

# Chapter three

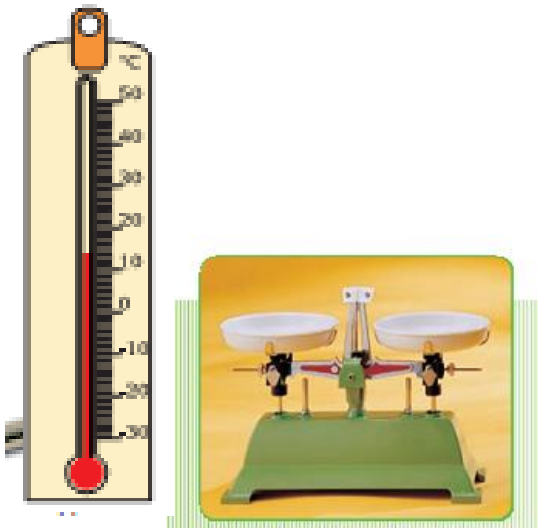
## Scalar and Vector Quantities

You will learn:

1. The principles of scalar and vector quantities
2. Mathematical combinations of vector quantities
3. Unit vectors

# Scalar Quantities:

- A **SCALAR** is a quantity of physics that has MAGNITUDE only, however, direction is not associated with it.
- Magnitude – A numerical value with units.
- Some examples of scalar quantities



Scalar Example	Magnitude
Speed	20 m/s
Distance	10 m
Age	15 years
Heat	1000 calories

# Vector Quantities:

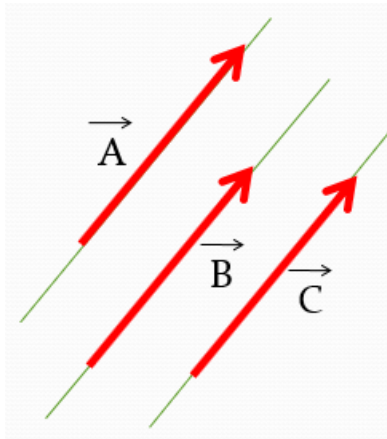
- A **VECTOR** is a quantity which has both **MAGNITUDE** and **DIRECTION**.
- Examples: force, displacement, velocity....

<b>Vector</b>	<b>Magnitude &amp; Direction</b>
Velocity	20 m/s, North
Acceleration	10 m/s/s, East
Force	5 N, West

**Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.**

# Some properties of Vectors

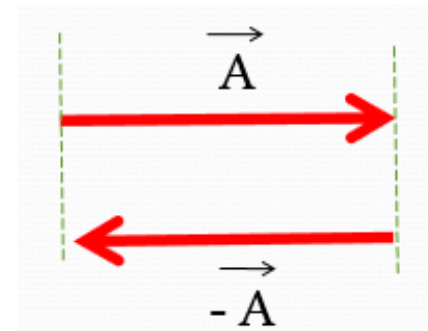
- **Equality of Two Vectors**
  - Two vectors are **equal** if they have **the same magnitude and the same direction**.



$$\vec{A} = \vec{B} = \vec{C}$$

- **Negative Vectors**

- Two vectors are **negative** if they have the same magnitude but are  $180^\circ$  apart (opposite directions)



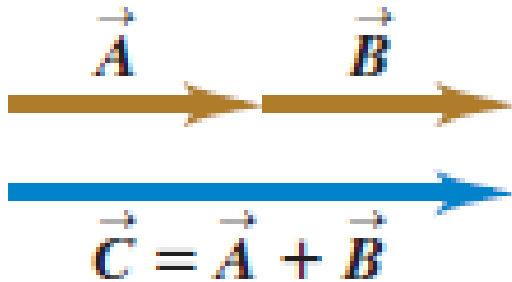
# Vector Applications

- Adding vectors
  - Subtracting vectors
  - Vector multiplication
- 
- There are two approaches of vector applications:
    1. Graphical Methods: the vectors are plotted.
      - Use scale drawings.
    2. Algebraic Methods: mathematics is used.
      - More convenient

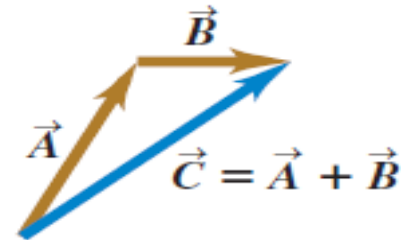
# Adding Vectors graphically

When adding vectors, their **directions must be taken into consideration.**

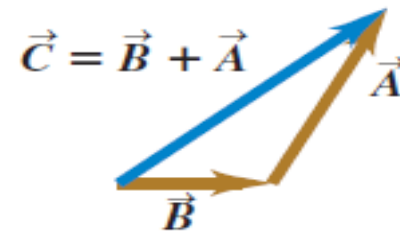
Units must be the same.



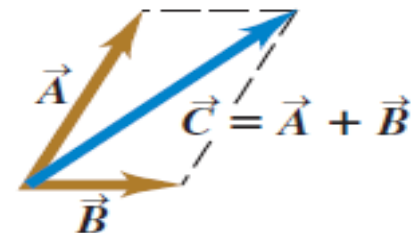
(a) We can add two vectors by placing them head to tail.



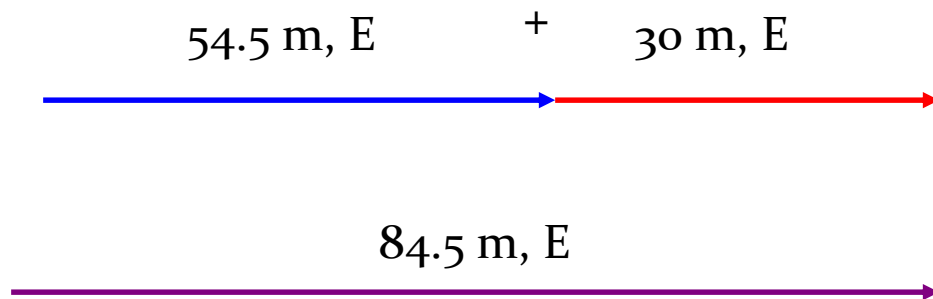
(b) Adding them in reverse order gives the same result.



(c) We can also add them by constructing a parallelogram.



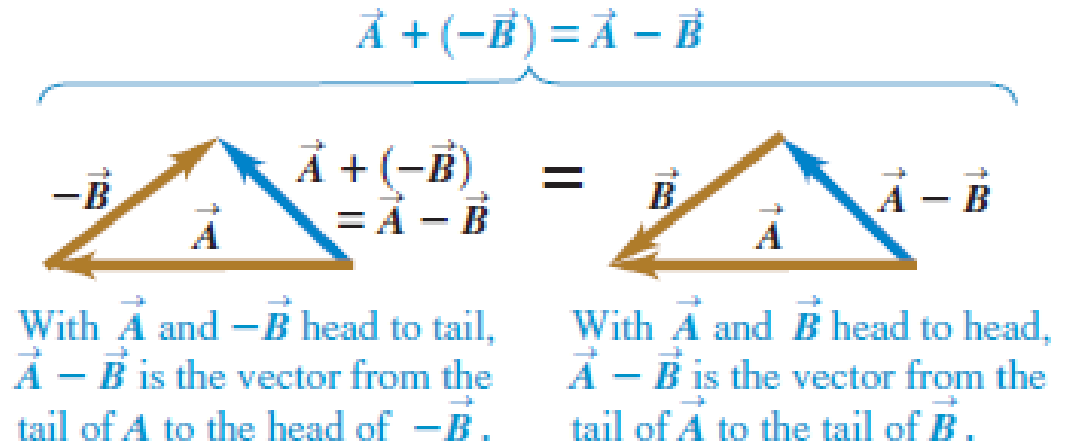
- **Example 1:** A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?



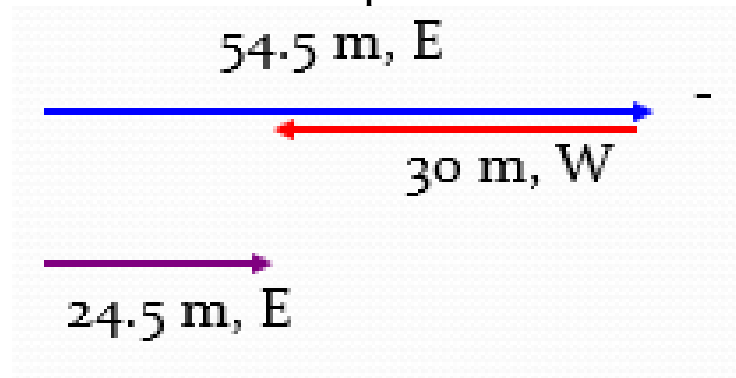
Notice that the **SIZE** of the arrow conveys **MAGNITUDE** and the way it was drawn conveys **DIRECTION**.

# Subtracting vectors graphically

- The vectors  $\vec{A}$  &  $-\vec{A}$  have the same magnitude but point in opposite directions. Therefore when we add negative vectors we get zero.



- Example 2:** A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started? •



Solution

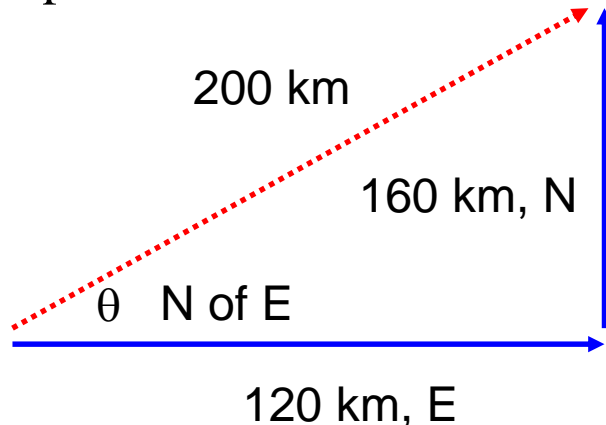


# Perpendicular vectors graphically

## NEED A VALUE – ANGLE!

- When two vectors are **perpendicular**, you can use the **Pythagorean theorem** if the **angle is not given**.
- Just putting N of E is not good enough (how far north of east ?).
- We need to find a numeric value for the direction.
- To find the value of the angle we use a Trig function called TANGENT.

**Example 3:** From the below diagram, find the angle between the resultant and east components.



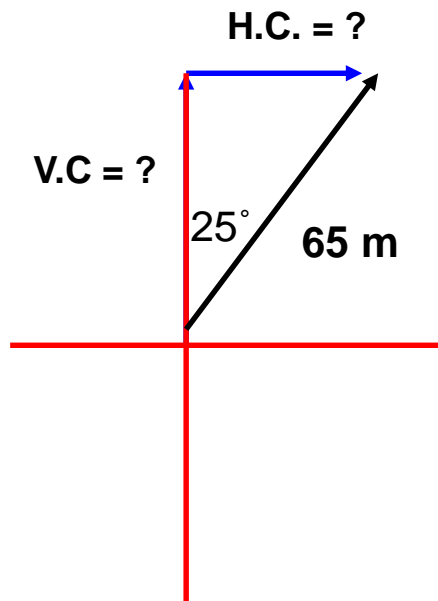
$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{160}{120} = 1.333$$

$$\theta = \tan^{-1}(1.333) = 53.1^\circ$$

So the COMPLETE final answer is : **200 km, 53.1 degrees North of East**

# What are your missing components?

**Example 4:** Suppose a person walked 65 m, 25 degrees East of North. What were his horizontal and vertical components?



The goal: **ALWAYS MAKE A RIGHT TRIANGLE!**

To solve for components, we often use the trig functions sine and cosine.

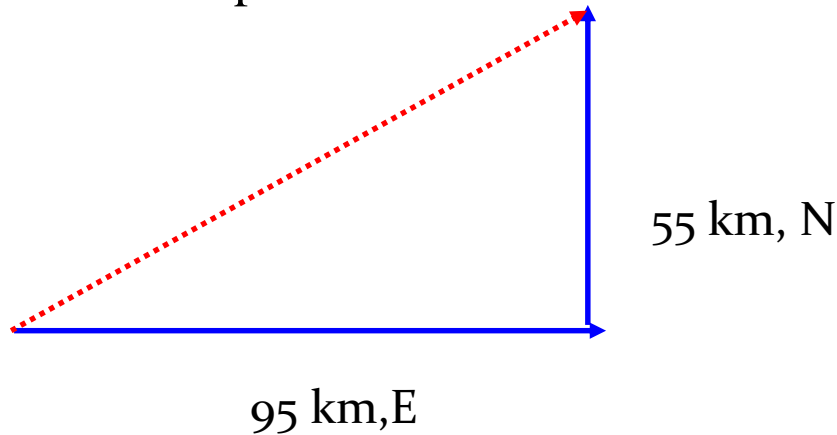
$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{adj} = \text{hyp} \cos \theta \quad \text{opp} = \text{hyp} \sin \theta$$

$$\text{adj} = \text{V.C.} = 65 \cos 25 = 58.91\text{m}, N$$

$$\text{opp} = \text{H.C.} = 65 \sin 25 = 27.47\text{m}, E$$

**Example 5:** A man walks 95 km East then 55 km north. Calculate his resultant displacement.

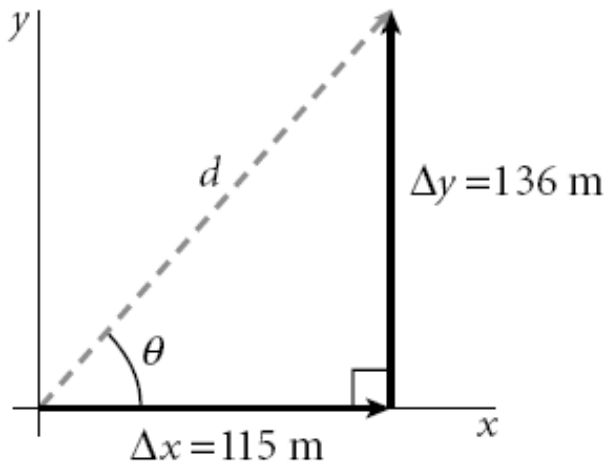


$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$

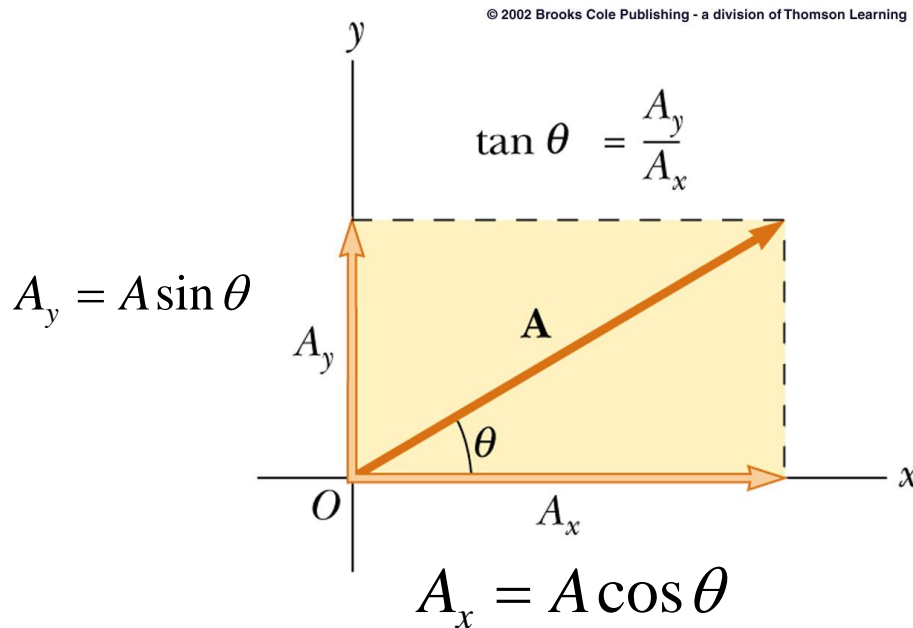
$$c = \sqrt{12050} = 109.8 \text{ km}$$

**Example 6: (HW)** From the diagram below, determine the value of  $d$ .



# Components of a Vector

- The projections of a vector on the x and y axis are called the components of the vector.



- *The y-component of a vector is the projection along the y-axis.*

*The x-component of a vector is the projection along the x-axis.*

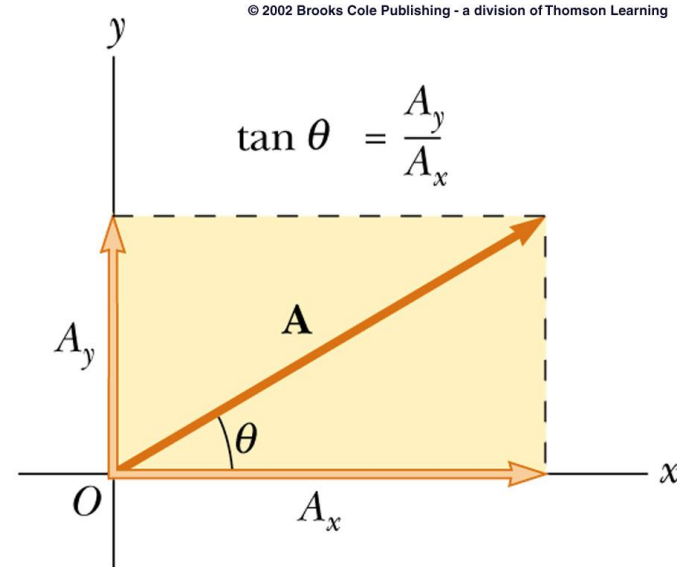
$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{A_y}{A_x}$$

# Vectors with two and three components

- Vectors with two components

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$$

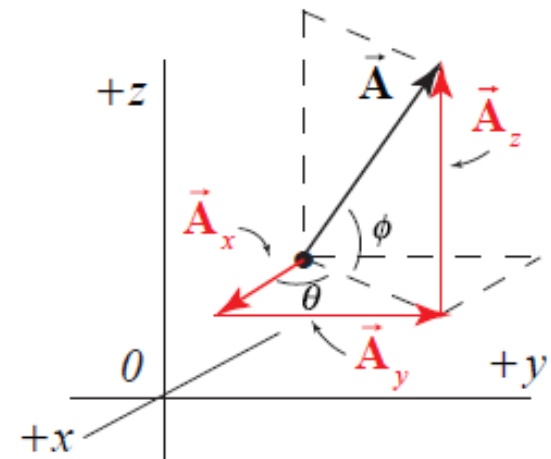
$$A = \sqrt{A_x^2 + A_y^2}$$



- Vectors with three components

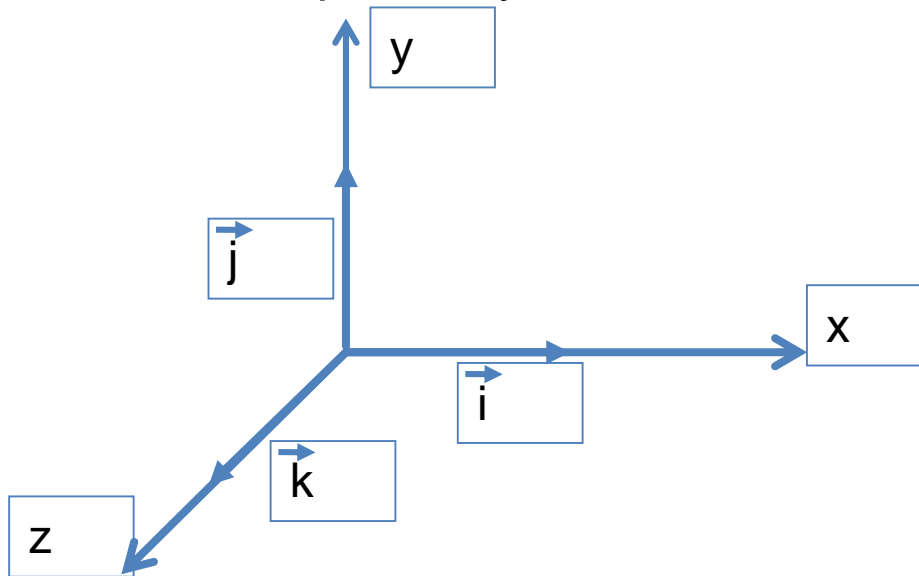
$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



# Unit Vectors

- ❑ Vector quantities are often expressed in terms of unit vector.
- ❑ A unit vector is a dimensionless vector having a magnitude of exactly one.
- ❑ Unit vectors are used to specify a given direction and have no other physical significance.
- ❑ Symbols **i**, **j** and **k** are used for unit vectors and pointing in the positive x, y and z direction respectively.



# Vector application (algebraic method)

## Add and subtracts vectors

**Example 7:** Using the two vectors below, determine:  $A+B$  and  $B+A$ ,  $A-B$  and  $B-A$ .

$$\text{Let } A = (3i - 4j - 5k)$$

$$\text{Let } B = (2i + 7j + 3k)$$

# Vector multiplication (algebraic method)

- Vector multiplications can be:
  1. Dot product
  2. Cross product
- Notes:
  - Dot product ( $\mathbf{A} \cdot \mathbf{B}$ ) multiplication results a scalar quantity.
  - Cross product ( $\mathbf{A} \times \mathbf{B}$ ) multiplication results a vector quantity.
  - When vectors are added, subtracted or multiplied, the unit vectors have to be considered.
  - As two vectors added or subtracted, the similar components (i with i, j with j and k with k) are combined or subtracted together, only.
  - When two vectors multiplied with dot ( $\cdot$ ), non-similar components multiplications are equal to zero.
  - We can also measure the angle between the two vectors using the below formular:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta$$



- Multiplying two vectors (for example A and B) sometimes gives you a SCALAR quantity which we call it the **SCALAR DOT PRODUCT**.

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

**Example 8:** Use the two vectors below to calculate A.B.

$$\text{Let } \mathbf{A} = (3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k})$$

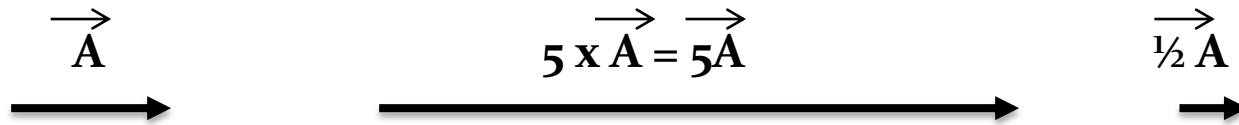
$$\text{Let } \mathbf{B} = (2\mathbf{i} + 7\mathbf{j} + 3\mathbf{k})$$

$$\text{Therefore, } \mathbf{A} \text{ "dot" } \mathbf{B} = (3)(2) + (-4)(7) + (-5)(3) = -37$$

# Multiplying or Dividing a Vector by a Scalar quantity

A vector can be multiplied or divided by a quantity (such as a number).

If the scalar is positive, the direction of the resultant is the same as of the original vector.

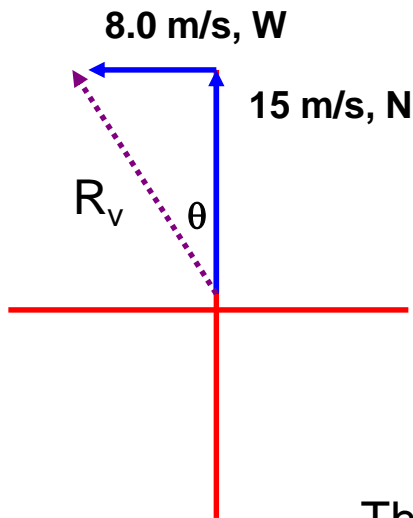


If the scalar is negative, the direction of the resultant is opposite that of the original vector.



## Example 9

A boat moves with a velocity of 15 m/s, N in a river which flows with a velocity of 8.0 m/s, west. Calculate the boat's resultant velocity with respect to due north.



$$R_v = \sqrt{8^2 + 15^2} = 17 \text{ m/s}$$

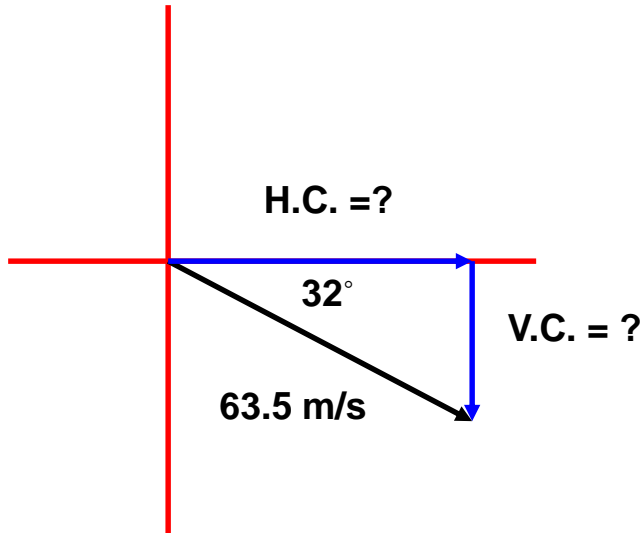
$$\tan \theta = \frac{8}{15} = 0.5333$$

$$\theta = \tan^{-1}(0.5333) = 28.1^\circ$$

The Final Answer : **17 m/s, @ 28.1 degrees West of North**

# Example 10

A plane moves with a velocity of **63.5 m/s** at **32 degrees South of East**.  
Calculate the plane's horizontal and vertical velocity components.



$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

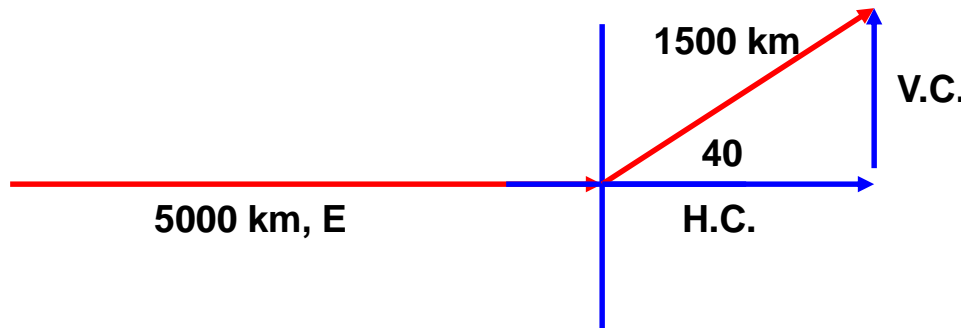
$$\text{adj} = \text{hyp} \cos \theta \quad \text{opp} = \text{hyp} \sin \theta$$

$$\text{adj} = H.C. = 63.5 \cos 32 = 53.85 \text{ m/s, E}$$

$$\text{opp} = V.C. = 63.5 \sin 32 = 33.64 \text{ m/s, S}$$

## Example 11

A storm system moves 5000 km due east, then shifts course at 40 degrees North of East for 1500 km. Calculate the storm's resultant displacement.

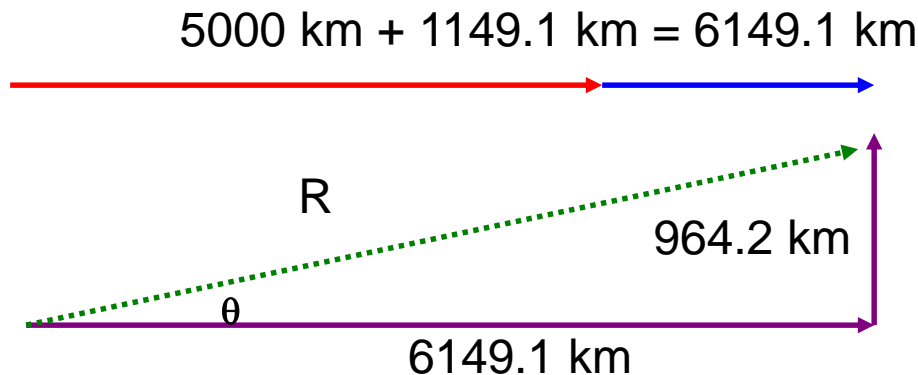


$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$adj = hyp \cos \theta \quad opp = hyp \sin \theta$$

$$adj = H.C. = 1500 \cos 40 = 1149.1 \text{ km, E}$$

$$opp = V.C. = 1500 \sin 40 = 964.2 \text{ km, N}$$



$$R = \sqrt{6149.1^2 + 964.2^2} = 6224.2 \text{ km}$$

$$\tan \theta = \frac{964.2}{6149.1} = 0.157$$

$$\theta = \tan^{-1}(0.157) = 8.92^\circ$$

The Final Answer: **6224.2 km @ 8.92 degrees, North of East**

End of chapter three