Chapter six Newton's Laws of Motion, & work

I. Law of Inertia

II. F=ma

III. Action-Reaction

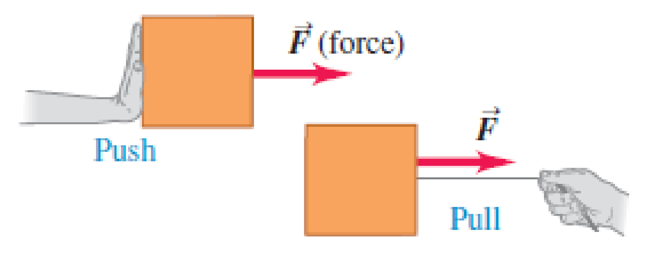
In this chapter, you learn the concept of:

- Newton's law
- Friction
- Force
- Hooke's Law
- Energy
- Work

Force and Interactions

- a **force is a push or a pull an object.**

- Abetter definition is that a force is an *interaction between two bodies or between a body and its environment*. That's why we always refer to the force that one body *exerts on a second body*.



- A force is a vector quantity, with magnitude and direction.

- The SI unit of the magnitude of force is the *Newton*.

Examples of contact and distance forces

- **Contact Forces**
  Frictional Force
  Tension Force
  Normal Force
  Spring Force

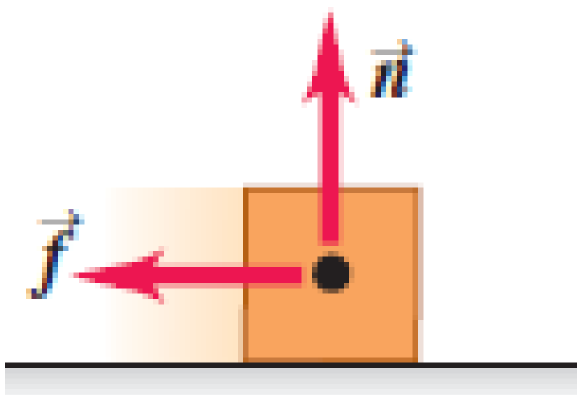
Action-at-a-Distance Forces
Gravitational Force
Electrical Force
Magnetic Force

Force and Interactions

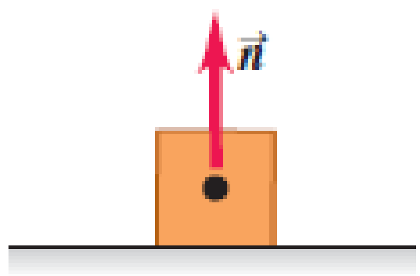
- When a force involves direct contact between two bodies, such as a push or pull that you exert on an object with your hand, we call it a **contact force.**
- **Normal force is exerted on an object by any surface with which it is in** contact.
- The adjective *normal means that the force always acts perpendicular to* the surface of contact, no matter what the angle of that surface.
- **friction force exerted on an object by a surface acts *parallel to the*** surface, in the direction that opposes sliding.
- The pulling force exerted by a stretched rope or cord on an object to which it's attached is called a **tension force.**
- When you tug on your dog's leash, the force that pulls on her collar is a tension force.

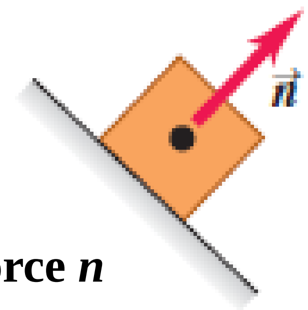
Force and Interactions

Weight *w*: ***The pull of gravity on an object*** is a long-range force (a force that acts over a distance).

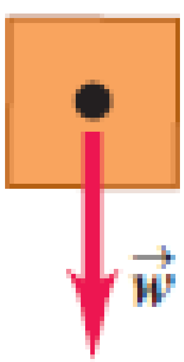


Friction force *f*:

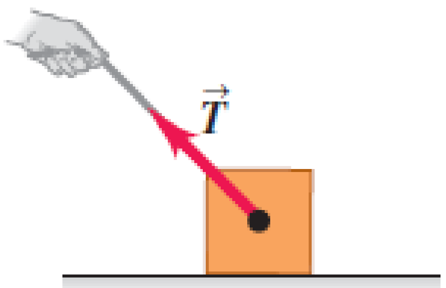




Normal force *n*



Weight

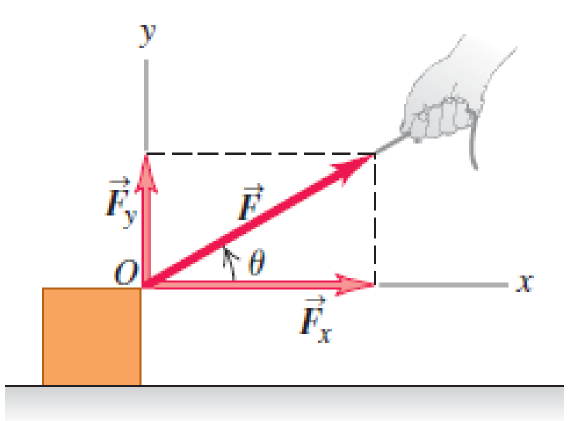


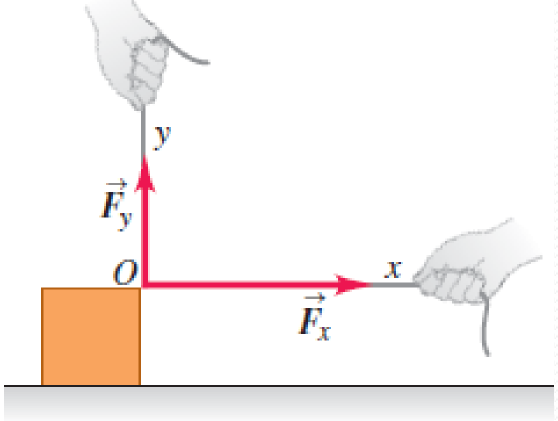
Tension force *T*:

Superposition of Forces

- The force , which acts at an angle from the *x-axis, may be replaced by its* rectangular component vectors F*x and Fy.*

- Component vectors: ***Fx and Fy*** Componentes: *Fx=F cos ϴ and Fy = F sin ϴ.*
- Component vectors ***Fx and Fy together*** have the same effect as original force ***F.***





EXAMPLE 1

Three professional wrestlers are fighting over a champion's belt. Figure 4.1 shows the horizontal force each wrestler applies to the belt, as viewed from above. The forces have magnitudes $F_1 = 250\ \text{N}$, $F_2 = 50\ \text{N}$, and $F_3 = 120\ \text{N}$. Find the x- and y-components of the net force on the belt, and find its magnitude and direction.

SOLUTION

$$F_{1x} = (250\ \text{N})\cos 127^\circ = -150\ \text{N}$$

$$F_{1y} = (250\ \text{N})\sin 127^\circ = 200\ \text{N}$$

$$F_{2x} = (50\ \text{N})\cos 0^\circ = 50\ \text{N}$$

$$F_{2y} = (50\ \text{N})\sin 0^\circ = 0\ \text{N}$$

$$F_{3x} = (120\ \text{N})\cos 270^\circ = 0\ \text{N}$$

$$F_{3y} = (120\ \text{N})\sin 270^\circ = -120\ \text{N}$$

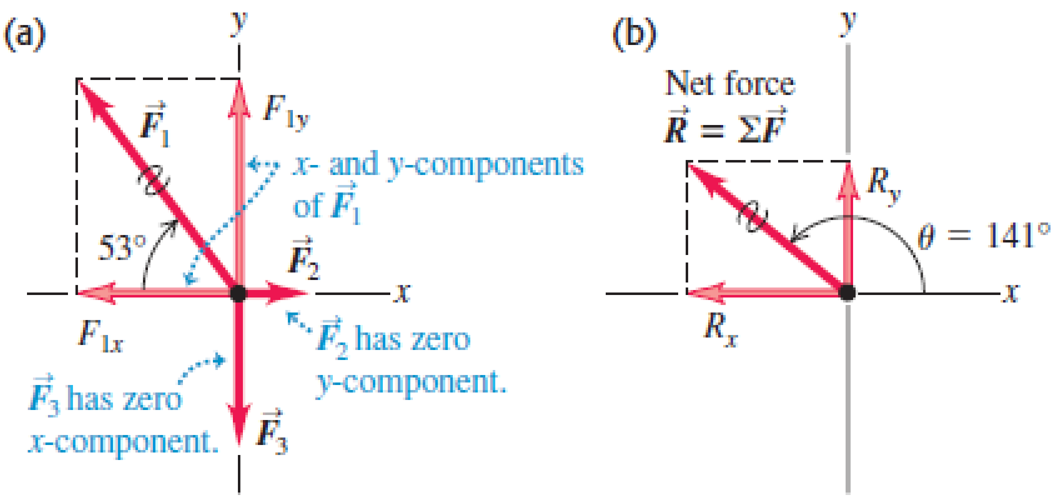


Figure 4.1

EXAMPLE 1 continue

the net force $\vec{R} = \sum \vec{F}$ has components

$$R_x = F_{1x} + F_{2x} + F_{3x} = (-150 \text{ N}) + 50 \text{ N} + 0 \text{ N} = -100 \text{ N}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 200 \text{ N} + 0 \text{ N} + (-120 \text{ N}) = 80 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-100 \text{ N})^2 + (80 \text{ N})^2} = 128 \text{ N}$$

Newton's Laws of Motion

- 1st Law – An object at rest will stay at rest, and an object in motion will stay in motion at constant velocity, unless acted upon by an unbalanced force.
- 2nd Law – Force equals mass times acceleration.
- 3rd Law – For every action there is an equal and opposite reaction.

1^{st} Law

- *Inertia is the tendency of an object to resist changes in its velocity: whether in motion or motionless.*

These pumpkins will not move unless acted on by an unbalanced force.

2^{nd} Law

- *The net force of an object is equal to the product of its mass and acceleration, or F=ma.*

- When mass is in kilograms and acceleration is in m/s/s, the unit of force is in Newton (N).

- One newton is equal to the force required to accelerate one kilogram of mass at one meter/second/second.

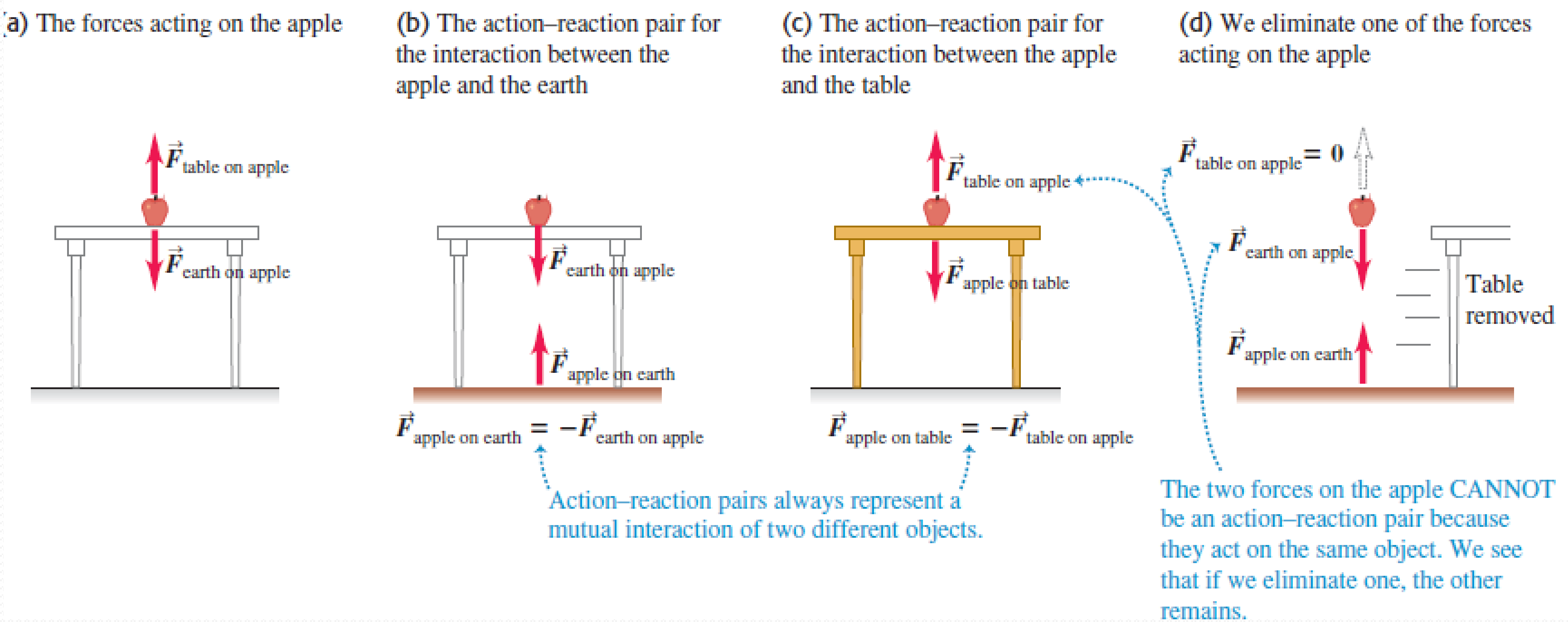
3rd Law

- For every action, there is an equal and opposite reaction .

- According to Newton, whenever objects A and B interact with each other, they exert forces upon each other. When you sit in your chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body.

- There are two forces resulting from this interaction - a force on the chair and a force on your body. These two forces are **called *action* and *reaction* forces. They have opposite directions.**

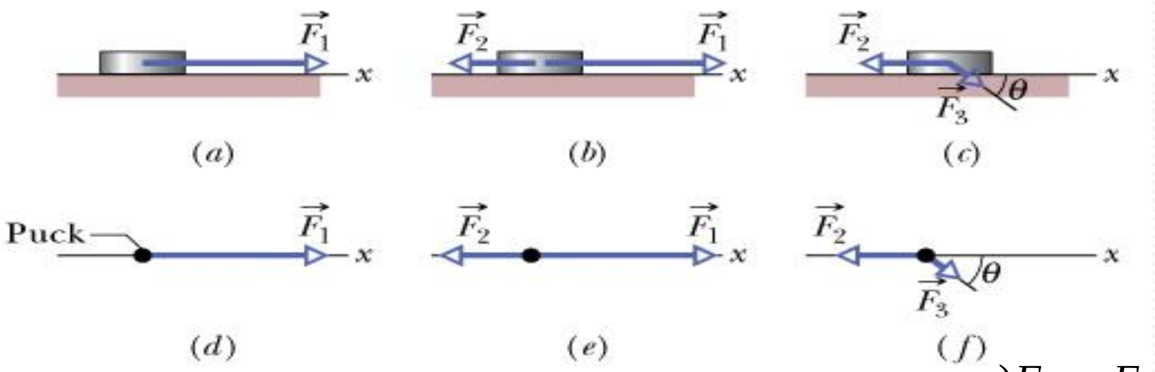
3rd Law



(a) The forces acting on the apple
(b) The action–reaction pair for the interaction between the apple and the earth
(c) The action–reaction pair for the interaction between the apple and the table
(d) We eliminate one of the forces acting on the apple
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{apple on earth}}$
$\vec{F}_{\text{apple on earth}} = -\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{apple on table}}$
$\vec{F}_{\text{apple on table}} = -\vec{F}_{\text{table on apple}}$
Action–reaction pairs always represent a mutual interaction of two different objects.
$\vec{F}_{\text{table on apple}} = \mathbf{0}$
$\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{apple on earth}}$
Table removed
The two forces on the apple CANNOT be an action–reaction pair because they act on the same object. We see that if we eliminate one, the other remains.

Sample 2

- One or two forces act on a puck that moves over frictionless ice along an *x* axis, in one-dimensional motion. The puck's mass is *m* = 0.20 kg. Forces **F₁** and **F₂** and are directed along the x axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\mathbf{F_3}$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?



$$F_{net,x} = ma_x$$

$$a)\ F_1 = ma_x$$

$$a_x = \frac{F_1}{m} = \frac{4.0\,\text{N}}{0.2\,\text{kg}} = 20\,\text{m/s}^2$$

$$b)\ F_1 - F_2 = ma_x$$

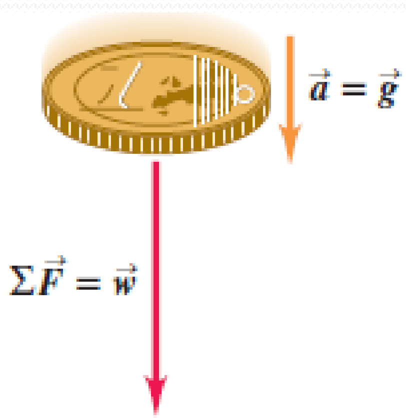
$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\,\text{N} - 2.0\,\text{N}}{0.2\,\text{kg}} = 10\,\text{m/s}^2$$

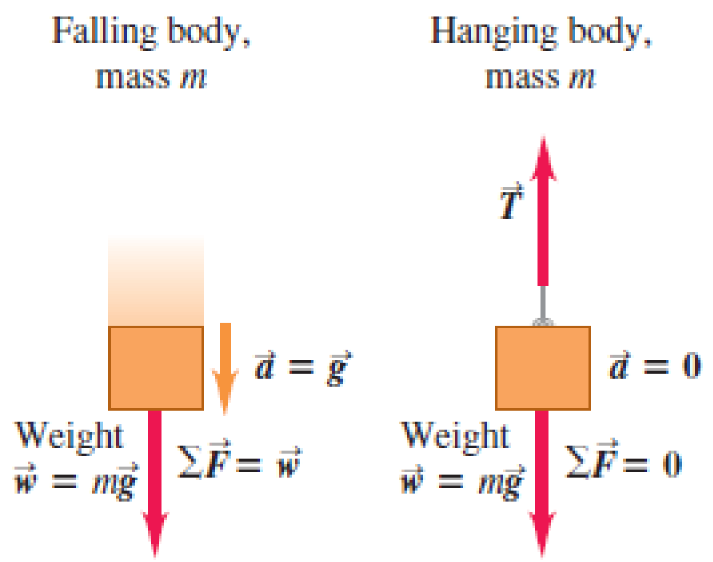
$$c)\ F_{3,x} - F_2 = ma_x \qquad F_{3,x} = F_3\cos\theta$$

$$a_x = \frac{F_3\cos\theta - F_2}{m} = \frac{1.0\,\text{N}\cos 30^\circ - 2.0\,\text{N}}{0.2\,\text{kg}} = -5.7\,\text{m/s}^2$$

Mass and Weight

- One of the most familiar forces is the *weight of a body, which is the gravitational* force that the earth exerts on the body.
- $w = mg$ (magnitude of the weight of a body of mass m).
- Hence the magnitude *w of a body's weight is directly proportional to its mass m.*





- The relationship of mass to weight: $\vec{w} = m\vec{g}$.
- This relationship is the same whether a body is falling or stationary.

Example 3

A 2.49×10^4 N Rolls-Royce Phantom traveling in the $+x$-direction makes an emergency stop; the x-component of the net force acting on it is -1.83×10^4 N. What is its acceleration?

SOLUTION

2.49×10^4 N is the car's *weight*,

ITE: The mass of the car is

$$m = \frac{w}{g} = \frac{2.49 \times 10^4 \text{ N}}{9.80 \text{ m/s}^2} = \frac{2.49 \times 10^4 \text{ kg} \cdot \text{m/s}^2}{9.80 \text{ m/s}^2}$$

$$= 2540 \text{ kg}$$

Then $\sum F_x = ma_x$ gives

$$a_x = \frac{\sum F_x}{m} = \frac{-1.83 \times 10^4 \text{ N}}{2540 \text{ kg}} = \frac{-1.83 \times 10^4 \text{ kg} \cdot \text{m/s}^2}{2540 \text{ kg}}$$

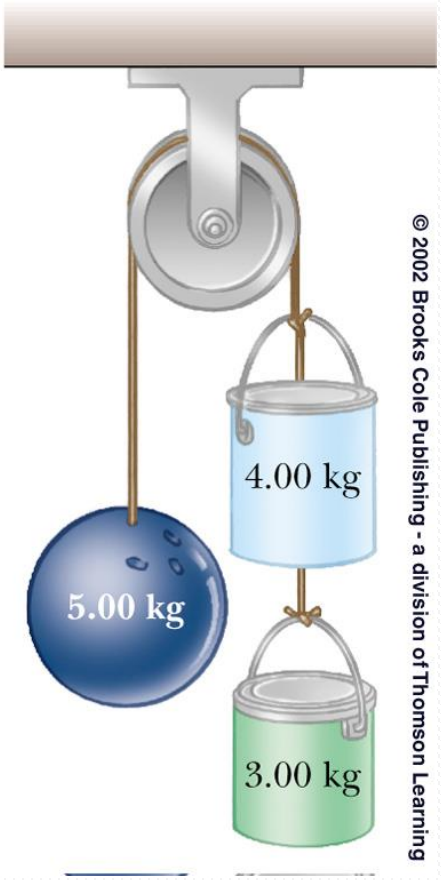
$$= -7.20 \text{ m/s}^2$$

Rolls-Royce Phantom

Rules for Ropes and Pulleys

Tension Force: *T*

- Force from rope points AWAY from object
  - (Rope can only pull)
- Magnitude of the force is Tension
  - Tension is same everywhere in the rope
- Tension does not change when going over pulley

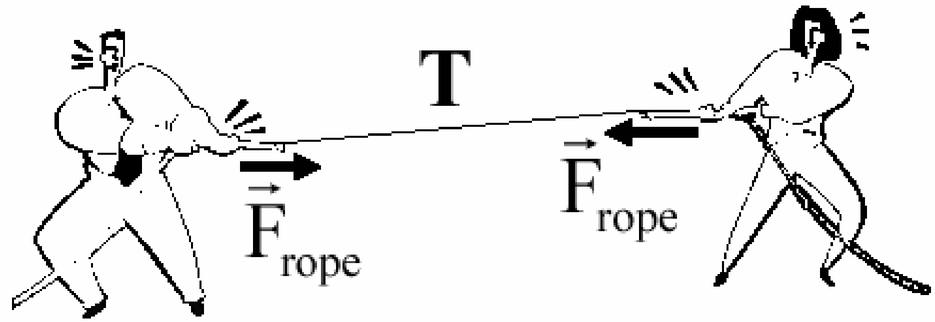


- **Tension Force (**F_{tens}**):** The tension force is the force that is transmitted through a string, rope, cable or wire when it is pulled tight by forces acting from opposite ends. The tension force is directed along the length of the wire and pulls equally on the objects on the opposite ends of the wire.

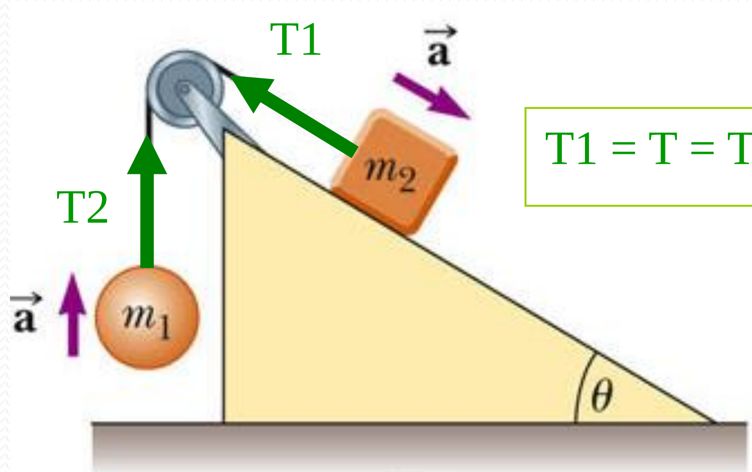
Approximations: Neglect mass of rope and pulley, neglect friction in pulley

Tension Force: *T*

- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ……) and away from the object
- Magnitude: depend on situation



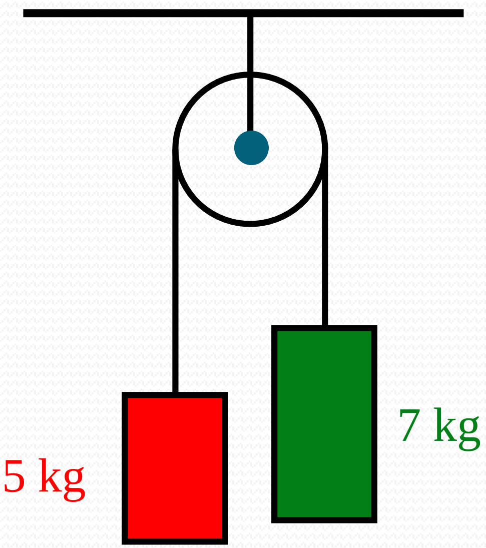
$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$



T1 = T = T2

Example 5 HW

a) Find acceleration
b) Find T, the tension in the string
c) Find force ceiling must exert on pulley



a) a = g/6 = 1.635 m/s^2
b) T = 57.2 N
c) F_{pulley}=2T = 114.5 N

Friction

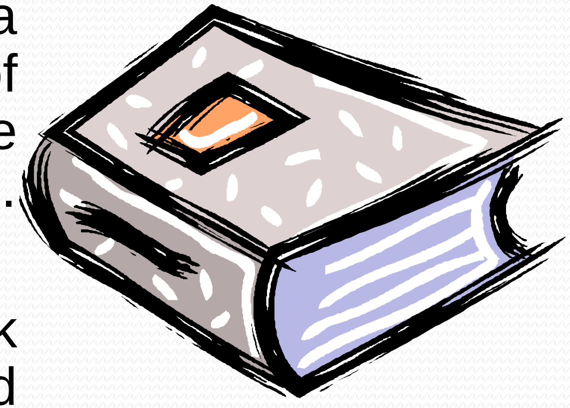
- *Objects on earth, unlike the frictionless space the moon travels through, are under the influence of friction.*

- **What is friction?**

  Friction is a force that resists motion. It occurs when one object is in contact with another object. When two surfaces are pressed together, the molecules of these surfaces come in contact and experience attractive forces between one another. In addition, rough materials can 'catch' on other materials, further impeding (stopping or slowing down) their motion.

- Friction is found almost everywhere.
- The only place where there is no friction is in a vacuum, as even air has friction.

- Slide a book across a table and watch it slide to a rest position. The book comes to a rest because of the *presence* of a force - that force being the force of friction - which brings the book to a rest position.



- In the absence of a force of friction, the book would continue in motion with the same speed and direction - forever! (Or at least to the end of the table top.)

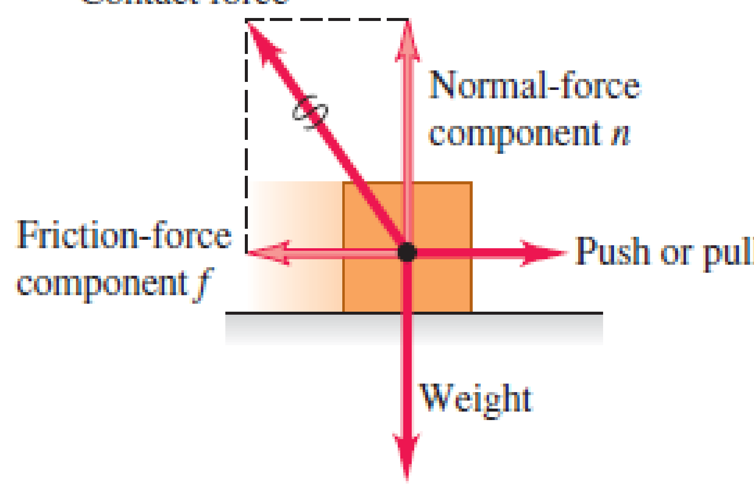
- The perpendicular component vector is

The normal force, denoted by (n).

- The component vector parallel to the surface

(and perpendicular to n is the **friction force (f).**

The friction and normal forces are really components of a single contact force.



- **Friction has two types: Kinetic and Static Friction**
- The kind of friction that acts when a body slides over a surface is called a **kinetic friction force.**

- **The adjective "kinetic" and the subscript "k" remind us** that the two surfaces are moving relative to each other.

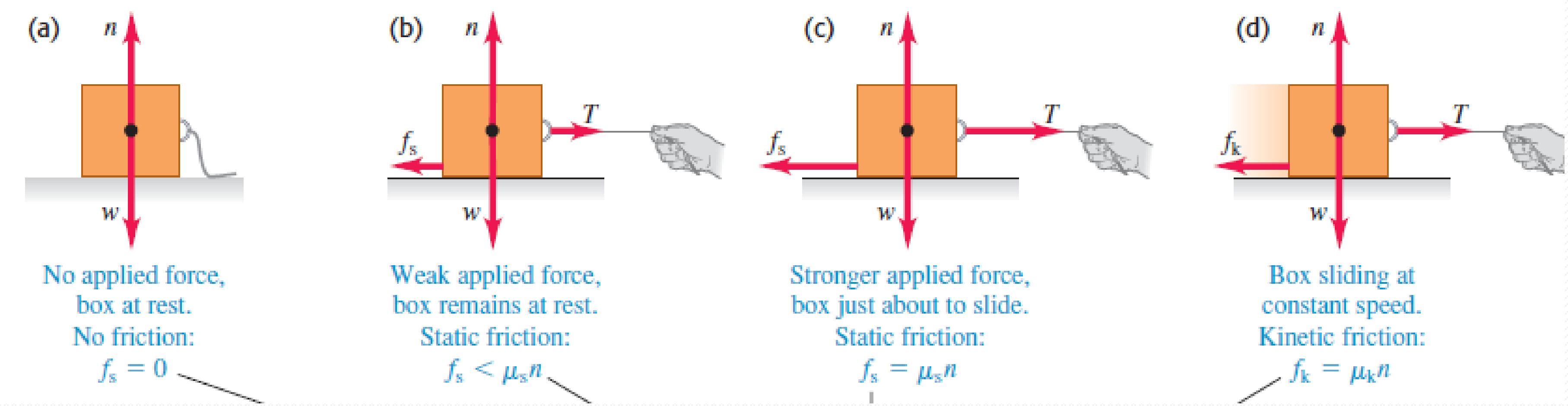
(magnitude of kinetic friction force) is

$$f_k = \mu_k n$$

- Where μ_k (pronounced "mu-sub-k") is a constant called the **coefficient of kinetic friction.**

- Friction forces may also act when there is *no relative motion. If you try to* slide a box across the floor, the box may not move at all because the floor exerts an equal and opposite friction force on the box. This is called a **static friction force.**
- (magnitude of kinetic friction force) is $f_s = \mu_s n$

5.19 (a), (b), (c) When there is no relative motion, the magnitude of the static friction force f_s is less than or equal to $\mu_s n$. (d) When there is relative motion, the magnitude of the kinetic friction force f_k equals $\mu_k n$. (e) A graph of the friction force magnitude f as a function of the magnitude T of the applied force. The kinetic friction force varies somewhat as intermolecular bonds form and break.



Example 6

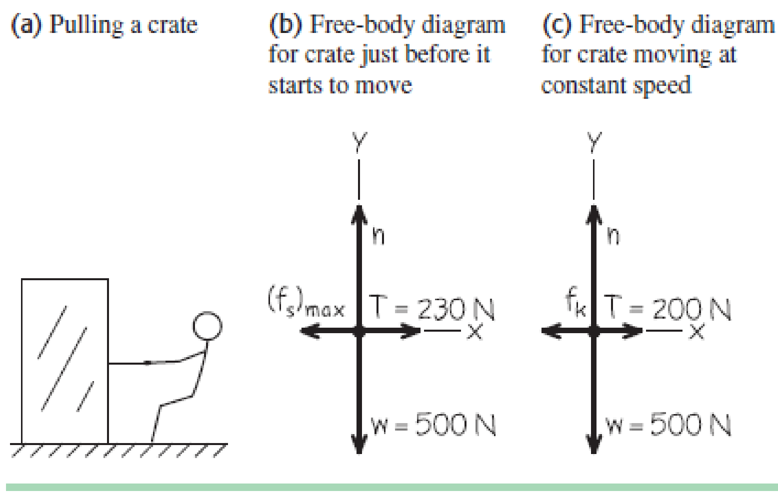
- You want to move a 500-N crate across a level floor. To start the crate moving, you have to pull with a 230-N horizontal force. Once the crate "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?
- Solution:

Just before the crate starts to move

$$\sum F_x = T + (-(f_s)_{max}) = 0 \quad \text{so} \quad (f_s)_{max} = T = 230\text{ N}$$
$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\text{ N}$$

Now we solve Eq. (5.6), $(f_s)_{max} = \mu_s n$, for the value of μ_s:

$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{230\text{ N}}{500\text{ N}} = 0.46$$



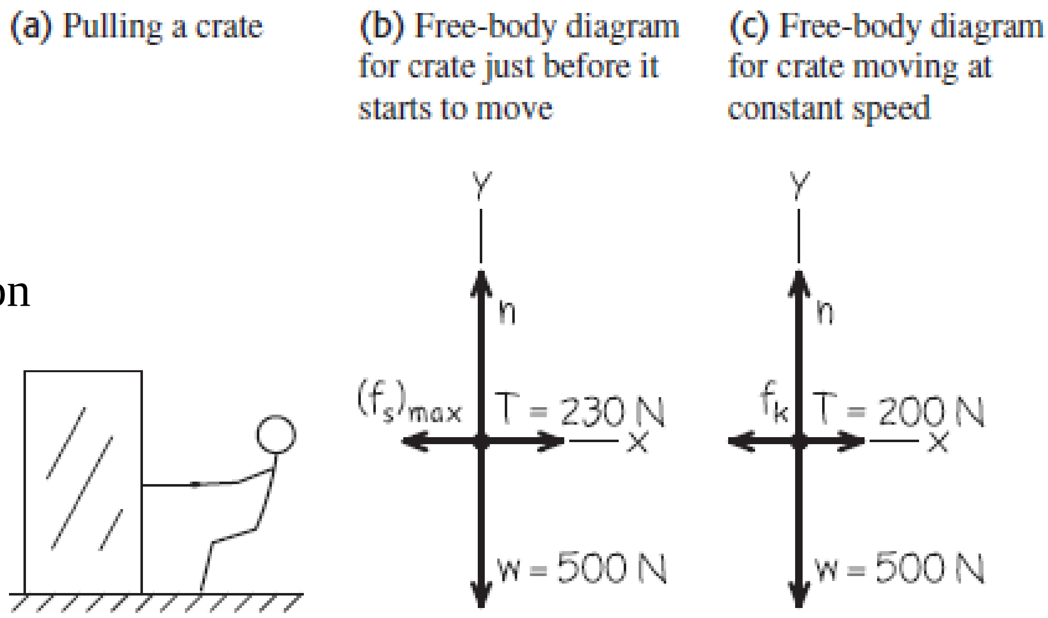
After the crate starts to move (Fig. 5.20c) we have

$$\sum F_x = T + (-f_k) = 0 \quad \text{so} \quad f_k = T = 200\ \text{N}$$
$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\ \text{N}$$

Using $f_k = \mu_k n$ from Eq. (5.5), we find

$$\mu_k = \frac{f_k}{n} = \frac{200\ \text{N}}{500\ \text{N}} = 0.40$$

As expected, the coefficient of kinetic friction is less than the coefficient of static friction.



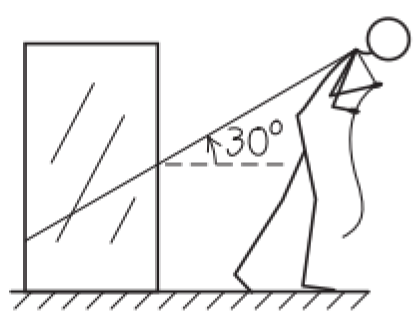
- In Example 6, suppose you move the crate by pulling upward on the rope at an angle of above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that $\mu_k = 0.40$.

$f_k = \mu_k n$, we have

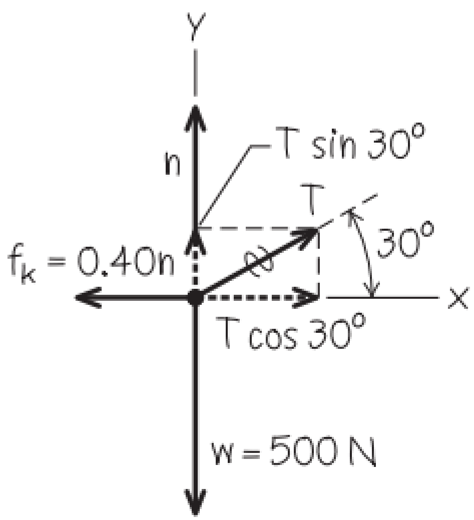
$$\sum F_x = T\cos 30° + (-f_k) = 0 \quad \text{so} \quad T\cos 30° = \mu_k n$$
$$\sum F_y = T\sin 30° + n + (-w) = 0 \quad \text{so} \quad n = w - T\sin 30°$$

$$T\cos 30° = \mu_k(w - T\sin 30°)$$
$$T = \frac{\mu_k w}{\cos 30° + \mu_k \sin 30°} = 188\ \text{N}$$

(a) Pulling a crate at an angle



(b) Free-body diagram for moving crate



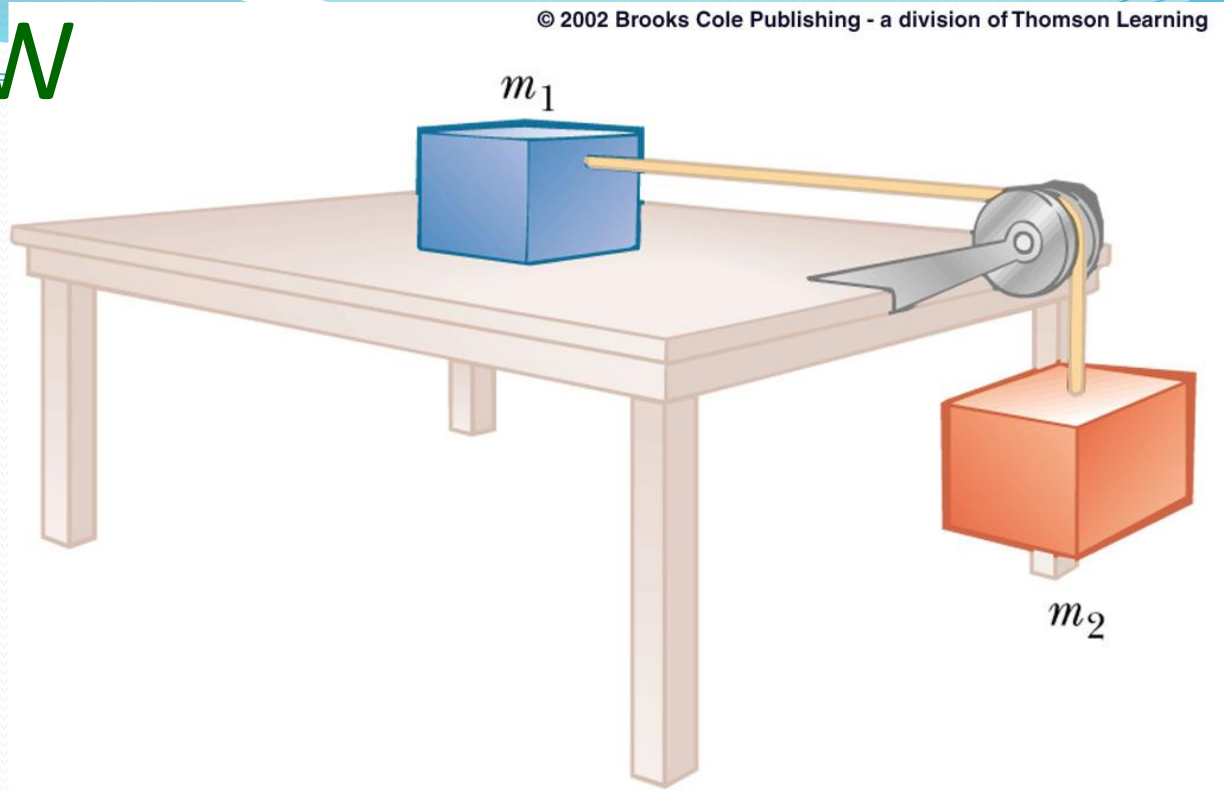
$$n = w - T\sin 30° = (500\ \text{N}) - (188\ \text{N})\sin 30° = 406\ \text{N}$$

Example 7 HW

The man pushes/pulls with a force of 200 N. The
child and sled combo has a mass of 30 kg and the coefficient of kinetic friction is 0.15. For each case:
What is the frictional force opposing his efforts?
What is the acceleration of the child?

f=59 N, a=3.80 m/s^2 / f=29.1 N, a=4.8 m/s^2

Example 8 HW



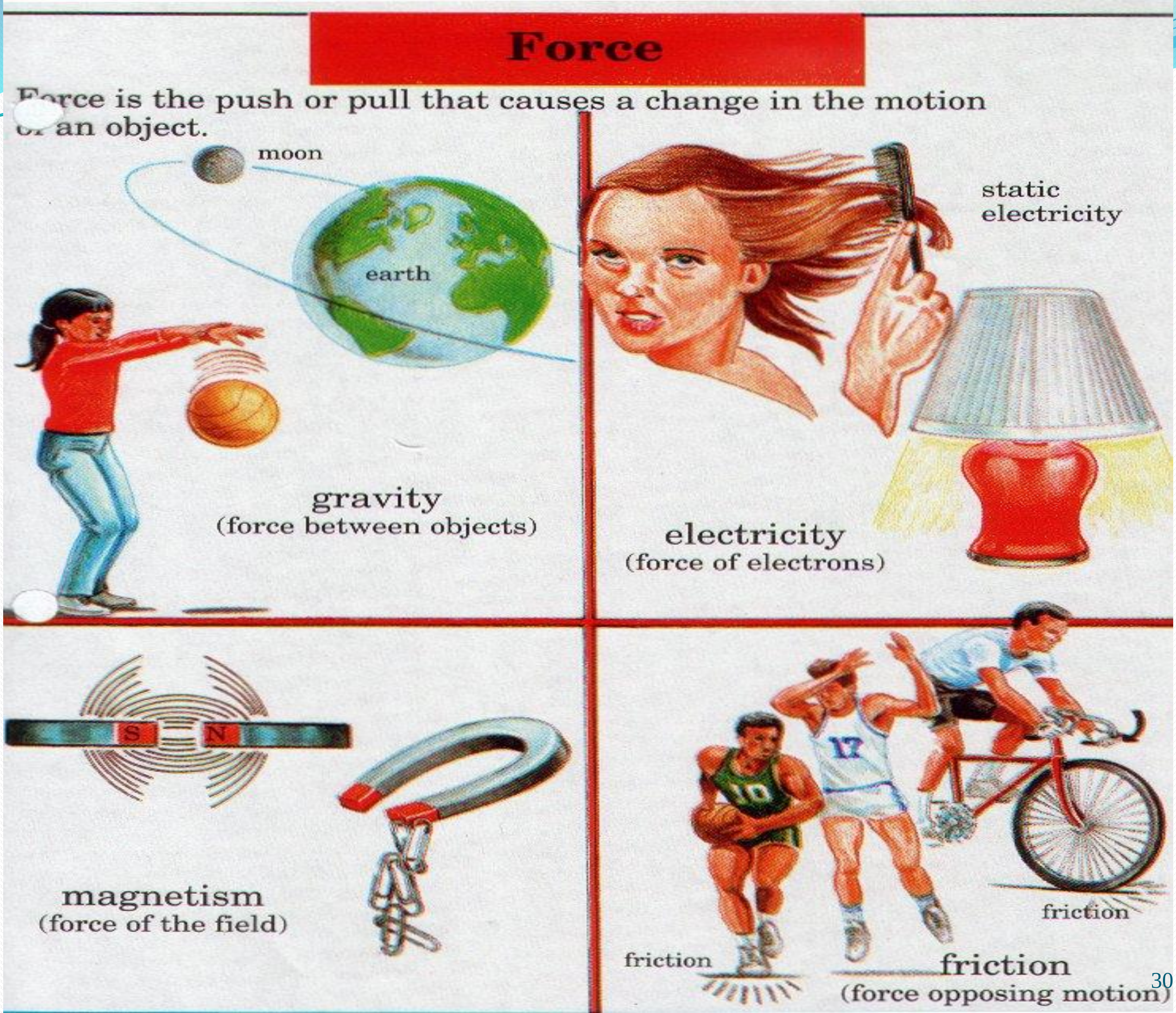
Given m_1 = 10 kg and m_2 = 5 kg:

a) What value of μ_s would stop the block from sliding?

b) If the box is sliding and μ_k = 0.2, what is the acceleration?

c) What is the tension of the rope?

a) μ_s = 0.5 b) a=1.96 m/s^2 c) 39.25 N

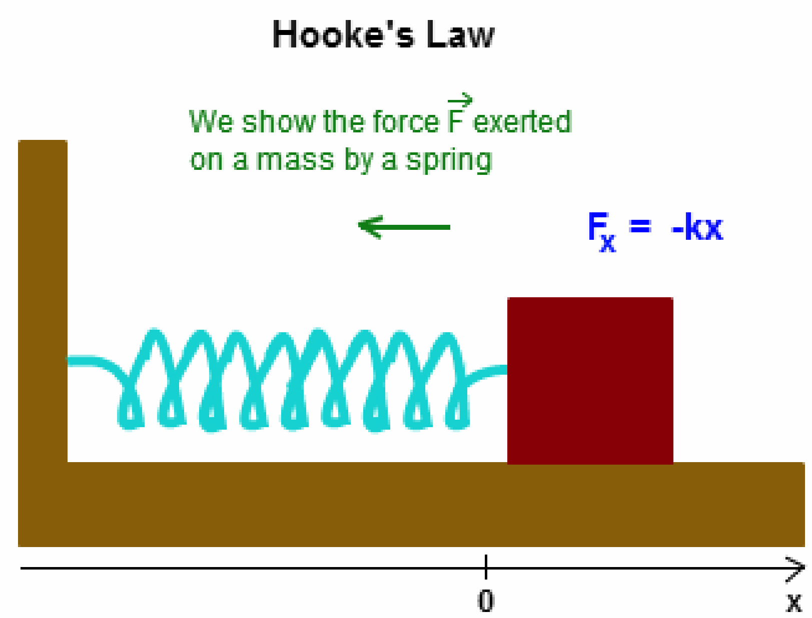

Force
Force is the push or pull that causes a change in the motion of an object.
moon
earth
gravity
(force between objects)
static electricity
electricity
(force of electrons)
S
N
magnetism
(force of the field)
17
10
friction
friction
friction
(force opposing motion)

Hooke's Law

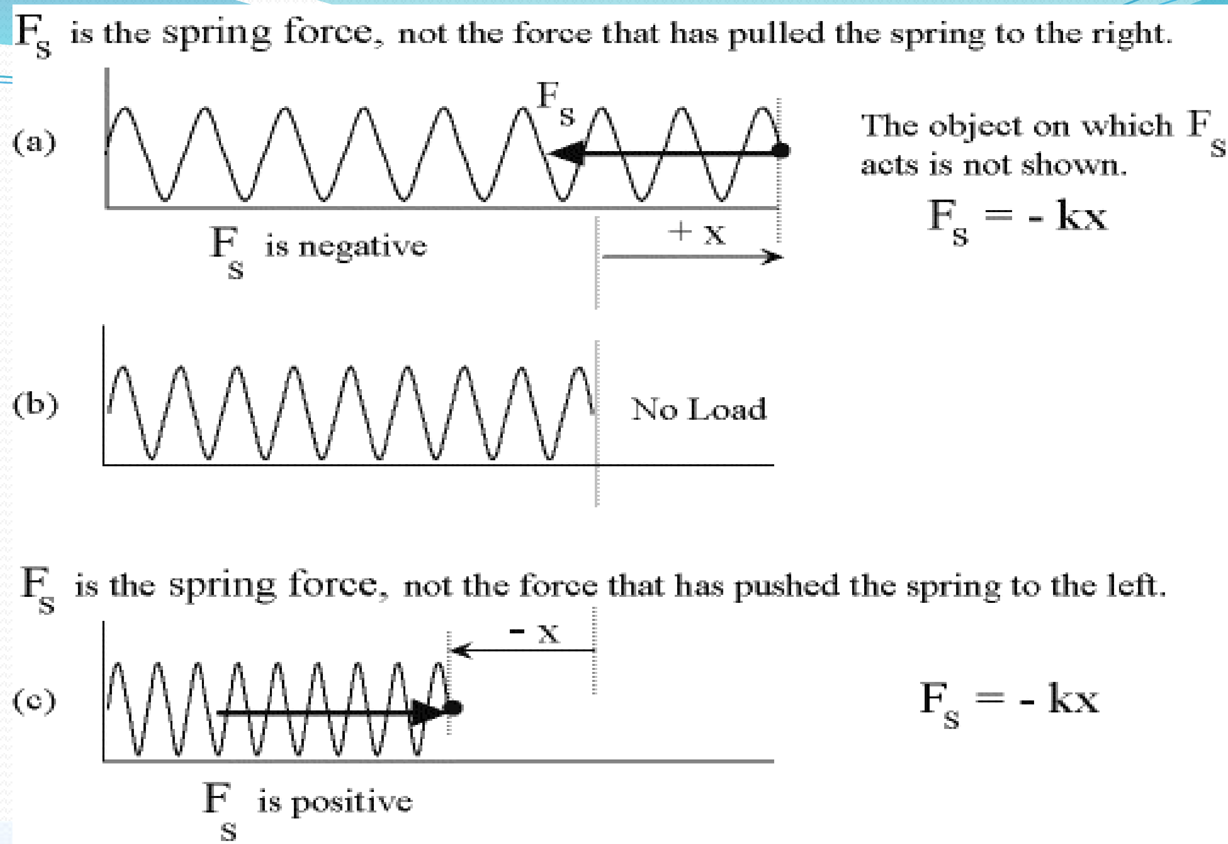
- **Hooke's law is a principle of physics that states that the force needed to extend or compress a spring by some distance is proportional to that distance. That is: where is a constant factor characteristic of the spring, its stiffness.**
- One of the properties of elasticity is that it takes about twice as much force to stretch a spring twice as far.
- Hooke's law is represented as below:

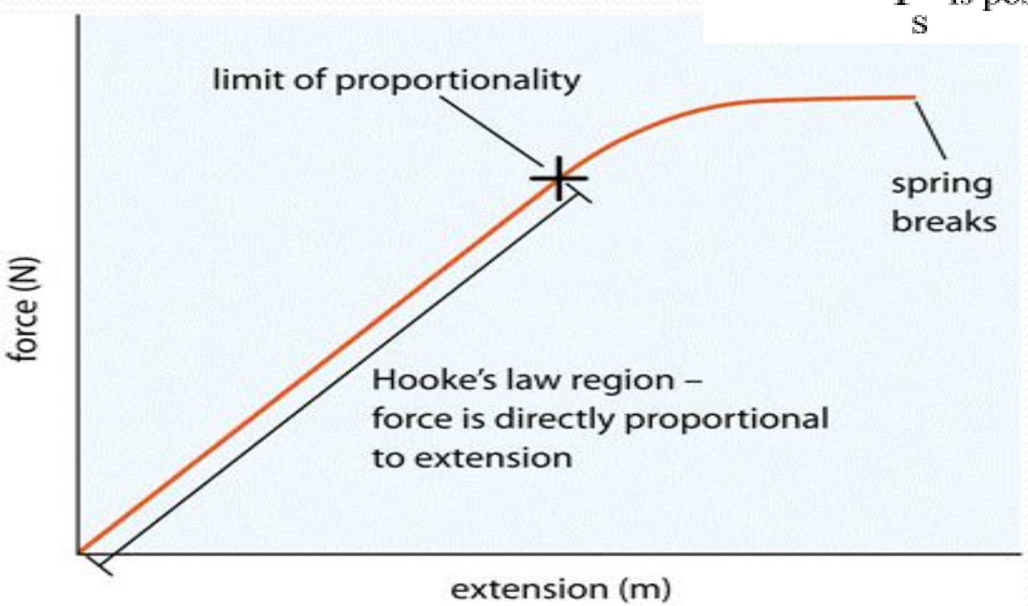
$$F = -kx$$

- Where,
  F is the amount of force applied in N,
  x is the displacement in the spring in m
  k is the spring constant or force constant.



- **Hooke's law formula is used to determine the force constant, displacement and force in a stretched spring.**


Fs is the spring force, not the force that has pulled the spring to the right.
Fs
(a)
The object on which Fs acts is not shown.
Fs = - kx
Fs is negative
+ x
(b)
No Load
Fs is the spring force, not the force that has pushed the spring to the left.
- x
(c)
Fs = - kx
Fs is positive


limit of proportionality
spring
breaks
force (N)
Hooke's law region –
force is directly proportional
to extension
extension (m)

Solved Examples

- **Question 9:** A spring is stretched by 5 cm and has force constant of 2 cm /dyne. Calculate the Force applied?
  Solution:
  Given: Force constant (k) = 2 cm/dyne, Extension (x) = 5 cm.
  The force applied is given by **F = - kx**
  = - 2cm/dyne × 5 cm = - 10 dyne.

- **Question 10:** A force of 100 N is stretching a spring by 0.2 m. Calculate the force constant?
  Solution:
  Given: Force (F)= 100 N, Extension (x) = 0.2 m.
  F = - kx
  The force constant is given by k = - F/x
  = - 100N/0.2m
  = - 500 N/m.

Energy and Work in physics

1. Energy

- A system possesses energy if it has the ability to do work. **Energy is a conservative quantity** (couldn’t be created or destroyed but It can be changed from one form to another).

- All forms of energy are either **kinetic or potential**. The energy associated with **motion is called kinetic energy**. The energy associated with **position is called potential energy**.

- Examples of **kinetic energy** such as mechanical energy, thermal energy and electrical energy.

- Examples of **potential energy such as** gravitational potential energy, electromagnetic potential energy.

Kinetic energy and potential energy

- *potential energy, is the* energy associated with the *position of a system rather than its motion (kinetic energy).*
- energy is *transformed from one form (potential energy) to* another (kinetic energy) or visa versa.
- a particle's kinetic energy is equal to the total work done on the particle by the forces that act on it.
- In many situations it seems as though energy has been stored in a system, potential energy, to be recovered later.
- weight *mg and the height y above the origin of coordinates, is* called the **gravitational potential energy,** *Ugrav:*

$$\boldsymbol{U_{grav}} = mgy$$

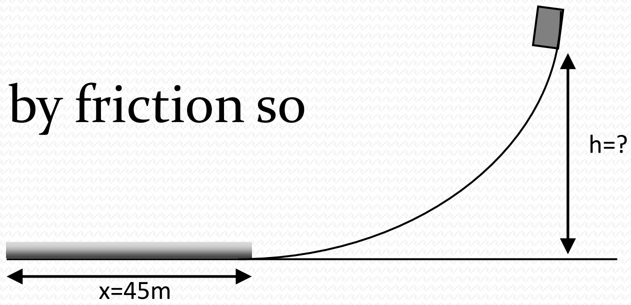
the total mechanical energy:

$$\mathbf{E = K + U_{grav} = constant}$$

Example 11

- An object with mass 6 kg slides down from an unknown height without initial velocity on a frictionless surface, and then it enters a frictional way with 45 m length when it reaches ground level. If the force of friction on the object is 12 N, what is the initial height? ($g=10 m/s^2$)

- Solution



The potential energy of the object will be spent by friction so

EP=W (in absolute value)

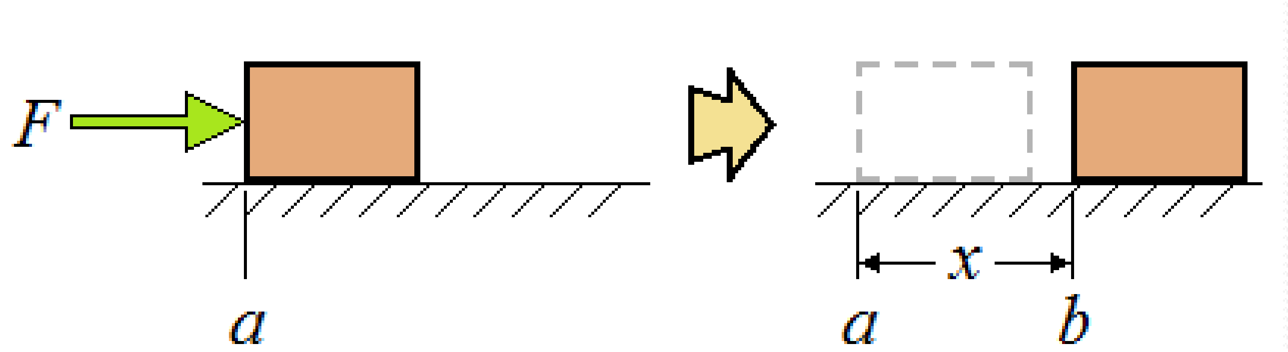
m.g.h = F.x

6 x 10 x h = 12 x 45

h = 9 m

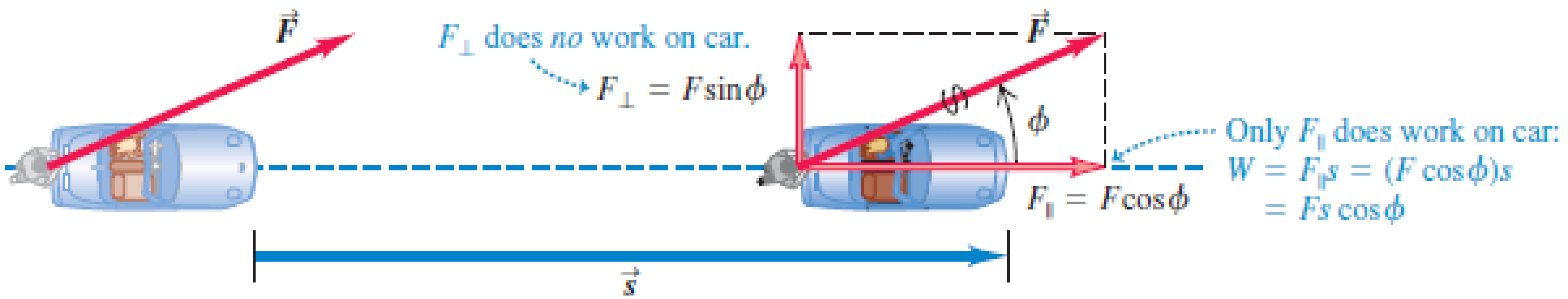
2. Work

- **Work is a force needed to move an object a displacement (d) from its equilibrium position. Work is measured by joule or erg**. Mathematically we can express work as below:
- **Work =force . distance**
- **W=F.d or**
- $W = \mathbf{F}\,\mathbf{d}$ ***if we take both F and d at the same direction***
- 1 joule = (1 Newton) (1 meter) or 1 J = 1 N . M
- Work is a *scalar quantity*



Example 12: (a) Steve exerts a steady **force of magnitude 210 N** on the stalled car in Fig. below as he pushes it a **distance of 18 m**. The car also has a flat tire, so to make the car track straight Steve must push at an **angle of 30°** to the direction of motion. **How much work does Steve do?**

(b) In a helpful mood, Steve pushes a second stalled car with a steady **force** (**F=160N i+ 40 N j**). The **displacement** of the car is (**d= 14m i+ 11m j**). **How much work does Steve do** in this case?



a $$W = Fs\cos\phi = (210\ \text{N})(18\ \text{m})\cos 30^\circ = 3.3 \times 10^3\ \text{J}$$

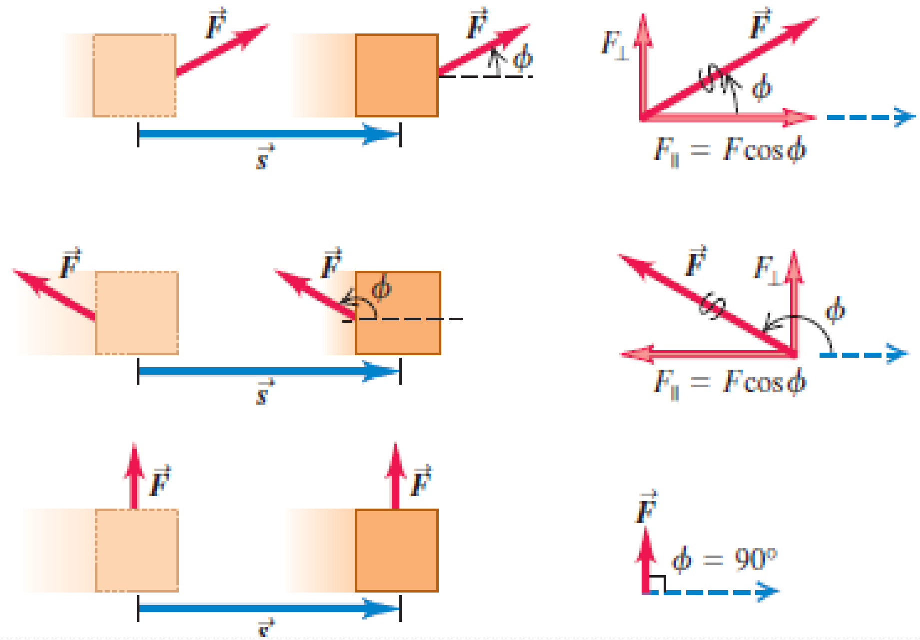
b $$W = \vec{F}\cdot\vec{s} = F_x x + F_y y$$
$$= (160\ \text{N})(14\ \text{m}) + (-40\ \text{N})(11\ \text{m})$$
$$= 1.8 \times 10^3\ \text{J}$$

Work can be either Positive, Negative, or Zero

(a) **Force $\vec{F}$ has a component in direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *positive*.

(b) **Force $\vec{F}$ has a component opposite to direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).

(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.



Example 13

- A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground. The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9 degree above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

Solution:

- We'll find the total work in two ways: (1) by adding the work done on the sled by each force and (2) by finding the work done by the net force on the sled.
- 1. The work done by the weight is zero because its direction is perpendicular to the displacement. For the same reason, the work done by the normal force is also zero.

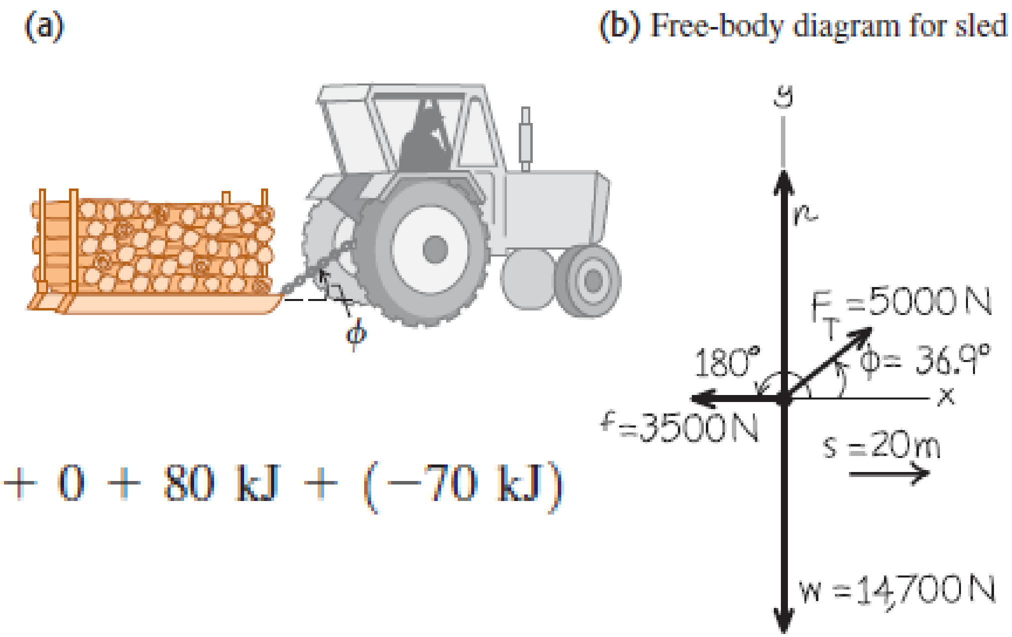
$$Ww = Wn = 0.$$

- That leaves the work done by the force exerted by the tractor and the work done by the friction force,

$$W_T = F_T s \cos\phi = (5000\ \text{N})(20\ \text{m})(0.800) = 80{,}000\ \text{N}\cdot\text{m}$$
$$= 80\ \text{kJ}$$

The friction force $\vec{f}$ is opposite to the displacement, so for this force $\phi = 180°$ and $\cos\phi = -1$.

$$W_f = f s \cos 180° = (3500\ \text{N})(20\ \text{m})(-1) = -70{,}000\ \text{N}\cdot\text{m}$$
$$= -70\ \text{kJ}$$



$$W_{tot} = W_w + W_n + W_T + W_f = 0 + 0 + 80\ \text{kJ} + (-70\ \text{kJ})$$
$$= 10\ \text{kJ}$$

- (2) In the second approach,

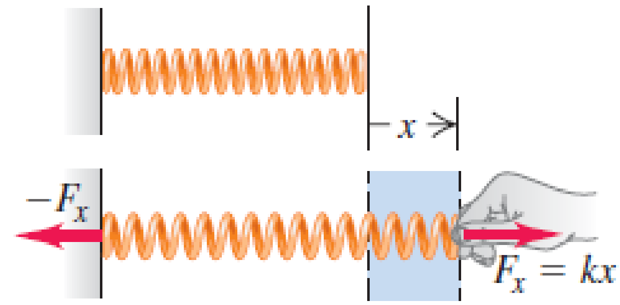
$$\sum F_x = F_T \cos\phi + (-f) = (5000\text{ N})\cos 36.9^\circ - 3500\text{ N}$$
$$= 500\text{ N}$$
$$\sum F_y = F_T \sin\phi + n + (-w)$$
$$= (5000\text{ N})\sin 36.9^\circ + n - 14{,}700\text{ N}$$

We don't need the second equation; we know that the *y-component* of force is perpendicular to the displacement, so it does no work.

$$W_{tot} = \left(\sum \vec{F}\right)\cdot\vec{s} = \left(\sum F_x\right)s = (500\text{ N})(20\text{ m}) = 10{,}000\text{ J}$$
$$= 10\text{ kJ}$$

POTENTIAL ENERGY OF A SPRING

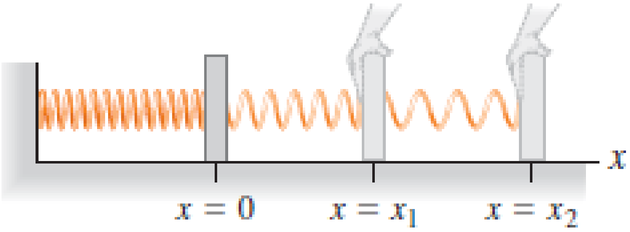
- According to Hooke's law for spring, the force needed to stretch the spring has magnitude of *k*x, where *k* is the spring's force constant ***of the spring***.



F= *k*x (Hooke's law)

- The units of *k are force divided by distance: N/ m*

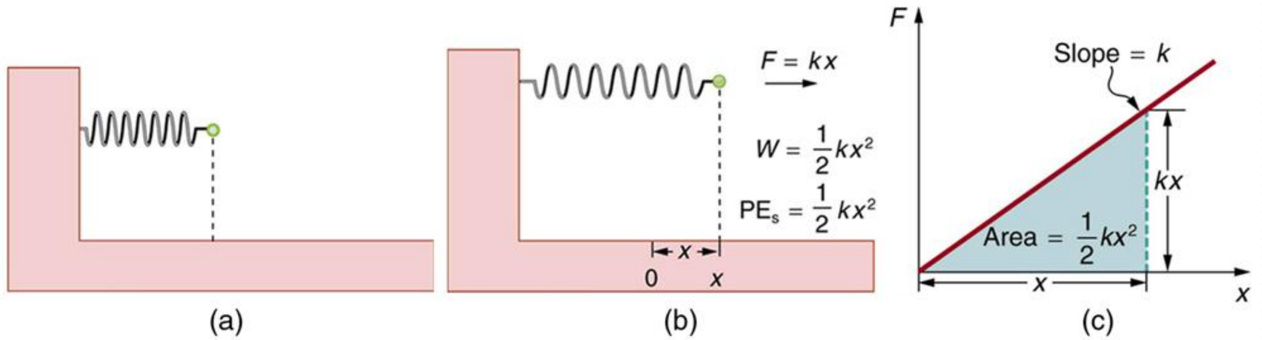
$$W = \int_{x_1}^{x_2} F_x \, dx = \int_{x_1}^{x_2} kx \, dx = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$



- We therefore define the ***potential energy of a spring***, **PE$_s$**, to be

$$PE_s = \frac{1}{2}kx^2,$$

The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.



Not: gravitational potential energy is $U_{grav} = mgy$

Example 14: A woman **weighing 600 N** steps on a bathroom scale that contains a stiff spring. In equilibrium, the spring is **compressed 1.0 cm** under her weight. Find the force constant of the spring and the **total work done** on it during the compression.

SOLVE

$$k = \frac{F_x}{x} = \frac{-600\ \text{N}}{-0.010\ \text{m}} = 6.0 \times 10^4\ \text{N/m}$$

Then, using $x_1 = 0$ and $x_2 = -0.010$ m

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 \quad = \tfrac{1}{2}(6.0 \times 10^4\ \text{N/m})(-0.010\ \text{m})^2 - 0 = 3.0\ \text{J}$$

CONSERVATION OF MECHANICAL ENERGY

- The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2 = \Delta KE.$$

If only conservative forces act, then

$$W_{net} = W_c$$

where W_c **is the total work done by all conservative forces**. Thus,

$$W_c = \Delta KE.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_c = -\Delta PE$. Therefore,

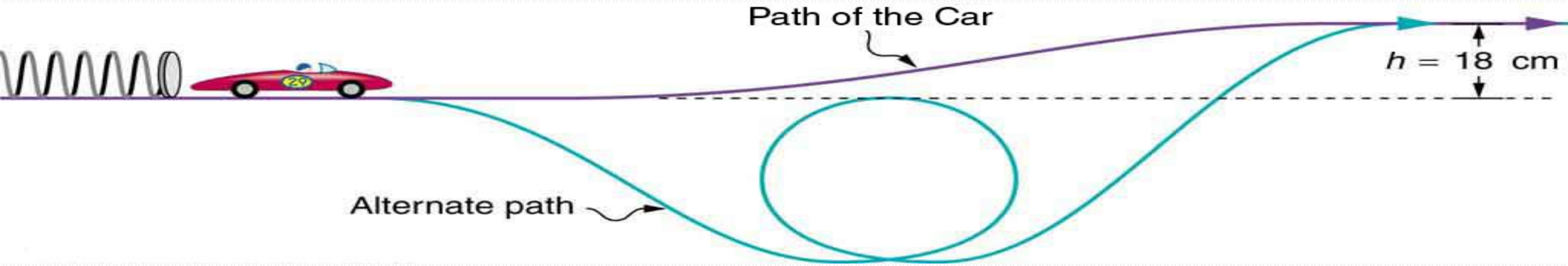
$$-\Delta PE = \Delta KE$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces.

$$\frac{1}{2}kx_i{}^2 = \frac{1}{2}mv_f{}^2.$$

EXAMPLE 15 . USING CONSERVATION OF MECHANICAL ENERGY TO CALCULATE THE SPEED OF A TOY CAR

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure below. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.



The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2,$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2.$$

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}}x_i \\ &= \sqrt{\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}}(0.0400\ \text{m}) \\ &= 2.00\ \text{m/s}. \end{aligned}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f.$$

$$\begin{aligned} v_f &= \sqrt{\frac{kx_i^2}{m} - 2gh_f} \\ &= \sqrt{\left(\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}\right)(0.0400\ \text{m})^2 - 2(9.80\ \text{m/s}^2)(0.180\ \text{m})} \\ &= 0.687\ \text{m/s}. \end{aligned}$$