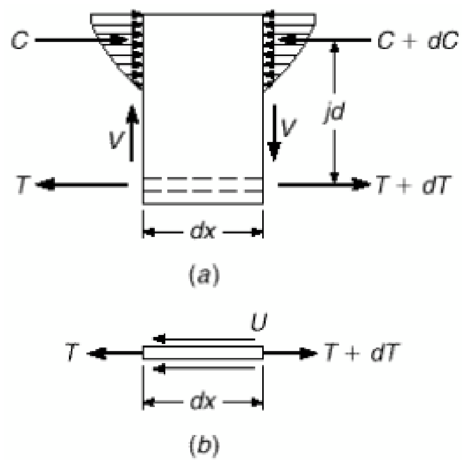
4 Bond, Anchorage and Development Length

For reinforced concrete to behave as intended, it is essential that bond forces be developed on the interface between concrete and steel, such as to prevent slip from occurring at that interface.
If plain bars (without surface deformations), initial bond strength is provided only by the relatively weak chemical adhesion and mechanical friction between steel and concrete. To improve this situation, deformed bars are now universally used (see Introduction chapter).

Bond Force Based on Simple Cracked Section Analysis

For the RC beam element shown in Fig. below:



$dT = dM / jd$
if U = bond force / unit length of the bar, then $U\,dx = dT$
Thus $U = dT/dx$
$U = (dM / dx) / jd$
$U = V / jd$

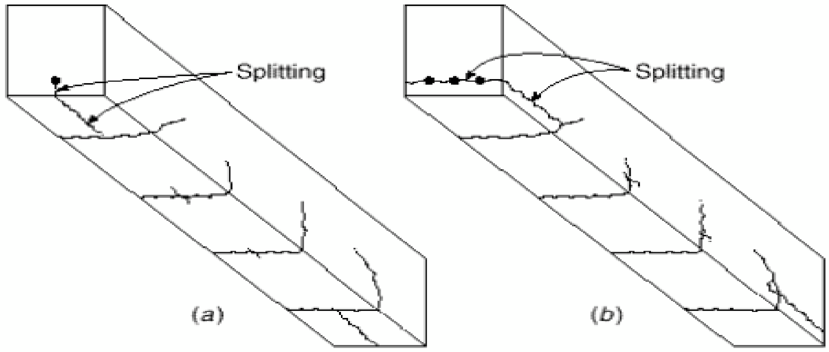
It indicates that bond force is proportional to the rate of change of bending moment, i.e., to the shear.

<u>Bond Strength and Development Length</u>

Two types of bond failure have been observed for reinforcing bars in tension;
The 1st is ***direct pullout*** of the bar, which occurs when ample confinement is provided by the surrounding concrete.
The 2nd is ***splitting*** of the concrete along the bar when cover, confinement, or bar spacing is insufficient to resist the lateral concrete tension from the wedging effect of the bar deformation.

Bond Strength

Bond failure resulting from splitting of the concrete is more common in beams than direct pullout. It may occur either in a vertical plane (fig. a below) or horizontally in the plane of the bars (fig. b)

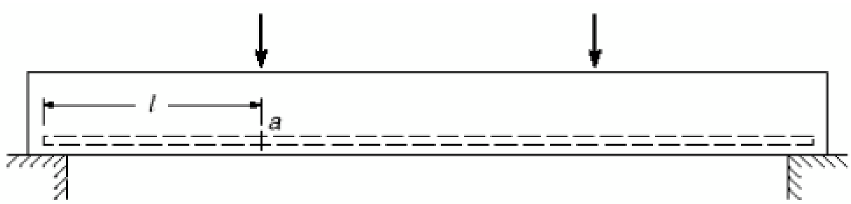


When pullout resistance is overcome or when splitting has spread all the way to the end of bar, complete bond failure occurs. Sliding of the steel relative to the concrete leads to immediate collapse of the beam.

Development Length

The ***development length*** is defined as the length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting. With reference to Fig. below, the moment, and therefore the steel stress, is maximum at point a and zero at supports. The total tension force $A_b f_s$ must be transferred from the bar to the concrete in the distance l by bond forces. To fully develop the strength of the bar $A_b f_s$, the distance l must be at least equal to the development length l_d of the bar.

In the Fig., <u>if $l \geq l_d$, no premature bond failure will occur. That is, the beam will fail in bending or shear rather than bond failure.</u>



Factors Influencing Development Length

Experimental research has identified the factors that influence l_d. the most basic factors are:

1. concrete tensile strength
2. cover distance
3. spacing of the reinforcing bars
4. transverse reinforcement

In addition, other influences have been identified: the *vertical location* of horizontal bars relative to beam depth, *epoxy-coating* bars, provided tensile reinforcement in *excess* over calculated, and *bar diameter* (smaller diameters require lower l_d)

ACI Code Provisions for l_d of Tension Reinforcement

In any case, $l_d \geq$ **300 mm**

Basic equation for l_d

According to ACI 25.4, for deformed bars or deformed wire,

$$\ell_d = \left(\frac{f_y}{1.1\,\lambda\sqrt{f'_c}} \frac{\Psi_t \Psi_e \Psi_s \Psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)} \right) d_b$$

In which the term $\{(c_b + K_{tr})/d_b\} \leq 2.5$

Table 25.4.2.5—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Reinforcement grade ψ_g	Grade 280 , Grade 420	1.0
	Grade 560	1.15
	Grade 700	1.3
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No 22 and larger bars	1.0
	No 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than (300 mm) fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1] The product $\psi_t\psi_e$ need not exceed 1.7.

c_b = spacing or cover dimension, mm
= lesser of: (a) the distance from center of a bar to nearest concrete surface, and
(b) One-half the center-to-center spacing of bars developed.
K_{tr} = transverse reinforcement index:

$$K_{tr} = 40A_{tr} / (sn)$$

Where
A_{tr} = total area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed, mm^2
s = maximum c.c. spacing of transverse reinforcement within l_d, mm
n = no. of bars or wires being developed along the plane of splitting.
It is permitted to use $K_{tr} = 0$ as a design simplification.

Simplified Equations for l_d

Table 25.4.2.3—Development length for deformed bars and deformed wires in tension

Spacing and cover	No.19 and smaller bars and deformed wires	No.22 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{1.1\lambda\sqrt{f_c'}}\right)d_b$

For example, in all members of normal-weight concrete ($\lambda = 1$), uncoated bars ($\psi_e = 1$), No.22 and larger bottom bars ($\psi_t = 1$) with $f_c' = 28$ MPa, and $f_y = 420$ MPa ($\psi_g = 1$), the expressions reduce to:
$l_d = \{420\,(1)(1)(1) / [1.7(1)\sqrt{28}]\}\times d_b = 47\ d_b$,
$l_d = \{420\,(1)(1)(1) / [1.1(1)\sqrt{28}]\}\times d_b = 72\ d_b$

Reduction of l_d

Regardless of whether l_d is calculated using the basic equation or the simplified equations, l_d may be reduced where reinforcement is in excess of that required by analysis. ACI Code 25.4.10 allows the reduction as follows:

$$\textbf{Reduced } l_d = (A_{s,\ required} / A_{s,\ provided})\ l_d$$

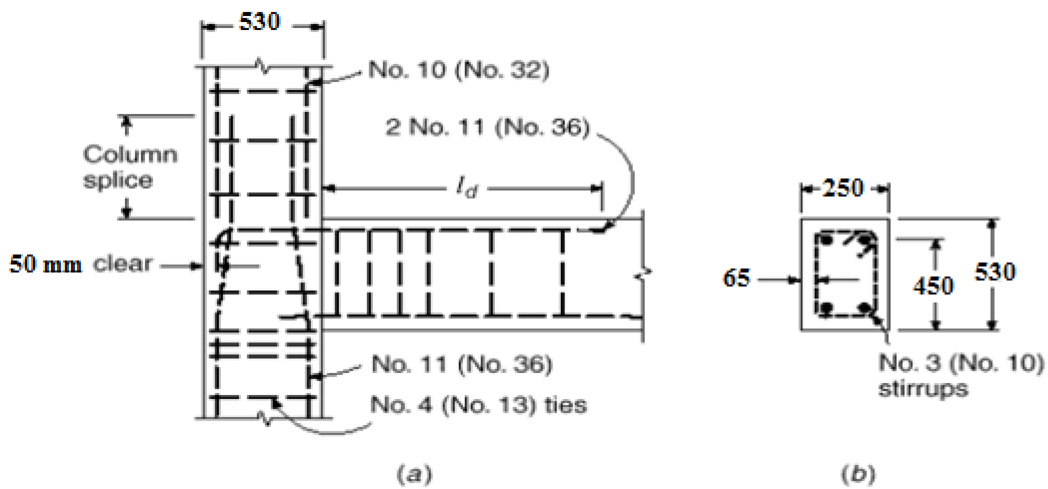
Reduction of l_d is not permitted at non-continuous supports, at locations where anchorage is required, and in seismic regions.

Example 1:

Figure below shows a beam-column joint. Based on frame analysis, at the end of beam the **–ve** $\boldsymbol{A_{s,\ required}}$ **= 1870 mm²**; **2 No.36** bars are used providing $\boldsymbol{A_s}$ **= 2012 mm²**. Beam dimensions are $\boldsymbol{b}$ **= 250 mm**, $\boldsymbol{d}$ **= 450 mm**, and $\boldsymbol{h}$ **= 530 mm**. The design will include **No.10** stirrups spaced **4 at 75 mm**, followed by constant **125 mm** spacing in the region of support, with **40 mm** clear cover. Normal-density concrete is used, with $\boldsymbol{f'_c}$ **= 28 MPa**. $\boldsymbol{f_y}$ **= 420 MPa**. Find the minimum distance $\boldsymbol{l_d}$ at which the negative bars can be cut off,

(a) using the simplified equations,

(b) using the basic equation.



Solution:

a) *simplified equations*

Check clear spacing between bars = 250 – 2(40 + 9.5 + 35.8) = 79 mm > 2d_b

Check clear cover to side face = 40 + 9.5 = 49.5 mm > d_b

Check clear cover to top face = 80 – 35.8/2 = 62.1 mm > d_b

These meet restrictions of 2nd row in table:

For No.22 and larger bars:

$\boldsymbol{l_d = [f_y\ \psi_t \psi_e\ \psi_g\ /\ (1.7\lambda\sqrt{f'_c})]\ d_b}$

ψ_t = 1.3 (top bars), ψ_e = 1.0 (uncoated), ψ_g = 1.0 (Grade 420) and **λ =** 1.0 (normal-weight conc.)

$\boldsymbol{l_d}$ = $\underline{61\ d_b}$ = 61 (35.8) = 2180 mm

Reduced $\boldsymbol{l_d = (A_{s,\ required} / A_{s,\ provided})\ l_d}$

= (1870 / 2012)(2180) = <u>2030 mm</u> final $\boldsymbol{l_d}$

b) *basic equation*

c.c. spacing bet bars = 250 – 2(40 + 9.5 + 35.8/2) = 115 mm, ½ = 57.5mm

side cover to bar = 40 + 9.5 + 35.8/2 = 67 mm, the top cover = 80 mm

$\boldsymbol{c}$ = 57.5 mm (the smallest controls)

$\boldsymbol{K_{tr} = 40A_{tr}/(sn)}$ = 40×71×2 / (125×2) = 22.7,

$\boldsymbol{(c_b + K_{tr})/d_b}$ = (57.7+22.7)/35.8 = 2.24 < 2.5.

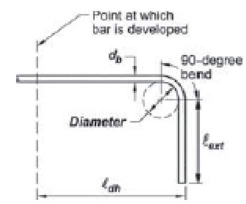
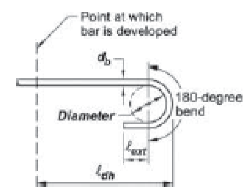
$\boldsymbol{l_d = \{f_y / (1.1\lambda\sqrt{f'_c})\}\ [\psi_t\psi_e\psi_s\ \psi_g /\ \{(c + K_{tr})/d_b\}]\ d_b}$ = $\underline{42\ d_b}$ = 1500 mm.

Reduced $\boldsymbol{l_d}$ = (1870 / 2012)(1500) = <u>1395 mm</u> final $\boldsymbol{l_d}$

Anchorage of Tension Bars by Hooks

a. Standard Dimensions

Table 25.3.1—Standard hook geometry for development of deformed bars in tension

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No.10 through No. 25	$6d_b$	$12d_b$	
	No 29 through No. 36	$8d_b$		
	No. 45 and No. 57	$10d_b$		
180-degree hook	No 10 through No. 25	$6d_b$	Greater of $4d_b$ and 65 mm	
	No 29 through No. 36	$8d_b$		
	No. 45 and No. 57	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

b. Development Length l_{dh} and Modification Factors for Hooked Bars

ACI Code 25.4.3 provisions for hooked bars in tension account for the combined contribution of bond along the straight bar leading to the hook, plus the hooked anchorage.

A total development length l_{dh} is measured from the critical section to the farthest point on the bar, parallel to the straight part of the bar.

In any case, $l_{dh} \geq 8\, d_b$, or 150 mm

1) Development length for hooked bars in tension

$$l_{dh} = \left(\frac{f_y \Psi_e \Psi_r \Psi_o \Psi_c}{23\sqrt{f'_c}}\right) d_b^{1.5}$$

2) Modification factors

Table 24.4.3.2 provides modification factors to l_{dh}

Table 25.4.3.2—Modification factors for development of hooked bars in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement ψ_r	For No. 11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or s[1] $\geq 6d_b$[2]	1.0
	Other	1.6
Location ψ_o	For No. 11 and smaller diameter hooked bars: (1) Terminating inside column core with side cover normal to plane of hook $\geq$ **65 mm** (2) With side cover normal to plane of hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength ψ_c	For $f_c' <$ **42 MPa**	$f_c'/$ **105** $+ 0.6$
	For $f_c' \geq$ **42 MPa**	1.0

[1] s is minimum center-to-center spacing of hooked bars.
[2] d_b is nominal diameter of hooked bar.

Reduction of l_{dh}

where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis

$$new\ l_{dh} = l_{dh}\ (A_{s,\ required} / A_{s,\ provided})$$

Area of ties or stirrups confining hooked bars = A_{th}

- At least 2 ties or stirrups shall be provided parallel to l_{dh} (*typ. in beam-column joints*), *Fig.25.4.3.3a.*

- At least 2 ties or stirrups shall be provided perpendicular to l_{dh} *(typ. in beam-girder joints), Fig.25.4.3.3b.*

-

Their details are described in Figs below:

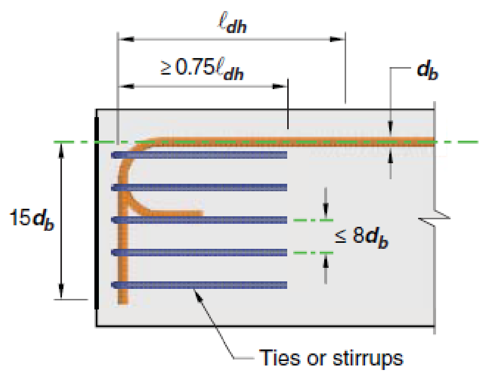


Fig. R25.4.3.3a—Confining reinforcement placed parallel to the bar being developed that contributes to anchorage strength of both 90- and 180-degree hooked bars.

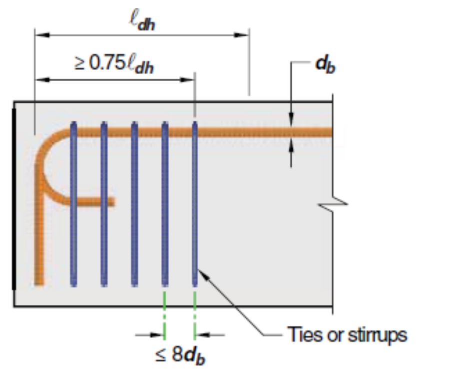


Fig. R25.4.3.3b—Confining reinforcement placed perpendicular to the bar being developed, spaced along the development length ℓ_{dh}, that contributes to anchorage strength of both 90- and 180-degree hooked bars.

At Discontinuous end with covers and side covers to hooks < 65 mm, the ties or stirrups shall satisfy the details in Fig below
This applies:

- at ends of simply-supported beams,
- at the free end of cantilevers, and
- at exterior joints for members framing into a joint where members do not extend beyond the joint.
-

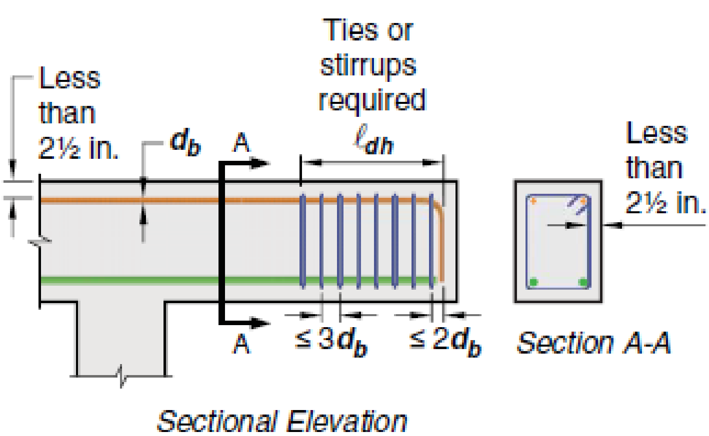


Fig. R25.4.3.4—Concrete cover according to 25.4.3.4.

Example 2:

A beam with **No.32 –ve bars** are to be extended into **600 mm** column and terminated in a **standard 90° hook**, keeping **50 mm** clear to the outside face of the column. Find the minimum length of embedment of the hook past the column face, and specify the hook details. Assume $A_{s,\ required} = A_{s,\ provided}$

Solution: l_{dh} =

$$\left(\frac{f_y \Psi_e \Psi_r \Psi_o \Psi_c}{23\sqrt{f'_c}}\right) d_b^{1.5}$$

Modification factors:
ψ_e = 1.0 (uncoated), ψ_r = 1.0 (No.36 bars or less), ψ_o = 1.0 (side cover > 65 mm), ψ_c = 28/105 + 0.6 = 0.87 (f'_c < 42 MPa) and **λ = 1.0** (normal-weight conc.)
l_{dh} = [420×1.0×1.0 ×1.0×0.87 / (23×λ√28)] $(32)^{1.5}$ = 544 mm

Other factor: **($A_{s,\,required}$ / $A_{s,\,provided}$) =** 1.0
Accordingly:
l_{dh} **=** 544 × 1.0 = 544 mm
Available distance = 600 – 50 = 550 mm > 544 mm, OK

Table 25.3.1, see p.6:
The hook will be bent a min **$8d_b$** = 8(32) = 256 mm
The bar will continue **$12d_b$** (384 mm) past the end of the bend in the vertical direction.

Development of Bars in Compression

In case of bars in compression, a part of the total force is transferred by bond along the embedded length, and a part is transferred by end bearing of the bars on the concrete. Because the surrounding concrete is relatively free of cracks and because of the beneficial effect of end bearing, shorter basic development lengths are permissible for compression bars than for tension bars.

ACI Code Provisions for l_d of Compression Reinforcement

In any case, **$l_{dc} \geq 200$ mm**
According to ACI Code 25.4.9, the development length in compression, l_{dc}, is the greater of (a) or (b):

$$\text{(a)}\quad \left(\frac{24 f_y \Psi_r}{100\lambda\sqrt{f'_c}}\right) d_b$$

$$\text{(b)}\quad 0.043\, f_y \Psi_r d_b$$

Modification factors:

Table 25.4.9.3—Modification factors for deformed bars and wires in compression

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement ψ_r	Reinforcement enclosed within (1), (2), (3), or (4): (1) a spiral (2) a circular continuously wound tie with $d_b \geq 1/4$ in. and pitch 4 in. (3) No. 4 bar or D20 wire ties in accordance with 25.7.2 spaced ≤ 4 in. on center (4) hoops in accordance with 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

It is permitted to take $\boldsymbol{\psi_r = 1.0}$

Reduction of l_{dc}

where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis

$$new\ l_{dc} = l_{dc}\ (A_{s,\ required} \ /\ A_{s,\ provided})$$

Bar Splices

It is frequently necessary to splice bars in the field.

Splices for No.36 bars and smaller are usually made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other.

The lapped bars are usually placed in contact and lightly ***wired*** so that they stay in position as the concrete is placed. Alternatively, splicing may be accomplished by ***welding*** or by ***sleeves or mechanical devices***.

Lapped splices are prohibited for bars larger than No.36, except that No.43 and No.57 bars may be lapped in compression with No.36 and smaller bars.

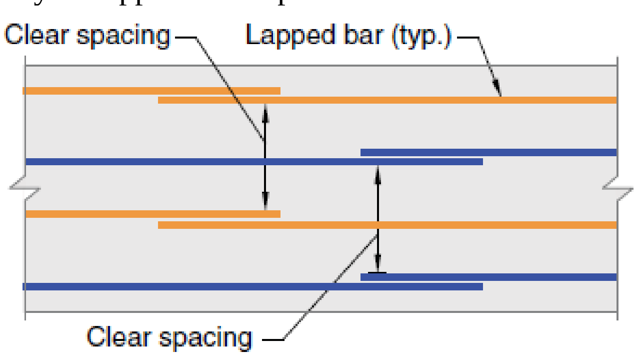
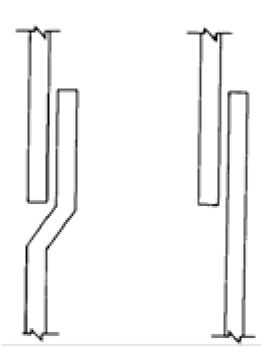


Fig. R25.5.2.1—Clear spacing of lap-spliced bars for determination of ℓ_d for staggered splices.



Lap splices

Lap Splices in Tension, ACI 25.5

Two different classifications are established:

Class A splice requires a lap of **1.0** $l_d \geq$ **300 mm**

Class B splice requires a lap of **1.3** $l_d \geq$ **300 mm**

Lap splices in general must be **class B**, except that **class A** splices are allowed when $A_{s,\ provided} = 2A_{s,\ required}$, and when $\leq \frac{1}{2} A_s$ is spliced.

Spiral reinforcement is spliced with a lap of $48d_b$.

Lap Splices in Compression, ACI 25.5

Reinforcing bars in compression are spliced mainly in columns, where bars are often terminated just above each floor or every other floor. This done partly for construction convenience, but it is also done to permit column steel area to be reduced in steps, as load become lighter at higher floors.

The minimum length of lap in compression:

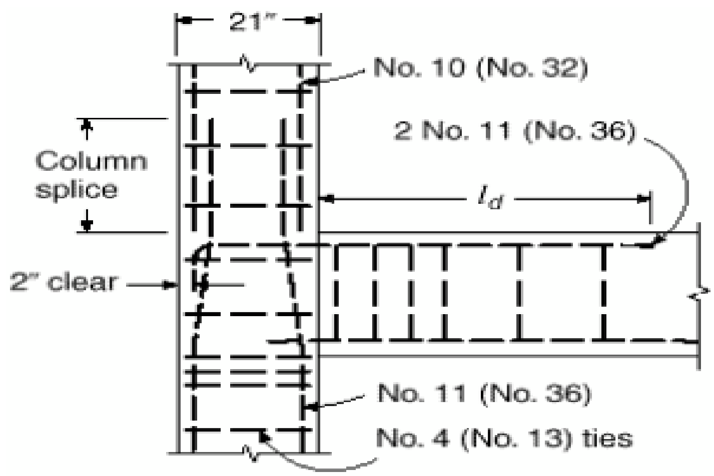
For bars with $f_y \leq 420$ MPa $\quad 0.071\, f_y\, d_b \quad \geq 300$ mm

For bars with $f_y > 420$ MPa $\quad (0.13\, f_y - 24)\, d_b \quad \geq 300$ mm

For $f'_c < 21$ **MPa, increase the lap by 33%**

Compression lap splices shall not be used for bars larger than **No.36**.

Example 3: For the Fig. below, **4 No.36** column bars from the floor below are to be lap spliced with **4 No.32** column bars from above, and the splice is to be made above a construction joint at floor level. The column, measuring **300 by 530 mm** in cross section, has **No.13 ties @ 400mm**. Calculate the required splice length. $f'_c = 28$ **MPa**. $f_y = 420$ **MPa**.



Solution:

The length of splice must be the larger of l_{dc} of No.36 bars and the splice length of the No.32 bars.

For No.36 bars: larger of $l_{dc} = [0.24 f_y\ /\ (\lambda\sqrt{f'_c})]\, d_b$ *and* $l_{dc} = [0.043 f_y]\, d_b$

$l_{dc} = [0.24(420)\ /\ \sqrt{28}]\, d_b$ *and* $l_{dc} = [0.043(420)]\, d_b$

= 682 mm and 647 mm. Thus 682 mm controls.

For No.32 bars: splice length = $0.071\, f_y\, d_b \geq 300$ **mm**

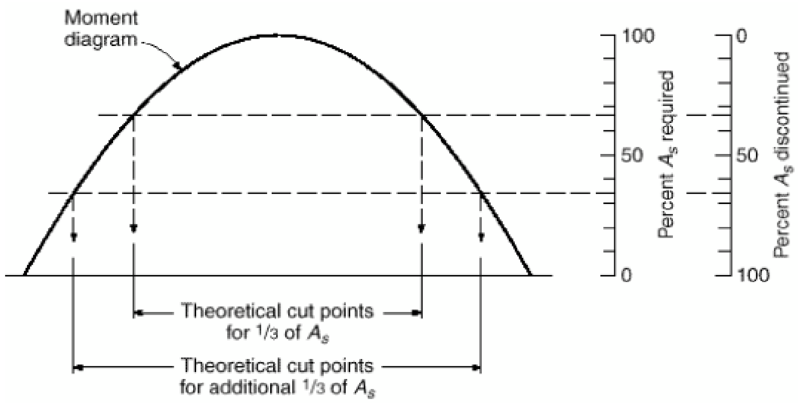
= 0.071(420)(32.3) = 963 mm > 682 mm. Thus <u>lap splice = 963 mm</u>

Bar Cutoff and Bend Points in Beams

The steel requirements in RC beams is easily varied in accordance with requirements for flexure, and it is common practice to cut off bars where they are no longer needed to resist stress or, sometimes in the case of continuous beams, to bend up the bottom steel (usually at 45°) so that it provides tensile reinforcement at the top of the beam over the supports.

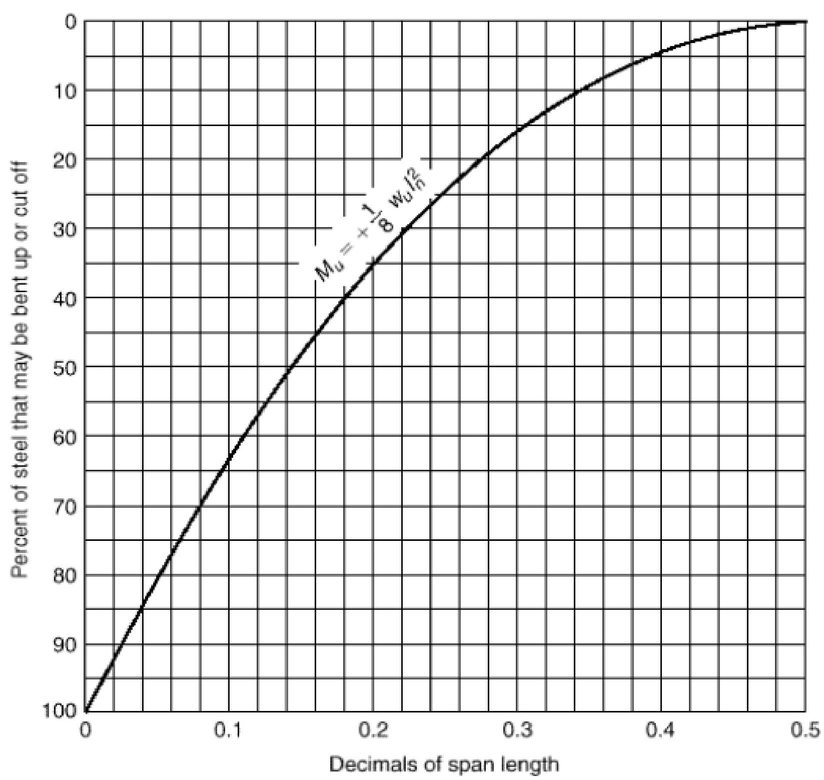
Simple Beams

The moment diagram for a uniformly loaded simple-span beam shown in Fig. below can be used as a steel-requirement diagram.



To facilitate the determination of cutoff or bend points for simple spans, Graph A.2 has been prepared:

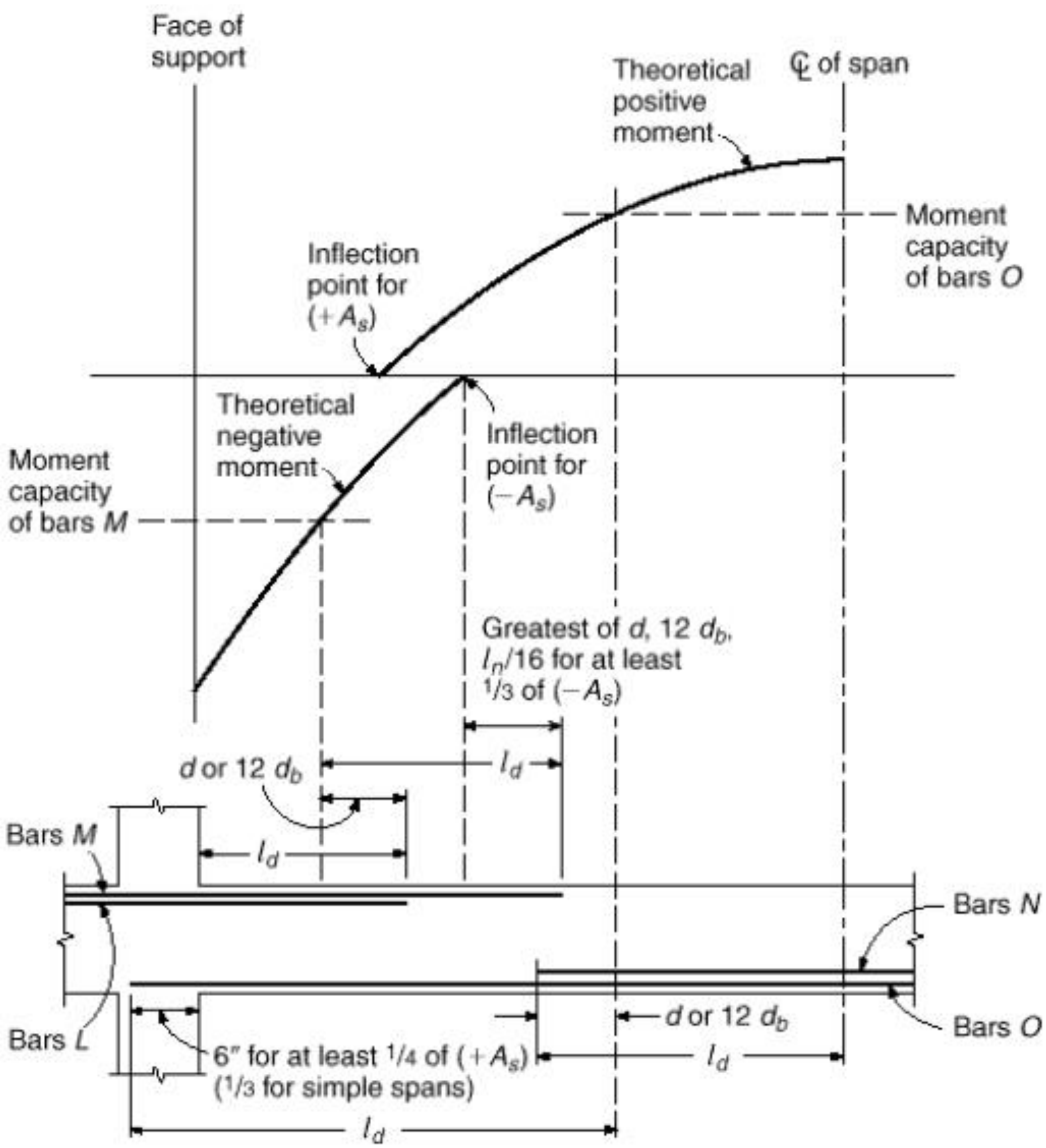
GRAPH A.2
Location of points where bars can be bent up or cut off for simply supported beams uniformly loaded.



Practical Considerations and ACI Requirements; ACI9.7, 25.4

- ACI Code requires that every bar should be continued at least a distance equal to ***d*** of the beam or **12*d*b** (whichever is larger) beyond the point at which it is theoretically no longer required to resist stress.
- Full l_d must be provided beyond critical sections at which peak stress exists in the bars.
- ACI Code requires that at least **⅓ +ve** A_s (**¼** in continuous spans) must be continued along the same face of the beam a distance at least **150 mm** into the support.
- ACI Code requires that at least **⅓ –ve** A_s at the support must be extended beyond the point of theoretically zero –ve M, not less than l_n**/16**, or ***d***, or **12*d*b**, whichever is larger.

Requirements of bar-cutoff or bend point locations are summarized in Fig. below:



For nearly equal spans, uniformly loaded, in which not more than about **½ A_s** is to be cut off or bent, the locations shown in Fig. below are satisfactory.

Note that the exterior support at the left is simple. If the beam is monolithic with exterior columns or walls at that end, use the details of interior spans for the end span.

Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.