Calculus

Lecture – 5 / Domain & Range

Functions

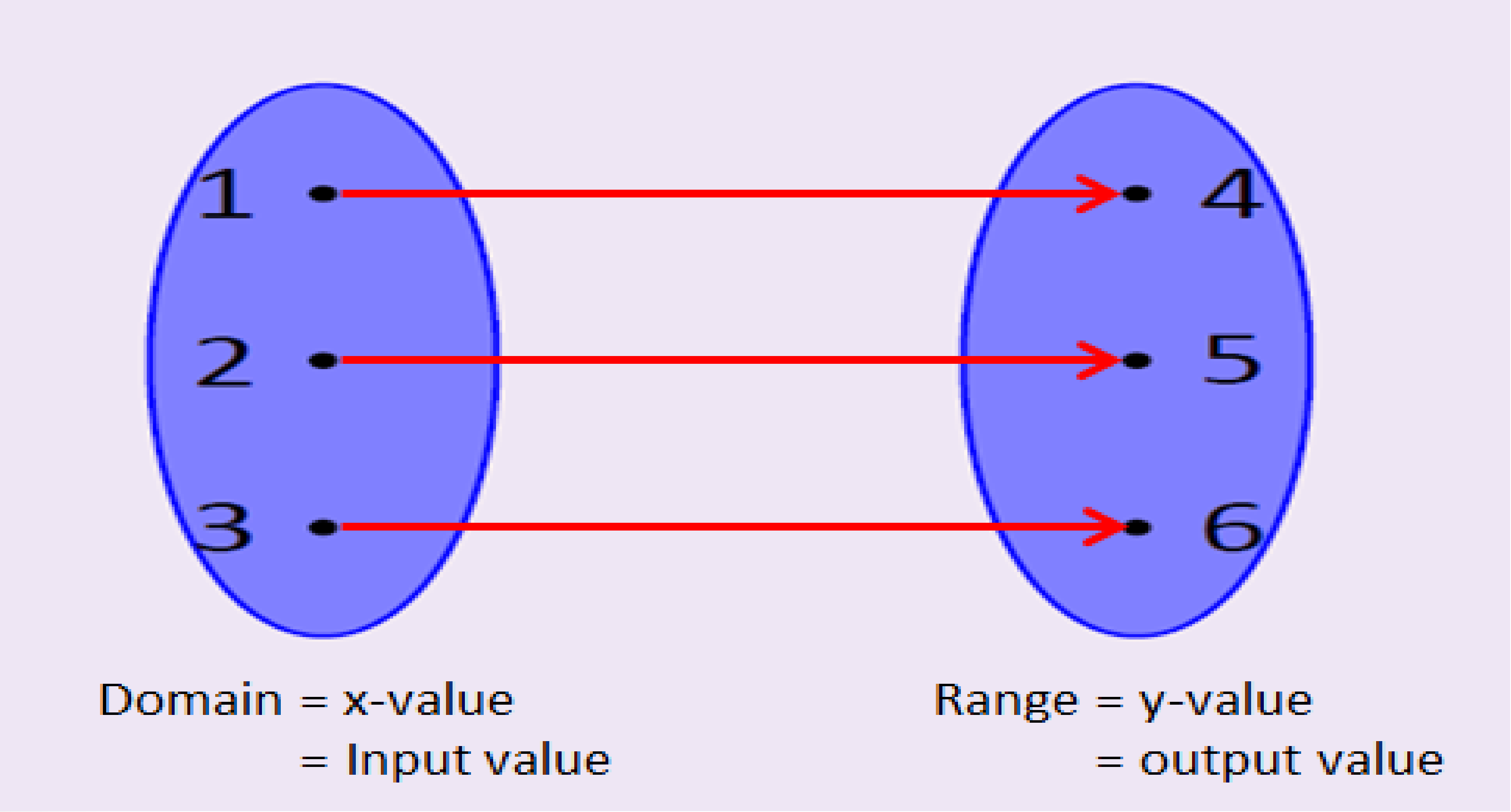
Lecture contents

- Domain and Range
- Notations for domain and range
- Domain restriction
- Domain of Purposely Restricted Functions
- Domain of rational functions
- Domain of radical functions
- Domain of both Rational and Radical Functions

Domain & Range

- **Definitions of Domain & Range:**
- **Domain:** is the set of all x values to which a function assigns a corresponding a real y value. Or the domain is the set of x-values where the function is defined (All input values of x).
- **Range:** is the set of all y values that result from x values. (All output values of y).

Domain & Range



Domain & Range

- **Notations for Domain and Range**
- There are three main notations to represent the domain or range.

1. **Roster Notation:**
2. **Set-Builder Notations:**
3. **Interval Notation:**

Domain & Range

- **Notations for Domain and Range**

1. **Roster Notation:**

For Example: Determine the domain and range of the following set of ordered pairs.

$$\{(-8,0),(6,4),(0,0),(-7,-3),(10,-5)\}$$

$\boldsymbol{Ans}.$

$\boldsymbol{Domain}$: $\{\mathbf{-8, 6, 0, -7, 10}\}$

$\boldsymbol{Range}$: $\{\mathbf{0, 4, -3, -5}\}$

Domain & Range

- **Notations for Domain and Range**

2. **Set-Builder Notations:**

$$D = \{x\!: discription\ of\ all\ included\ values\}$$

- **For Example:**

$$Domain\ = \ \{x:\ x\ is\ all\ real\ numbers\}$$

$$Domain\ = \ \{x:\ x\ \in \mathbb{R}\}$$

$$Domain\ = \ \{x:\ x\ \in \mathbb{R}, x \geq 0\}$$

Domain & Range

- **Notations for Domain and Range**

3. **Interval Notation:**

(x, y)

- Open brackets ( )

(2, 8) = 3, 4, 5, 6, 7

- Closed brackets [ ]

[2, 8] = 2, 3, 4, 5, 6, 7, 8

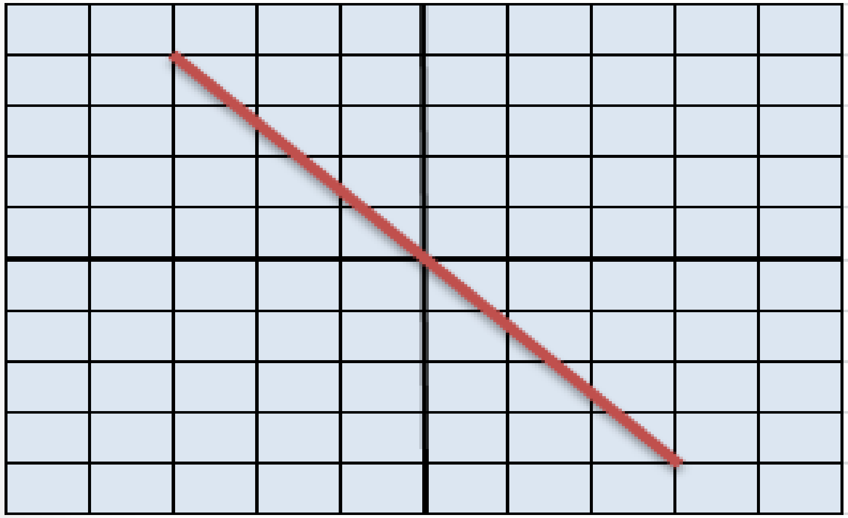
- One side open the other side is closed ( ] or [ )

(2, 8] = 3, 4, 5, 6, 7, 8

or [2, 8) = 2, 3, 4, 5, 6, 7

Domain & Range

Example – 1: Determine the Domain and Range for the following graphs of functions.



$$\boldsymbol{y = -x}$$

$x = -1 \quad \rightarrow \quad y = 1$

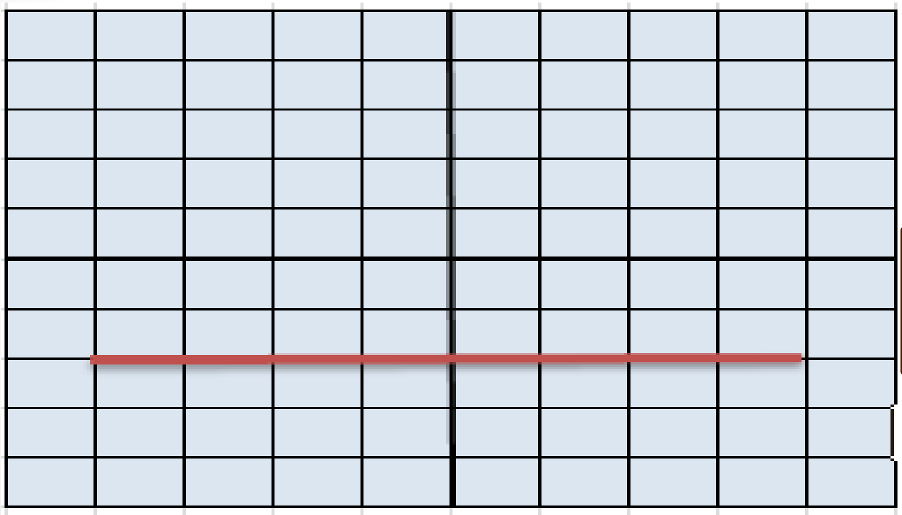
$x = 0 \quad \rightarrow \quad y = 0$

$x = 1 \quad \rightarrow \quad y = -1$

- $\boldsymbol{Domain = \{x : x \text{ is all real numbers}\}}$
- $\boldsymbol{Range = \{y : y \text{ is all real numbers}\}}$

Domain & Range

Example – 2: Determine the Domain and Range for the following graphs of functions.

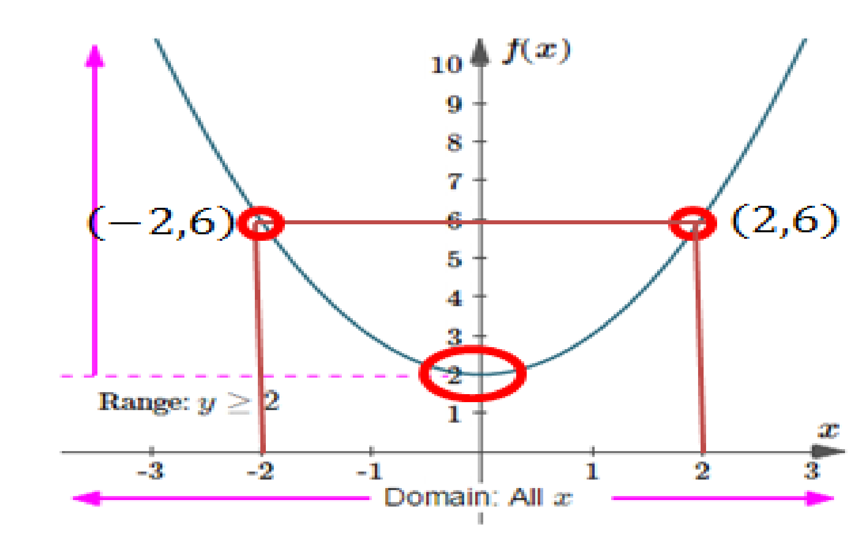


$$y = -2$$

- $\text{Domain} = \{x : x \text{ is all real numbers}\}$
- $Range = \{y = -2\}$

Domain & Range

Example – 3: Determine the Domain and Range for the following graphs of functions.



$$\boxed{y = x^2 + 2}$$

$$y = (-2)^2 + 2 \quad = 6$$

$$y = (0)^2 + 2 \quad = 2$$

$$y = (2)^2 + 2 \quad = 6$$

- $\textit{Domain} = \{x : x \in \mathbb{R}\}$
- $\textit{Range} = \{y : y \in \mathbb{R},\ y \geq 2\}$

Domain & Range

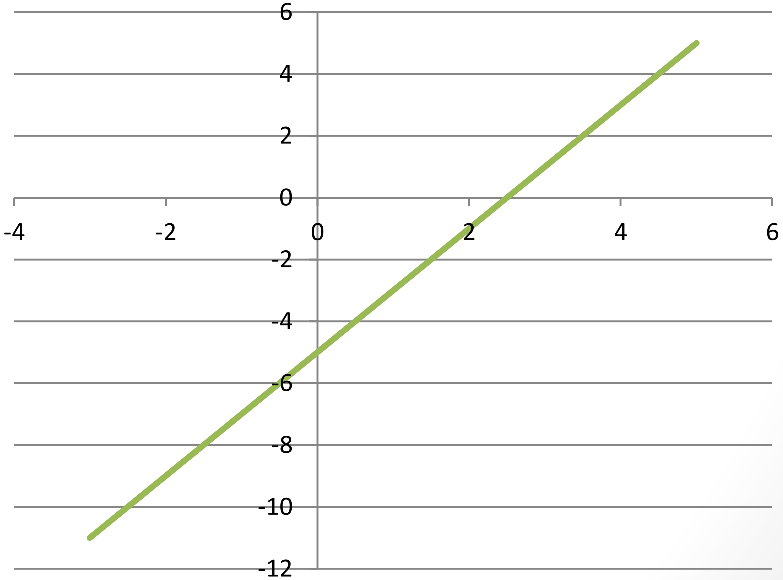
Example – 4: Determine the domain and range of the following functions.

$f(x) = 2x - 5$

$X = -3, \quad f(-3) = 2(-3) - 5 = -11$

$X = 0, \quad f(0) = 2(0) - 5 = -5$

$X = 3, \quad f(3) = 2(3) - 5 = 1$



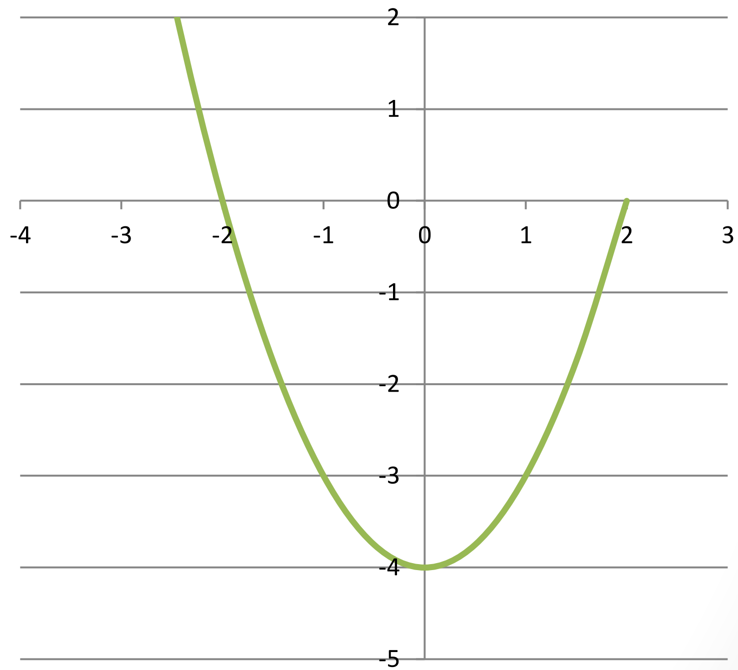
$Domain = \{x : x \in \mathbb{R}\}$

$Range = \{y : y \in \mathbb{R}\}$

Domain & Range

Example – 5: Determine the domain and range of the following functions.

$$f(x) = x^2 - 4$$



$$X = -2 \quad f(-2) = (-2)^2 - 4 = 0$$

$$X = 0 \quad f(0) = (0)^2 - 4 = -4$$

$$X = 2 \quad f(2) = (2)^2 - 4 = 0$$

$$Domain = \{x : x \in \mathbb{R}\}$$

$$Range = \{y : y \in \mathbb{R},\ y \le -4\}$$

Domain & Range

There are four main types of domain restriction

1. Domain of Purposely Restricted Functions
2. Domain of Rational Functions
3. Domain of Radical Functions
4. Domain of both Rational and Radical Functions
    - Denominator is radical function
    - Numerator is radical function

Domain & Range

Example – 6: Determine the range of the following function

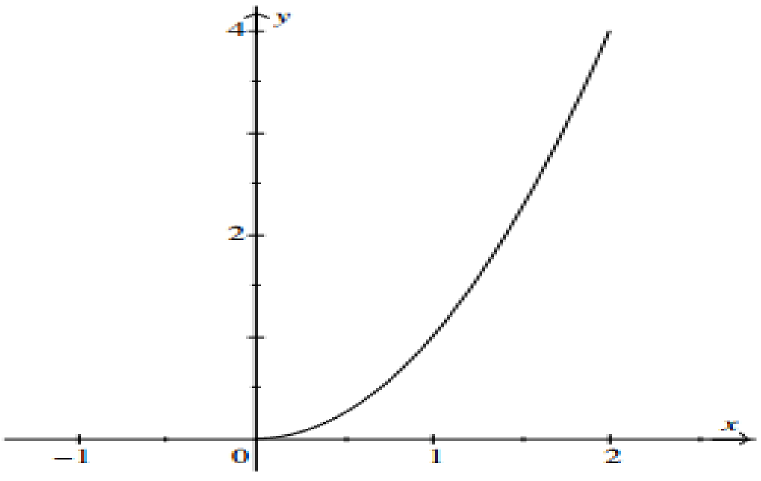
- $y = x^2$ $\quad$ *The domain is given as* $0 \le x \le 2$. So,

what is the range?

$y = (0)^2 = 0$

$y = (2)^2 = 4$

The Range = [0, 4]



The graph of $y = x^2$ for $0 \le x \le 2$.

Domain & Range

Example – 7: Determine the range of the following function

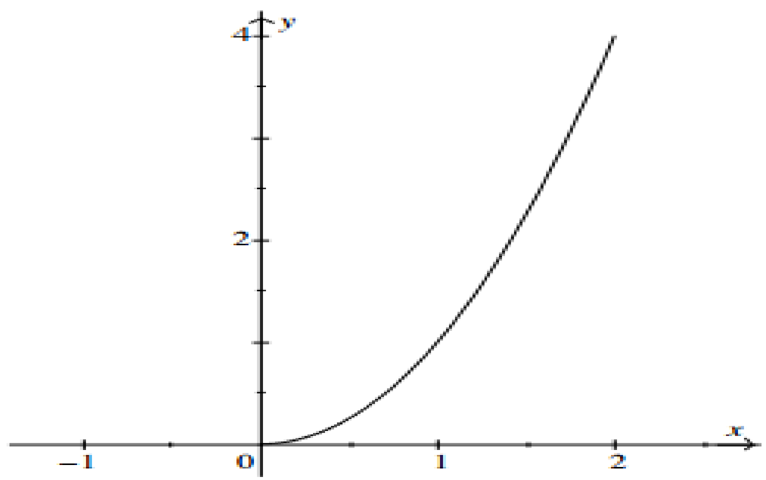
- $y = x^2$ $\quad The\ domain\ is\ given\ as\ -4 \le x \le 2$. So,

what is the range?

$$y = (0)^2 = 0$$

$$y = (-4)^2 = 16$$

The Range = [0, 16]



The graph of $y = x^2$ for $0 \le x \le 2$.

Domain & Range

Example – 8: Determine the range of the following function

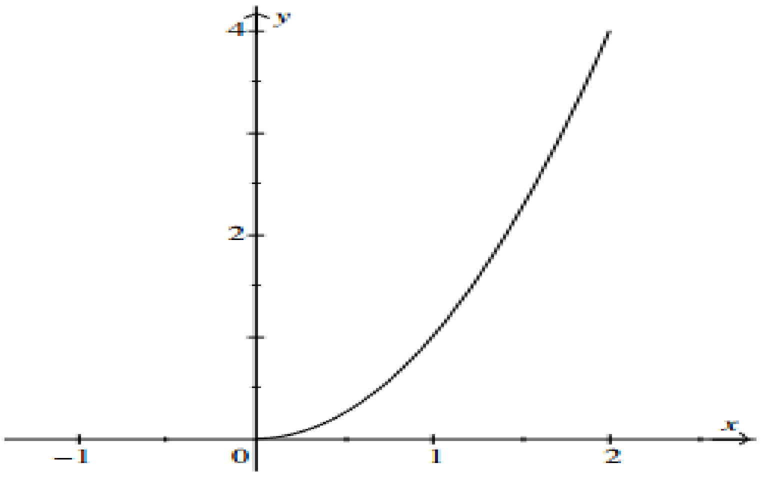
- $\boldsymbol{y = x^2 + 2}$ *The domain is given as* $\boldsymbol{-4 \le x \le 4}$.

So, what is the range?

$$\boldsymbol{y = (0)^2 + 2 = 2}$$

$$\boldsymbol{y = (4)^2 + 2 = 18}$$

The Range = [2, 18]



The graph of $y = x^2$ for $0 \le x \le 2$.

Domain & Range

Example – 8: Determine the range of the following function

- $y = x + 2$ *The domain is given as* $-4 \leq x \leq 4$. So, what is the range?

$y = -4 + 2 = -2$

$y = 4 + 2 = 6$

The Range = [-2, 6]

Domain & Range

Domain of Rational Functions: A rational function is a quotient of two polynomial functions. Rational functions are undefined for any values of x that cause the denominator to be 0. These values are restricted from the domain of the function.

Domain & Range

Example – 7: Determine the domain of the following function

$$y = \frac{1}{x+3}$$

We don't want denominator = 0

$$x + 3 = 0$$

$$x = -3$$

That's mean if x = -3 *denominator = 0*

$$Domain = \{x : x \in \mathbb{R},\ x \neq -3\}$$

Domain & Range

Example – 8: Determine the range of the following function

$$y = \frac{1}{x+3}$$

- $y(x+3) = 1$
- $xy + 3y = 1$
- $x = \frac{1-3y}{y}$ **We don't want denominator = 0**
- y = 0 **That's mean if y = 0 denominator = 0**
- $Range = \{y \in \mathbb{R},\ y \neq 0\}$

Domain & Range

Example – 9: Determine the domain of the following function

$$y = \frac{2x}{7x + 28}$$

We don't want denominator = 0

- $7x + 28 = 0$
- $7x = -28$
- $x = -4$ **That's mean if x = - 4 denominator = 0**

$$Domain = \{x : x \in \mathbb{R},\ x \neq -4\}$$

Domain & Range

Example – 10: Determine the range of the following function

$$y = \frac{2x}{7x+28}$$

$7xy + 28y = 2x$

$7xy - 2x = -28y$

$x\,(7y - 2) = -28y$

Range $= \{y \in \mathbb{R}, y \neq 2/7\}$

$x = \frac{-28y}{7y-2}$

We don't want denominator = 0

$7y - 2 = 0$

$y = \frac{2}{7}$

That's mean if y = $\frac{2}{7}$ ***denominator = 0***

Domain & Range

Domain of Radical Functions: A radical function is a function that contains the square root of a variable expression. Rational functions are undefined for any values of x that cause the radicand to be negative. These values are restricted from the domain of the function.

Domain & Range

Example – 11: Determine the domain and range of the following function

$$f(x) = \sqrt{x + 7}$$

We don't want radical value be < 0

$x + 7 \geq 0$

$x \geq -7$ **That's mean if x < -7** *radical = __negative value__*

Domain = {x : x ∈ ℝ, x ≥ – 7}

Range = {y : y ∈ ℝ, y ≥ 0}

Domain & Range

Example – 12: Determine the domain and range of the following function

$$f(x) = \sqrt{2x + 3}$$

We don't want radical value be < 0

$$2x + 3 \geq 0$$

$$2x \geq -3$$

$$x \geq -1.5$$

That's mean if x < -1.5 ***radical = <u>negative value</u>***

Domain = {x : x ∈ ℝ, x ≥ – 1.5}

Range = {y : y ∈ ℝ, y ≥ 0}

Domain & Range

Example – 13: Determine the domain and range of the following function

$$f(x) = -\sqrt{\mathbf{10 - 2}\boldsymbol{x}}$$

We don't want radical value be < 0

- $\mathbf{10 - 2}\boldsymbol{x} \geq 0$
- $-2x \geq -10$
- $2x \leq 10$
- $x \leq 5$ **That's mean if x < 5,** ***radical = <u>negative value</u>***
- **Domain = {x : x ∈ $\mathbb{R}$, x $\leq$ 5}**
- **Range = {y : y ∈ $\mathbb{R}$, y $\leq$ 0}**

Domain & Range

Example – 14: Determine the domain and range of the following function

$$y = \sqrt{x - 1} + 3$$

We don't want radical value be < 0

$$x - 1 \geq 0$$

$x \geq 1$ **That's mean if x < 1,** ***radical = <u>negative value</u>***

- **Domain = {x : x ∈ $\mathbb{R}$, x ≥ 1}**
- **Range = {y : y ∈ $\mathbb{R}$, y ≥ 3}**

Domain & Range

Domain of both Rational and Radical Functions: in this type of restriction we must follow both rules of domain of rational functions and domain of radical functions at the same time. It has two different types.

- **Denominator is Radical:** because this is a rational function, so the denominator must not be zero**.** However, the denominator is also a radical function so the denominator must not be negative**.** Therefore, the denominator must be > 0.

Domain & Range

Example – 15: Determine the domain of the following function

$$y = \frac{8x-1}{\sqrt{5x-45}}$$

Be aware the denominator is radical

We don't want denominator = 0

We don't want denominator < 0

Why ?

$5x - 45 > 0$

$5x > 45$

$x > 9$

Because denominator must not be zero and because denominator is radical so the value under the root square must not be negative

$$\textit{Domain} = \{x : x \in \mathbb{R},\ x > 9\}$$

Domain & Range

- **Numerator is Radical:** because this is a rational function, so the denominator must not be zero. However, numerator is also a radical function so the numerator must not be negative**.**

Therefore,

- The denominator must be $\neq 0$
- The numerator must not be negative

Domain & Range

Example – 16: Determine the domain of the following function

$$f(x) = \frac{\sqrt{x+3}}{2-x}$$

Be aware the numerator is radical

1. **First work with numerator $\sqrt{x+3}$:**

We don't want radical < 0

$$x + 3 \geq 0$$

$$x \geq -3$$

Domain & Range

Example – 17: Determine the domain of the following function

$$f(x) = \frac{\sqrt{x+3}}{2-x}$$

2. Then, work with denominator 2 – x:

$$2 - x = 0$$
$$x = 2$$

Combine the two restrictions $x \geq -3 \; and \; x = 2$ together.

$$\boldsymbol{Domain} = \{x : x \in \mathbb{R},\ x \geq -3 \text{ but } x \neq 2\}$$