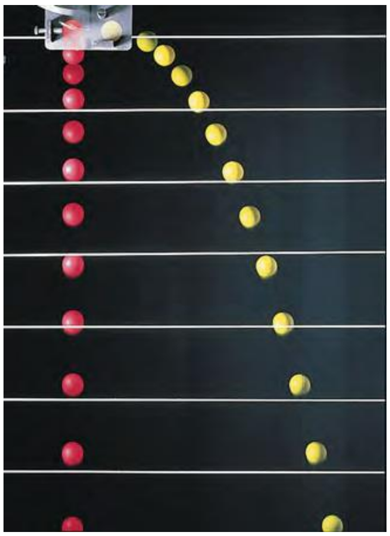
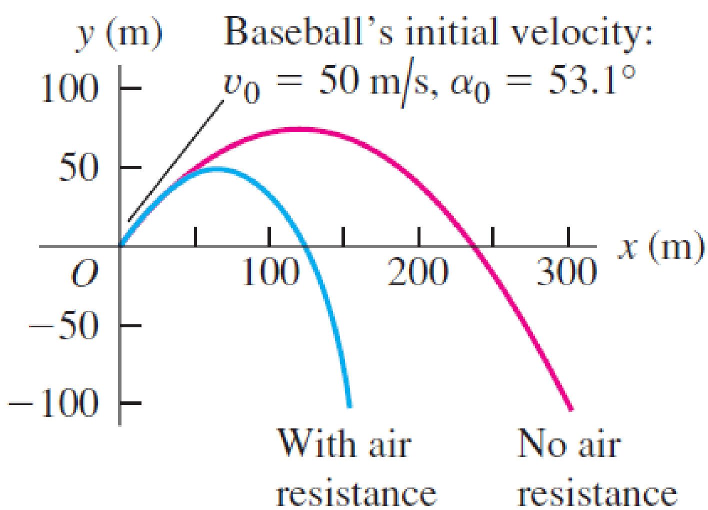
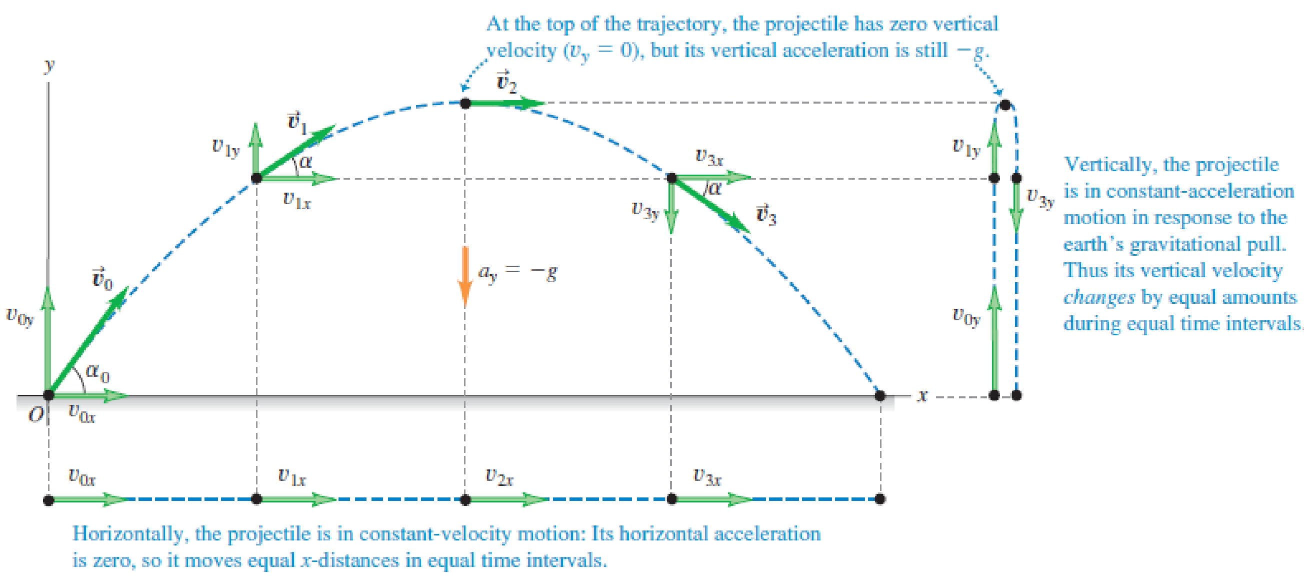
Chapter 8: projectile motion

Projectile Motion

A projectile is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its trajectory.







Projectile motion

I. Rolling Down From height

General Equations:

$d = v t$ (constant acceleration)

$v_F = v_0 + at$

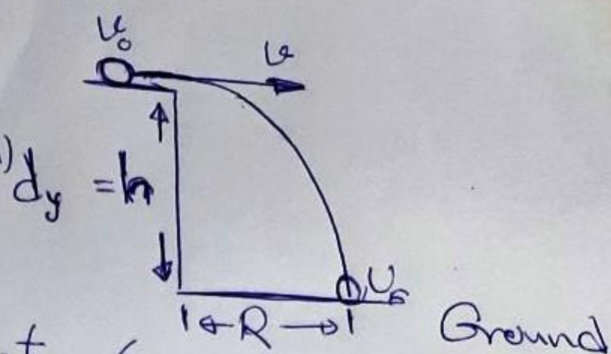
$v_F^2 = v_0^2 + 2ad$

$d = v_{av} t = \frac{1}{2}(v_0 + v_F) \cdot t$

$d = v_0 t + \frac{1}{2} a t^2$

$d = x_F - x_0$

$d = y_F - y_i$



* $d_y = v_{y_0} t + \frac{1}{2} a_y t^2$ (at y direction)

$v_{y_0} = 0$

$\therefore h = \frac{1}{2} a t^2$ when $a = g$

$\boxed{h = \frac{1}{2} g t^2}$ --- 1 to measure the Height

* $d = vt$

$d_x = v_x t$ at x-direction:

d_x is R (the Range of the Ground)

$\therefore \boxed{R = v_x t}$ -- 2 to Measure the Range

* $v_{Fy} = v_{0y} + a_y t$ when $v_{0y} = 0$

$v_{Fy} = a_y t$ as $a = g$

$\boxed{v_{Fy} = gt}$ to measure the Final velocity --- 3

* also:

$v = \sqrt{v_x^2 + v_y^2}$ --- 4

$\theta = \tan^{-1} \frac{v_y}{v_x}$ --- 5

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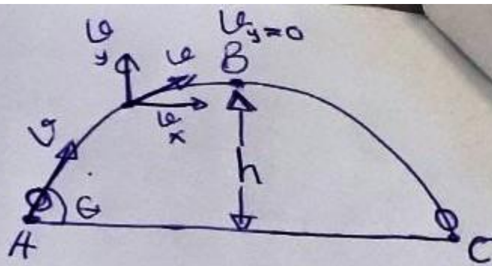
II Symmetric



* $v_{yf} = v_{y0} + a_y t$

$v_{yf} = 0$ at Point B

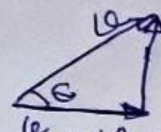
$v_{y0} = v\sin\theta$

$\therefore -v\sin\theta = gt$

$$\boxed{t_{A\to B} = -\frac{v\sin\theta}{g}}$$

where $g = -9.8\,m/s^2$

$$\boxed{t_{A-C} = 2\left(-\frac{v\sin\theta}{g}\right)}$$



$v_y = v\sin\theta$

$v_x = v\cos\theta$

$\theta = \tan^{-1}\frac{v_y}{v_x}$

* height at Point B

$v_{yf}^2 = v_{y0}^2 + 2a_y d_y$

$0 = (v\sin\theta)^2 + 2gh$

$$\therefore \boxed{h = -\frac{v^2\sin^2\theta}{2g}}$$

* Range:

$$\boxed{R_{A\to C} = v_x t} \quad \text{or} \quad \boxed{R = -\frac{v^2\sin 2\theta}{g}}$$

also:

$r = \sqrt{x^2 + y^2}$

$v = \sqrt{v_x^2 + v_y^2}$

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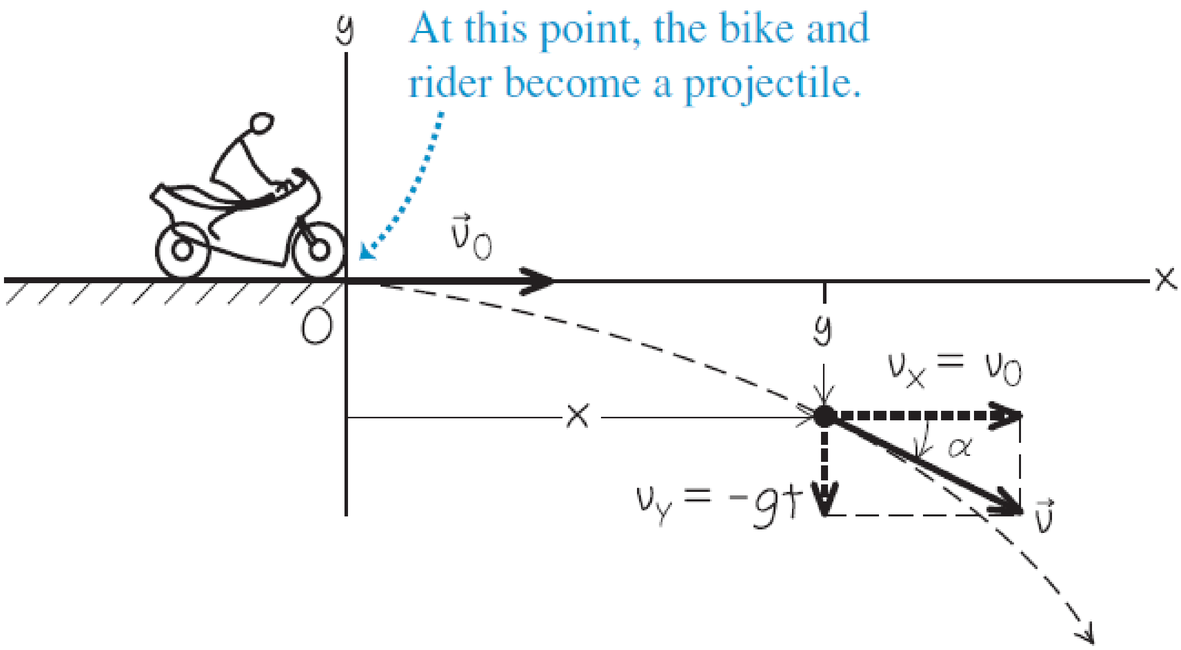
Examples 1

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9 m/s Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff.

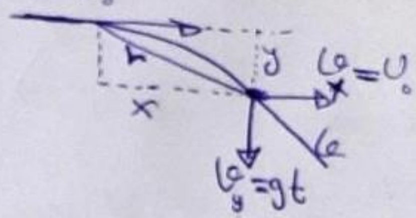


Ex 1

Sol:

$v_0 = 9\ m/s$

$t = 0.5\ sec$



$x = v_{0x} t$

$= 9 * 0.5 = 4.5\ m$

$y = v_{0y} t + \frac{1}{2} g t^2$

$= 0 + \frac{1}{2}(-9.8)(0.5)^2$

$y = -12\ m$

$r = \sqrt{x^2 + y^2} = \sqrt{(4.5)^2 + (-12)^2}$

$r = 4.7\ m$

$v_x = v_{0x} = 9\ m/s$

$v_y = v_{0y} + gt = 0 + (-9.8) * 0.5$

$v_y = -4.9\ m/s$

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{9^2 + 4.9^2} = 10.2\ m/s$

$\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{-4.9}{9}$

$\theta = -29^\circ$

Example 2

A batter hits a baseball so that it leaves the bat at speed 37 m/s at an angle $\theta = 53.1^0$. (a) Find the position of the ball and its velocity (magnitude and direction) at t=2 sec (b) Find the time when the ball reaches the highest point of its flight, and its height h at this time. (c) Find the horizontal range R—that is, the horizontal distance from the starting point to where the ball hits the ground.

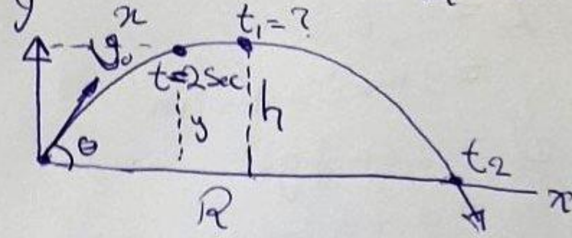
H

or, solution

Ex2

$v_0 = 37\,m/s$, $\theta = 53.1$

$t = 2\,Sec$



a)

$v_{ox} = v_0 \cos\theta$

$= 37 \cos 53.1 = 22.2\,m/s$

$v_{oy} = v_0 \sin\theta$

$= 37 \sin 53.1 = 29.6\,m/s$

$x = v_{ox} t = 22.2 * 2 = 44.4\,m$

$y = v_{oy} t + \frac{1}{2} g t^2 = 29.6 * 2 - \frac{1}{2}(9.8) * 2^2$

$y = 39.6\,m$

$r = \sqrt{x^2 + y^2} = \sqrt{44.4^2 + 39.6^2}$

$v_x = v_{ox} + a_x t \quad a_x = 0 \quad \therefore \; v_x = v_{xo} = 22.2\,m/s$

$v_y = v_{oy} + g t = 29.6 - 9.8 * 2 = 10\,m/s$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{22.2^2 + 10^2} = 24.4\,m/s$

$\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{10}{22.2} = 24.2^\circ$

b)

$v_y = v_{oy} + g t_1 = 0 \;\therefore\; t = \frac{-v_{oy}}{g} = -\frac{29.6}{-9.8} = 3.02\,Sec$

$h = v_{oy} t + \frac{1}{2} g t_1^2$

$= 29.6 * 3.02 - \frac{1}{2} 9.8 (3.02)^2$

$h = 44.7\,m$

$v_y = v_{oy} - g t_2$

$= 29.6 - 9.8 * 6.04$

$= -29.6\,m/s$

c) $R = v_{ox} t_2 = 22.2 * 6.04 = 134\,m$

But , $t_2 = 2 * t_1 = 6.04\,Sec$

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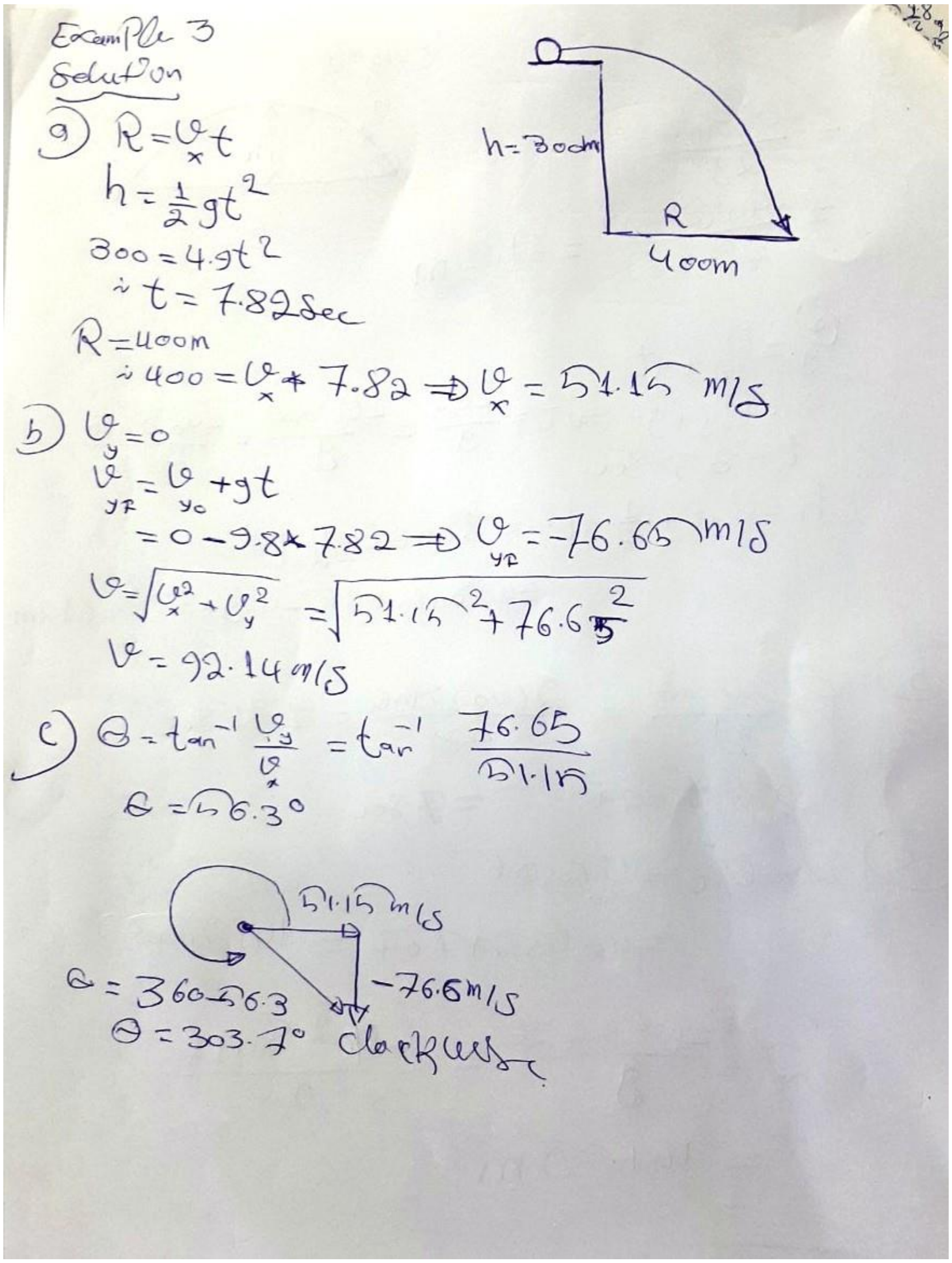
Example 3

A ball is kicked horizontally from the roof of a building that is 300m tall and lands about 400m from the base of the building.

a) Calculate the initial speed of the ball
b) Calculate the final speed of the ball just before it hits the ground.
c) Find the angle of the ball relative to the positive x-axis.



Example 3
Solution

h=300m
R
400m

a) $R = v_x t$

$h = \frac{1}{2} g t^2$

$300 = 4.9 t^2$

$\therefore t = 7.82\,sec$

$R = 400m$

$\therefore 400 = v_x * 7.82 \Rightarrow v_x = 51.15\ m/s$

b) $v_y = 0$

$v_{yf} = v_{yo} + gt$

$= 0 - 9.8 * 7.82 \Rightarrow v_{yf} = -76.65\ m/s$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{51.15^2 + 76.65^2}$

$v = 92.14\ m/s$

c) $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{76.65}{51.15}$

$\theta = 56.3^\circ$

51.15 m/s
−76.6 m/s

$\theta = 360 - 56.3$

$\theta = 303.7^\circ$ clockwise

Example 4

A ball is kicked off the ground at 40m/s at an angle of 60^0. Find

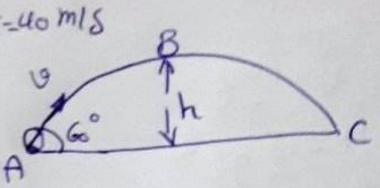
a) Maximum height

b) Time it takes to hit the ground

c) Range of the ball

Solve Ex4

$v_0 = 40\ m/s$



a)

$$h = \frac{-v_0^2 \sin^2\theta}{2g}$$

$$= \frac{-40^2(\sin 60)^2}{2(-9.8)} = 61.2\ m$$

or

$$v_y = v_{0y} + gt$$

$$0 = v_{0y} + gt \Rightarrow t = \frac{-v_{0y}}{g} = \frac{-v_0 \sin\theta}{g} = \frac{-40 \sin 60}{-9.8}$$

$$t = 3.5\ \text{Sec}$$

$$h = v_{0y}t + \frac{1}{2}gt^2 = v_0 \sin\theta\, t + \frac{1}{2}gt^2$$

$$= 40 \sin 60 * 3.5 - 4.9 * 3.5^2 = 61.2m$$

b) $t = \dfrac{2v_0 \sin\theta}{g} = \dfrac{2(40)\sin 60}{9.8} = 7.07\ \text{Sec}$

or: $t = 2 * 3.5 = 7\ \text{Sec}$

c) $R = v_x t = v_{x0} \cos\theta\, t$

$$= 40 \cos 60 * 7.07 = 141.4m$$

or

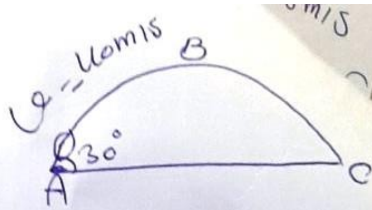
$$R = \frac{-v_0^2 \sin 2\theta}{g} = \frac{-40^2 \sin 120}{-9.8}$$

$$= 141.39\ m$$

Example 5

A ball is kicked from the ground at a speed 40m/s at an angle of 30^0. Calculate the horizontal and vertical velocity and acceleration components when the ball was kicked and when it reaches its maximum height.

Ex.5 Solution



(a) at point A

$v_x = v\cos\theta$

$= 40\cos 30$

$= 34.6 m/s$

$v_y = v\sin\theta$

$= 40\sin 30 = 20 m/s$

$a_x = 0$

$a_y = g = -9.8 m/s^2$

(b) at B.

$v_x = 3.4 m/s$

$v_y = 0$

$a_x = 0$

$a_y = -9.8 m/s^2$

(c) at C

$v_x = 34.6 m/s$

$a_x = 0$

$a_y = g = -9.8 m/s^2$

$v_y = -20 m/s$

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