Degree of freedom

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Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as mechanism. It may be used for transmitting or transforming motion e.g. engine indicators, typewriter etc.

A mechanism with four links is known as simple mechanism, and the mechanism with more than four links is known as compound mechanism. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine. In such cases, the various links or
Degrees of Freedom

• It is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering purpose.

• It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of joints which it includes.
Consider a four bar chain, as shown in (a). A little consideration will show that only one variable such as \( \theta \) is needed to define the relative positions of all the links. In other words, we say that the number of degrees of freedom of a four bar chain is one. Now, let us consider a five bar chain, as shown in (b). In this case two variables such as \( \theta_1 \) and \( \theta_2 \) are needed to define completely the relative positions of all the links. Thus, we say that the number of degrees of freedom is two.

(a) Four bar chain. 

(b) Five bar chain.
Video describing DOF
In order to develop the relationship in general, consider two links $AB$ and $CD$ in a plane motion as shown in Fig. 2.

The link $AB$ with coordinate system $OXY$ is taken as the reference link (or fixed link). The position of point $P$ on the moving link $CD$ can be completely specified by the three variables, i.e., the
• the number of degrees of freedom of a mechanism is given by

\[ n = 3(l - 1) - 2j - h \]

• This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

• What is Kutzbach criterion for degree of freedom?

• The Kutzbach criterion is also called the mobility formula, because it computes the number of parameters that define the configuration of a linkage from the number of links and joints and the degree of freedom at each joint.
Consider a four bar chain, as shown in Fig 3.

As we know that each link of a mechanism has 3 degrees of freedom before it is connected to any other link. (2 translator & 1 rotational)

Look at Fig 4 (a):

When a link is connected to a fixed link by a turning pair (i.e. lower pair), two degrees of freedom are destroyed. So it may be clearly understood that the resulting four bar mechanism has one degree of freedom (i.e. n = 1) as shown in Fig 2 (e).
Finding the number of the degree of freedom for the following mechanism

Application of Kutzbach Criterion to Plane Mechanisms

How to apply Kutzbach Criterion to find number of degrees of freedom for mechanisms having no higher pair?

(a) Three-bar mechanism.  
(b) Four bar mechanism.  
(c) Five bar mechanism.  
(d) Five bar mechanism.  
(e) Six bar mechanism.
How to decide DOF?

Example - 1

Consider the arrangement of three links AB, BC and CA as shown in Fig.

- Number of links: \( l = 3 \)
- Number of binary joints: \( j = 3 \)

From equation (1),
\[
  n = 3 (l - 1) - 2j \\
  = 3 (3 - 1) - 2 \cdot 3 \\
  \therefore n = 0
\]

Here \( n = 0 \), means the mechanism forms a structure and no relative motion between the links is possible.
How to decide DOF? Example - 2

Consider the arrangement of four links AB, BC, CD and DA as shown in Fig.

- Number of links: \( l = 4 \)
- Number of binary joints: \( j = 4 \)

From equation (1),
\[
 n = 3 (l - 1) - 2j \\
= 3 (4 - 1) - 2 \cdot 4 \\
\therefore n = 1
\]

Here \( n = 1 \), means the mechanism can be driven by a single input motion.
How to decide DOF? Example - 3

Consider the arrangement of four links AB, BC, CD, DE and EA as shown in Fig.

- Number of links: \( l = 5 \)
- Number of binary joints: \( j = 5 \)

\[
 n = 3 (l - 1) - 2j \\
= 3 (5 - 1) - 2 \times 5 \\
\therefore n = 2
\]

From equation (1),

Kutzbach Criterion

Here \( n = 2 \), means two separate input motions are necessary to produce constrained motion for the mechanism.
How to decide DOF? Example - 4

Consider the arrangement of four links AB, BC, CD and DA as shown in Fig.

- Number of links: \( l = 5 \)
- Number of binary joints: \( j = 6 \)

From equation (1),
\[
n = 3 (l - 1) - 2j
\]
\[
= 3 (5 - 1) - 2 \times 6
\]
\[
= 0
\]

This equation gives \( n = 0 \), meaning the mechanism forms a structure and no relative motion between the links is possible.

Kutzbach Criterion

- There are 2 binary joints at B & D, and 2 ternary joints at A & C.
- So \( j = 2 + 2(2) = 6 \).
Fig. 5 Plane mechanisms.

The number of degrees of freedom or movability \((n)\) for some simple mechanisms having no higher pair \((i.e. h = 0)\), as shown in Fig. 5 are determined as follows:
1. The mechanism, as shown in Fig. 5.16 (a), has three links and three binary joints, \( i.e. \) \( l = 3 \) and \( j = 3 \).

\[
\therefore \quad n = 3 (3 - 1) - 2 \times 3 = 0
\]

2. The mechanism, as shown in Fig. 5.16 (b), has four links and four binary joints, \( i.e. \) \( l = 4 \) and \( j = 4 \).

\[
\therefore \quad n = 3 (4 - 1) - 2 \times 4 = 1
\]

3. The mechanism, as shown in Fig. 5.16 (c), has five links and five binary joints, \( i.e. \) \( l = 5 \), and \( j = 5 \).

\[
\therefore \quad n = 3 (5 - 1) - 2 \times 5 = 2
\]

4. The mechanism, as shown in Fig. 5.16 (d), has five links and six equivalent binary joints (because there are two binary joints at \( B \) and \( D \), and two ternary joints at \( A \) and \( C \)), \( i.e. \) \( l = 5 \) and \( j = 6 \).

\[
\therefore \quad n = 3 (5 - 1) - 2 \times 6 = 0
\]

5. The mechanism, as shown in Fig. 5.16 (e), has six links and eight equivalent binary joints (because there are four ternary joints at \( A, B, C \) and \( D \)), \( i.e. l = 6 \) and \( j = 8 \).

\[
\therefore \quad n = 3 (6 - 1) - 2 \times 8 = -1
\]
Examples

\[
F = 3(N-1) - 2j - 1h
\]

\[
\begin{align*}
N &= 4 \\
j &= 4 \\
h &= 0 \\
F &= 3(4-1)-(2*4)-0=1
\end{align*}
\]

\[
F = 3(N-1) - 2j - 1h
\]

\[
\begin{align*}
N &= 5 \\
j &= 6 \\
h &= 0 \\
F &= 3(5-1)-(2*6)-0=0
\end{align*}
\]
Finding degree of freedom

N = number of the link

$N = 7$
$\text{j} = 8$
$\text{h} = 1$

$F = 3(N-1) - 2\text{j} - 1\text{h}$
$F = 3(7-1) - 2\cdot8 - 1\cdot1 = 1$

$N = 8$
$\text{j} = 10$
$\text{h} = 0$

$F = 3(N-1) - 2\text{j} - 1\text{h}$
$F = 3(8-1) - 2\cdot10 - 0 = 1$
It may be noted that

(a) When $n = 0$, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Fig. 5.16 (a) and (d).

(b) When $n = 1$, then the mechanism can be driven by a single input motion, as shown in Fig. 5.16 (b).

(c) When $n = 2$, then two separate input motions are necessary to produce constrained motion for the mechanism, as shown in Fig. 5.16 (c).

(d) When $n = -1$ or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure, as shown in Fig. 5.16 (e).