Calculus

Lecture – 2 / Functions

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Functions

Definition:

- A function is a special relationship where each input has a single output.
- A function f is a rule that assigns to each element x in a set A exactly one element, called f(x); in a set B.
- The set A is called the domain of f: The range of f is the set of all possible values of f(x), as x varies throughout the domain.

Functions

First, what exactly is a function?

$$y = f(x)$$

Output — Function notation — Input

f(x) read as f of x

$$f(x) = 3x + 1$$

Functions

What is function notation?

Function notation is nothing more than a fancy way of writing the y in a function that will allow us to simplify notation and some of our work a little.

- $y = 2x^2 - 5x + 3$

$f(x) = 2x^2 - 5x + 3$ $\qquad$ $g(x) = 2x^2 - 5x + 3$

$h(x) = 2x^2 - 5x + 3$ $\qquad$ $R(x) = 2x^2 - 5x + 3$

$w(x) = 2x^2 - 5x + 3$ $\qquad$ $y(x) = 2x^2 - 5x + 3$

Functions

So, why is this useful?

Some times we have more than one function, using function notation will help us to distinguish between them.

- If x = 3 then using function notation we write it as
- $f(x) = 2x^2 - 5x + 3 \rightarrow \mathrm{f}(3) = 2(3)^2 - 5(3) + 3 = 6$
- $\mathrm{g}(\mathrm{x}) = 2x^2 - 5x + 3 \rightarrow \mathrm{g}(3) = 2(3)^2 - 5(3) + 3 = 6$
- $h(x) = 2x^2 - 5x + 3 \rightarrow \mathrm{h}(2) = 2(2)^2 - 5(2) + 3 = 6$

Functions

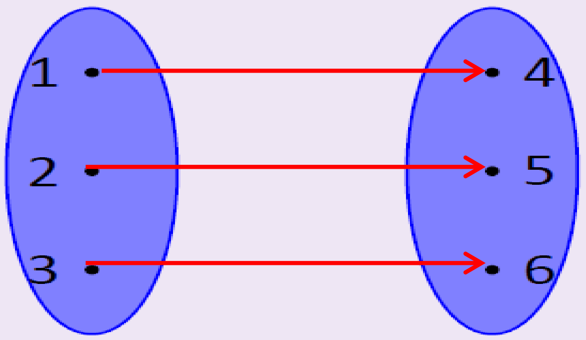
Example-1: Determine which of the following are functions?

Example

Is this a function? If so, what is the range?

x	$f(x)$
1	4
2	5
3	6

Solution



Each input has only one outputs, so its a function

Functions

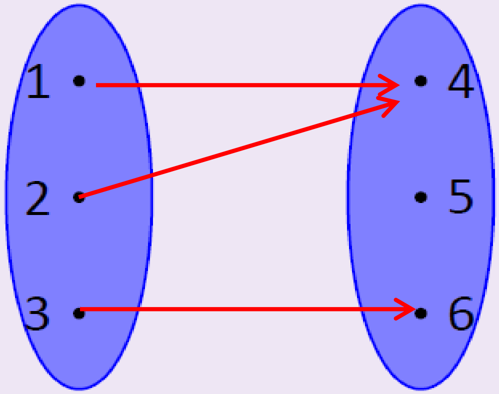
Example-2: Determine which of the following are functions?

Example

Is this a function? If so, what is the range?

x	$f(x)$
1	4
2	4
3	6

Solution



Each input has only one outputs even two of them has the same output, but its still a function

Functions

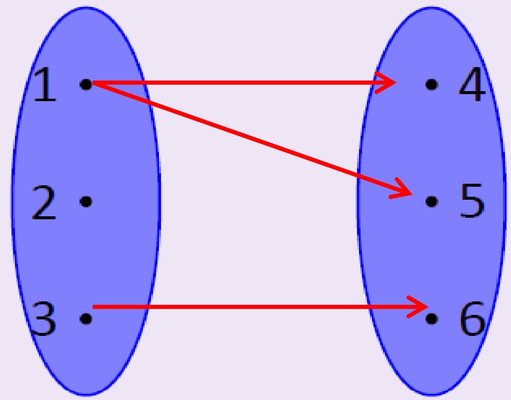
Example-3: Determine which of the following are functions?

Example

Is this a function? If so, what is the range?

x	$f(x)$
1	4
1	5
3	6

Solution



One in put has two outputs, so its not a function

Functions

Example – 4: Determine if each of the following is a function ?

- $y = x^2 + 1$
- $y^2 = x + 1$

$y = x^2 + 1$

$For\ x = -2 \rightarrow y = (-2)^2 + 1 = 5$

$For\ x = 0 \rightarrow y = (0)^2 + 1 = 1$

$For\ x = 2 \rightarrow y = (2)^2 + 1 = 5$

Functions

Example – 5: Determine if each of the following is a function ?

- $y = x^2 + 1$
- $y^2 = x + 1$

$y^2 = x + 1$

$For\ x = -2 \rightarrow y^2 = (-2) + 1 = -1 \quad \rightarrow y = \sqrt{-1}$ **!**

$For\ x = 0 \rightarrow y^2$ … $\rightarrow y = \pm 1$

$For\ x = 2 \rightarrow y$ … $3 \quad \rightarrow y = \pm\sqrt{3}$

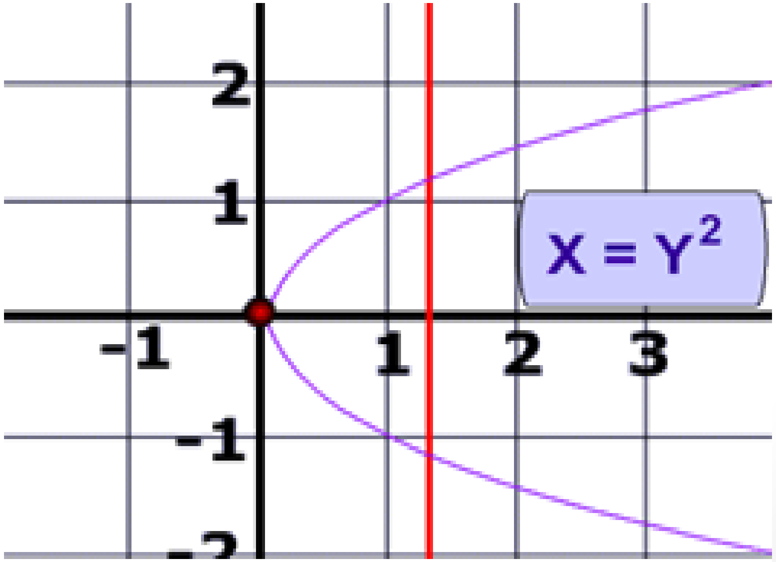
What's this ?

Imaginary No.

Functions

- All vertical lines are not functions.
- **Examples : x = 2 or x = 5**
- Any equation that has y^2 within it should be examined very carefully.

- Any parabola that opens horizontally is not a function because it will never pass the vertical line test.



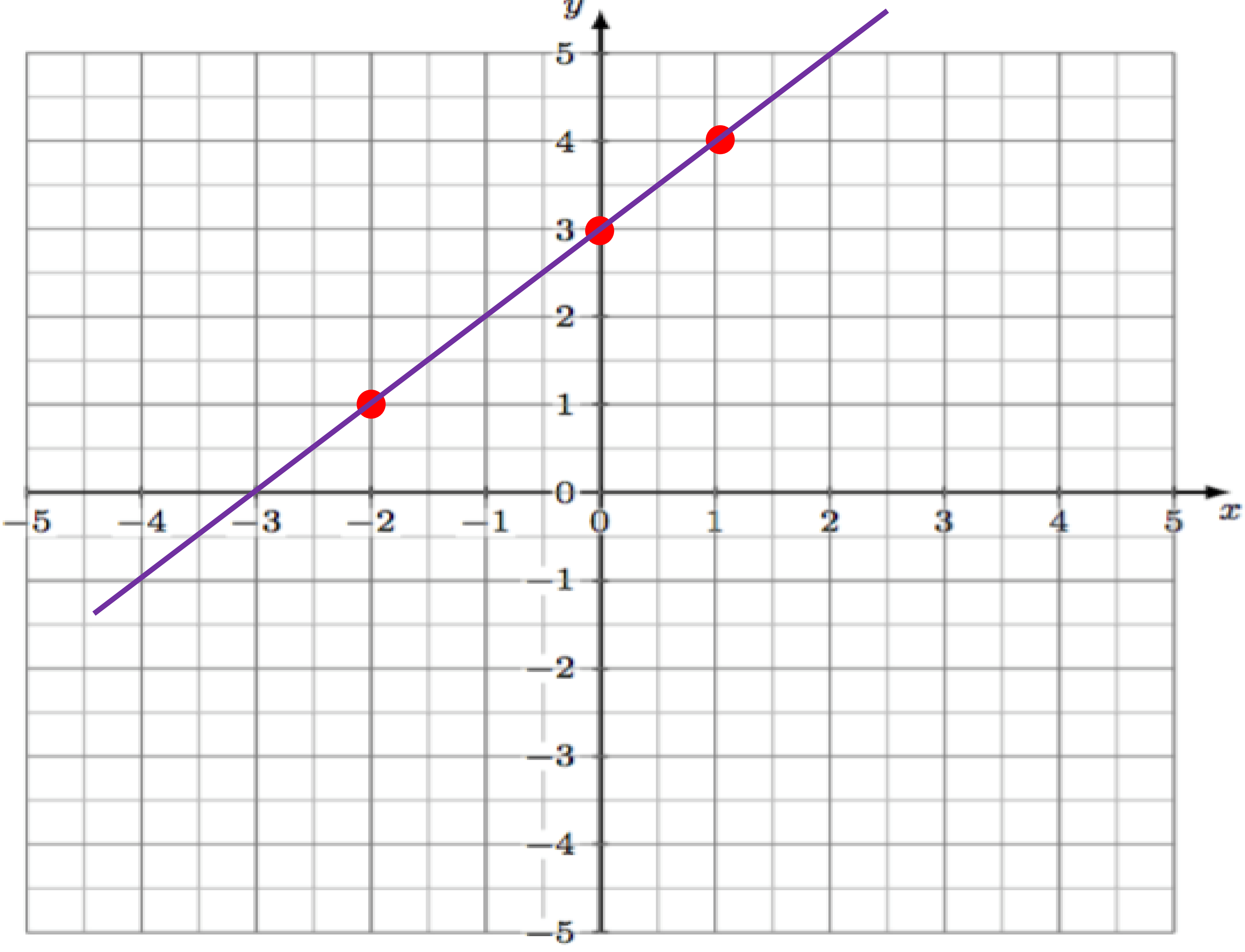
Graphing of Linear Equation

Example – 3: $y = x + 3$

Use table to draw the graph for this linear equation

First choose three values of x lets say (-2, 0, 1).

x	y
-2	1
0	3
1	4


y
x
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5

Functions

Example – 6: Determine which of the following equations is a function?

- $\boldsymbol{y = 5x + 1}$
- $\boldsymbol{y = x^2 + 1}$
- $\boldsymbol{y^2 = x + 1}$
- $\boldsymbol{y^2 - x^2 = 4}$

Solutions

$\boldsymbol{y = 5x + 1}$

$x = -4$: $y = 5(-4) + 1 = -20 + 1 = -19$

$x = 0$: $y = 5(0) + 1 = 0 + 1 = 1$

$x = 4$: $y = 5(4) + 1 = 20 + 1 = 21$

As we can see that for each input value there is only one output value, so its function

Functions

Example – 6: Determine which of the following equations is a function?

- $\boldsymbol{y = 5x + 1}$
- $\boldsymbol{y = x^2 + 1}$
- $\boldsymbol{y^2 = x + 1}$
- $\boldsymbol{y^2 - x^2 = 4}$

Solutions

$\boldsymbol{y = x^2 + 1}$

$x = -3$: $\quad y = (-3)^2 + 1 = 9 + 1 = 10$

$x = 3$: $\quad y = (3)^2 + 1 = 9 + 1 = 10$

$x = 5$: $\quad y = (5)^2 + 1 = 25 + 1 = 26$

As we can see that for each input value there is only one output value, so its function

Functions

Example – 6: Determine which of the following equations is a function?

- $\boldsymbol{y = 5x + 1}$
- $\boldsymbol{y = x^2 + 1}$
- $\boldsymbol{y^2 = x + 1}$
- $\boldsymbol{y^2 - x^2 = 4}$

Solutions

$\boldsymbol{y^2 = x + 1}$

$x = 3: \quad y^2 = 3 + 1 = 4 \rightarrow y = \pm 2$

$x = -1: \quad y^2 = -1 + 1 = 0$

$x = 10: \quad y^2 = 10 + 1 = 11 \rightarrow y = \pm\sqrt{11}$

As we can see that for inputs (3 and 10) there are two output values for each one For (3) +2 and -2. For (10) $+\sqrt{11}$ and $-\sqrt{11}$

Functions

Example – 6: Determine which of the following equations is a function?

- $\boldsymbol{y = 5x + 1}$
- $\boldsymbol{y = x^2 + 1}$
- $\boldsymbol{y^2 = x + 1}$
- $\boldsymbol{y^2 - x^2 = 4}$

Solutions

$\boldsymbol{y^2 - x^2 = 4} \quad \rightarrow \quad \boldsymbol{y^2 = x^2 + 4}$

$x = 0: \quad y^2 = (0)^2 + 4 = 4 \rightarrow \; y = \pm 2$

For any given value of x there are 2 outputs. So, this is obviously not a function.

Functions

Example-7: Given $\mathbf{f}(x) = -x^2 + 6x - 11$

Find each of the following.

- $f(2)$ $\quad \mathbf{f(2) = -(2)^2 + 6(2) - 11 = -3}$
- $f(-10)$
- $f(t)$
- $f(x - 3)$
- $f(4x - 1)$

Functions

Example-7: Given $\mathbf{f(x) = -x^2 + 6x - 11}$

Find each of the following.

- $f(2)$
- $f(-10)$
- $f(t)$
- $f(x-3)$
- $f(4x-1)$

$$\mathbf{f(-10) = -(-10)^2 + 6(-10) - 11}$$
$$\mathbf{= -100 - 60 - 11 = -171}$$

Be careful when squaring negative numbers!

Functions

Example-7: **Given** $\mathbf{f(x) = -x^2 + 6x - 11}$

Find each of the following.

- $f(2)$
- $f(-10)$
- $\boldsymbol{f(t)}$
- $f(x-3)$
- $f(4x-1)$

$$\mathbf{f(t) = -t^2 + 6t - 11}$$

Just change the x by t

Functions

Example-7: $\mathbf{Given\ f(x) = -x^2 + 6x - 11}$

Find each of the following.

- $f(2)$
- $f(-10)$
- $f(t)$
- $f(x-3)$
- $f(4x-1)$

$$f(x-3) = -(x-3)^2 + 6(x-3) - 11$$
$$= -(x^2 - 6x + 9) + (6x - 18) - 11$$
$$= -x^2 + 12x - 38$$

Functions

Example-7: Given $\mathbf{f(x) = -x^2 + 6x - 11}$

Find each of the following.

- $f(2)$
- $f(-10)$
- $f(t)$
- $f(x-3)$
- $f(4x-1)$

$$\mathbf{f(4x-1) = -(4x-1)^2 + 6(4x-1) - 11}$$

$$\mathbf{= -(16\,x^2 - 8x + 1) + (24x - 6) - 11}$$

$$\mathbf{= -16x^2 + 32x - 18}$$

Functions

Example-8: **Given $\mathbf{g}(x) = x^2 - 3x + 7$**

Find each of the following.

- $g(10)$ $\quad\quad \mathbf{g(10) = (10)^2 - 3(10) + 7 = 77}$
- $g(a+1)$
- $g(a^2)$
- $g(x+h)$

Functions

Example-8: Given $g(x) = x^2 - 3x + 7$

Find each of the following.

- $g(10)$
- $g(a+1)$
- $g(a^2)$
- $g(x+h)$

$$g(a+1) = (a+1)^2 - 3(a+1) + 7$$

$$= (a^2 + 2a + 1) + (-3a - 3) + 7$$

$$= a^2 - a + 5$$

Functions

Example-8: **Given** $\mathbf{g(x) = x^2 - 3x + 7}$

Find each of the following.

- $g(10)$
- $g(a+1)$
- $g(a^2)$
- $g(x+h)$

$$g(a^2) = (a^2)^2 - 3(a^2) + 7$$

$$= a^4 - 3a^2 + 7$$

Functions

Example-8: **Given $g(x) = x^2 - 3x + 7$**

Find each of the following.

- $g(10)$
- $g(a+1)$
- $\boldsymbol{g(a^2)}$
- $\boldsymbol{g(x+h)}$

$$\mathbf{g}(x+h) = (x+h)^2 - 3(x+h) + 7$$
$$= (x^2 + 2hx + h^2) + (-3x - 3h) + 7$$
$$= x^2 + 2hx + h^2 - 3x - 3h + 7$$

Functions

Example-9: Given $Let\, f\left(x\right)=\frac{x-5}{x^{2}+4}$

Find each of the following.

- $f(2)$
- $f(5)$
- $f(a+1)$
- $f(a^2)$

$$f\left(x\right)=\frac{2-5}{2^{2}+4}=\frac{-3}{8}$$

Functions

Example-9: Given $Let\ f(x)=\dfrac{x-5}{x^2+4}$

Find each of the following.

- $f(2)$
- $f(5)$

$$f(x)=\frac{5-5}{5^2+4}=\frac{0}{29}=0$$

- $f(a+1)$
- $f(a^2)$

Functions

Example-9: Given $Let\ f(x) = \dfrac{x-5}{x^2+4}$

Find each of the following.

- $f(2)$
- $f(5)$
- $f(a+1)$
- $f(a^2)$

$$f(x) = \frac{(a+1)-5}{(a+1)^2+4} = \frac{a-4}{a^2+2a+1+4} = \frac{a-4}{a^2+2a+5}$$

Functions

Example-9: Given $Let\ f(x) = \frac{x-5}{x^2+4}$

Find each of the following.

- $f(2)$
- $f(5)$
- $f(a+1)$
- $f(a^2)$

$$f(a^2) = \frac{a^2-5}{(a^2)^2+4} = \frac{a^2-5}{a^4+4}$$

Functions

Example-10: Given

- $$g(x) = \begin{cases} 3x^2 + 4 & if\ x \leq -4 \\ 10 & if - 4 < x \leq 20 \\ 1 - 6x & if\ x > 20 \end{cases}$$

Evaluate each of the following.

- **g(-6)**
- g(-4)
- g(1)
- g(20)
- g(21)

Since, $-6 < -4$

$Therefore, -6\ satisifies\ the\ first\ inequality$

$3(-6)^2 + 4 = 112$

Functions

Example-10: Given

- $g(x) = \begin{cases} 3x^2 + 4 & if\ x \leq -4 \\ 10 & if -4 < x \leq 20 \\ 1 - 6x & if\ x > 20 \end{cases}$

Evaluate each of the following.

- g(-6)
- **g(-4)**
- g(1)
- g(20)
- g(21)

Since, $-4 = -4$

$Therefore, -4\ satisifies\ the\ first\ inequality$

$3(-4)^2 + 4 = 52$

Functions

Example-10: Given

- $g(x) = \begin{cases} 3x^2 + 4 & if\ x \leq -4 \\ 10 & if -4 < x \leq 20 \\ 1 - 6x & if\ x > 20 \end{cases}$

Evaluate each of the following.

- g(-6)
- g(-4)
- **g(1)**
- g(20)
- g(21)

Since, $-4 < 1 < 20$

$Therefore, 1\ satisifies\ the\ second\ inequality$

g(1) = 10

Functions

Example-10: Given

- $$g(x) = \begin{cases} 3x^2 + 4 & if\ x \le -4 \\ 10 & if -4 < x \le 20 \\ 1 - 6x & if\ x > 20 \end{cases}$$

Evaluate each of the following.

- g(-6)
- g(-4)
- g(1)
- **g(20)**
- g(21)

Since, $-4 < 20 \le 20$

$Therefore, 20\ satisifies\ the\ 2nd\ \ inequality$

g(20) = 10

Functions

Example-10: Given

- $g(x) = \begin{cases} 3x^2 + 4 & if\ x \le -4 \\ 10 & if -4 < x \le 20 \\ 1 - 6x & if\ x > 20 \end{cases}$

Evaluate each of the following.

- g(-6)
- g(-4)
- g(1)
- g(20)
- **g(21)**

Since, $20 < 21$

$Therefore, 21\ satisifies\ the\ 3rd\ inequality$

$1 - 6(21) = -125$