Calculus

Polynomials & Linear equations

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1st Stage

Introduction to Polynomials

- **Monomial:** A number, a variable or the product of a number and one or more variables. (ax^n with n is a non-negative integers, a is a real number)
- **Examples:** $3x$, -3, or $4xy^2z$
- **Polynomial:** A monomial or a sum of monomials.
- **Binomial:** A polynomial with exactly two terms.
- **Examples:** $3x - 5$, or $4xy^2z + 3ab$
- **Trinomial:** A polynomial with exactly three terms.
- **Examples:** $4x^2 + 2x - 3$

Introduction to Polynomials

Identify the parts of monomial

$$7x^3$$

Coefficient

Variable

Exponent

$$\frac{1}{2}a^9$$

Coefficient

Variable

Exponent

Introduction to Polynomials

- **Coefficient:** A numerical factor in a term of an algebraic expression, such as number 4 in term 4x.
- **Degree of a monomial:** The sum of the exponents of all of the variables in the monomial.
- **Degree of a polynomial:** is equal the highest exponent (if the term has more than 1 variable, then add all exponents of that term)
- **Standard form:** When the terms of a polynomial are arranged from the largest exponent to the smallest exponent in decreasing order.

Degree of Monomials

- **What is the degree of the monomial?**

$$5x^3y^2$$

- The degree of a monomial is the sum of the exponents of the variables in the monomial.
- The exponents of each variable are 3 and 2. $\mathbf{3+2=5}$
  - The degree of the monomial is 5.
  - The monomial can be referred to as a fifth degree monomial.

Degree of Monomials

- A polynomial is a monomial or the sum of monomials
- **Examples:** $2x^2,\ \ 3x^2+2x,\ \ x^2+x^2-10$
- Each monomial in a polynomial is a term of the polynomial.
  - The number factor of a term is called the coefficient.
  - The coefficient of the first term in a polynomial is the lead coefficient.
- A polynomial with two terms is called a binomial.
- A polynomial with three terms is called a trinomial.

Degree of Monomials

- The degree of a polynomial in one variable is the largest exponent of that variable.
- **4** is a constant has no variable. It's called a 0 degree polynomial.
- $\mathbf{2x+3}$ This is a 1st degree polynomial. 1st degree polynomials are linear.
- $\mathbf{2x^2+3}$ This is a 2nd degree polynomial. 2nd degree polynomials are quadratic.
- $\mathbf{2x^3-5}$ This is a 3rd degree polynomial. 3rd degree polynomials are cubic.

Degree of Monomials

- Are these polynomials or not polynomials?

$\frac{3}{x}$		
$\frac{1}{2}x$		
$\lvert x - 3 \rvert$		
$\sqrt{x}$		
xy		
x^{-2}		
Sinx+8		

Degree of Monomials

Classify the polynomials by degree and number of terms.

Polynomial	Degree	Classify by degree	Classify by terms
3	Zero	Constant	Monomial
$x - 5$	First	Linear	Binomial
$x^2 + x - 4$	Second	Quadratic	Trinomial
$4x^3 - 6$	Third	Cubic	Binomial
$x^4 + 2$	Fourth	Quartic	Binomial
$x3 - 6x^5 + 2$	Fifth	Quantic	Trinomial

Standard Forms

- To rewrite a polynomial in standard form, rearrange the terms of the polynomial starting with the largest degree term and ending with the lowest degree term.
- The ***leading coefficient***, the coefficient of the first term in a polynomial written in standard form, should be positive.

Standard Forms

Examples: Write the polynomials in standard forms

- $3x + 2x^3 - 5 - x^2$

$2x^3 - x^2 + 3x - 5$

- $-2x^2 + x + x^4 - 7 + 5x^3$

$x^4 + 5x^3 - 2x^2 + x - 7$

- $x^3 + x^4 - x + 9 + 3x^5$

$3x^5 + x^4 + x^3 - x + 9$

- $7x^3 + 2x^4 + 6 - x^5$

$-x^5 + 2x^4 + 7x^3 + 6$

Standard Forms

Examples: Write the polynomials in standard forms

- $7x^3 + 2x^4 + 6 - x^5$
- $-x^5 + 2x^4 + 7x^3 + 6$
- $-1(-x^5 + 2x^4 + 7x^3 + 6)$
- $x^5 - 2x^4 - 7x^3 - 6$

Remember: The lead coefficient should be **positive** in standard form.

To do this, multiply the polynomial by -1 using the distributive property.

Standard Forms

Examples: Write the polynomials in standard forms

- $x^4 - x^7 - x^5 - 1$
- $-x^7 + x^5 + x^4 - 1$
- $-1(-x^7 + x^5 + x^4 - 1\ )$
- $x^7 - x^5 - x^4 + 1$

- $x^2 - 2x^6 - x^4 - 4 + x$
- $-2x^6 - x^4 + x^2 + x - 4$
- $-1(-2x^6 - x^4 + x^2 + x - 4)$
- $2x^6 + x^4 - x^2 - x + 4$

Solve Problems

Examples: Write down one example of each of the following types of polynomial function:

(a) Cubic

(b) Linear

(c) Quartic

(d) Quadratic

In Class Activity

Solve Problems

Write the polynomials in standard form and identify the polynomial by degree, number of terms and leading coefficient.

- $5 + x^2 + x$
- $3x + 2x^3 - 7x^5 + 4 - x^2$
- $8 - 7x^5 - 5x - 6x^9 - x^3$
- $-6x^8 - 2 - x^6 - x^4 - x^2$

Solve Problems

- $5 + x^2 + x$
- $x^2 + x + 5$
- This is a 2nd degree polynomial (quadratic) with binomial terms
- Leading Coefficient = 1

Solve Problems

- $3x + 2x^3 - 7x^5 + 4 - x^2$
- $-7x^5 + 2x^3 - x^2 + 3x + 4$
- $-1(-7x^5 + 2x^3 - x^2 + 3x + 4)$
- $7x^5 - 2x^3 + x^2 - 3x - 4$
- This is a 5th degree polynomial with 5 terms
- Leading Coefficient = 7

Solve Problems

- $8 - 7x^5 - 5x - 6x^9 - x^3$
- $-6x^9 - 7x^5 - x^3 - 5x + 8$
- $-1(-6x^9 - 7x^5 - x^3 - 5x + 8)$
- $6x^9 + 7x^5 + x^3 + 5x - 8$
- This is a 9th degree polynomial with 5 terms
- Leading Coefficient = 6

Solve Problems

- $-3x^8 - 2 - x^6 - x^4$
- $-3x^8 - x^6 - x^4 - 2$
- $-1(-3x^8 - x^6 - x^4 - 2\ )$
- $3x^8 + x^6 + x^4 + 2$
- This is a 8th degree polynomial with 4 terms
- Leading Coefficient = 3

Linear Equations

- Linear equations are first degree polynomial equations.

- A linear equation with one variable can be written in the form of

$$ax + b = c$$

- Where, a, b and c are all real numbers and $a \neq 0$

Linear Equations

Distinguish between expressions and equations.

- Equations and inequalities compare algebraic expressions.
- An equation is a statement that two algebraic expressions are equal.
- An equation always contains an equals symbol, while an expression does not.

$$3x - 7$$

Expression
(to simplify or evaluate)

$$\underbrace{3x - 7}_{\text{Left side}} = \underbrace{2}_{\text{Right side}}$$

Equation (to solve)

Linear Equations with One Variable

So, can you tell these equations are linear equation with one variable or not?

$\boldsymbol{x + 2y = 10}$ NO, because there are 2 variables x and y

$\boldsymbol{(x + 3)^2 = 6}$

$x^2 + 6x + 3 = 0$ NO, because x^2 is not a linear equation

$\boldsymbol{\frac{5}{2x} + 3 = x - 5}$ NO, because there is variable at denominator

Linear Equations with One Variable

Strategy for solving Linear Equations. Isolate variable terms on one side and constant terms on the other side.

Problem-1. Solve the following Linear Equations.

- $x - 2 = 8$
- $2x + 4 = 0$
- $8 - 2x = 6$

Solve Linear Equations with One Variable

Example – 1: Solve the following Linear Equations.

$$\boldsymbol{x - 2 = 8}$$

$$x = 8 + 2$$

$$x = 10$$

Check for x

$$\mathbf{10 - 2 = 8}$$

$$\mathbf{8 = 8}$$

$$\boldsymbol{2x + 4 = 0}$$

$$2x = -4$$

$$x = -2$$

Check for x

$$\mathbf{2(-2) + 4 = 0}$$

$$\mathbf{-4 + 4 = 0}$$

Solve Linear Equations with One Variable

Example – 1: Solve the following Linear Equations.

$$\mathbf{8-2x=6}$$

$$-2x=6-8$$

$$-1(-2x=-2\ )$$

$$2x=2$$

$$x=1$$

Check for x

$$\mathbf{8-2x=6}$$

$$\mathbf{8-2(1)=6}$$

$$\mathbf{8-2=6}$$

$$\mathbf{6=6}$$

Solve Linear Equations with One Variable

To generate equivalents equations you can use the following operations.

1. Simplify an expression by removing grouping symbols and combining like terms.
2. Add/subtract the same real number/expression on both sides of the equation.
3. Multiply/divide on both sides of the equation by the same nonzero quantity
4. Interchange two sides of the equation.

Solve Linear Equations with One Variable

Example – 2: Solve the following linear equation.

$$\boldsymbol{x - 4 = 3(x - 5)}$$

$$x - 4 = 3x - 15$$

Multiply number with inside parenthesis

$$x - 3x = 4 - 15$$

Put all variables at the left side and numbers at the right side.

$$-2x = -11$$

Add or subtract variables with same variables and number with number if there is any.

$$-1(-2x = -11)$$

Multiply both side by -1

$$2x = 11$$

Divide both side by 2 to get the value of x

$$x = 5.5$$

Solve Linear Equations with One Variable

Example – 2: Solve the following linear equation.

$$\boldsymbol{x - 4 = 3(x - 5)}$$

Check for x

Substitute the value of x = 5.5 in the equation, if left side equals right side then value of x is correct

$$x - 4 = 3x - 15$$

$$5.5 - 4 = 3(5.5) - 15$$

$$1.5 = 16.5 - 15$$

$$1.5 = 1.5$$

Solve Linear Equations with One Variable

Example – 3: Solve the following linear equation.

$$x - 10 = 5[2 - 5(2x - 2)]$$

$$x - 10 = 5[2 - 10x + 10]$$

$$x - 10 = 10 - 50x + 50$$

$$x + 50x = 10 + 50 + 10$$

$$51x = 70$$

$$x = \frac{70}{51}$$

Solve Linear Equations with One Variable

Example – 4: Solve the following linear equation.

$$2x - 1 = x - 5[2x - 2(3x - 3)]$$

$$2x - 1 = x - 5[2x - 6x + 6]$$

$$2x - 1 = x - 10x - 30x + 30$$

$$2x - x + 10x + 30x = 30 + 1$$

$$41x = 31$$

$$x = \frac{31}{41}$$

Identification of Linear Equation

How to Determine

the Solution Sets of Linear Equations

Identification of Linear Equation

Identify conditional equations, contradictions, and identities.

Type of Linear Equation	Number of Solutions	Indication when Solving
Conditional	One	Final line is x = a number.
Identity	Infinite; solution set {all real numbers}	Final line is true, such as 0 = 0.
Contradiction	None; solution set $\varnothing$	Final line is false, such as $0 = -20$.

Identification of Linear Equation

Example – 5: Solve each equation, Decide whether it is conditional, identity, or contradiction.

1. $\mathbf{5x - 9 = 4(x - 3)}$

$$5x - 9 = 4x - 12$$

$$5x - 4x = 9 - 12$$

$$x = -3$$

So, this equation is Conditional

And solution set = {-3}

Identification of Linear Equation

Example – 5: Solve each equation, Decide whether it is conditional, identity, or contradiction.

2. $\mathbf{5x - 15 = 5(x - 3)}$

$$5x - 15 = 5x - 15$$

$$5x - 5x = 15 - 15$$

$$0 = 0$$

So, this equation is Identity

And solution set = {real numbers}

Identification of Linear Equation

Example – 5: Solve each equation, Decide whether it is conditional, identity, or contradiction.

3. $\mathbf{5x - 15 = 5(x - 4)}$

$$5x - 15 = 5x - 20$$

$$5x - 5x = 15 - 20$$

$$0 = -5$$

So, this equation is Contradiction

And solution set = { } or no solution

Identification of Linear Equation

Example – 6: Solve each equation, Decide whether it is conditional, identity, or contradiction.

$$\frac{x+2}{2}-\frac{x}{3}=5$$

$$6\left(\frac{x+2}{2}-\frac{x}{3}=5\right)$$

$$3x+6-2x=30$$

$$x=24$$

So, this equation is Conditional
And solution set = {24}

Home Work

Example – 7: Solve each equation, Decide whether it is conditional, identity, or contradiction.

1. $3(2x+3)-2(2x+2)=2x+4$

2. $\frac{6x+4}{3}+\frac{x}{3}=2x+\frac{x}{3}+\frac{4}{3}$

3. $\frac{x}{4}+\frac{4}{6}=\frac{x}{2}$