Calculus

Lecture – 6 / Non-Linear Quadratic Functions

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Non Linear Equations

- The aim of this lecture is to enable students to distinguish between linear and non linear functions.
- To better understand nonlinear function we have to review the linear functions first.

Linear Equations

- A **linear** equation is an equation for a **straight line**
- The highest exponent of variables must be 1.
- There should be no radicals in the equation.
- The variables are added or subtracted.
- The variables can NOT be divided or multiplied by other variables.
- Every linear equation graphs as a line.
- Any Other functions are considered to be **Non Linear** functions.

Non Linear Equations

- A table is linear if the rate of change is constant. There is a common difference.
- A graph is linear if it is a straight line.
- An equation is linear if the power of x is either 1 or 0 and it appears in the numerator.

Non Linear Equations

- **Are the rates of change constant?**

x	y
2	50
4	35
6	20
8	5

x	y
1	1
4	16
7	49
10	100

For any given table it could be linear only if satisfy the following equation

$$Linear = \frac{change\ in\ y}{change\ in\ x} = always\ constant$$

Non Linear Equations

- Are the rates of change constant?

x	y
2	50
4	35
6	20
8	5

+2, +2, +2 (x); -15, -15, -15 (y)

- linear

x	y
1	1
4	16
7	49
10	100

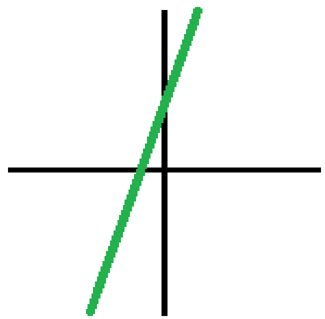
+3, +3, +3 (x); +15, +33, +51 (y)

- nonlinear

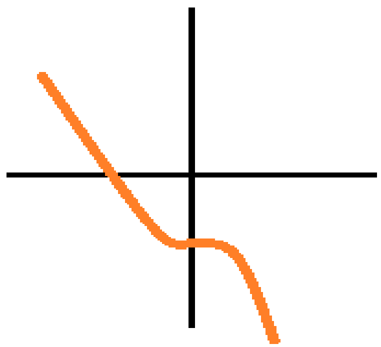
Non Linear Equations

- **Are these graphs straight lines?**

linear



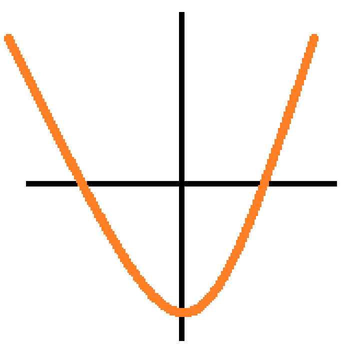
$y = 4 + 5x$



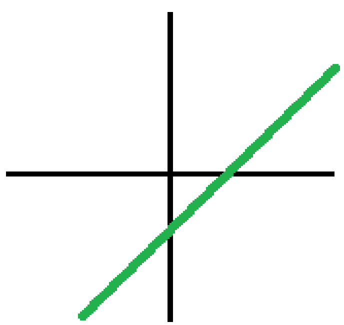
$y = x^3 - 6$

nonlinear

nonlinear



$y = x^2$



$y = x - 3$

linear

Non Linear Equations

- Does the power of x is equal 1 or 0 ?

$y = x + 4$

linear

$y = 6/x$

nonlinear

$y = x^3 + 1$

nonlinear

$y = \frac{1}{2} x$

linear

$y = 4$

linear

Non Linear Equations

Example – 1: Identifying Functions from Tables

Does the table represent a linear or nonlinear function? Explain.

a.

x: +3, +3, +3

x	3	6	9	12
y	40	32	24	16

y: −8, −8, −8

As x increases by 3, y decreases by 8. The rate of change is constant. So, the function is linear.

b.

x: +2, +2, +2

x	1	3	5	7
y	2	11	33	88

y: +9, +22, +55

As x increases by 2, y increases by different amounts. The rate of change is not constant. So, the function is nonlinear.

Non Linear Equations

- **Example – 2: Standardized Test Practice**

Which equation represents a nonlinear function?

Ⓐ $y = 4.7$ Ⓑ $y = \pi x$

Ⓒ $y = \frac{4}{x}$ Ⓓ $y = 4(x - 1)$

- The equations $\mathbf{y = 4.7}$, $\mathbf{y = \pi x}$, and $\mathbf{y = 4(x - 1)}$ can be rewritten in slope-intercept form. So, they are linear functions.
- The equation $\boldsymbol{y = \frac{4}{x}}$ cannot be rewritten in slope-intercept form. So, it is a nonlinear function.
- **The correct answer is © .**

Non Linear Equations

Example – 3: The table shows the volume V (in cubic feet) of a cube with a side length of x feet. Does the table represent a linear or nonlinear function? Explain.

Side Length, *x*	1	2	3	4	5	6	7	8
Volume, *V*	1	8	27	64	125	216	343	512

Answer:

Use $\frac{\Delta y}{\Delta x}$ equation if the rate is constant its linear equation if the rate is changed its non linear equation.

Non Linear Equations

- **Answer:**

Side Length, x	1	2	3	4	5	6	7	8
Volume, V	1	8	27	64	125	216	343	512

- $\frac{\Delta x}{\Delta v} = \frac{2-1}{8-1} = \frac{1}{7}$
- $\frac{\Delta x}{\Delta v} = \frac{3-2}{27-8} = \frac{1}{19}$
- $\frac{\Delta x}{\Delta v} = \frac{4-3}{64-27} = \frac{1}{37}$

The x rate is constant however the V rate is not constant so the table represent nonlinear function

Non Linear Equations

Example – 4: Find the missing value in the table.

x	1	2	3	4	5
y	2	4	6	**?**	10

Answer:

- $4 - 2 = 2$
- $6 - 4 = 2$
- We know the rate of increase is 2
- To find the missing value add the rate of increase with the value before missing value

  $6 + 2 = 8$

Non Linear Equations

Example – 5: Find the missing value in the table.

x	1	2	3	⋯	8
y	1.5	3	4.5	⋯	?

$$missing\ value = Rate\ of\ change \times No.\ opposite\ to\ missing\ value$$
$$Missing\ value = 1.5 \times 8$$
$$Missing\ value = 12$$

Answer:

- $3 - 1.5 = 1.5$
- $4.5 - 3 = 1.5$
- We know the rate of increase is 1.5
- To find the missing value multiply the rate of increase by 5 and add it with the value before missing value
  $4.5 + 1.5(5) = 12$

Non Linear Equations

Example – 6: SUNFLOWER SEEDS. The table shows the cost y (in dollars) of x pounds of sunflower seeds.

What is the missing y-value that makes the table represent a linear function?

Pound (x)	Cost (y)
2	2.8
3	?
4	5.6

Find the missing value

$5.6 - 2.8 = 2.8$

$2.8/2 = 1.4$

$2.8 + 1.4 = 4.2$

So, the missing value = 4.2

Quadratic Equation

Quadratic Equation: is a simple Non Linear equation it is written in the Standard Form

$$Ax^2 + Bx + C = 0$$

Where;

- A, B, and C are real numbers and only $A \neq 0$
- The highest power of quadratic equation is 2.

Quadratic Equation

Remember! quadratic equation can be in any of these forms.

$$1.\ \mathbf{Ax^2 + Bx + C = 0}$$

$$2.\ \mathbf{Ax^2 + 0x + C = 0}$$

$$\mathbf{Ax^2 + C = 0}$$

Only coefficient of x^2 can't be zero

$$3.\ \mathbf{Ax^2 + Bx + 0 = 0}$$

$$\mathbf{Ax^2 + Bx = 0}$$

Zero Product Property

Zero Factor Property:

- If a and b are real numbers and,
- If $a \times b = 0$
- Then, either a or b one of them must be zero

Example; if $(\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}+\mathbf{3})=\mathbf{0}$ then either,

- $(\boldsymbol{x}+\mathbf{2})=\mathbf{0}$ or
- $(\boldsymbol{x}+\mathbf{3})=\mathbf{0}$

Zero Product Property

Use Zero Factor Property to solve these examples.

Example – 7: $(x - 10)(3x - 6) = 0$

$(x - 10) = 0$

$x - 10 + 10 = 0 + 10$

$x = 10$

$(3x - 6) = 0$

$3x - 6 + 6 = 0 + 6$

$3 \div\ 3x = 6\ \ \div 3$

$x = 2$

Zero Product Property

Use Zero Factor Property to solve these examples.

Example – 8: 2x $(x + 5) = 0$

$2x = 0$

$x = 0$

$(x + 5) = 0$

$x + 5 - 5 = 0 - 5$

$x = -5$

Solve Quadratic Equation by Factoring

Procedure

- Write the equation in a standard form
- Factor the equation completely
- Set each factor equal to zero
- Solve each equation independently
- Check the solution in the original equation

Solve Quadratic Equation by Factoring

Use factoring to solve the following examples.

Example – 9: $x^2 - 3x = 18$

Step one: write the equation in a standard form

$$x^2 - 3x - 18 = 0$$

Step two: Factor the equation completely

$x \times -6 = -6x$

$3 \times -6 = -18$

$$(x + 3)(x - 6) = 0$$

$x \times x = x^2$

$x \times 3 = 3x$

$x^2 + 3x - 6x - 18$

$x^2 - 3x - 18$

OK

Solve Quadratic Equation by Factoring

Step Three: Set each factor equal to zero

- $(x + 3) = 0$
- $(x - 6) = 0$

Step Four: Solve each equation independently

- $x + 3 = 0 \quad \rightarrow \quad x = -3$
- $x - 6 = 0 \quad \rightarrow \quad x = 6$

Solve Quadratic Equation by Factoring

Step Five: Check the solution in the original equation

Original Equation is: $x^2 - 3x = 18$

- $x = -3$

$(-3)^2 - 3(-3) = 18$

$9 + 9 = 18$

- $x = 6$

$(6)^2 - 3(6) = 18$

$36 - 18 = 18$

Solve Quadratic Equation by Factoring

Example – 10: $x(x-4)=5$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$(x-5)=0$

$x=5$

$(x+1)=0$

$x=-1$

$x(x-4)=5$

$x=5$

$5(5-4)=5$

$25-20=5$ OK

$x(x-4)=5$

$x=-1$

$-1(-1-4)=5$

$1+4=5$ OK

Solve Quadratic Equation by Factoring

Example – 11: $x(3x+7)=6$

$$3x^2+7x-6=0$$

$$(x+3)(3x-2)=0$$

$$(x+3)=0$$

$$x=-3$$

$$(3x-2)=0$$

$$x=\frac{2}{3}$$

$$x(3x+7)=6$$

$$x=-3$$

$$-3(3(-3)+7)=6$$

$$27-21=6 \quad \underline{\text{OK}}$$

$$x(3x+7)=6$$

$$x=\frac{2}{3}$$

$$\frac{2}{3}\left(3\left(\frac{2}{3}\right)+7\right)=6$$

$$\frac{4}{3}+\frac{14}{3}=6 \quad \underline{\text{OK}}$$

Solve Quadratic Equation by Factoring

Example – 12: $x(3x + 7) = 6$

$$3x^2 + 7x - 6 = 0$$

$$(x + 3)(3x - 2) = 0$$

$$(x + 3) = 0$$

$$x = -3$$

$$(3x - 2) = 0$$

$$x = \frac{2}{3}$$

$$x(3x + 7) = 6$$

$$x = -3$$

$$-3(3(-3) + 7) = 6$$

$$27 - 21 = 6$$ **OK**

$$x(3x + 7) = 6$$

$$x = \frac{2}{3}$$

$$\frac{2}{3}\left(3\left(\frac{2}{3}\right) + 7\right) = 6$$

$$\frac{4}{3} + \frac{14}{3} = 6$$ **OK**

Solve Quadratic Equation by Factoring

Example – 13: $2y^2 + 29y + 14 = 0$

$(2y + 1)(y + 14) = 0$

$(2y + 1) = 0$

$y = -\frac{1}{2}$

$(y + 14) = 0$

$y = -14$

$2y^2 + 29y + 14 = 0$

$y = -\frac{1}{2}$

$2\left(-\frac{1}{2}\right)^2 + 29\left(-\frac{1}{2}\right) + 14 = 0$

$\frac{1}{2} - \frac{29}{2} + 14 = 0$ OK

$2y^2 + 29y + 14 = 0$

$y = -14$

$2(-14)^2 + 29(-14) + 14 = 0$

$392 - 406 + 14 = 0$ OK

Prime Results

There are expressions that cant be factored, so we call them Prime:

Example – 14: $x^2 + 4 = 0$

$(x + 2)(x - 2) = 0$

$x^2 - 4$ **Wrong**

$(x - 2)(x - 2) = 0$

So, this is a Prime

$x^2 - 4x + 4$ **Wrong**

Solve Quadratic Equation by Square Root Method

Use square root method to solve a quadratic equation with one variable (means b = 0).

Example – 15: $x^2 + 4 = 0$

$x^2 = -4$ **Square root both sides**

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm\sqrt{-4}$

Imaginary number, no real solution

Quadratic Formula

Use Quadratic Formula to solve quadratic equation

Example – 16: $x^2 + 6x + 3 = 0$

$x^2 + 6x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(3)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{36 - 12}}{6}$$

Quadratic Formula

Use Quadratic Formula to solve quadratic equation

Example – 17: $x^2 + 6x + 3 = 0$

$$x = \frac{-6 \pm \sqrt{24}}{6}$$

$$x = \frac{-6 \pm 2\sqrt{6}}{6}$$

$$x = -3 \pm \sqrt{6}$$

Quadratic Formula

The use of Quadratic Formula lead to the following three results:

1. The "stuff" under the square root can be positive and we'd get two unequal real solutions $b^2 - 4ac > 0$
2. The "stuff" under the square root can be zero and we'd get one solution (called a repeated or double root because it would factor into two equal factors, $b^2 - 4ac = 0$
3. The "stuff" under the square root can be negative and we'd get no real solutions. $b^2 - 4ac < 0$

Summary of Solving Quadratic Equations

- Get the equation in standard form: $Ax^2 + Bx + C = 0$
- If there is no middle term (**b = 0**) then get the x^2 alone and square root both sides (if you get a negative under the square root there are no real solutions).
- If there is no constant term (**c = 0**) then factor out the common x and use the null factor law to solve (set each factor = 0).
- If it doesn't factor or is hard to factor, use the quadratic formula to solve (if you get a negative under the square root there are no real solutions).