NAME OF THE SUBJECT : Mathematics – I

SUBJECT CODE : MA8151

NAME OF THE MATERIAL : Formula Material

REGULATION : R2017

Unit – I (Differential Calculus)

1. **Limit of a Function**

   The limit of $f(x)$, as x approaches a, equals ℓ if we can make the value of $f(x)$ arbitrarily close to ℓ by taking x to be sufficiently close to a but not equal to a. i.e. $\lim_{x \to a} f(x) = \ell$

2. **Continuity**

   A function f is continuous at a number a if $\lim_{x\to a} f(x) = f(a)$.

3. **Derivatives**

   The derivative of a function $f(x)$ at $x = a$ denoted by $f'(a)$, is $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ if this limit exists.

   (or) $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

4. **Table of derivative of the functions:**

Sl.No.	y	$\frac{dy}{dx}$
1.	Constant	0
2.	x^n	nx^{n-1}
3.	x	1
4.	$\frac{1}{x^n}$ $\frac{1}{x}$	$\frac{-n}{x^{n+1}}$ $\frac{-1}{x^2}$

5.	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
6.	$e^{(ax+b)}$ e^x a^x	$ae^{(ax+b)}$ e^x $a^x \log a$
7.	$\log(ax+b)$ $\log x$ $\log_{10} x$	$\frac{a}{ax+b}$ $\frac{1}{x}$ $\frac{1}{x}\log_{10} e$
8.	$\sin(ax+b)$ $\sin x$	$a\cos(ax+b)$ $\cos x$
9.	$\cos(ax+b)$ $\cos x$	$-a\sin(ax+b)$ $-\sin x$
10.	$\tan(ax+b)$ $\tan x$	$a\sec^2(ax+b)$ $\sec^2 x$
11.	$\operatorname{cosec}(ax+b)$ $\operatorname{cosec} x$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$ $-\operatorname{cosec} x \cot x$
12.	$\sec(ax+b)$ $\sec x$	$a\sec(ax+b)\tan(ax+b)$ $\sec x \tan x$
13.	$\cot(ax+b)$ $\cot x$	$-a\operatorname{cosec}^2(ax+b)$ $-\operatorname{cosec}^2 x$
14.	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
15.	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$

16.	$\tan^{-1} x$	$\frac{1}{1+x^2}$

5. Special Formulae:

i) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

ii) $\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + vw\frac{du}{dx} + wu\frac{dv}{dx}$

iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

6. Equation of tangent line: $y - y_1 = m(x - x_1)$

7. Equation of normal line: $y - y_1 = \frac{-1}{m}(x - x_1)$

8. If the tangent line parallel to x-axis (horizontal) then $\frac{dy}{dx} = 0$.

9. If the tangent line parallel to y-axis (vertical) then $\frac{dx}{dy} = 0$.

10. **Increasing and Decreasing Function**

Let f be a function defined on the interval $[a,b]$ and have a finite derivative inside the segment, then

(i) f is increasing if and only if $f'(x) \geq 0$ for all x in $[a,b]$.

(ii) f is decreasing if and only if $f'(x) \leq 0$ for all x in $[a,b]$.

11. **Monotonic Functions**

If a function f is completely increasing or completely decreasing in an interval $[a,b]$, then the function f is called monotonic function in $[a,b]$.

12. **Critical Number**

A critical number of a function f is a number c in the domain of f such that $f'(c) = 0$.

13. Maxima and Minima by First Derivative Test

Consider $x = a$ be a critical point of a continuous function $f(x)$.

i) If $f'(x)$ changes from positive to negative at $x = a$, then $f(x)$ has a maximum at $x = a$.

ii) If $f'(x)$ changes from negative to positive at $x = a$, then $f(x)$ has a minimum at $x = a$.

14. Maxima and Minima by Second Derivative Test

Consider $x = a$ be a critical point of a continuous function $f(x)$.

i) If $f''(a) < 0$, then $f(x)$ has a maximum at $x = a$.

ii) If $f''(a) > 0$, then $f(x)$ has a minimum at $x = a$.

15. Concavity Test

Suppose $f(x)$ is twice differentiable on an interval I.

i) If $f''(x) > 0$ for all x in I, then the graph of $f(x)$ is concave upward on I.

ii) If $f''(x) < 0$ for all x in I, then the graph of $f(x)$ is concave downward on I.

16. Point of Inflection

A point P on a curve is called a point of inflection if the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Unit – II (Functions of Several Variables)

1. Euler's Theorem:

If f is a homogeneous function of x and y in degree n, then

(i) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$ (first order)

(ii) $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$ (second order)

2. If $u = f(x, y, z)$, $x = g_1(t), y = g_2(t), z = g_3(t)$ then $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$.

3. If $u = f(x, y), x = g_1(r,\theta), y = g_2(r,\theta)$ then

(i) $\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$ (ii) $\frac{\partial u}{\partial \theta}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$

4. **Maxima and Minima :**

Working Rules:

Step: 1 Find f_x and f_y. Put $f_x=0$ and $f_y=0$. Find the value of x and y.

Step: 2 Calculate $r=f_{xx}, s=f_{xy}, t=f_{yy}$. Now $\Delta=rt-s^2$

Step: 3 i. If $\Delta>0$, then the function have either maximum or minimum.

1. If $r<0 \Rightarrow f$ has maximum
2. If $r>0 \Rightarrow f$ has minimum

ii) If $\Delta<0$, then the function is neither Maximum nor Minimum, it is called Saddle Point.

iii) If $\Delta=0$, then the test is inconclusive.

5. Maxima and Minima of a function using Lagrange's Multipliers:

Let $f(x,y,z)$ be given function and $g(x,y,z)$ be the subject to the condition.

Form $F(x,y,z)=f(x,y,z)+\lambda g(x,y,z)$, Putting $F_x=F_y=F_z=F_\lambda=0$ and then find the value of x,y,z.

6. **Jacobian:**

Jacobian of two dimensions: $J\left(\frac{u,v}{x,y}\right)=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}$

7. The functions u and v are called functionally dependent if $\frac{\partial(u,v)}{\partial(x,y)}=0$.

8. $\frac{\partial(u,v)}{\partial(x,y)}\times\frac{\partial(x,y)}{\partial(u,v)}=1$

9. **Taylor's Expansion:**

$$f(x,y)=f(a,b)+\frac{1}{1!}\{hf_x(a,b)+kf_y(a,b)\}+\frac{1}{2!}\{h^2f_{xx}(a,b)+2hkf_{xy}(a,b)+k^2f_{yy}(a,b)\}$$
$$+\frac{1}{3!}\{h^3f_{xxx}(a,b)+3h^2kf_{xxy}(a,b)+3hk^2f_{xyy}(a,b)+k^3f_{yyy}(a,b)\}+\ldots$$

where $h=x-a$ and $k=y-b$

Unit – III (Integral Calculus)

1. $\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1}+c$ $\int x^n dx = \frac{x^{n+1}}{n+1}+c$

2. $\int e^{ax+b}dx = \frac{e^{ax+b}}{a}+c$ $\int e^x dx = e^x+c$

3. $\int \frac{1}{ax+b}dx = \frac{1}{a}\log(ax+b)+c$ $\int \frac{1}{x}dx = \frac{1}{x}+c$

   $\int \frac{1}{x^n}dx = \frac{-1}{(n-1)x^{n-1}}+c$

4. $\int \sqrt{x}dx = \frac{2x^{3/2}}{3}+c$

5. $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$ $\int \sin x\,dx = -\cos x+c$

6. $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c$ $\int \cos x\,dx = \sin x+c$

7. $\int \sec x\,dx = \log(\sec x+\tan x)+c$

8. $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec}x-\cot x)+c$

9. $\int \tan(ax+b)dx = \frac{1}{a}\log\sec(ax+b)+c$ $\int \tan x\,dx = \log\sec x+c$

10. $\int \cot(ax+b)dx = \frac{1}{a}\log\sin(ax+b)+c$ $\int \cot x\,dx = \log\sin x+c$

11. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$ $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x+c$

12. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+c$ (or) $\log\left[x+\sqrt{x^2-a^2}\right]+c$

13. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+c$ (or) $\log\left[x+\sqrt{x^2+a^2}\right]+c$

14. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ $\int \frac{dx}{1+x^2} = \tan^{-1}x+c$

15. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$

16. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$

17. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

18. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)+c$

    (or)

$$= \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left[x+\sqrt{a^2+x^2}\right]+c$$

19. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)+c$

(or)

$$= \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left[x+\sqrt{x^2-a^2}\right]+c$$

20. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$

21. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$

22. Reduction Formulae

$$\int_0^{\frac{\pi}{2}} \cos^n x\,dx \text{ (or) } \int_0^{\frac{\pi}{2}} \sin^n x\,dx = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\frac{2}{3}.1 \qquad [\text{if } n \text{ is odd}]$$

$$= \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\frac{1}{2}.\frac{\pi}{2} \qquad [\text{if } n \text{ is even}]$$

$$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x\,dx = \frac{\left[(m-1)(m-3)\ldots\right]\left[(n-1)(n-3)\ldots\right]}{(m+n)(m+n-2)(m+n-4)\ldots}$$

$$= \frac{\left[(m-1)(m-3)\ldots\right]\left[(n-1)(n-3)\ldots\right]}{(m+n)(m+n-2)(m+n-4)\ldots} \qquad [\text{Both m and n are even}]$$

23. $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \qquad [\text{if } f(x) \text{ is an even function}]$

$$= 0 \qquad [\text{if } f(x) \text{ is an odd function}]$$

24. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

25. $\int_a^b f(x)\,dx = -\int_b^a f(a-x)\,dx$

26. Integration by Parts: $\int u\,dv = uv - \int v\,du$

27. Bernoulli's Formulae: $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$

Unit – IV (Multiple Integrals)

1. $\int_a^b \int_0^x f(x,y)dxdy$ $x: a \text{ to } b$ and $y: 0 \text{ to } x$ (Here the first integral is w.r.t. y)
2. $\int_a^b \int_0^y f(x,y)dxdy$ $x: 0 \text{ to } y$ and $y: a \text{ to } b$ (Here the first integral is w.r.t. x)
3. Area $= \iint_R dxdy$ (or) $\iint_R dydx$

   To change the polar coordinate $x = r\cos\theta$, $y = r\sin\theta$ and $dxdy = rdrd\theta$.
4. Volume $= \iiint_V dxdydz$ (or) $\iiint_V dzdydx$

Unit – V (Differential Equations)

1. **ODE with constant coefficients:** Solution $y = \mathbf{C.F} + \mathbf{P.I}$

Complementary functions:

Sl.No.	Nature of Roots	C.F
1.	$m_1 = m_2$	$(Ax+B)e^{mx}$
2.	$m_1 = m_2 = m_3$	$\left(Ax^2+Bx+c\right)e^{mx}$
3.	$m_1 \neq m_2$	$Ae^{m_1x} + Be^{m_2x}$
4.	$m_1 \neq m_2 \neq m_3$	$Ae^{m_1x} + Be^{m_2x} + Ce^{m_3x}$
5.	$m_1 = m_2,\ m_3$	$(Ax+B)e^{mx} + Ce^{m_3x}$
6.	$m = \alpha \pm i\beta$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$
7.	$m = \pm i\beta$	$A\cos\beta x + B\sin\beta x$

Particular Integral:

Type-I

If $f(x) = 0$ then $P.I = 0$

Type-II

If $f(x) = e^{ax}$ (or) $\sinh ax$ (or) $\cosh ax$

$$P.I = \frac{1}{\phi(D)} e^{ax}$$

Replace D by a. If $\phi(D) \neq 0$, then it is P.I. If $\phi(D) = 0$, then diff. denominator w.r.t D and multiply x in numerator. Again replace D by a. If you get denominator again zero then do the same procedure.

Type-III

Case: i If $f(x) = \sin ax \ (or) \ \cos ax$

$$P.I = \frac{1}{\phi(D)} \sin ax \ (\text{or}) \ \cos ax$$

Here you have to replace only for D^2 not for D. D^2 is replaced by $-a^2$. If the denominator is equal to zero, then apply same procedure as in Type – I.

Case: ii If $f(x) = Sin^2 x \ (\text{or}) \ \cos^2 x \ (\text{or}) \ \sin^3 x \ (\text{or}) \ \cos^3 x$

Use the following formulas $Sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$,

$\sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$, $\cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos 3x$ and separate $P.I_1$ & $P.I_2$

Case: iii If $f(x) = \sin A \cos B \ (or) \ \cos A \sin B \ (or) \ \cos A \cos B \ (or) \ \sin A \sin B$

Use the following formulas:

$$(i) \quad \sin A \cos B = \frac{1}{2}\left(sin(A+B) + \sin(A-B)\right)$$

$$(\text{ii}) \quad \cos A \sin B = \frac{1}{2}\left(Sin(A+B) - \sin(A-B)\right)$$

$$(iii) \quad \cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$(iv) \quad \sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

Type-IV

If $f(x) = x^m$

$$P.I = \frac{1}{\phi(D)} x^m = \frac{1}{1+g(D)} x^m = \left(1+g(D)\right)^{-1} x^m$$

Here we can use Binomial formula as follows:

i) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$

ii) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$

iii) $(1+x)^{-2}=1-2x+3x^2-4x^3+\ldots$

iv) $(1-x)^{-2}=1+2x+3x^2+4x^3+\ldots$

Type-V

If $f(x)=e^{ax}V$ where $V=\sin ax, \cos ax, x^m$

$$P.I=\frac{1}{\phi(D)}e^{ax}V \;=e^{ax}\frac{1}{\phi(D+a)}V$$

Type-VI

If $f(x)=x^nV$ where $V=\sin ax, \cos ax$

$\sin ax=\text{I.P of } e^{iax}$

$\cos ax=\text{R.P of } e^{iax}$

Type-VII

If $f(x)=\sec ax \text{ (or) } \operatorname{cosec} ax \text{ (or) } \tan ax$

$$P.I=\frac{1}{D-a}f(x)=e^{ax}\int e^{-ax}f(x)dx$$

1. **ODE with variable co-efficient: (Euler's Method)**

The equation is of the form $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=f(x)$

Implies that $(x^2D^2+xD+1)y=f(x)$

To convert the variable coefficients into the constant coefficients

Put $z=\log x$ implies $x=e^z$

$xD=D'$

$x^2D^2=D'(D'-1)$ where $D=\frac{d}{dx}$ and $D'=\frac{d}{dz}$

$x^3D^3=D'(D'-1)(D'-2)$

The above equation implies that $\left(D'(D'-1)+D'+1\right)y=f(x)$ which is O.D.E with constant coefficients.

2. **Legendre's Linear differential equation:**

The equation if of the form $(ax+b)^2\frac{d^2y}{dx^2}+(ax+b)\frac{dy}{dx}+y=f(x)$

Put $z=\log(ax+b)$ implies $(ax+b)=e^z$

$$(ax+b)D = aD'$$

$$(ax+b)^2 D^2 = a^2 D'(D'-1) \qquad \text{where } D = \frac{d}{dx} \text{ and } D' = \frac{d}{dz}$$

$$(ax+b)^3 D^3 = a^3 D'(D'-1)(D'-2)$$

3. Method of Variation of Parameters:

The equation is of the form $\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)}$

$C.F = Ay_1 + By_2$ and

$P.I = Py_1 + Qy_2$

where $P = -\int \frac{y_2 f(x)}{y_1 y_2' - y_1' y_2} dx$ and

$Q = \int \frac{y_1 f(x)}{y_1 y_2' - y_1' y_2} dx$

Textbook for Reference:

"ENGINEERING MATHEMATICS - I"

Publication: Sri Hariganesh Publications **Author: C. Ganesan**

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