Chapter two

Coordinate Systems

You will learn:

1. **Types of coordinators**
2. **Using coordinators to determine dimensions**

Coordinate Systems

- A coordinate system is **used to determine** each point uniquely in a **plane.**

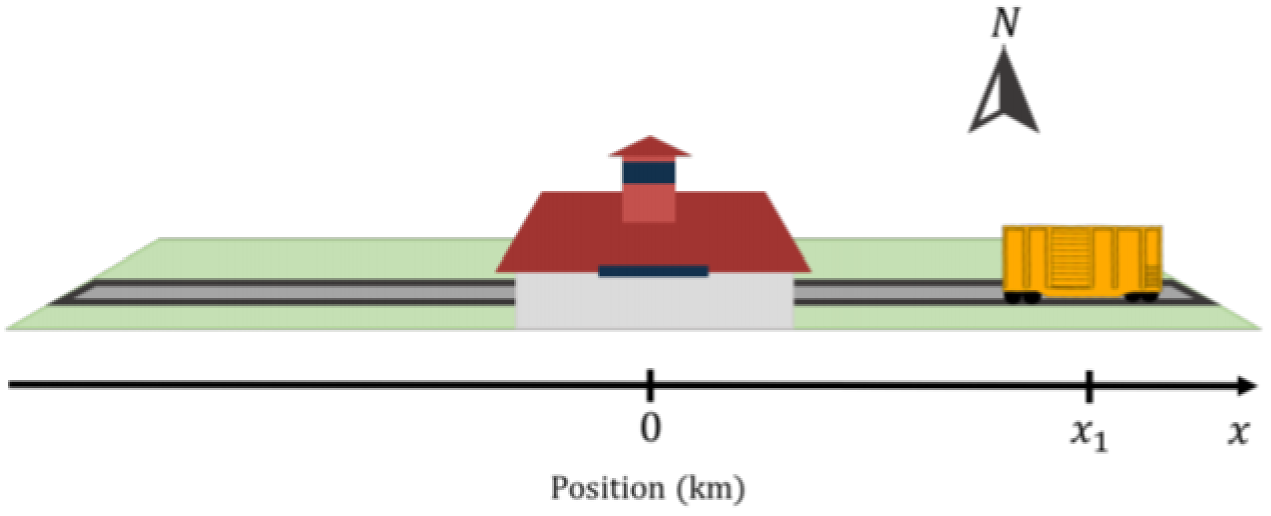
- A coordinate system is an artificial mathematical tool that we construct in order to describe the **position of a real object**.

- Coordinate system (**frame**) consists of:
  - a fixed **reference** point called the origin.
  - specific axes with scales and **labels.**
  - instructions on how to label a point relative to the origin and the axes.

- **Coordinating system can be**:
  - **1D Coordinate systems**
  - **2D Coordinate systems**
  - **3D Coordinate systems**

1D Coordinate systems

- The easiest coordinate system to construct is one that we can use to describe the location of objects in one dimensional space.
- For example, we may wish to describe the location of a train along a straight section of track that runs in the East-West direction.
- First define an "origin", (x=0) which is the reference point of our coordinate system.
- We can describe the position of the train by specifying how far it is from the train station (the origin), using a single real number, say X-direction.



2D Coordinate systems

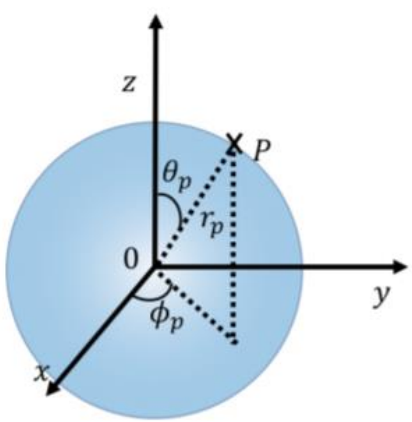
- To **describe** the **position** of an object in **two dimensions**
- We need to specify two numbers to define two axes, x and y, whose origin and direction we must define.
- **Examples of 2D** coordinating system:
  - **‘’Cartesian” coordinate system, and**
  - **“Polar” coordinate system**

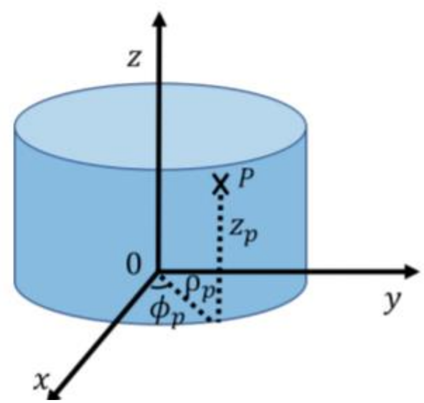
3D Coordinate systems

- In three dimensions, we need to specify three numbers to describe the position of an object (e.g. **a bird flying** in the air).

- In a three dimensional Cartesian coordinate system, we simply add a third axis, **z,** that is mutually perpendicular to both **x and y**.

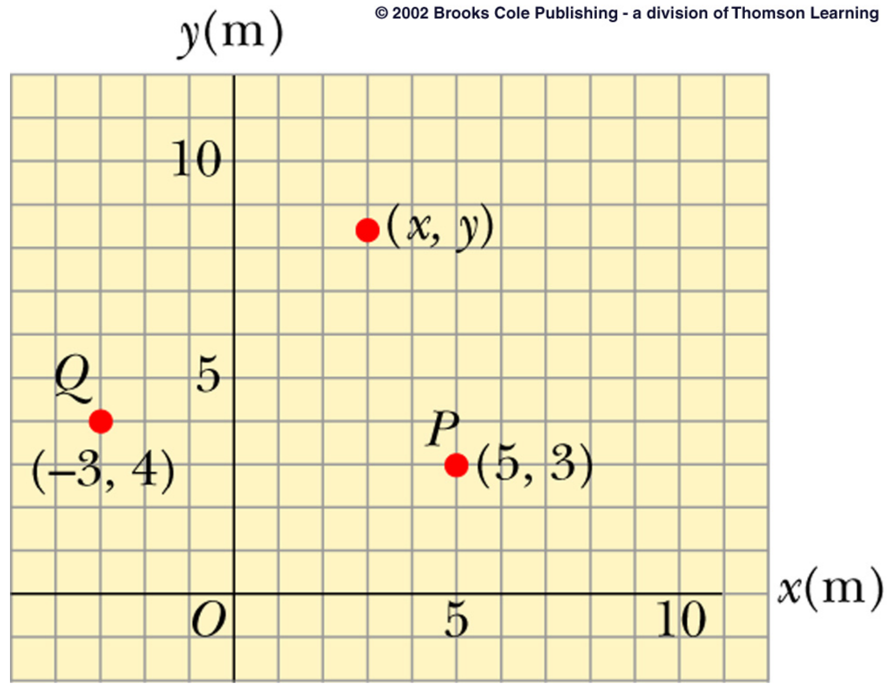
- **Examples of three dimensions:**
  - **“cylindrical” coordinates system and,**
  - **“spherical” coordinates system**





Cartesian Coordinate Systems

- It is also called rectangular coordinate system
- x- and y- axes
- Points are labeled as (x,y)



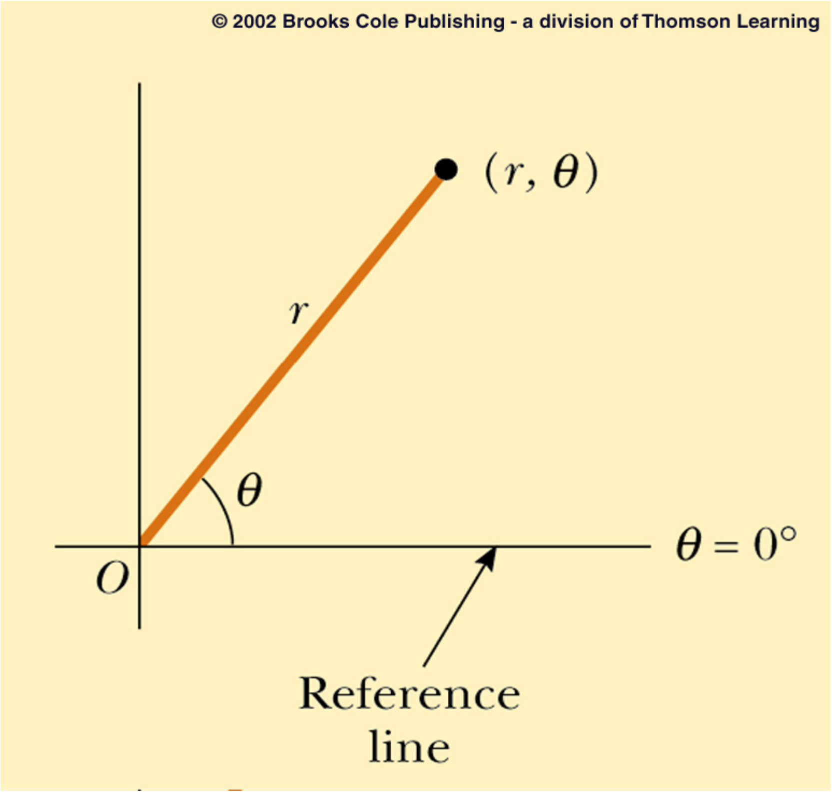
Plane polar coordinate system

Here:

- The origin and reference line are noted.

- The point **(r, θ)** is a distance (r) from the origin in the direction of angle θ.

- The points are labeled as (r, θ)



Conversion between the two systems

- On a polar coordinating system, the values of r of which corresponded to the x and y-axis can be measured as following:

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

- If x, and y given, the value of r is determined as : $r = \sqrt{x^2 + y^2}$
- The **angle** which the vector r makes with an original axis is measured as below:

$$\tan(\theta) = \frac{y}{x}$$

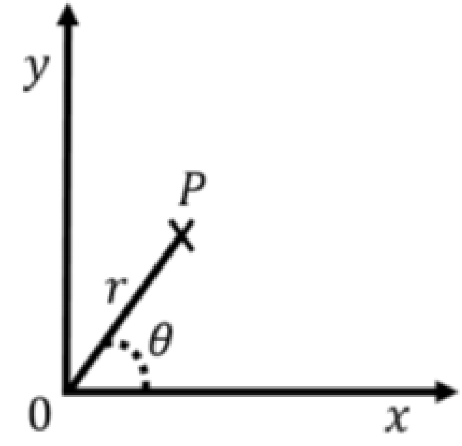
- Other approaches can contribute to find out one of the four parameters (x, y, r, and θ) on the below diagram.

- These approaches are:

1) sine, cosine and tan function or

2) Pythagorean Theorem

- **Note**: These principles are mostly applied to solve mathematical problems of physics.

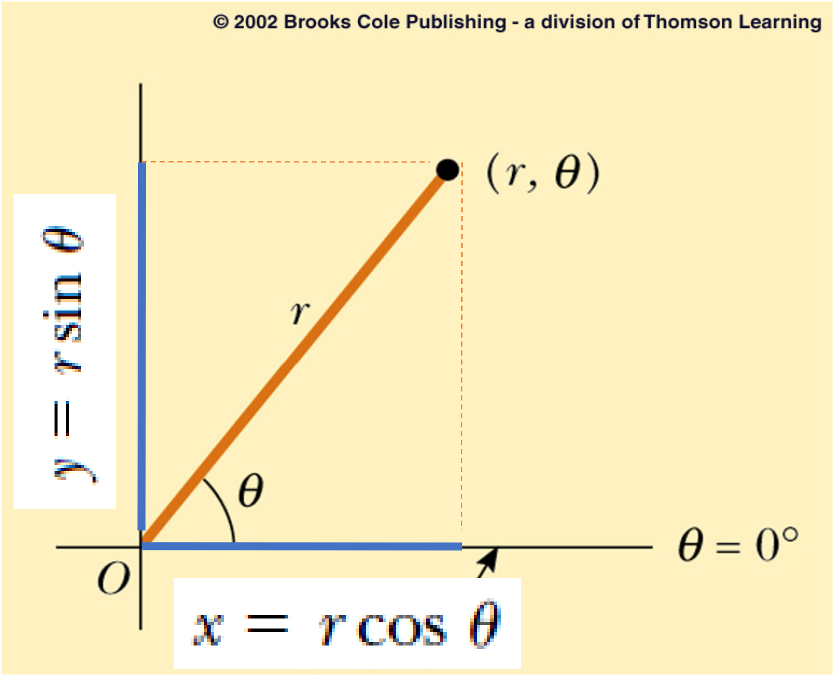


Plane polar coordinate system

In this diagram, there are four parameters: x, y, r and θ. If two of them are given, the rest can be found easily using one of the discussed approaches.

Note:

1. If the angle is not given, think of an approach that does not contain the angle such as **Pythagorean.**
2. **There may be more than one approach to determine one single parameter, feel free to use any appropriate method.**



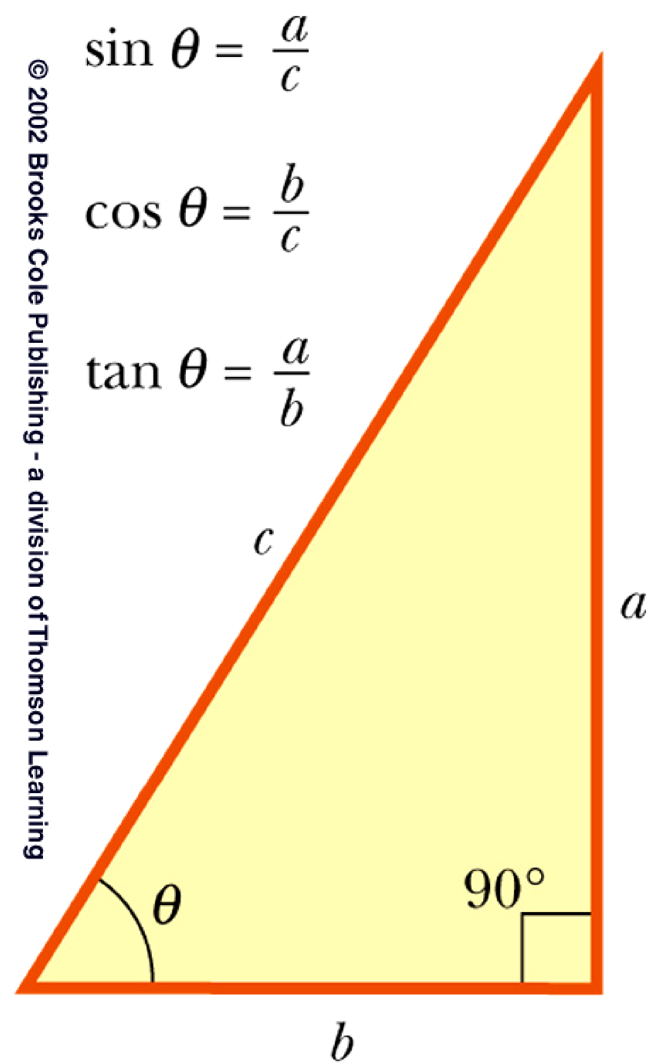
$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$r = \sqrt{x^2 + y^2}$$
$$\tan(\theta) = \frac{y}{x}$$

❑ Math Review: Trigonometry

$$\sin\theta = \frac{opposite\ side}{hypotenuse}$$

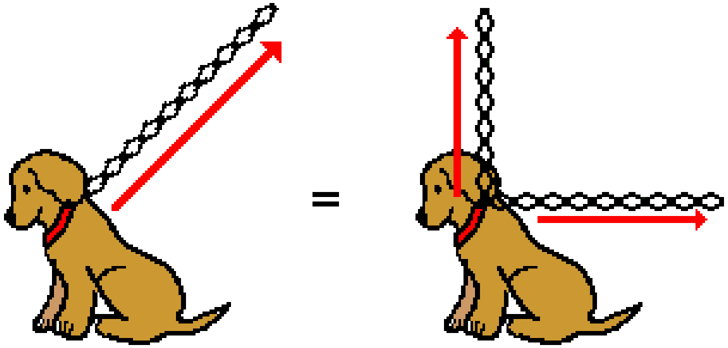
$$\cos\theta = \frac{adjacent\ side}{hypotenuse}$$

$$\tan\theta = \frac{opposite\ side}{adjacent\ side}$$

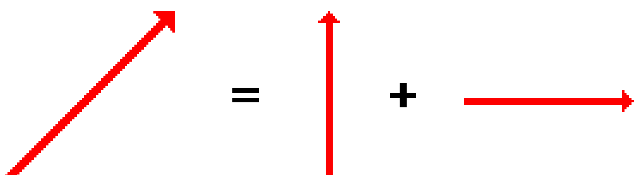


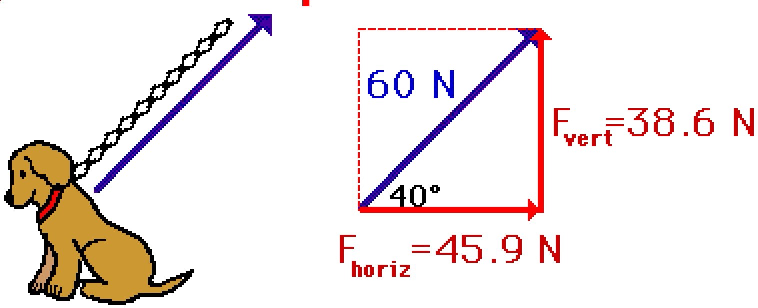
❑ Pythagorean Theorem

$$c^2 = a^2 + b^2$$



The upward and rightward force of the chain is equivalent to
an upward force and a rightward force by two chains.

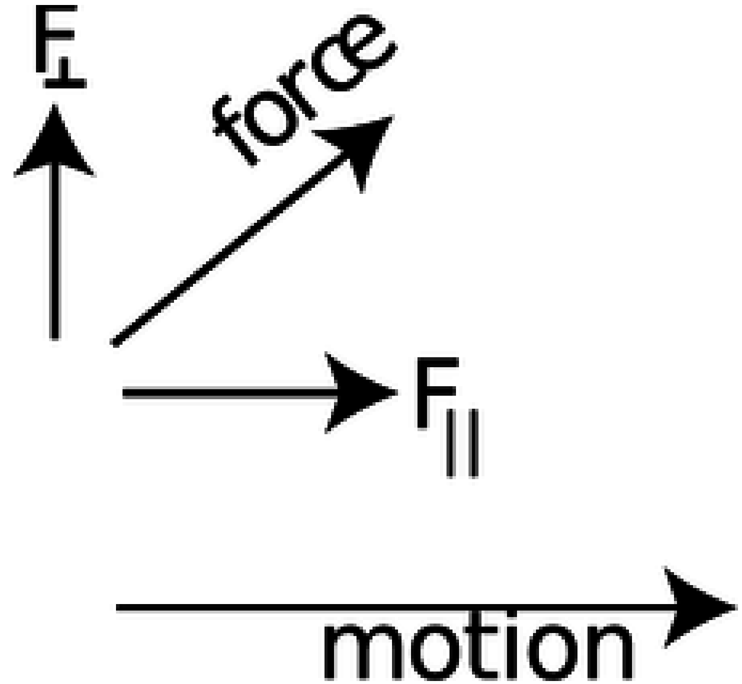




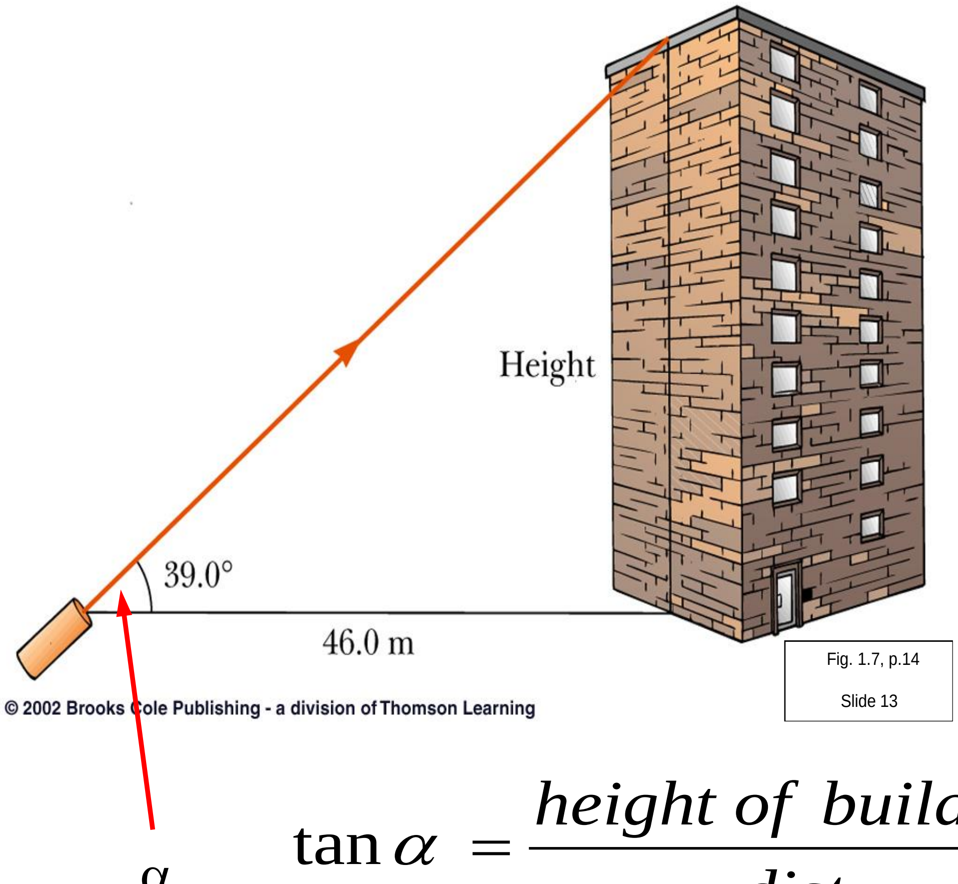
$$\sin 40^\circ = \frac{F_{vert}}{60\ N} \qquad \cos 40^\circ = \frac{F_{horiz}}{60\ N}$$

$$F_{vert} = 60\ N \times \sin 40^\circ \qquad F_{horiz} = 60\ N \times \cos 40^\circ$$

$$F_{vert} = 38.6\ N \qquad F_{horiz} = 45.9\ N$$



Example 1: How high is the building?



Known: angle and one side
Find: another side

Key: tangent is defined via two sides!

$$\tan\alpha = \frac{height\ of\ building}{dist.},$$

$$height = dist. \times \tan\alpha = (\tan 39.0^\circ)(46.0\ m) = 37.3\ m$$

Example 2: Finding Polar Coordinates

If the rectangular coordinates of a point are given by (**3**, **y**) and its polar coordinates are (**r**, **60^o**), determine **y** and **r**.

Note : sin 30 = cos 60 = ½ sin 60 = cos 30 =√3 / 2

Solution:

Y= r sinɵ, x = r cosɵ, x=3

Thus: 3 = r cos60, r = 3/cos60 = 3*2 = 6 m

Y = r sin 60 = 6* √3 /2=3 √3 m