Calculus

Solving system of linear equations

Zaid Al Hamdany
Architectural Engineering Department
1^{st} Stage

System of Linear Equations

After completing this tutorial, you should be able to:

1. Know if an ordered pair is a solution to a system of linear equations in two variables or not.
2. Solve a system of linear equations in two variables by **graphing**.
3. Solve a system of linear equations in two variables by the **substitution method**.
4. Solve a system of linear equations in two variables by the **elimination method**.

System of Linear Equations

A system of linear equations is two or more linear equations that are being solved simultaneously.

- In this lecture, we will be looking at systems that have only two linear equations and two unknowns.

System of Linear Equations

In general, a solution of a system in two variables is an ordered pair that makes BOTH equations true.

- In other words, it is where the two graphs intersect, what they have in common. So if an ordered pair is a solution to one equation, but not the other, then it is NOT a solution to the system.
- A **consistent system** is a system that has **at least one solution**.
- An **inconsistent system** is a system that **has no solution**.

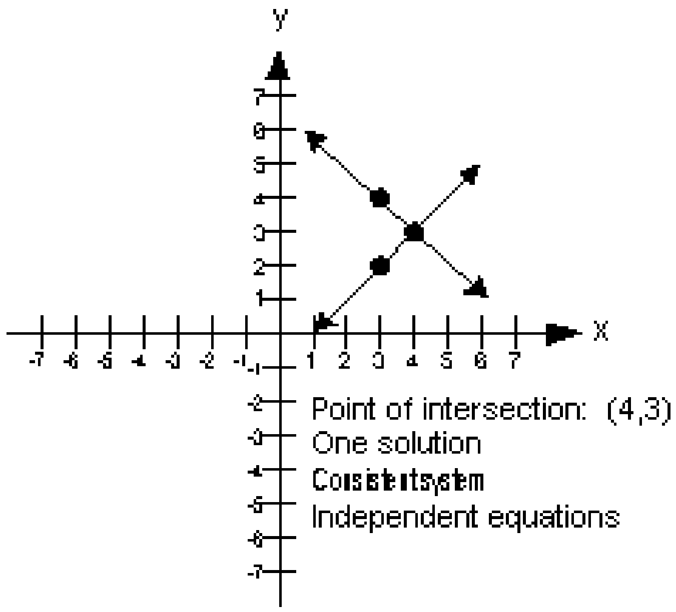
System of Linear Equations

There are three possible outcomes that you may encounter when working with these systems:

- **One Solution (Conditional)**
- **No Solution (Contradiction)**
- **Infinite Solutions (Identity)**

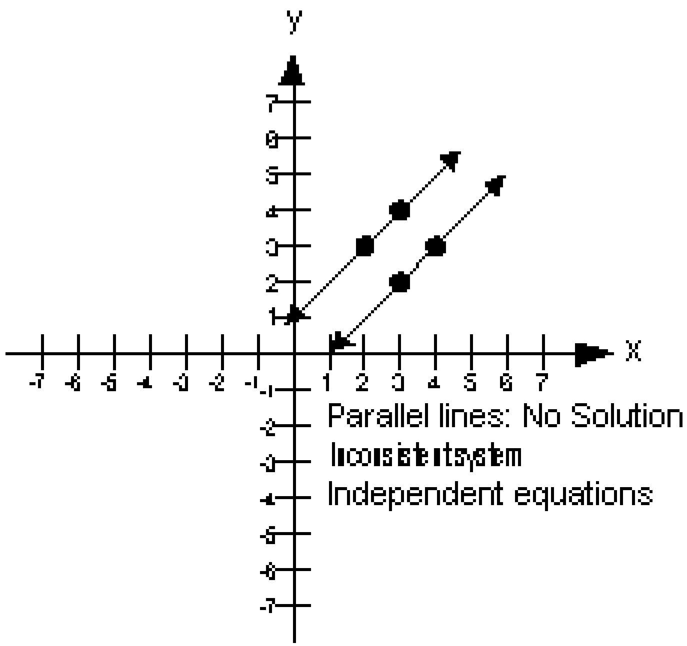
System of Linear Equations

- **One Solution:** If the system in two variables has one solution, it is an ordered pair that is a solution to BOTH equations. In other words, when you plug in the values of the ordered pair it makes BOTH equations TRUE.



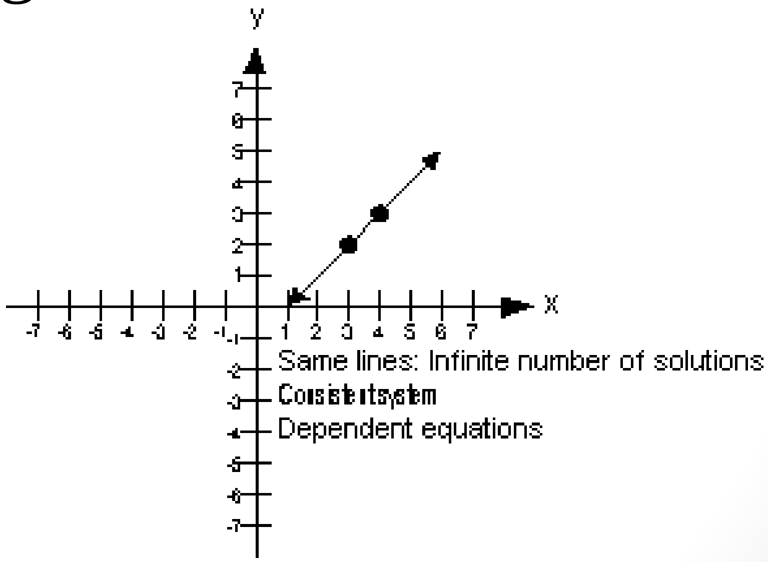
System of Linear Equations

- **No Solution:** If the two lines are parallel to each other, they will never intersect. This means they do not have any points in common. In this situation, you would have no solution.



System of Linear Equations

- **Infinite Solutions: If the two lines end up lying on top of each other, then there is an infinite number of solutions.** In this situation, they would end up being the same line, so any solution that would work in one equation is going to work in the other.



System of Linear Equations

Example 1: Determine whether each ordered pair is a solution of the system. $(3, -1)$ and $(0,2)$

$$\begin{cases} x + y = 2 \\ x - y = 4 \end{cases}$$

System of Linear Equations

Let's check the ordered pair (3, -1) in the first equation:

$$x + y = 2$$

$$3 + (-1) = 2$$

$$3 - 1 = 2$$

$$2 = 2$$

True statement

Now, let's check (3, -1) in the second equation:

$$x - y = 4$$

$$3 - (-1) = 4$$

$$3 + 1 = 4$$

$$4 = 4$$

True statement

System of Linear Equations

Let's check the ordered pair (0, 2) in the first equation:

$$x + y = 2$$

$$0 + 2 = 2$$

$$2 = 2$$

True statement

Now, let's check (0, 2) in the second equation:

$$x - y = 4$$

$$0 - 2 = 4$$

$$-2 = 4$$

False statement

System of Linear Equations

There are Three Ways to Solve Systems of Linear Equations in Two Variables

1. **Graphing**
2. **Substitution Method**
3. **Elimination Method**

Solve by Graphing

Step 1: Graph the first equation.

- you can use any "legitimate" way to graph the line

Step 2: Graph the second equation on the same coordinate system as the first.

- You must graph the second line on the same coordinate system as the first.

Solve by Graphing

Step 3: Find the solution.

- If the two lines intersect at one place, then the point of intersection is the solution to the system.

- If the two lines are parallel, then they never intersect, so there is no solution.

- If the two lines lie on top of each other, then they are the same line and you have an infinite number of solutions. In this case you can write down either equation as the solution to indicate they are the same line.

Solve by Graphing

Step 4: Check the proposed ordered pair solution in BOTH equations.

- You can plug in the proposed solution into BOTH equations. If it makes BOTH equations true then you have your solution to the system.
- If it makes at least one of them false, you need to go back and redo the problem.

Solve by Graphing

Example 2: Solve the system of equations by graphing.

$$x + y = 3$$

$$x - y = 1$$

Step 1: Graph the first equation.

***x*-intercept**

$$x + y = 3$$

$$x + 0 = 3$$

$$x = 3$$

Plug in 0 for *y* for *x*-intercept

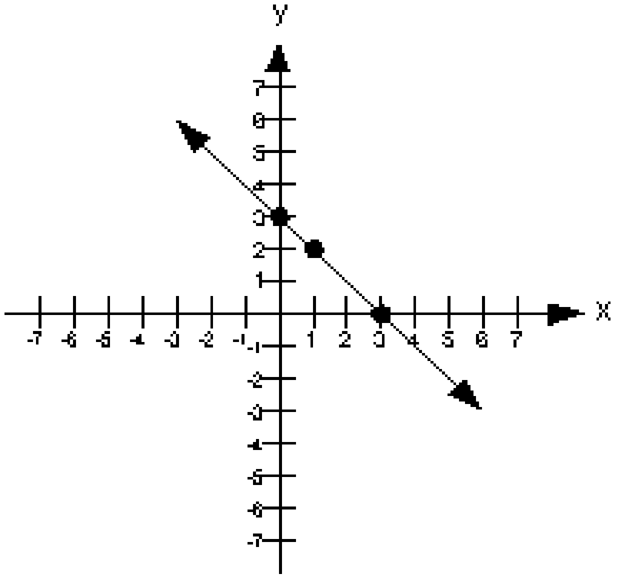
The *x*-intercept is (3, 0)

Solve by Graphing

Example 2: Solve the system of equations by graphing.

$x + y = 3$

$x - y = 1$



Step 1: Graph the first equation.

***y*-intercept**

$x + y = 3$

$0 + y = 3$

$y = 3$

Plug in 0 for *x* for *y*-intercept

The *y*-intercept is (0, 3).

Solve by Graphing

Example 2: Solve the system of equations by graphing.

$$x + y = 3$$
$$x - y = 1$$

Step 2: Graph the second equation on the same coordinate system as the first.

x-intercept

$$x - y = 1$$
$$x - 0 = 1$$
$$x = 1$$

Plug in 0 for y for x-intercept

The x-intercept is (1, 0)

Solve by Graphing

Example 2: Solve the system of equations by graphing.

$x + y = 3$
$x - y = 1$

Step 2: Graph the second equation on the same coordinate system as the first.

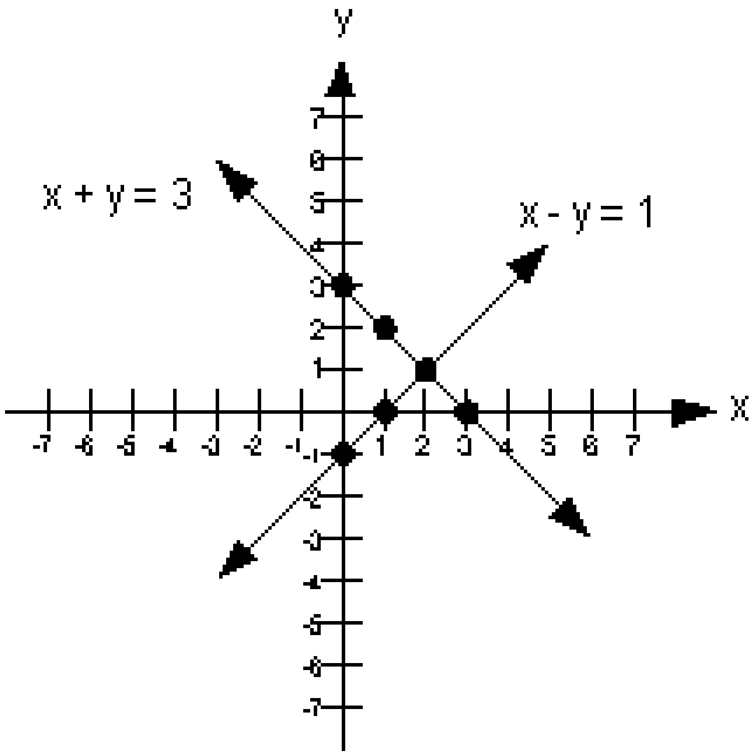
***y*-intercept**

$$x - y = 1$$
$$0 - y = 1$$
$$-y = 1$$
$$\frac{-y}{-1} = \frac{1}{-1}$$
$$y = -1$$

Plug in 0 for x for *y*-intercept

The *x*-intercept is (0, -1)

Solve by Graphing



Step 3: **Find the solution.**

We need to ask ourselves, is there any place that the two lines intersect, and if so, where?

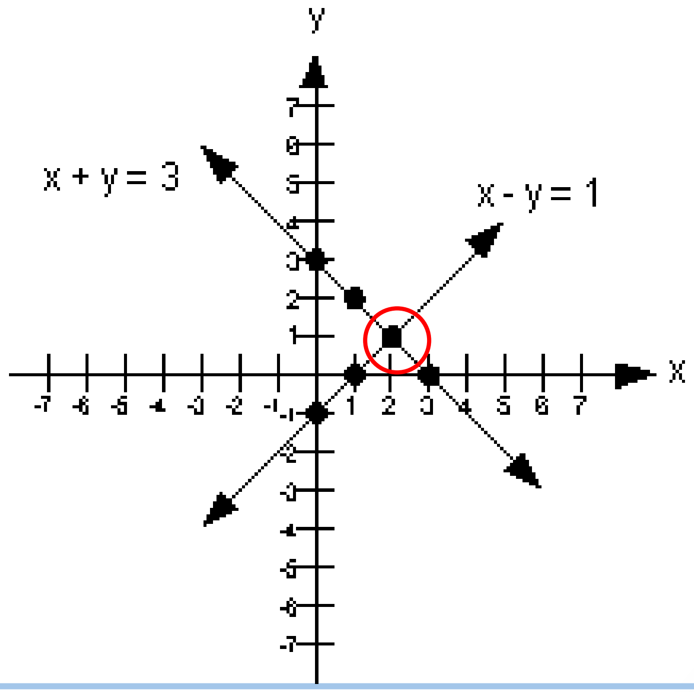
The answer is yes, they intersect at (2, 1).

Solve by Graphing

Step 4: Check the proposed ordered pair solution in BOTH equations.

You will find that if you plug the ordered pair (2, 1) into BOTH equations of the original system, that this is a solution to BOTH of them.

The solution to this system is (2, 1).



Solve by Graphing

Example 3: Solve the system of equations by graphing.

$x + y = 5$

$y = -x + 3$

Step 1: Graph the first equation.

***x*-intercept**

$x + y = 5$

$x + 0 = 5$

$x = 5$

Plug in 0 for *y* for *x*-intercept

The *x*-intercept is (5, 0)

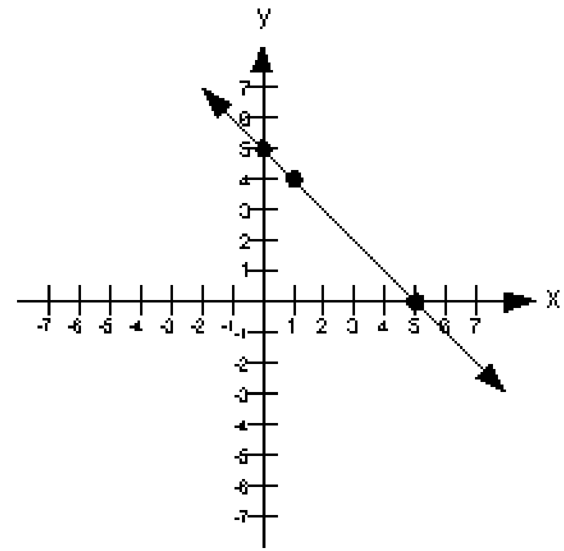
Solve by Graphing

Example 3: Solve the system of equations by graphing.

$x + y = 5$

$y = -x + 3$

Step 1: **Graph the first equation.**



***y*-intercept**

$$x + y = 5$$

$$0 + y = 5$$

$$y = 5$$

Plug in 0 for *x* for *y*-intercept

The *y*-intercept is (0, 5).

Solve by Graphing

Example 3: Solve the system of equations by graphing.

$x + y = 5$
$y = -x + 3$

Step 2: **Graph the second equation on the same coordinate system as the first.**

***x*-intercept**

$$y = -x + 3$$
$$0 = -x + 3$$
$$0 - 3 = -x + 3 - 3$$
$$-3 = -x$$
$$\frac{-3}{-1} = \frac{-x}{-1}$$
$$3 = x$$

Plug in 0 for y for *x*-intercept

The *x*-intercept is (3, 0)

Solve by Graphing

Example 3: Solve the system of equations by graphing.

$x + y = 5$
$y = -x + 3$

Step 2: Graph the second equation on the same coordinate system as the first.

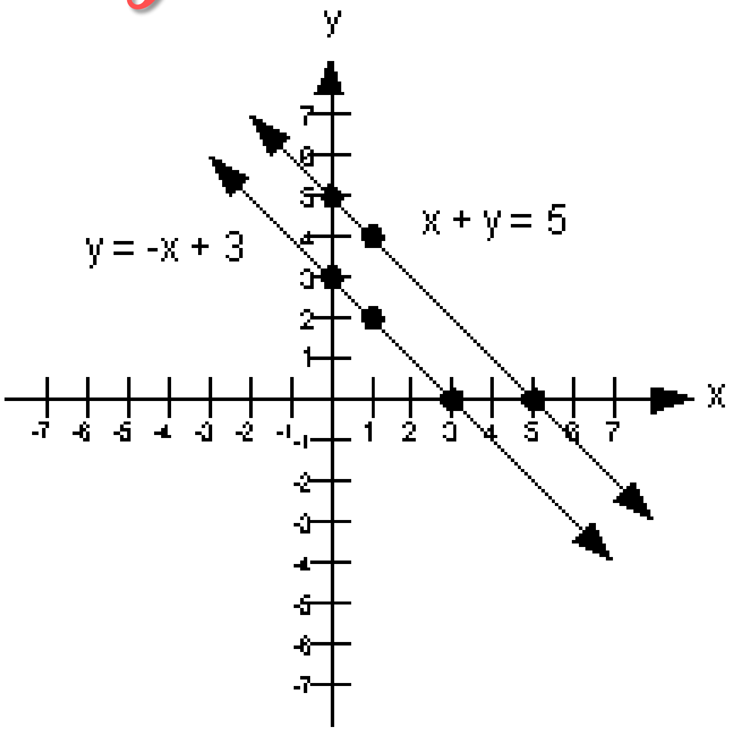
y-intercept

$$y = -x + 3$$
$$y = -0 + 3$$
$$y = 3$$

Plug in 0 for x for y-intercept

The x-intercept is (0, -1)

Solve by Graphing



Step 3: **Find the solution.**

We need to ask ourselves, is there any place that the two lines intersect, and if so, where?

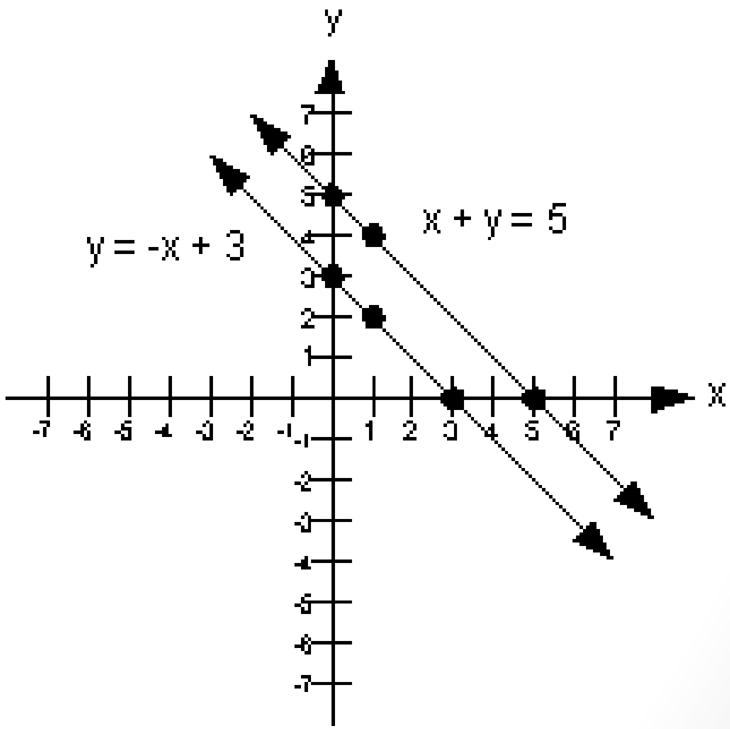
The answer is no, they do not intersect. We have two parallel lines.

Solve by Graphing

Step 4: Check the proposed ordered pair solution in BOTH equations.

There are no ordered pairs to check.

The answer is no solution.



Solve by the Substitution Method

Example 4: Solve the system of equations by the substitution method.

$$\begin{cases} 3x - 5y = 15 \\ y = 2x + 4 \end{cases}$$

Step 1: Simplify if needed.

Both of these equations are already simplified. No work needs to be done here.

Step 2: Solve one equation for either variable.

the second equation is already solved for *y*. We can use that one for this step.

$$y = 2x + 4$$

Solve by the Substitution Method

Example 4: Solve the system of equations by the substitution method.

$$\begin{cases} 3x - 5y = 15 \\ y = 2x + 4 \end{cases}$$

Step 3: Substitute what you get for step 2 into the other equation .

$$3x - 5y = 15$$

$$3x - 5(2x + 4) = 15$$

$$3x - 10x - 20 = 15$$

Solve by the Substitution Method

Step 4: **Solve for the remaining variable**

$$3x - 5y = 15$$

$$3x - 5(2x + 4) = 15$$

$$3x - 10x - 20 = 15$$

$$-7x - 20 = 15$$

$$-7x - 20 + 20 = 15 + 20$$

$$-7x = 35$$

$$\frac{-7x}{-7} = \frac{35}{-7}$$

$$x = -5$$

Solve by the Substitution Method

Step 5: Solve for second variable

Plug in -5 for *x* into the equation in step 2 to find *y*'s value.

$$y = 2x + 4$$
$$y = 2(-5) + 4$$
$$y = -10 + 4$$
$$y = -6$$

Solve by the Substitution Method

Step 6: Check the proposed ordered pair solution in BOTH original equations.

You will find that if you plug the ordered pair (-5, -6) into BOTH equations of the original system, that this is a solution to BOTH of them.

(-5, -6) is a solution to our system.

Solve by the Substitution Method

Example 5: Solve the system of equations by the substitution method.

$$\begin{cases} x - 2y = 5 \\ 2x - 4y = 1 \end{cases}$$

Step 1: Simplify if needed.

Both of these equations are already simplified. No work needs to be done here.

Step 2: Solve one equation for either variable.

$$x - 2y = 5$$

$$x - 2y + 2y = 5 + 2y$$

$$x = 5 + 2y$$

Solve by the Substitution Method

Step 3: **Substitute what you get for step 2 into the other equation .**

$$2x - 4y = 1$$

$$2(5 + 2y) - 4y = 1$$

Step 4: **Solve for the remaining variable**

$$2x - 4y = 1$$

$$2(5 + 2y) - 4y = 1$$

$$10 + 4y - 4y = 1$$

$$10 = 1$$

Solve by the Substitution Method

Step 3: Substitute what you get for step 2 into the other equation .

$$2x - 4y = 1$$

$$2(5 + 2y) - 4y = 1$$

Step 4: Solve for the remaining variable

$$2x - 4y = 1$$

$$2(5 + 2y) - 4y = 1$$

$$10 + 4y - 4y = 1$$

$$10 = 1$$

Note: the system is contradiction there is no solution for this system

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

$$\begin{cases} \frac{1}{3}x + \frac{1}{5}y = 2 \\ \frac{1}{3}x + \frac{1}{2}y = -\frac{1}{2} \end{cases}$$

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 1: Simplify and put both equations in the form A*x* + B*y* = C if needed

$$\begin{cases} (15)\left(\dfrac{1}{3}x + \dfrac{1}{5}y\right) = (15)(2) \\ (6)\left(\dfrac{1}{3}x + \dfrac{1}{2}y\right) = (6)\left(-\dfrac{1}{2}\right) \end{cases}$$

$$\begin{cases} 5x + 3y = 30 \\ 2x + 3y = -3 \end{cases}$$

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 2: Multiply one or both equations by a number that will create opposite coefficients for either *x* or *y* if needed

$$\begin{cases} 5x + 3y = 30 \\ (-1)(2x + 3y) = (-1)(-3) \end{cases}$$

Mult. both sides of 2nd eq. by -1

$$\begin{cases} 5x + 3y = 30 \\ -2x - 3y = 3 \end{cases}$$

***y*'s have equal and opposite coefficients**

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 3: Add equations

$$\begin{cases} 5x + 3y = 30 \\ \underline{-2x - 3y = 3} \end{cases}$$

$$3x \qquad = 33$$

Note that *y*'s dropped out

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 4: Solve for remaining variable

$$3x = 33$$

$$\frac{3x}{3} = \frac{33}{3}$$

$$x = 11$$

Inverse of mult. by 3 is div. by 3

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 5: Solve for second variable

$$5x + 3y = 30$$

$$5(11) + 3y = 30$$ **Plug in 11 for *x***

$$55 + 3y = 30$$

$$55 + 3y - 55 = 30 - 55$$ **Subtract both side by 55**

$$3y = -25$$

$$\frac{3y}{3} = \frac{-25}{3}$$ **Divide both side by 3**

$$y = -\frac{25}{3}$$

Solve by the Elimination Method

Example 6: Solve the system of equations by the elimination method.

Step 6: Check the proposed ordered pair solution in BOTH original equations

You will find that if you plug the ordered pair (11, -25/3) into BOTH equations of the original system, that this is a solution to BOTH of them.

(11, -25/3) is a solution to our system.