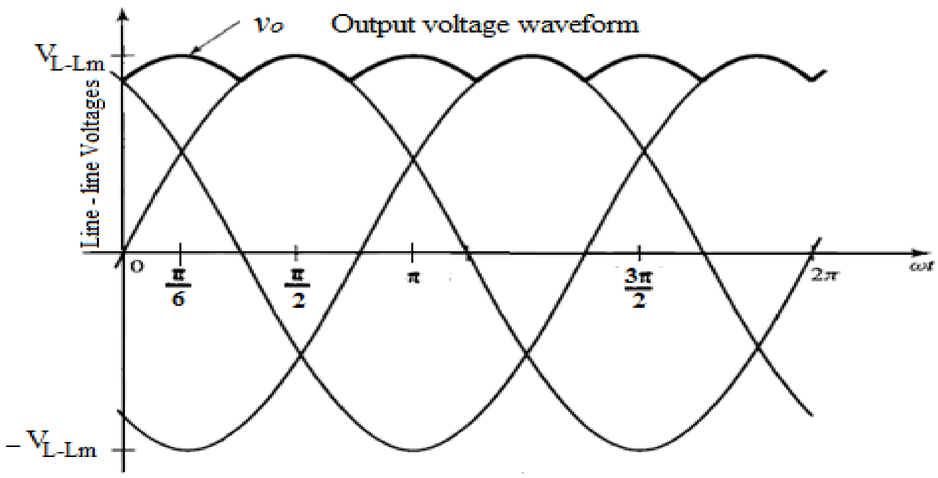


Fig.2.24 The line to line supply voltage and the output voltage waveforms.

hence

$$v_{ab}(\omega t) = \sqrt{3}\, V_m sin\left(\omega t + \frac{\pi}{6}\right) \tag{2.64}$$

Similarly,

$$v_{bc}(\omega t) = \sqrt{3}\; V_m sin\left(\omega t - \frac{\pi}{2}\right) \tag{2.65}$$

$$v_{ca}(\omega t) = \sqrt{3}\; V_m sin\left(\omega t - \frac{7\pi}{6}\right) \tag{2.66}$$

The resultant output voltage waveform appears across the load is the line to line values of the three-phase supply as shown in Fig.2.25.

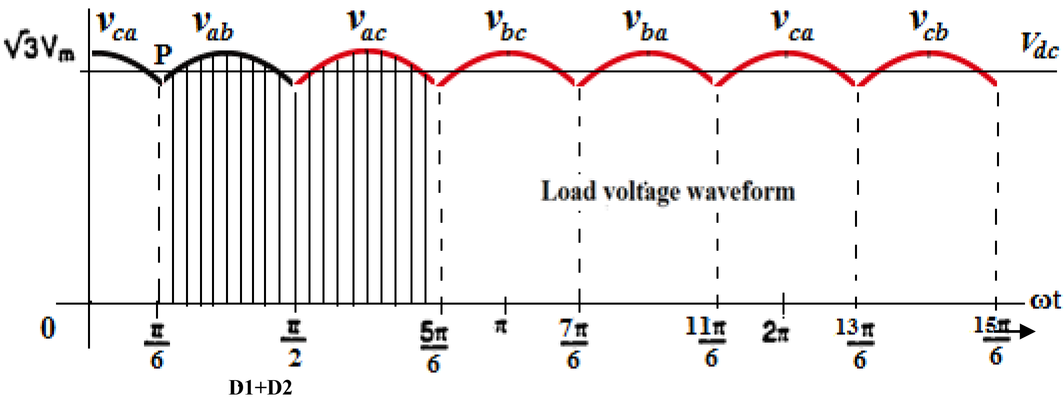


Fig.2.25 The d.c. output voltage waveform.

The average value V_{dc} of the output voltage can be calculated by integrating over 1/6 of a cycle,

$$V_{dc} = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} v_{ab}(\omega t)\, d\omega t$$

$$= \frac{3}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{3}V_m sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m(\sin\omega t\cos\frac{\pi}{6} - \cos\omega t\sin\frac{\pi}{6})d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m\left(\sin\omega t\ \frac{\sqrt{3}}{2} + \cos\omega t\frac{1}{2}\right) d\omega t$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[\left(-\cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{6}\right)\right)\frac{\sqrt{3}}{2} + (\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right))\frac{1}{2}\right]$$

$$\therefore\ V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \qquad (2.67)$$

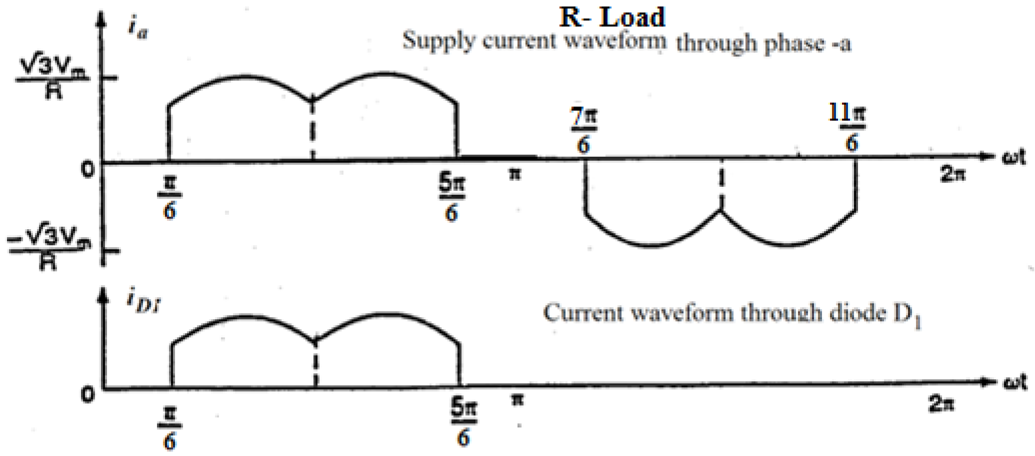
The load current I_{dc} is:

$$I_{dc} = \frac{3\sqrt{3}V_m}{\pi R} \qquad (2.68)$$

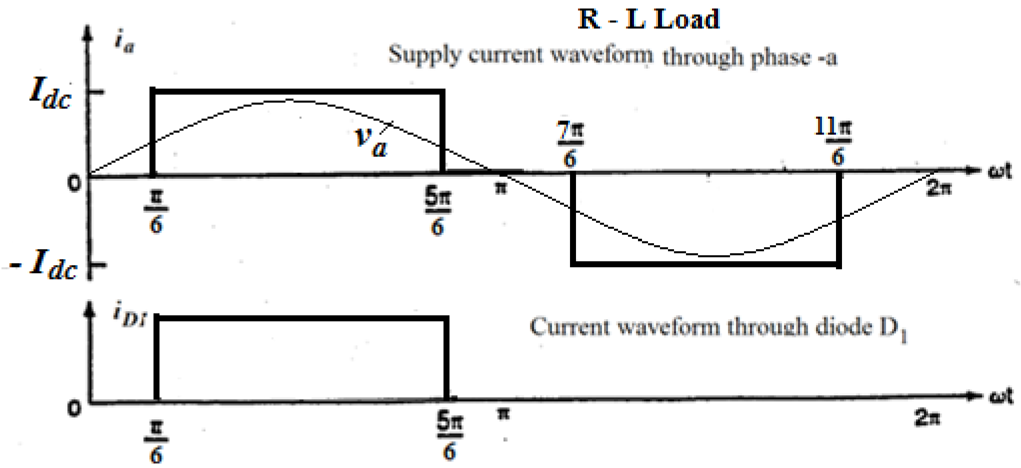
The average power is:

$$P_{dc} = V_{dc} I_{dc} = \frac{3\sqrt{3}V_m}{\pi} \times \frac{3\sqrt{3}V_m}{\pi R} = \frac{27\, V_m^2}{\pi^2 R} \tag{2.69}$$

The waveforms of the supply line current through phase-a and the current through diode D_1 are shown in Fig.2.26 for resistive and inductive loads .



(a)



(b)

Fig.2.26 Waveforms of the supply line current through phase-a and the current through diode D_1 in a three-phase full-wave rectifier bridge: (a)With resistive load, and (b)With highly inductive load.

To calculate the power factor, for phase - a for example:

$$I_{arms} = \sqrt{\frac{1}{\pi}\int_{\pi/6}^{5\pi/6} I_{dc}^2 \; d\omega t}$$

$$I_{arms} = \sqrt{\frac{I_{dc}^2}{\pi}[\frac{5\pi}{6} - \frac{\pi}{6}]} = \sqrt{\frac{2}{3}}\; I_{dc} \qquad (2.70)$$

$$P = \frac{1}{\pi}\int_{\pi/6}^{5\pi/6} I_{dc} V_m sin\omega t \; d\omega t = \frac{\sqrt{3}}{\pi} V_m I_{dc} \qquad (2.71)$$

$$PF = \frac{P}{V_{rms}\, I_{rms}} = \frac{\left[\frac{\sqrt{3}}{\pi} V_m I_{dc}\right]}{\left[\frac{V_m}{\sqrt{2}}\sqrt{\frac{2}{3}}\; I_{dc}\right]} = \frac{3}{\pi} \approx 0.96 \qquad (2.72)$$

Compare to 0.91 for the single-phase full-wave bridge rectifier. Also the three-phase bridge rectifier uses 6 diodes, while single-phase bridge rectifier uses only 4 diodes.
Regarding the ripple voltage for the three-phase bridge rectifier it is at 6*f* and is small in magnitude, while for single-phase bridge rectifier it is at 2*f*. This means that the three-phase bridge rectifier output voltage is easier to filter.

2.5.3 Six-Phase (Hexa-Phase) Uncontrolled Rectifier

To get more smooth output voltage waveform, higher phase numbers than three may be used. Poly-phase systems with more than three phases are easily accommodated into bridge rectifier scheme. Once poly-phase power is available, it may be converted to any desired number of phases with a suitable arrangement of transformers. Common practice for rectifier installations and in HVDC converters is to provide six phases, with 60 degree phase spacing, to reduce harmonic generation in the a.c. supply system and to provide smoother direct current. Six-phase power has the same voltage between adjacent phases as between phase and neutral.

Six-phase half-wave (6-puls) rectifier circuit

A six-phase supply is obtained from a three-phase system using transformer with centre tapped secondary winding as shown in the Fig.2.27. This circuit is, in fact, a six-phase, half-wave rectifier. Obviously, each diode in this circuit conducts for one-sixth of a cycle.

Hence, the conduction angle of each diode will be 60° only. The supply and output voltage waveforms are shown in Fig.2.28.

Referring to Fig.2.27, the average output voltage may be given by,

$$V_{dc} = \frac{3}{\pi}\int_{\pi/3}^{2\pi/3} V_m \sin \omega t \, d\omega t = \frac{3V_m}{\pi}[\cos 120^{\circ} - cos60^{\circ}]$$

$$\therefore \quad V_{dc} = \frac{3V_m}{\pi} \qquad (2.73)$$

The output voltage has a mean value of 0.955 V_m. Ripple has a very small value and a fundamental frequency six times the supply frequency.

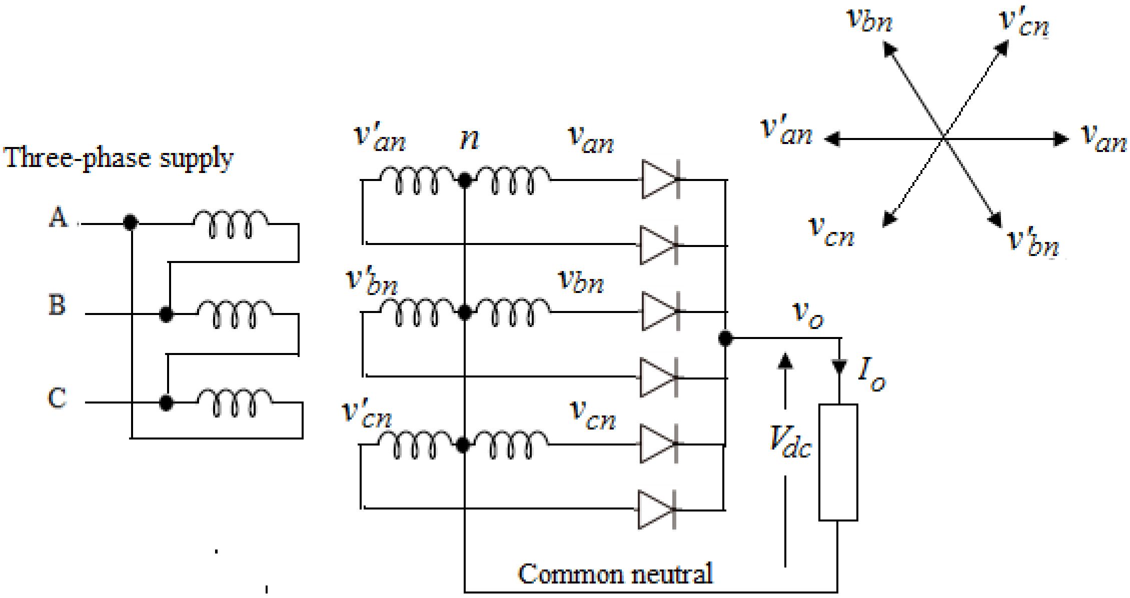


Fig.2.27 Six-phase rectifier.

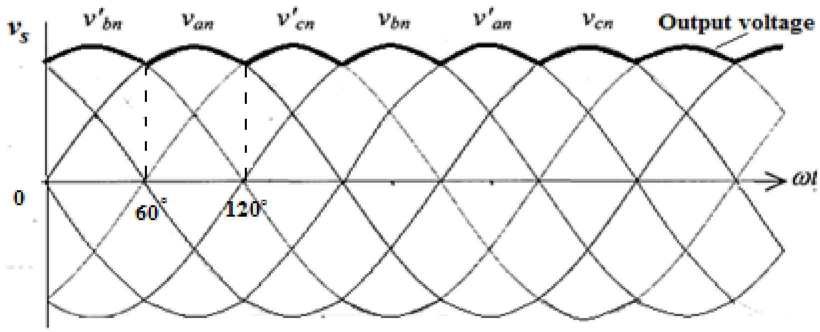


Fig.2.28 Six-phase rectifier: Input and output voltage waveforms .

Six-phase full-wave (12-puls) rectifier circuit

In many applications, it is necessary to reduce ripple in the output d.c. voltage waveform as much as possible. For example the ripple of a three-phase full-wave rectifier is about 4.2%. This can be reduced to about 1% by increasing the ripple frequency to twelve times the input frequency by using six-phase full-wave rectifier circuit. To construct a six-phase full-wave rectifier circuit, two three-phase full-wave bridges are connected in parallel as shown in Fig. 2.29.

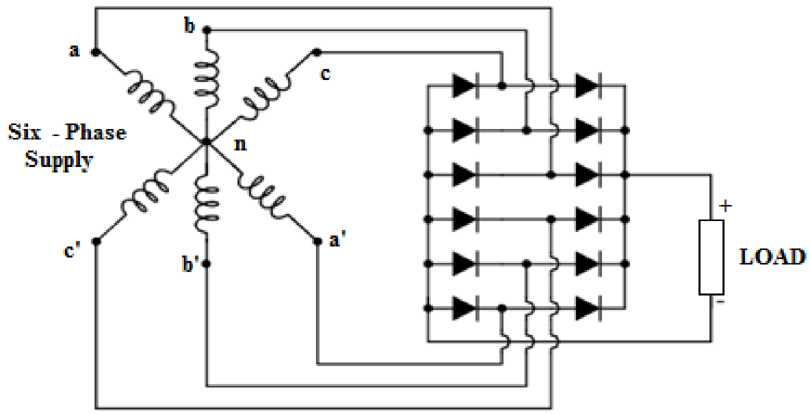


Fig.2.29 Six-phase full-wave rectifier.

The conduction period for each two diodes will be 30° in this case. The six voltages to neutral will be:

$$v_a = V_m \sin \omega t \quad , \quad v_{a'} = V_m \sin(\omega t - 180°)$$

$$v_b = V_m \sin(\omega t - 60°) \quad , \quad v_{b'} = V_m \sin(\omega t - 240°)$$

$$v_c = V_m \sin(\omega t - 120°) \quad , \quad v_{c'} = V_m \sin(\omega t - 300°)$$

$$v_{ab} = V_m \sin \omega t - V_m \sin(\omega t - 60°)$$

$$v_{ab} = V_m [\sin \omega t - (\sin \omega t \cos 60° - \cos \omega t \sin 60°)]$$

$$v_{ab} = V_m [\frac{1}{2} \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t]$$

$$= V_m [\sin \omega t \sin 30° + \cos \omega t \cos 30°]$$

$$\therefore \quad v_{ab} = V_m \cos(\omega t - 30°) \qquad (2.74)$$

2.6 GENERAL FORMULA FOR THE OUTPUT VOLTAGE OF A P-PULSE UNCONTROLLED RECTIFIER

A general expression for the average d.c. output voltage can be derived for any number of phases (p-pulse) when the load is connected between the cathode rail of the converter and the neutral of the transformer secondary.
Let us consider a p-phase rectifier having maximum voltage V_m / phase. The waveform of the output voltage will be as shown in Fig.2.30.

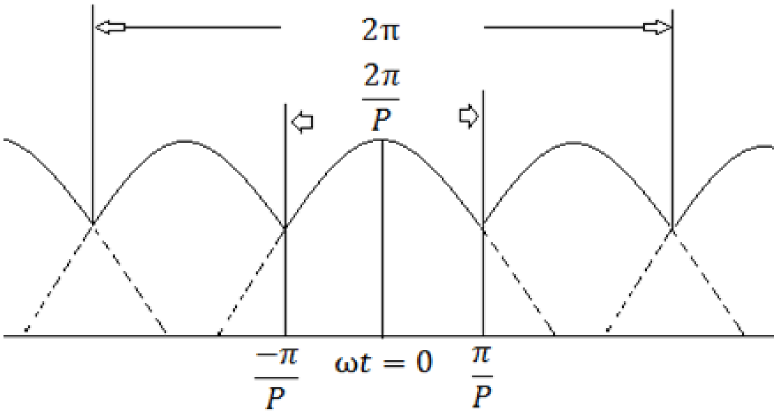


Fig.2.30 P-phase rectifier voltage waveforms .

The average d.c. voltage V_{dc} is,

$$V_{dc} = \frac{1}{\frac{2\pi}{p}} \int_{(-\frac{\pi}{p})}^{(+\frac{\pi}{p})} V_m \cos \omega t \, d\omega t$$

$$= \left(\frac{p}{2\pi}\right) V_m \left[\sin\left(\frac{\pi}{p}\right) - \sin(-\frac{\pi}{p})\right] \quad = \frac{P\,V_m}{2\pi}\left[2 \sin \frac{\pi}{P}\right]$$

$$\therefore V_{dc} = \frac{p}{\pi} V_m \sin\frac{\pi}{p} \qquad (2.75)$$

For a three-phase, half-wave circuit, p = 3, hence,

$$V_{dc} = \frac{3}{\pi} V_m \sin\frac{\pi}{3} = \frac{3\sqrt{3}V_m}{2\pi}$$

which is the same result as that given in Eq. (2.62).

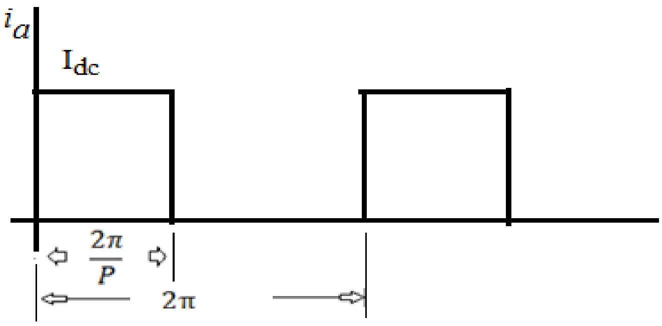
2.6.1 Output Current of P-Pulse Converter

The current in p-pulse rectifier is supplied by each phase of the transformer secondary windings for a period of 2π/p only of each cycle. Hence the secondary current is a train of pulses as shown in Fig.2.31.

The mean current per phase - *a*

$$I_{d(av)} = \frac{1}{2\pi}\int_0^{2\pi/p} I_{dc}\, d\omega t$$

Fig.2.31.



$$I_{d(av)} = \frac{I_{dc}}{p} \tag{2.76}$$

The *rms* current, I_{rms} , per phase is,

$$I_{arms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/p} I_{dc}^2\ d\omega t} \quad = \frac{I_{dc}}{\sqrt{p}} \tag{2.77}$$

Hence for 6 – pulse converter

$$I_{arms} = \frac{I_{dc}}{\sqrt{6}} \tag{2.78}$$

For 12 – pulse converter

$$I_{arms} = \frac{I_{dc}}{\sqrt{12}} = \frac{I_{dc}}{\sqrt{2}\sqrt{6}} \tag{2.79}$$

So for the 12- pulse converter I_{arms} decreases by factor of $\sqrt{2}$, but twice as many devices , each device carries the full current for half the time; and ohmic loss in devices, transformers lines depends on the *rms* value of the current itself.

2.6.2 Power Factor of a P-Pulse Rectifier

The power factor of a rectifier is , as defined before, is the ratio of power output in watt and the volt-amperes drawn from the supply, hence,

$$PF = \frac{\text{Watts output}}{\text{Volt Amperes supplied}} = \frac{\mathrm{V_{dc}I_{dc}}}{\mathrm{p}\frac{\mathrm{V_m}}{\sqrt{2}}\mathrm{I_{arms}}}$$

$$\therefore \qquad PF \quad = \frac{\frac{p}{\pi}V_m\sqrt{p}\ I_{arms}\sin\frac{\pi}{p}}{p\frac{V_m}{\sqrt{2}}I_{arms}} = \frac{\sqrt{2p}}{\pi}\sin\frac{\pi}{p} \qquad (2.80)$$

Example 2.7

A three-phase (3-pulse) half-wave uncontrolled rectifier supplies a load of 5 kW at 240 V. Detrmine the rating of the secondary winding of the supply transformer and the input power factor.

Solution

For 3-pulse rectifier, the average output d.c. voltage of the rectifier V_{dc} is,

$$V_{dc} = \frac{3}{\pi}V_m\ \sin\frac{\pi}{3} = \frac{3\sqrt{3}V_m}{2\pi} = 240\text{ V}$$

$$240 = \frac{3\sqrt{3}V_m}{2\pi} \quad \rightarrow \quad V_m = \frac{2\pi \times 240}{3\sqrt{3}} = 290\text{ V}$$

The *rms* value of the supply voltage is

$$V_s = \frac{V_m}{\sqrt{2}} = 205.14\text{ V}$$

$$\text{Output current} \quad I_{dc} = \frac{5000}{240} = 20.8\text{ A}$$

The *rms* value of the secondary phase current, I_{arms} ,

$$I_{arms} = \frac{I_{dc}}{\sqrt{p}} = \frac{20.8}{\sqrt{3}} = 12\text{ A}$$

Transformer rating per phase = $V_s I_{arms} = 205.14 \times 12 = 2461.68$ VA

Total transformer rating in kVA = $2461.68 \times 3 = 7.385$

$$PF = \frac{\sqrt{2p}}{\pi}\sin\frac{\pi}{p} = \frac{\sqrt{2\times 3}}{\pi}\sin\frac{\pi}{3} = 0.675$$

2.7 HARMONIC ANALYSIS OF P-PULSE UNCONTROLLED RECTIFIER

The output voltage waveform $v_o(\omega t)$ of a p-pulse converter shown in Fig.2.30 may be represented in terms of its harmonic components using Fourier series as,

$$v_o(\omega t) = a_0 + \sum_{n=1}^{\infty} a_0 \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

where , a_o , a_n and b_n are the Fourier coefficient that may be written as:

$$a_0 = \frac{1}{\omega t}\int_{-\frac{\omega t}{2}}^{\frac{\omega t}{2}} v(\omega t)d\omega t$$

,

$$a_n = \frac{2}{\omega t}\int_{\frac{-\omega t}{2}}^{\frac{\omega t}{2}} v(\omega t)\cos(\omega t)\,d\omega t$$

$$b_n = \frac{2}{\omega t}\int_{\frac{-\omega t}{2}}^{\frac{\omega t}{2}} v(\omega t)\sin(\omega t)\,d\omega t = 0$$

for function $f(\omega t)$ symmetrical about $\omega t = 0$ (even function harmonic series only).

$$a_0 = V_m \frac{p}{\pi} sin\frac{\pi}{p} \qquad \therefore V_{dc} = a_0 = \frac{p}{\pi} V_m \sin\frac{\pi}{p} \qquad (2.81)$$

This result is the same as that given in Eq.(2.75).
For evaluating, a_n, regain integration productivity

$\int \cos(\omega t)\cos(n\omega t)\,d\omega t.$

Now $\int udv = vu - \int v.du$ (integration by parts)
$\int \cos(\theta)\cos(n\theta)\,d\omega t.$ cos (θ) =dv, $\cos(n\theta) = $ u

$$\int \cos(\theta)\cos(n\theta)\,d\omega t. = \sin(\theta)\cos(n\theta) + n\int sin\,(\theta)\sin(n\theta)\,.\,d\theta$$

Also $\sin(\theta) = dv$, $\sin(n\theta) = u$
So

$$\int sin\,(\theta)\sin(n\theta)\,.\,d\theta = -\cos(\theta)\sin(n\theta) - n\int(-\cos(\theta))\cos(n\theta)\,.\,d\theta$$

$$\therefore \int \cos(\theta)\cos(n\theta)\,d\omega t = \sin(\theta)\cos(n\theta) + n[-\cos(\theta)\sin(n\theta)$$
$$+\int \mathrm{n}\cos(n\theta)\cos(\,\theta).\,d\theta$$

$$\therefore \int \cos(\theta)\cos(n\theta)\,d\omega t = \frac{1}{n^2-1}[ncos(\theta)\sin(n\theta) - \sin(\theta)\cos(\theta)]$$

$$\therefore a_n = \frac{p}{\pi}\int_{\frac{-\pi}{P}}^{\frac{\pi}{P}} v_m \cos(\omega t)\cos(n\omega t)\,.\,d\omega t$$

$$= \frac{V_m p}{\pi(n^2-1)}[ncos(\omega t)\sin(\omega t) - \sin(\omega t)\cos(\omega t)]_{\frac{-\pi}{P}}^{\frac{\pi}{P}}$$

$$= \frac{V_m p}{\pi(n^2-1)}[ncos\left(\frac{\pi}{P}\right)\sin\left(\frac{\pi}{P}\right) - \sin\left(\frac{\pi}{P}\right)\cos\left(\frac{\pi}{P}\right) - \sin\left(\frac{\pi}{P}\right)\cos(n\left(\frac{\pi}{P}\right)$$
$$-ncos\left(-\frac{\pi}{P}\right)\sin\left(n\left(-\frac{\pi}{P}\right) + sin\left(-\frac{\pi}{P}\right)cos\left(n\left(-\frac{\pi}{P}\right)\right)\right]$$

Now n = cp , i.e. a multiple of p.

$$\therefore \sin\left(\frac{n\pi}{P}\right) = 0, \qquad \cos\left(\frac{n\pi}{p}\right) = \pm 1$$

$$a_n = \frac{V_m p}{\pi(n^2-1)}[-\sin\left(\frac{\pi}{P}\right)\cos\left(n\frac{\pi}{P}\right) - \sin\left(\frac{\pi}{P}\right)\cos\left(n\frac{\pi}{P}\right)]$$

$$= -\frac{P}{\pi}V_m \sin\left(\frac{\pi}{P}\right)\frac{2\cos\left(\frac{n\pi}{P}\right)}{(n^2-1)} \qquad (2.82)$$

The amplitude c_n of the n^{th} order harmonic is:

$$c_n = \sqrt{{a_n}^2 + {b_n}^2} = \sqrt{{a_n}^2 + 0} = a_n \qquad (2.83)$$

The phase angle ψ_n of the n^{th} order harmonic is:

$$\psi_n = \tan^{-1}\frac{a_n}{b_n} = \tan^{-1}\frac{a_n}{0} = \infty \qquad \therefore \psi_n = 90°. \tag{2.84}$$

Application of the general equation to single-phase full wave:

For P =2 full wave single-phase converter, applying Eq. (2.75) yields

$$a_0 = V_m\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi}{2}\right) = \frac{2V_m}{\pi}$$

.

$$a_n = -\frac{2P}{\pi}\left(\frac{V_m}{n^2-1}\right)\sin\left(\frac{\pi}{P}\right)\cos\left(n\frac{\pi}{P}\right)$$

$$= \frac{-4}{\pi}\left(\frac{V_m}{n^2-1}\right)\sin\left(\frac{\pi}{2}\right)\cos\left(n\frac{\pi}{2}\right)$$

and $c_n = \dfrac{-4V_m}{(n^2-1)}$

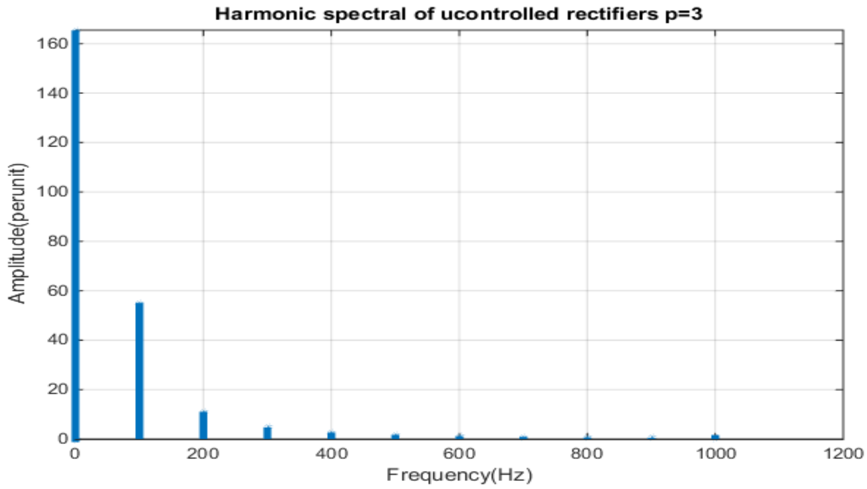
$$\therefore v_0(\omega t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos(2\omega t) + \frac{V_m}{15T}\cos(4\omega t) - \frac{4V_m}{35\pi}\cos(6\omega t) + \cdots \tag{2.85}$$

which is the same as that obtained in Eq.(2.55). The amplitude of the fundamental component is: c_1= 0, since the function contains even harmonics only. Harmonic spectra for the output voltage waveform for 3- pulse (p = 3) , and 6-pulse (p = 6) converters are shown in Fig. 2.32 . It is clear that as the number of pulses is increased, the value of the output d.c. voltage is increased. The second order harmonic is also increased significantly, however, the higher order even harmonics are increased also but their increasing is small compared with the second harmonic.

Now if the number of pulse is increased to 12 , the d.c. value will not increase a lot as expected, and the second harmonic remains the dominant one in the spectrum beside the d.c. component. This shown in Fig.33 which gives the frequency spectra of the output voltage for 12-pulse converter.

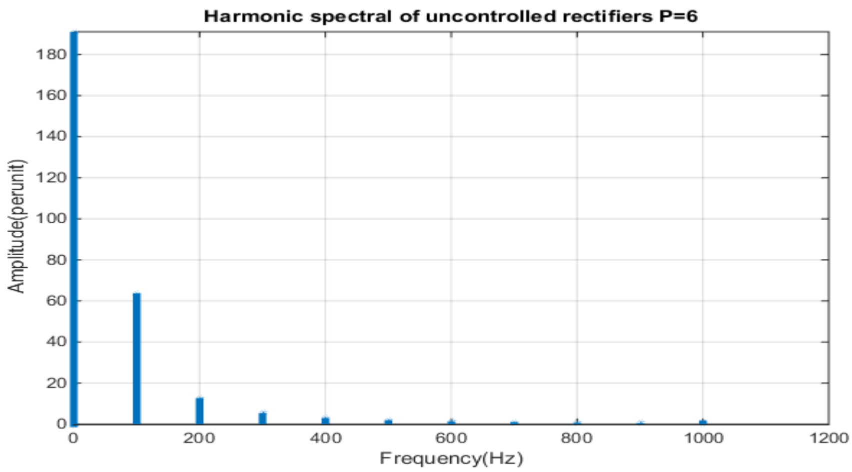
2.8 USES OF POLY-PHASE UNCONTROLLED RECTIFIERS

Uncontrolled rectifiers are extensively used in a number of power electronic based converters. In most cases, they used to provide an intermediate unregulated d.c. voltage source, which is further processed to obtain a regulated d.c. or a.c. output. They have, in general, been proved efficient and robust power stages. However, they suffer from few distinct



n	0	2	4	6	8	10	12	16	20
Hz	0	100	200	300	400	500	600	800	1000
c(n)	165.372	55.1241	11.0248	4.72492	2.62496	1.67043	1.15645	0.64852	0.41447

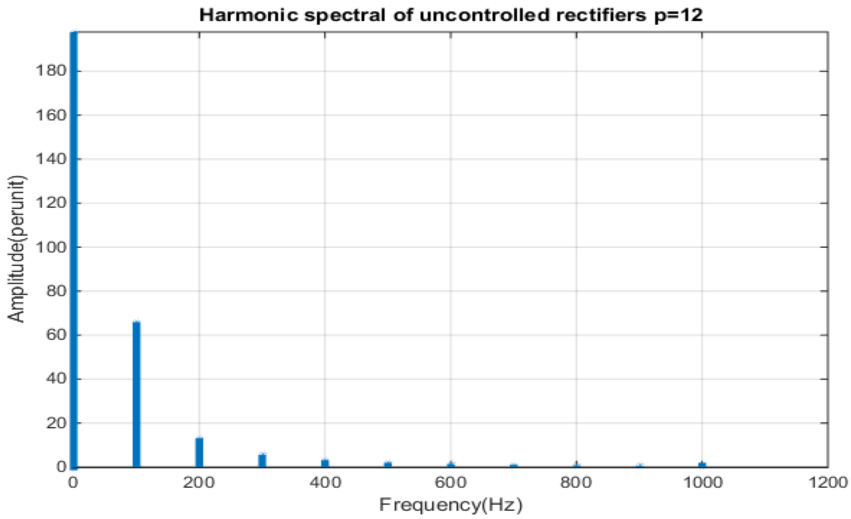
(a)



n	0	2	4	6	8	10	12	16	20
Hz	0	100	200	300	400	500	600	800	1000
c(n)	190.979	63.6596	12.7319	5.45654	3.03141	1.92908	1.33552	0.74894	0.47864

(b)

Fig.2.32 Harmonic frequency spectra for the output voltage waveform for: (a) 3-pulse (p =3), and (b) 6-pulse (p = 6) converters.



n	0	2	4	6	8	10	12	16	20
Hz	0	100	200	300	400	500	600	800	1000
c(n)	197.721	65.9071	13.1814	5.64918	3.13843	1.99719	1.38267	0.77538	0.49554

Fig.2.33 Harmonic spectra for the output voltage waveform for the 12-pulse converters (p = 12).

disadvantages. The main among them is their inability to control the output d.c. voltage / current magnitude when the input a.c. voltage and load parameters remain fixed. They are also unidirectional in the sense that they allow electrical power to flow from the a.c. side to the d.c. side only. These two disadvantages are the direct consequences of using power diodes in these converters, which can block voltage only in one direction.

It has been shown in the analysis carried on in this chapter that, the output d.c. voltages for half-wave and full-wave three-phase rectifier circuits in terms of the *rms* value of the supply phase voltages are 1.17 V_s and 2.34V_s respectively. Apart from greatly increasing the output voltage, using three-phase rectification reduces the magnitude of the ripple voltage and increases its frequency to either three (for half-wave) or six (for the full-wave) times the supply frequency.

Hence, poly-phase rectifiers are only used where the efficiency of rectific- ation is more important than the cost of the rectifier itself. This is generally the case when the output d.c. power of the rectifier exceeds about 11kW. Many industrial plants, broadcast, and television stations requiring up to 20,000 volts d.c. at peak currents of 10A or more utilize poly-phase rectifiers. Poly-phase circuits have also the advantage over single-phase types of developing fairly steady output d.c. voltages that requires little filtering.

2.9 THE FREEWHEELING DIODE

The operation of both single-phase and poly-phase circuits is normally modified by including a freewheeling diode across the load whenever the load is inductive. This diode blockes the flow of rectifier current but allows the load current to circulate during period of negative supply voltage. Fig.2.34 shows a simple single-phase, half-wave rectifier with inductive load and freewheeling diode. When the freewheeling diode

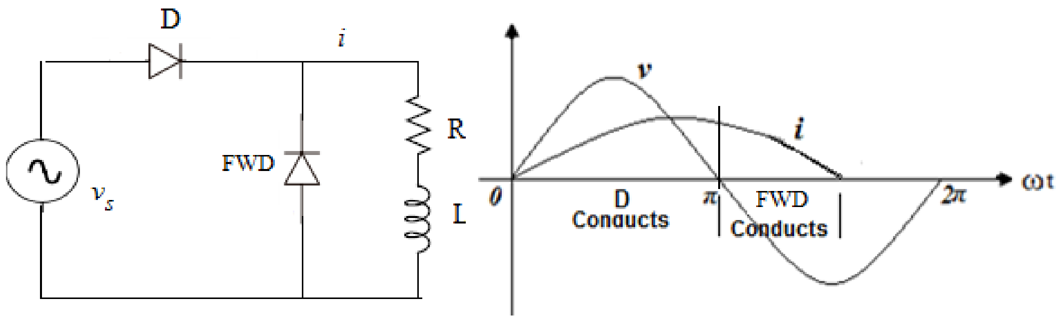


Fig.2.34 The freewheel diode FWD.

FWD is removed from the circuit, the inductive load maintains a continuous current and forces rectifier current flow during periods of negative output voltage. When the freewheeling diode is connected to the circuit, it immediately comes into conduction whenever the supply voltage reverses.This allows the load current which is maintained by the load *emf* to circulate during period of negative supply voltage at the load and the rectifier current becomes zero. The load is effectively short circuited by the freeeheeling diode until suplly voltage becomes positive.

For all the above circuits the d.c. output voltage is directly proportional to the a.c. supply voltage and it cannot be regulated or controlled. By replacing the simple diodes by thyristors (SCRs) such control becomes possible.

PROBLEMS

2.1 A circuit consists of a resistor R ,inductor L and diode D in series , supplied from an ideal source of instantaneous value $v = V_m$ sinωt.

(a) Derive an expression for the time variation i(ωt) of the instantaneous current in terms of V_m , (Z=$\sqrt{R^2+X^2}$) and $\emptyset = \tan^{-1} \omega L/R$.

(b) Sketch roughly to scale ,consistent time variations of v , i and the inductor induced *emf* v_L if $\emptyset = 60°$.

(c) Show that , with a load time constant $\tau = L/R$, the instant of the cycle when the inductor *emf* is zero is given by the transcendental equation

$$\cos(\omega t - \emptyset) = \cos\emptyset\, e^{-t/\tau}$$

2.2 For the half-wave uncontrolled rectifier circuit supplying a series resistive-inductive load shown in Fig.2.35. The supply voltage is $v = \sqrt{2} \times 220 \sin\omega t$ and the supply frequency is 50 Hz and the load parameter values are: $R = 10\ \Omega$, and $X_L = 20\ \Omega$. The current i for the conduction interval $\omega t = 0$ to $\omega t = \beta$ is given by:

$$i(\omega t) = \frac{V_m}{|z|}[\sin(\omega t - \theta) + sin\theta e^{-cot\theta\,.\omega t}]$$

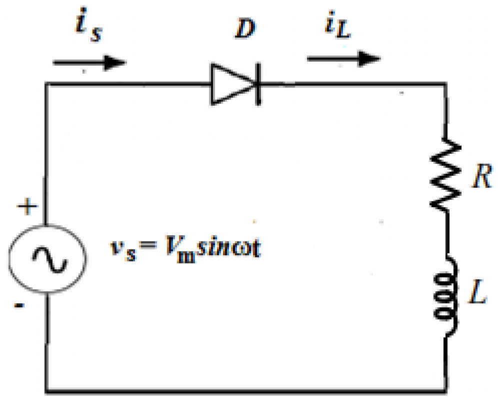
where $|z| = \sqrt{R^2 + (\omega L)^2}$ and $\tan\theta = \omega L/R$

(a) Sketch the load voltage and current waveforms.

(b) Calculate the current extinction angle β.

(c) Calculate the average d.c. load voltage V_{dc}.

Fig.2.35.

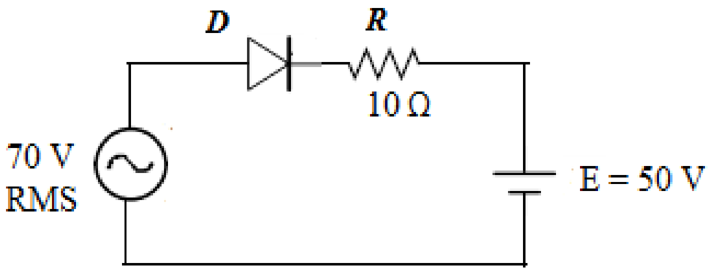


[Ans: β =249.25° , V_{dc} = 64.69 V]

2.3 A single-phase half-wave uncontrolled rectifier supplied from 70 V a.c. supply is used to charge a 50 V battery through a 10 Ω resistance as shown in Fig.2.36. Determine the average value of the current in the circuit.

[Ans: 1.053 A]

Fig.2.36.



2.4 An ideal single-phase source, 230 V, 50 Hz, supplies power to a load resistor $R = 40\ \Omega$ via a single ideal diode. Calculate the average and *rms* values of the current and the power dissipation. What must be the rating of the diode?

[Ans: I_{dc} =2.59 A, I_{Lrms} = 2.875 A , P_L = 330.6 W , I_D = 3 A , V_D = 400 V]

2.5 A single-phase, full-wave diode bridge is used to supply power to a resistive load of value $R = 75\ \Omega$. If the supply voltage is sinusoidal, $v_s = 400 \sin \omega t$, calculate the average and rms values of the load current. Calculate the ripple factor for the load current and compare this with half-wave operation.

[Ans : 7.63A , I_L = 8.5 A , RF = 0.487 compared with 1.21 for half-wave operation]

2.6 Derive expressions for the average and *rms* currents for a single-phase, full-wave uncontrolled diode bridge circuit loaded with pure resistive load. Show that the ripple factor is less than the value for half-wave uncontrolled rectifier circuit with resistive load. What is the ideal value of ripple factor?

2.7 A single-phase supply $v_s = V_m \sin \omega t$ supplies power to a resistor R through an ideal diode. Show that the load voltage $v_L(\omega t)$ can be represented by the Fourier series

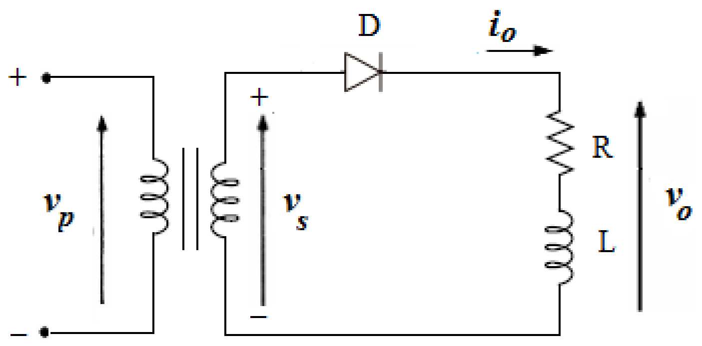
$$v_L(\omega t) = V_m \left(\frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t + \cdots . \right)$$

2.8 A single-phase supply $v_s = \sqrt{2} \times 240 \sin \omega t$, 50 Hz, supplies power to an R-L through an ideal diode as shown in Fig.2.37.

(a) Sketch , approximately to scale for at least two cycles , the waveforms for v_s , v_o and i_o given that the current i_o conducts for 222° before it goes zero. Neglect source impedances.

(b) Show that average value of the output voltage is: V_o = 95.4V.

Fig.2.37.



2.9 For a single-phase, half-wave diode rectifier circuit with resistive load and supply voltage $v_s = V_m \sin \omega t$, obtain an expressions for the Fourier coefficients a_1, b_1 and c_1 of the fundamental component of the load current.

2.10 For the circuit shown in Fig.2.38 ,

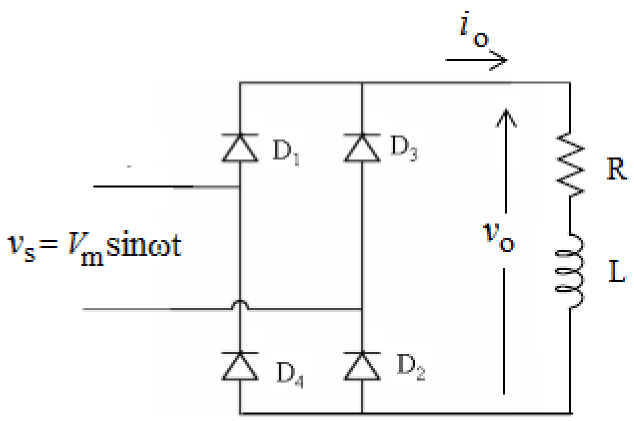
(a) If the current is assumed to be continuous, prove that the load current i_o is given by

$$i_o = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \left[\sin(\omega t - \theta) + \frac{2}{1 - e^{-(R/L)(\frac{\pi}{\omega})}} \sin\theta \, e^{-(R/L)t} \right]$$

where

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

Fig. 2.38.



(b) If the current is assumed to be discontinuous starting from zero , prove that the load current i_o is given by

$$i_o = \frac{V_m}{\sqrt{R^2 + X^2}} \left[\sin(\omega t - \theta) + \sin\theta \, e^{-(R/L)t} \right]$$

2.11 A single-phase full-wave uncontrolled bridge rectifier is supplying a highly inductive load (*L/R* ratio is very large), the load current is assumed to be smooth and ripple-free. If the supply voltage is 220 V, 50 Hz, and the inductor load resistance $R = 22\ \Omega$, Calculate:

(a) The average output voltage V_{dc} and current I_{dc}.
(b) The *rms* value of the output voltage V_{orms} and current I_{orms} .
(c) The *rms* value of the diode current I_{Drms} and the *PRV* of each diode.

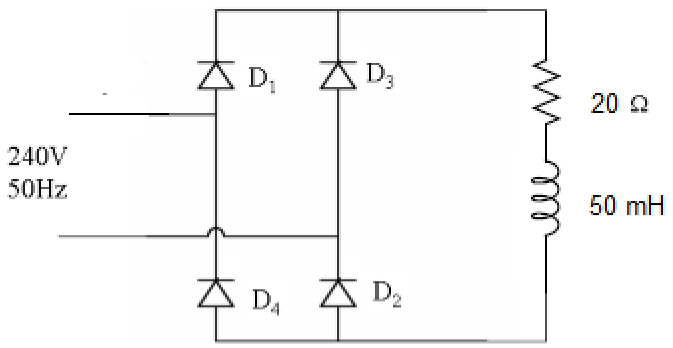
[Ans: (a) 198.1 V, 9 A , (b) 220 V, 10 A , (c) 4.5 A, 155.5 V]

2.12 Repeat problem (2.11) with Bi-phase rectifier type and compare your results.

2.13 A single-phase, full-wave, uncontrolled diode bridge rectifier is used to supply to an inductive load with a resistive component of 20 Ω and inductance of 50 mH from a 240 V (*rms*), 50 Hz supply. The electrical diagram is as shown in Fig.2.39. Use the current formula obtained in Problem 2.10,

(a) Calculate the average output current and voltage supplied to the load, ignore the voltage drops across the diodes.
(b) Calculate the new value of the average output current if the load resistor is halved while retaining the same circuit parameters as before. Comment on the results.

Fig.2.39.



[Ans: (a) 10.8A, (b) 5.4A]

2.14 An electroplating bath is to be supplied from a single-phase full wave rectifier. The load requires 10 A at 100 V. The a.c. supply is from the 230 V mains. Produce the design details of using bi-phase and bridge rectifier circuits and compare the two designs. Assume the load is highly inductive such that the current is continuous and ripple free, and the voltage drop across the diode is neglected.

[Ans: Bi-Phase rectifier: PRV = 314V, Transformer kVA = 1570, I_{Drms} = 7.07A, I_{Ddc} = 5A, Bridge rectifier: PRV = 157V, I_{Drms} = 7.07A, Transformer kVA = 1110]

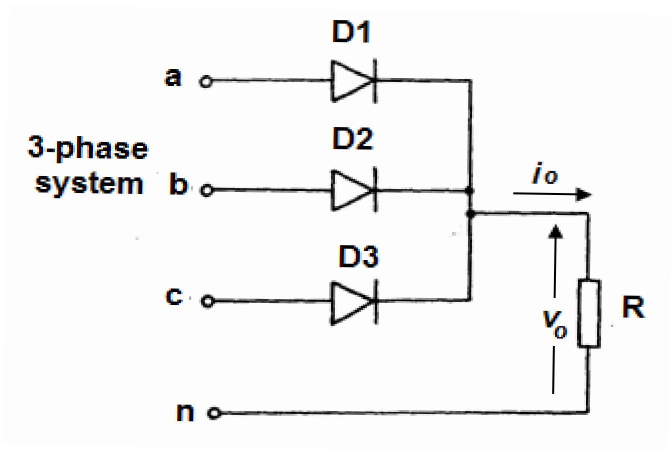
2.15 Develop expressions for average and *rms* load voltages for a three-phase half-wave uncontrolled rectifier connected to a resistive load.

2.16 Compare the power factor of the 3-phase bridge rectifier supplied from a Y-connected supply with that of a single-phase bridge rectifier. Assume ripple-free load current in each case.

2.17 A three-phase, half-wave uncontrolled rectifier shown in Fig.2.40 contains three ideal diodes and is fed from an ideal three-phase voltage source of 380V (*rms*, line-to-line), 50 Hz. The load is resistive with R =15 Ω. Calculate:

(a) The average output voltage and average load current of the rectifier.
(b) The d.c. and a.c. power dissipations in the load.
(c) The diode ratings.
(d) What is the lowest ripple frequency of the output voltage waveform.

Fig.2.40.



[Ans: (a) 256.5V, 17.1A, (b) P_{dc}= 4388W, P_{ac} = 4525.6W, (c) I_D= 10A, V_D = PRV= √3 V_m= 537.3 (say 600V), (d) Lowest ripple = 3f=150Hz]

2.18 Using notation of Fig.2.30 show that for the case of uncontrolled rectification (α = 0) the harmonic of the d.c. voltage produced by an a.c. to d.c. converter with continuous d.c. current flow are contained in the following expression for d.c. voltage :

$$v(\omega t) = V_m \frac{p}{\pi} \sin\frac{\pi}{p}\left[1 - \sum_{n=1}^{n=\infty} \frac{2\cos n\pi/p}{n^2-1}\cos n\omega t\right]$$

where V_m is the peak value of the transformer secondary voltage per-phase.
p is the pulse number.
N= cp with c is a positive integer.

2.19 Power is supplied to a load resistor R from a three-phase, zero-impedance supply of balanced sinusoidal voltages, using full-wave uncontrolled bridge circuit of Fig.2.41. The six diodes may be considered as ideal switches.

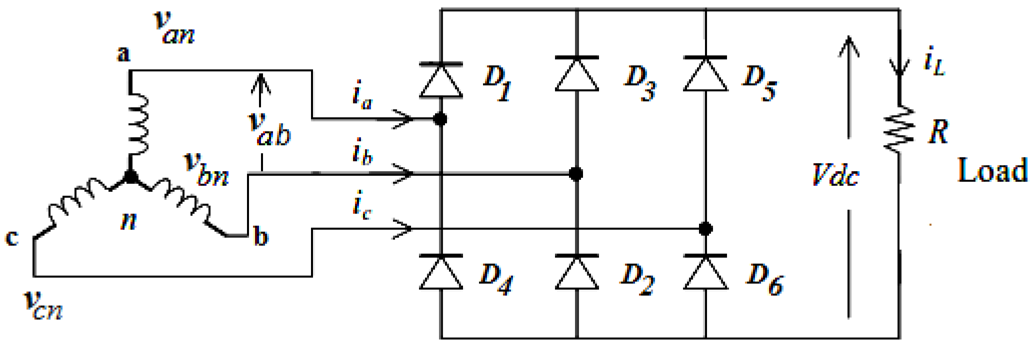


Fig.2.41.

(a) If the supply voltage for phase- a is $v_{an} = V_m \sin \omega t$, and the line- to -line voltage is

$$v_{ab}(\omega t) = \sqrt{3}\, V_m sin\left(\omega t + \frac{\pi}{6}\right) ,$$

show that the average value of the load current is given by :

$$I_{dc} = \frac{3\sqrt{3}V_m}{\pi R}$$

(b) Calculate V_{dc}, I_{dc} and P_{dc} if $V_m = 300$ V, $R = 50\ \Omega$.

[Ans : (b) V_{dc} = 496.43 V, I_{dc} = 9.92 A , P_{dc} = 4928.8 W]

2.20 A three-phase, full-wave uncontrolled bridge rectifier contains six ideal diodes and is fed from an ideal three-phase voltage source of 240 V, 50 Hz. The load is resistive with R =10 Ω. Calculate the average load voltage and the power dissipation. Also calculate the required voltage and current ratings of the diodes.

[Ans: V_{dc} = 324 V, P_{dc} = 10.497 kW, I_{Drms} = 18.7 A, V_D =339.4 V]

Test Questions

Mark the correct answer:

2.21 In a single phase uncontrolled half-wave rectifier, if the *rms* value of the input voltage is V_s = 220 V, the average d.c. voltage is

(a) 104V (b) 120V (c) 99V (d) 85V

2.22 In a single-phase full-wave uncontrolled bridge rectifier supplying resistive load, if the input voltage is v_s=311sinωt, then the average output voltage is

(a) 209 (b) 106 V (c) 198 V (d) 99 V

2.23 Each diode of a three-phase half-wave uncontrolled rectifier conducts for:

(a) 60 (b) 120° (c) 180° (d) 90°.

2.24 In a Bi-phase full-wave uncontrolled rectifier, if per-phase *rms* input voltage is 230 V, then the average output voltage is

(a) 257.3 V (b) 116.95 V (c) 207.00 V (d) 101.28 V

2.25 The supply voltage for a single-phase half-wave uncontrolled rectifier is v_s = 220√2 sin314t. The rectifier is loaded with resistive load R = 10 Ω. The d.c. power absorbed by the load is

(a) 980.1 W (b) 875.9 W (c) 1073.0 W (d) 790.9 W

2.26 In a single-phase full-wave uncontrolled rectifier supplying resistive load, the input voltage has an *rms* value of 250 V, if the load is resistive with R = 25 Ω , then the average current I_{dc} is

(a) 4A (b) 9A (c) 15 A (d) 20A

2.27 In a three-phase full-wave uncontrolled bridge rectifier, if per-phase peak input voltage is V_m, the average output voltage is given by

(a) $V_m/2\pi$ (b) $\sqrt{3}V_m/2\pi$ (c) $3\sqrt{3}V_m/4\pi$ (d) $3\sqrt{3}V_m/\pi$

2.28 Each diode of a hexa-phase uncontrolled diode rectifier conducts for:

(a) 60° (b) 120° (c) 180° (d) 90°

2.29 In a three-phase half-wave uncontrolled rectifier, if per-phase peak input voltage is V_m, then the average output voltage is given by

(a) $V_m/2\pi$ (b) $\sqrt{3}V_m/2\pi$ (c) $3\sqrt{3}V_m/\pi$ (d) $3\sqrt{3}V_m/2\pi$

[Ans : 2.21(**c**) , 2.22 (**c**) , 2.23 (**b**), 2.24 (**c**) , 2.25 (**a**) , 2.26(**b**) , 2.27 (**d**), 2.28 (**a**), 2.29 (**d**)]