Control Systems

Time Response Analysis

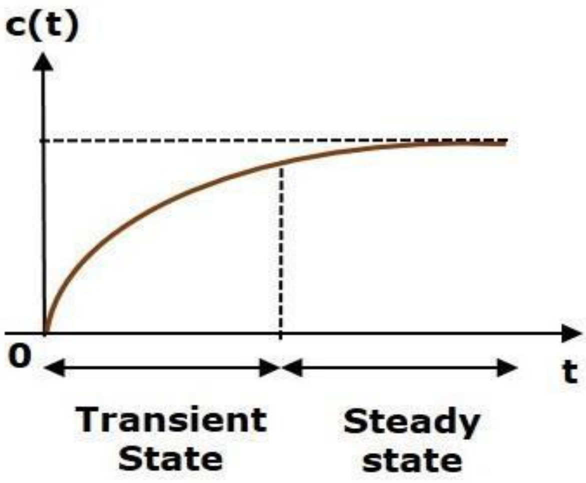
We can analyze the response of the control systems in both the time domain and the frequency domain. We will discuss frequency response analysis of control systems in later chapters. Let us now discuss about the time response analysis of control systems. What is Time Response?

If the output of control system for an input varies with respect to time, then it is called the time response of the control system. The time response consists of two parts.

Transient response

Steady state response

The response of control system in time domain is shown in the following figure.



Here, both the transient and the steady states are indicated in the figure. The responses corresponding to these states are known as transient and steady state responses. Mathematically, we can write the time response c(t) as

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

Where,

$c_{tr}(t)$ is the transient response

$c_{ss}(t)$ is the steady state response

Transient Response

After applying input to the control system, output takes certain time to reach steady state. So, the output will be in transient state till it goes to a steady state. Therefore, the response of the control system during the transient state is known as transient response.

The transient response will be zero for large values of ‘t’. Ideally, this value of ‘t’ is infinity and practically, it is five times constant. Mathematically, we can write it as

$\lim_{t\rightarrow\infty} ctr(t)= 0$

The part of the time response that remains even after the transient response has zero value for large values of ‘t’ is known as steady state response. This means, the transient response will be zero even during the steady state.

For an example, Let us find the transient and steady state terms of the time response of the control system $c(t)=10+5e^{-t}$

Here, the second term $5e^{-t}$ will be zero as t denotes infinity. So, this is the transient term. And the first term 10 remains even as t approaches infinity. So, this is the steady state term.

Standard Test Signals

The standard test signals are impulse, step, ramp and parabolic. These signals are used to know the performance of the control systems using time response of the output.

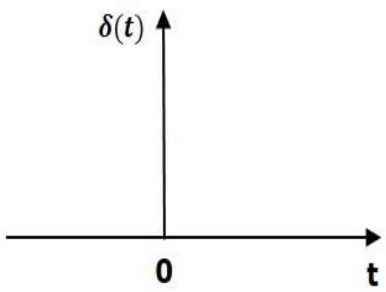
Unit Impulse Signal

A unit impulse signal, δ(t) is defined as

$\delta(t)=0$ for $t\neq0$

and $\int_{0-}^{+0} \delta(t)dt=1$

The following figure shows unit impulse signal.



So, the unit impulse signal exists only at ‘t’ is equal to zero. The area of this signal under small interval of time around ‘t’ equal to zero is one. The value of unit impulse signal is zero for all other values of ‘t’.

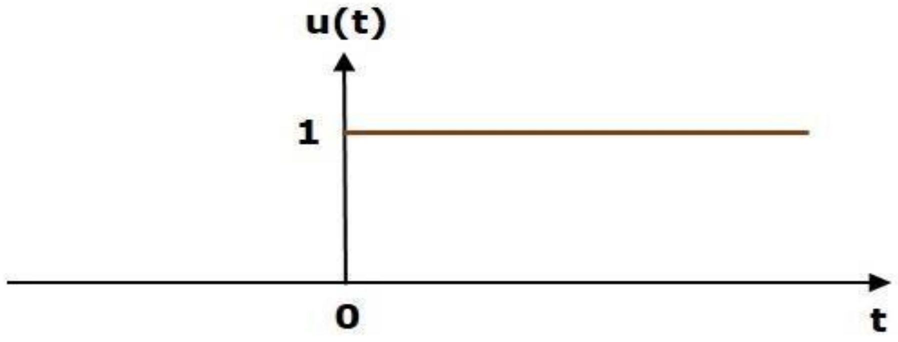
Unit Step Signal

A unit step signal, u(t) is defined as

$u(t)=1; t\geq 0$

$=0; t<0$

Following figure shows unit step signal.



So, the unit step signal exists for all positive values of 't' including zero. And its value is one

during this interval. The value of the unit step signal is zero for all negative values of 't'.

Unit Ramp Signal

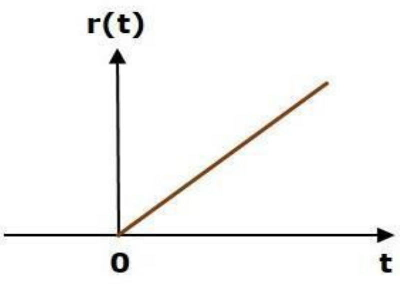
A unit ramp signal, r(t) is defined as

$r(t)= t$ …… $t\geq 0$

$=0$ ……………….. $t< 0$

We can write unit ramp signal, $r(t)$ in terms of unit step signal, $u(t)$ as $r(t)= t\,u(t)$

The following figure shows unit ramp signal



So, the unit ramp signal exists for all positive values of 't' including zero. And its value increases linearly with respect to 't' during this interval. The value of unit ramp signal is zero for all negative values of 't'.

Unit Parabolic Signal

A unit parabolic signal, p(t) is defined as,

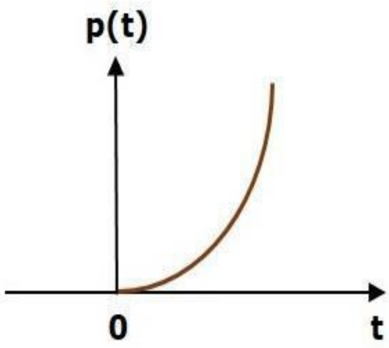
$p(t)=t^2/2$ ….. $t \geq 0$

$=0$ … … $t<0$

We can write unit parabolic signal, $p(t)$ in terms of the unit step signal, $u(t)$ as,

$p(t)=t^2/2\ u(t)$

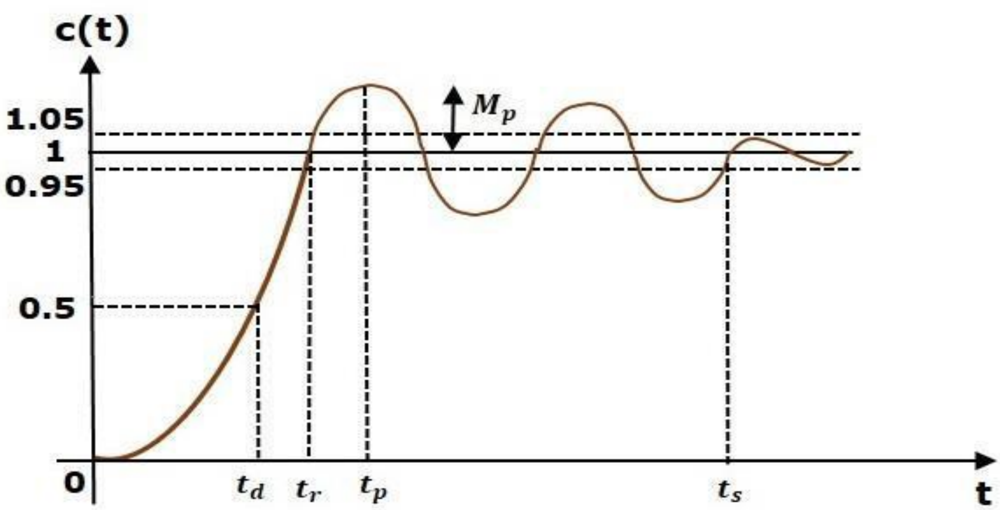
The following figure shows the unit parabolic signal.



So, the unit parabolic signal exists for all the positive values of 't' including zero. And its value increases non-linearly with respect to 't' during this interval. The value of the unit parabolic signal is zero for all the negative values of 't'.

Time Domain Specifications

In this lecture, let us discuss the time domain specifications of the second order system. The step response of the second order system for the under damped case is shown in the following figure:



All the time domain specifications are represented in this figure. The response up to the settling time is known as transient response and the response after the settling time is known as steady state response.

Delay Time

It is the time required for the response to reach half of its final value from the zero instant. It is denoted by t_d.

The final value of the step response is equal to one, Therefore at $t = t_d$, the value of the step response will be 0.5.

Rise Time

It is the time required for the response to rise from 0% to 100% of its final value. This is applicable for the under-damped systems. For the over-damped systems, consider the duration from 10% to 90% of the final value. Rise time is denoted by $\mathbf{t_r}$.

At $t = t_1 = 0$, $c(t) = 0$.

We know that the final value of the step response is one. Therefore, at $t=t_2$, the value of step response is one.

Peak Time

It is the time required for the response to reach the peak value for the first time. It is denoted by ***tp***. At $t=t_p$, the first derivate of the response is zero.

Peak overshoot M_p

is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum overshoot.

Mathematically, we can write it as

$M_p = c(t_p) - c(\infty)$

Where,

$c(t_p)$ is the peak value of the response.

$c(\infty)$ is the final (steady state) value of the response.

Settling time

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. The settling time is denoted by ***ts***.

The settling time for 5% tolerance band is -

$T_s = 3\tau$

The settling time for 2% tolerance band is -

$T_s = 4\tau$

Where, $\boldsymbol{\tau}$ is the time constant of the system