CHAPTER 1

Introduction—Concept of Stress

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1.1 INTRODUCTION

The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load-bearing structures.

Both the analysis and the design of a given structure involve the determination of *stresses* and *deformations*. This first chapter is devoted to the concept of *stress*.

Section 1.2 is devoted to a short review of the basic methods of statics and to their application to the determination of the forces in the members of a simple structure consisting of pin-connected members. Section 1.3 will introduce you to the concept of *stress* in a member of a structure, and you will be shown how that stress can be determined from the *force* in the member. After a short discussion of engineering analysis and design (Sec. 1.4), you will consider successively the *normal stresses* in a member under axial loading (Sec. 1.5), the *shearing stresses* caused by the application of equal and opposite transverse forces (Sec. 1.6), and the *bearing stresses* created by bolts and pins in the members they connect (Sec. 1.7). These various concepts will be applied in Sec. 1.8 to the determination of the stresses in the members of the simple structure considered earlier in Sec. 1.2.

The first part of the chapter ends with a description of the method you should use in the solution of an assigned problem (Sec. 1.9) and with a discussion of the numerical accuracy appropriate in engineering calculations (Sec. 1.10).

In Sec. 1.11, where a two-force member under axial loading is considered again, it will be observed that the stresses on an *oblique* plane include both *normal* and *shearing* stresses, while in Sec. 1.12 you will note that *six components* are required to describe the state of stress at a point in a body under the most general loading conditions.

Finally, Sec. 1.13 will be devoted to the determination from test specimens of the *ultimate strength* of a given material and to the use of a *factor of safety* in the computation of the *allowable load* for a structural component made of that material.

1.2 A SHORT REVIEW OF THE METHODS OF STATICS

In this section you will review the basic methods of statics while determining the forces in the members of a simple structure.

Consider the structure shown in Fig. 1.1, which was designed to support a 30-kN load. It consists of a boom AB with a 30×50-mm rectangular cross section and of a rod BC with a 20-mm-diameter circular cross section. The boom and the rod are connected by a pin at B and are supported by pins and brackets at A and C, respectively. Our first step should be to draw a *free-body diagram* of the structure by detaching it from its supports at A and C, and showing the reactions that these supports exert on the structure (Fig. 1.2). Note that the sketch of the structure has been simplified by omitting all unnecessary details. Many of you may have recognized at this point that AB and BC are *two-force members*. For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at A and C are unknown. Each of these

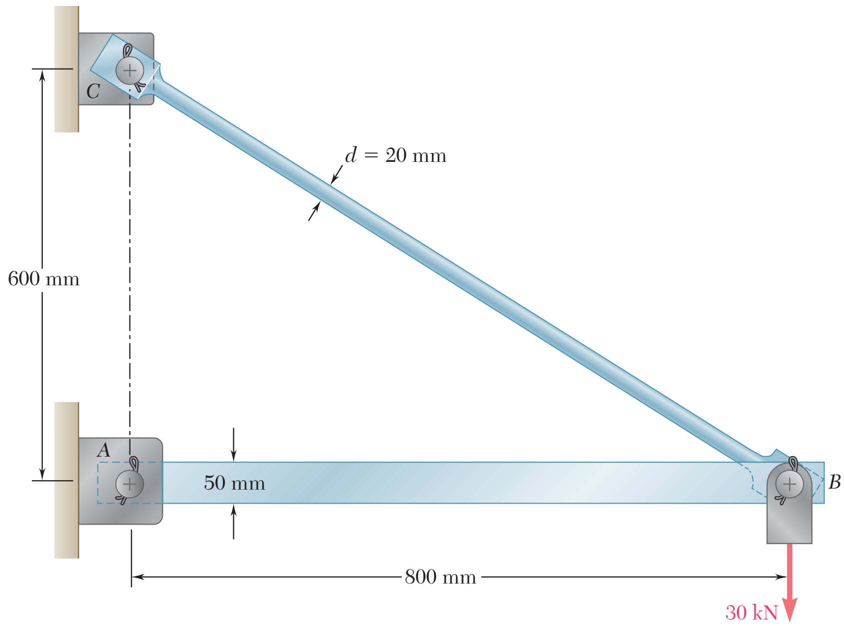


Fig. 1.1 Boom used to support a 30-kN load.

reactions, therefore, will be represented by two components, $\mathbf{A}_x$ and $\mathbf{A}_y$ at A, and $\mathbf{C}_x$ and $\mathbf{C}_y$ at C. We write the following three equilibrium equations:

$$+\circlearrowleft \Sigma M_C = 0: \qquad A_x(0.6 \text{ m}) - (30 \text{ kN})(0.8 \text{ m}) = 0$$
$$A_x = +40 \text{ kN} \tag{1.1}$$
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x + C_x = 0$$
$$C_x = -A_x \qquad C_x = -40 \text{ kN} \tag{1.2}$$
$$+\uparrow \Sigma F_y = 0: \qquad A_y + C_y - 30 \text{ kN} = 0$$
$$A_y + C_y = +30 \text{ kN} \tag{1.3}$$

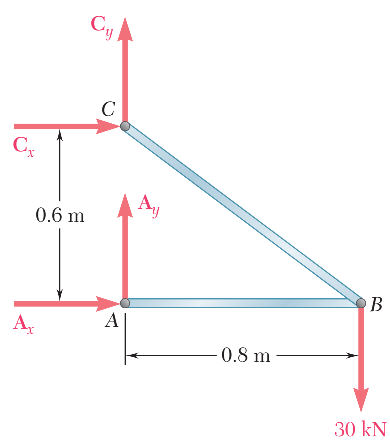


Fig. 1.2

We have found two of the four unknowns, but cannot determine the other two from these equations, and no additional independent equation can be obtained from the free-body diagram of the structure. We must now dismember the structure. Considering the free-body diagram of the boom AB (Fig. 1.3), we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(0.8 \text{ m}) = 0 \qquad A_y = 0 \tag{1.4}$$

Substituting for A_y from (1.4) into (1.3), we obtain $C_y = +30$ kN. Expressing the results obtained for the reactions at A and C in vector form, we have

$$\mathbf{A} = 40 \text{ kN} \rightarrow \qquad \mathbf{C}_x = 40 \text{ kN} \leftarrow, \mathbf{C}_y = 30 \text{ kN} \uparrow$$

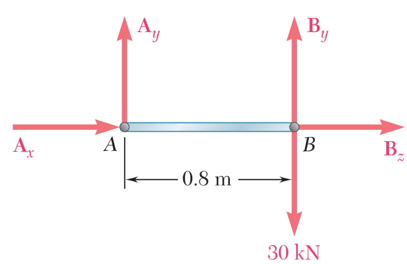


Fig. 1.3

We note that the reaction at A is directed along the axis of the boom AB and causes compression in that member. Observing that the components C_x and C_y of the reaction at C are, respectively, proportional to the horizontal and vertical components of the distance from B to C, we conclude that the reaction at C is equal to 50 kN, is directed along the axis of the rod BC, and causes tension in that member.

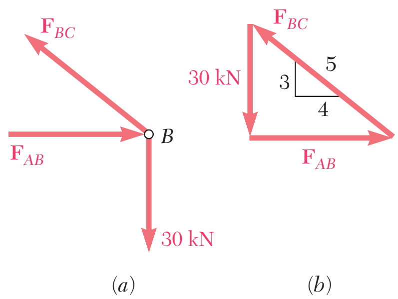


Fig. 1.4

These results could have been anticipated by recognizing that AB and BC are two-force members, i.e., members that are subjected to forces at only two points, these points being A and B for member AB, and B and C for member BC. Indeed, for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points. Using this property, we could have obtained a simpler solution by considering the free-body diagram of pin B. The forces on pin B are the forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ exerted, respectively, by members AB and BC, and the 30-kN load (Fig. 1.4*a*). We can express that pin B is in equilibrium by drawing the corresponding force triangle (Fig. 1.4*b*).

Since the force $\mathbf{F}_{BC}$ is directed along member BC, its slope is the same as that of BC, namely, 3/4. We can, therefore, write the proportion

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

from which we obtain

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

The forces $\mathbf{F}'_{AB}$ and $\mathbf{F}'_{BC}$ exerted by pin B, respectively, on boom AB and rod BC are equal and opposite to $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ (Fig. 1.5).

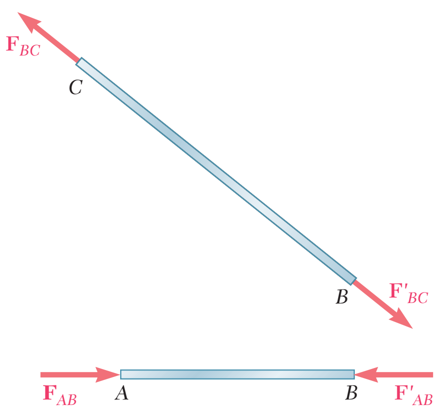


Fig. 1.5

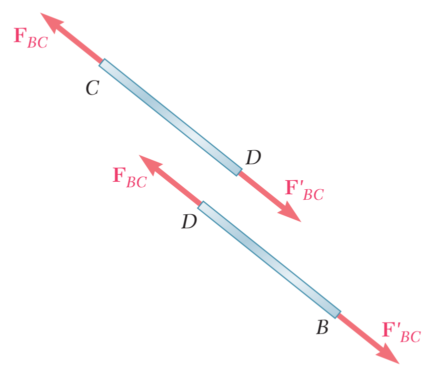


Fig. 1.6

Knowing the forces at the ends of each of the members, we can now determine the internal forces in these members. Passing a section at some arbitrary point D of rod BC, we obtain two portions BD and CD (Fig. 1.6). Since 50-kN forces must be applied at D to both portions of the rod to keep them in equilibrium, we conclude that an internal force of 50 kN is produced in rod BC when a 30-kN load is applied at B. We further check from the directions of the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ in Fig. 1.6 that the rod is in tension. A similar procedure would enable us to determine that the internal force in boom AB is 40 kN and that the boom is in compression.

1.3 STRESSES IN THE MEMBERS OF A STRUCTURE

While the results obtained in the preceding section represent a first and necessary step in the analysis of the given structure, they do not tell us whether the given load can be safely supported. Whether rod *BC*, for example, will break or not under this loading depends not only upon the value found for the internal force F_{BC}, but also upon the cross-sectional area of the rod and the material of which the rod is made. Indeed, the internal force F_{BC} actually represents the resultant of elementary forces distributed over the entire area *A* of the cross section (Fig. 1.7) and the average intensity of these distributed forces is equal to the force per unit area, F_{BC}/A, in the section. Whether or not the rod will break under the given loading clearly depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces. It thus depends upon the force F_{BC}, the cross-sectional area *A*, and the material of the rod.

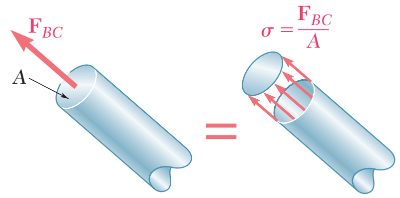


Fig. 1.7

The force per unit area, or intensity of the forces distributed over a given section, is called the *stress* on that section and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area *A* subjected to an axial load **P** (Fig. 1.8) is therefore obtained by dividing the magnitude *P* of the load by the area *A*:

$$\sigma = \frac{P}{A} \tag{1.5}$$

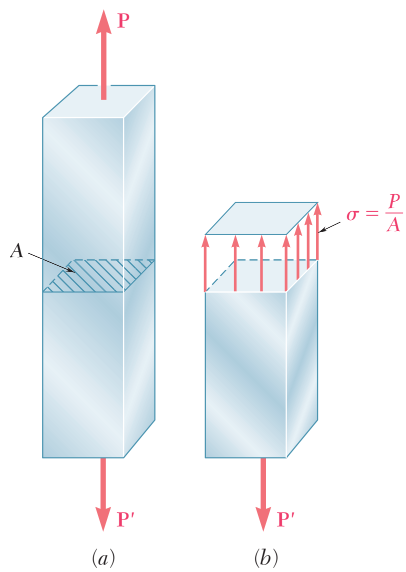


Fig. 1.8 Member with an axial load.

A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression).

Since SI metric units are used in this discussion, with *P* expressed in newtons (N) and *A* in square meters (m^2), the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, one finds that the pascal is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa). We have

$$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$$

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2$$

$$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$$

When U.S. customary units are used, the force *P* is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area *A* in square inches (in^2). The stress σ will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

†The principal SI and U.S. customary units used in mechanics are listed in tables inside the front cover of this book. From the table on the right-hand side, we note that 1 psi is approximately equal to 7 kPa, and 1 ksi approximately equal to 7 MPa.

1.4 ANALYSIS AND DESIGN

Considering again the structure of Fig. 1.1, let us assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{\text{all}} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was found earlier to be 50 kN. Recalling that the diameter of the rod is 20 mm, we use Eq. (1.5) to determine the stress created in the rod by the given loading. We have

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$
$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$
$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since the value obtained for σ is smaller than the value σ_{all} of the allowable stress in the steel used, we conclude that rod BC can safely support the load to which it will be subjected. To be complete, our analysis of the given structure should also include the determination of the compressive stress in boom AB, as well as an investigation of the stresses produced in the pins and their bearings. This will be discussed later in this chapter. We should also determine whether the deformations produced by the given loading are acceptable. The study of deformations under axial loads will be the subject of Chap. 2. An additional consideration required for members in compression involves the *stability* of the member, i.e., its ability to support a given load without experiencing a sudden change in configuration. This will be discussed in Chap. 10.

The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions. Of even greater importance to the engineer is the *design* of new structures and machines, that is, the selection of appropriate components to perform a given task. As an example of design, let us return to the structure of Fig. 1.1, and assume that aluminum with an allowable stress $\sigma_{\text{all}} = 100$ MPa is to be used. Since the force in rod BC will still be $P = F_{BC} = 50$ kN under the given loading, we must have, from Eq. (1.5),

$$\sigma_{\text{all}} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{\text{all}}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

and, since $A = \pi r^2$,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} \text{ m}^2}{\pi}} = 12.62 \times 10^{-3} \text{ m} = 12.62 \text{ mm}$$

$$d = 2r = 25.2 \text{ mm}$$

We conclude that an aluminum rod 26 mm or more in diameter will be adequate.

1.5 AXIAL LOADING; NORMAL STRESS

As we have already indicated, rod BC of the example considered in the preceding section is a two-force member and, therefore, the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ acting on its ends B and C (Fig. 1.5) are directed along the axis of the rod. We say that the rod is under *axial loading*. An actual example of structural members under axial loading is provided by the members of the bridge truss shown in Photo 1.1.

Photo 1.1 This bridge truss consists of two-force members that may be in tension or in compression.

Returning to rod BC of Fig. 1.5, we recall that the section we passed through the rod to determine the internal force in the rod and the corresponding stress was perpendicular to the axis of the rod; the internal force was therefore normal to the plane of the section (Fig. 1.7) and the corresponding stress is described as a *normal stress*. Thus, formula (1.5) gives us the *normal stress in a member under axial loading*:

$$\sigma = \frac{P}{A} \tag{1.5}$$

We should also note that, in formula (1.5), σ is obtained by dividing the magnitude P of the resultant of the internal forces distributed over the cross section by the area A of the cross section; it represents, therefore, the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section.

To define the stress at a given point Q of the cross section, we should consider a small area ΔA (Fig. 1.9). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, we obtain the average value of the stress over ΔA. Letting ΔA approach zero, we obtain the stress at point Q:

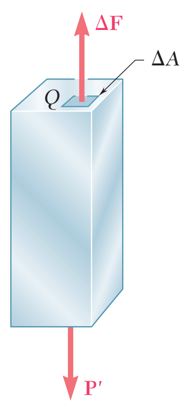


Fig. 1.9

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

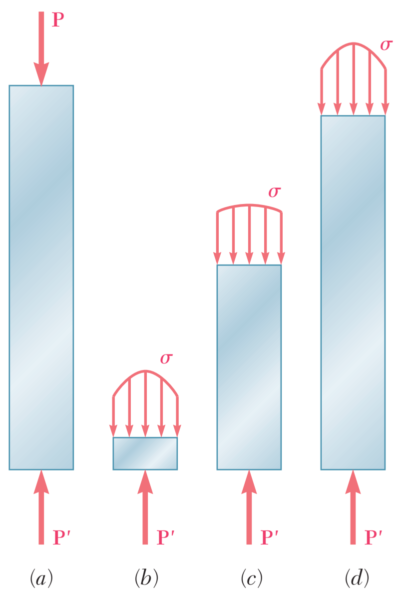


Fig. 1.10 Stress distributions at different sections along axially loaded member.

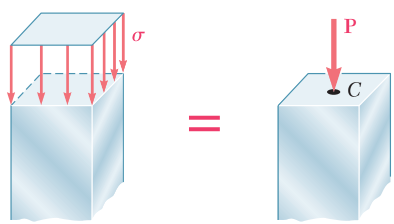


Fig. 1.11

In general, the value obtained for the stress σ at a given point Q of the section is different from the value of the average stress given by formula (1.5), and σ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads $\mathbf{P}$ and $\mathbf{P}'$ (Fig. 1.10*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.10*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.10*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma\, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.10 require that this magnitude be equal to the magnitude P of the concentrated loads. We have, therefore,

$$P = \int dF = \int_A \sigma\, dA \tag{1.7}$$

which means that the volume under each of the stress surfaces in Fig. 1.10 must be equal to the magnitude P of the loads. This, however, is the only information that we can derive from our knowledge of statics, regarding the distribution of normal stresses in the various sections of the rod. The actual distribution of stresses in any given section is *statically indeterminate*. To learn more about this distribution, it is necessary to consider the deformations resulting from the particular mode of application of the loads at the ends of the rod. This will be discussed further in Chap. 2.

In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value σ of the stress is then equal to σ_{ave} and can be obtained from formula (1.5). However, we should realize that, when we assume a uniform distribution of stresses in the section, i.e., when we assume that the internal forces are uniformly distributed across the section, it follows from elementary statics† that the resultant $\mathbf{P}$ of the internal forces must be applied at the centroid C of the section (Fig. 1.11). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* $\mathbf{P}$ *and* $\mathbf{P}'$ *passes through the centroid of the section considered* (Fig. 1.12). This type of loading is called *centric loading* and will be assumed to take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1. However, if a two-force member is loaded axially, but *eccentrically* as shown in Fig. 1.13*a*, we find from the conditions of equilibrium of the portion of member shown in Fig. 1.13*b* that the internal forces in a given section must be

†See Ferdinand P. Beer and E. Russell Johnston, Jr., *Mechanics for Engineers*, 5th ed., McGraw-Hill, New York, 2008, or *Vector Mechanics for Engineers*, 9th ed., McGraw-Hill, New York, 2010, Secs. 5.2 and 5.3.

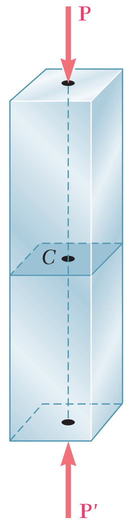
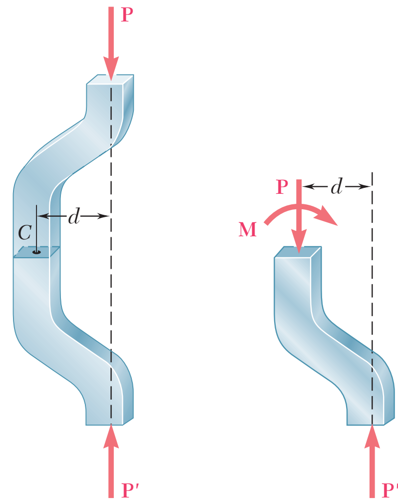


Fig. 1.12



(a) (b)

Fig. 1.13 Eccentric axial loading.

equivalent to a force **P** applied at the centroid of the section and a couple **M** of moment $M = Pd$. The distribution of forces—and, thus, the corresponding distribution of stresses—*cannot be uniform*. Nor can the distribution of stresses be symmetric as shown in Fig. 1.10. This point will be discussed in detail in Chap. 4.

1.6 SHEARING STRESS

The internal forces and the corresponding stresses discussed in Secs. 1.2 and 1.3 were normal to the section considered. A very different type of stress is obtained when transverse forces **P** and **P′** are applied to a member AB (Fig. 1.14). Passing a section at C between the points of application of the two forces (Fig. 1.15*a*), we obtain the diagram of portion AC shown in Fig. 1.15*b*. We conclude that internal forces must exist in the plane of the section, and that their resultant is equal to **P.** These elementary internal forces are called *shearing forces*, and the magnitude P of their resultant is the *shear* in the section. Dividing the shear P by the area A of the cross section, we

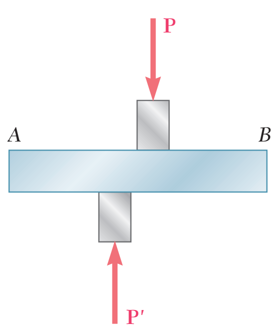


Fig. 1.14 Member with transverse loads.

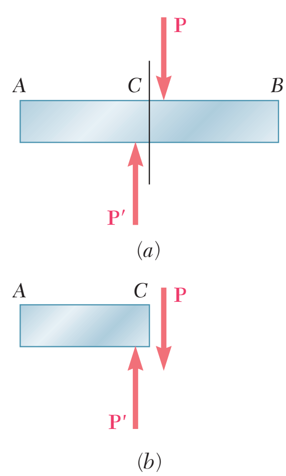


Fig. 1.15

obtain the *average shearing stress* in the section. Denoting the shearing stress by the Greek letter τ (tau), we write

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

It should be emphasized that the value obtained is an average value of the shearing stress over the entire section. Contrary to what we said earlier for normal stresses, the distribution of shearing stresses across the section *cannot* be assumed uniform. As you will see in Chap. 6, the actual value τ of the shearing stress varies from zero at the surface of the member to a maximum value $\tau_{\max}$ that may be much larger than the average value τ_{ave}.

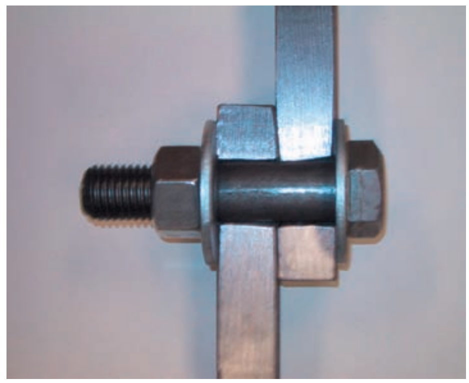


Photo 1.2 Cutaway view of a connection with a bolt in shear.

Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components (Photo 1.2). Consider the two plates A and B, which are connected by a bolt CD (Fig. 1.16). If the plates are subjected to tension forces of magnitude F, stresses will develop in the section of bolt corresponding to the plane EE'. Drawing the diagrams of the bolt and of the portion located above the plane EE' (Fig. 1.17), we conclude that the shear P in the section is equal to F. The average shearing stress in the section is obtained, according to formula (1.8), by dividing the shear $P = F$ by the area A of the cross section:

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

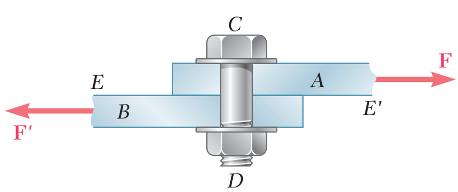


Fig. 1.16 Bolt subject to single shear.

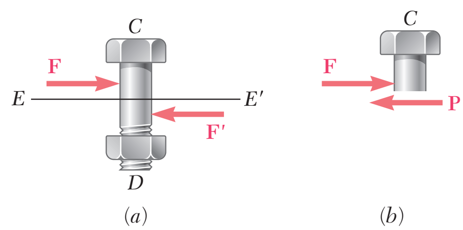


Fig. 1.17

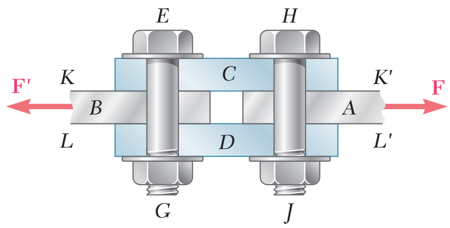


Fig. 1.18 Bolts subject to double shear.

The bolt we have just considered is said to be in *single shear*. Different loading situations may arise, however. For example, if splice plates C and D are used to connect plates A and B (Fig. 1.18), shear will take place in bolt HJ in each of the two planes KK' and LL' (and similarly in bolt EG). The bolts are said to be in *double shear*. To determine the average shearing stress in each plane, we draw free-body diagrams of bolt HJ and of the portion of bolt located between the two planes (Fig. 1.19). Observing that the shear P in each of the sections is $P = F/2$, we conclude that the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

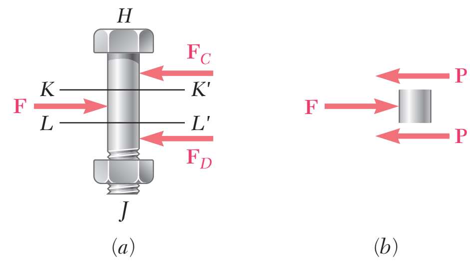


Fig. 1.19

1.7 BEARING STRESS IN CONNECTIONS

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface*, or surface of contact. For example, consider again the two plates A and B connected by a bolt CD that we have discussed in the preceding section (Fig. 1.16). The bolt exerts on plate A a force $\mathbf{P}$ equal and opposite to the force $\mathbf{F}$ exerted by the plate on the bolt (Fig. 1.20). The force $\mathbf{P}$ represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter d and of length t equal to the thickness of the plate. Since the distribution of these forces—and of the corresponding stresses—is quite complicated, one uses in practice an average nominal value σ_b of the stress, called the *bearing stress*, obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1.21). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

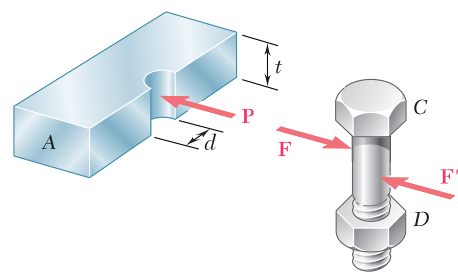


Fig. 1.20

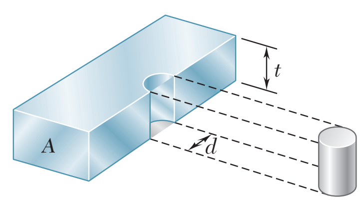


Fig. 1.21

1.8 APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES

We are now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and, thus, to design such structures.

As an example, let us return to the structure of Fig. 1.1 that we have already considered in Sec. 1.2 and let us specify the supports and connections at A, B, and C. As shown in Fig. 1.22, the 20-mm-diameter rod BC has flat ends of 20×40-mm rectangular cross section, while boom AB has a 30×50-mm rectangular cross section and is fitted with a clevis at end B. Both members are connected at B by a pin from which the 30-kN load is suspended by means of a U-shaped bracket. Boom AB is supported at A by a pin fitted into a double bracket, while rod BC is connected at C to a single bracket. All pins are 25 mm in diameter.

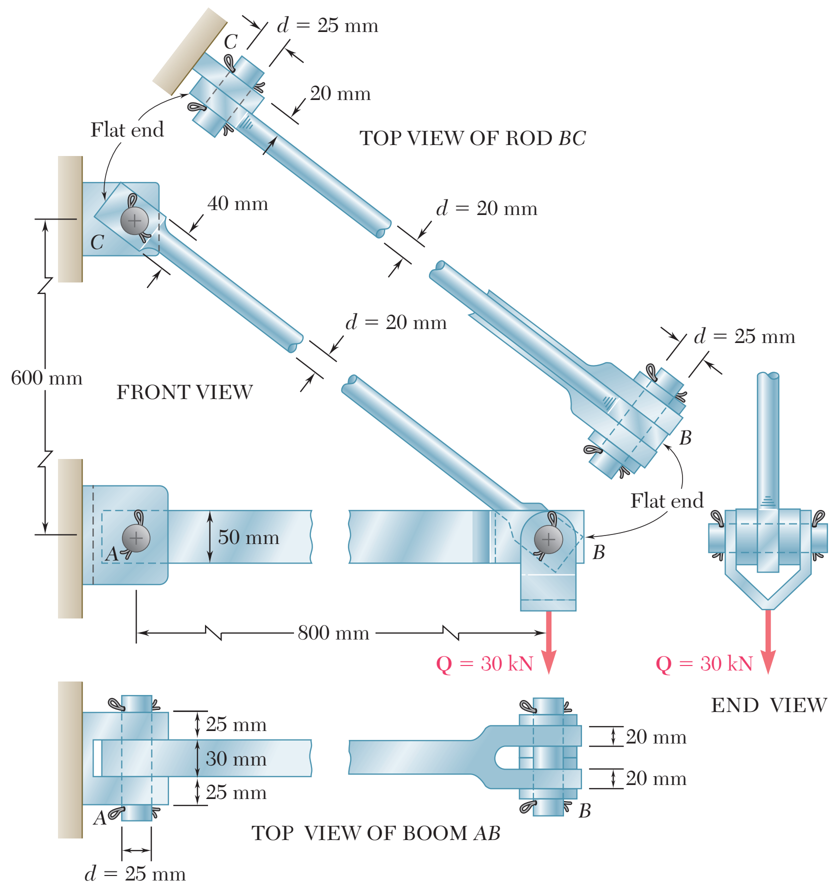


Fig. 1.22

a. Determination of the Normal Stress in Boom *AB* and Rod *BC*. As we found in Secs. 1.2 and 1.4, the force in rod BC is $F_{BC} = 50$ kN (tension) and the area of its circular cross section is $A = 314 \times 10^{-6}$ m^2; the corresponding average normal stress is $\sigma_{BC} = +159$ MPa. However, the flat parts of the rod are also under tension and at the narrowest section, where a hole is located, we have

$$A = (20 \text{ mm})(40 \text{ mm} - 25 \text{ mm}) = 300 \times 10^{-6} \text{ m}^2$$

The corresponding average value of the stress, therefore, is

$$(\sigma_{BC})_{\text{end}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{300 \times 10^{-6} \text{ m}^2} = 167 \text{ MPa}$$

Note that this is an *average value*; close to the hole, the stress will actually reach a much larger value, as you will see in Sec. 2.18. It is clear that, under an increasing load, the rod will fail near one of the holes rather than in its cylindrical portion; its design, therefore, could be improved by increasing the width or the thickness of the flat ends of the rod.

Turning now our attention to boom AB, we recall from Sec. 1.2 that the force in the boom is $F_{AB} = 40$ kN (compression). Since the area of the boom's rectangular cross section is $A = 30 \text{ mm} \times 50 \text{ mm} = 1.5 \times 10^{-3} \text{ m}^2$, the average value of the normal stress in the main part of the rod, between pins A and B, is

$$\sigma_{AB} = -\frac{40 \times 10^3 \text{ N}}{1.5 \times 10^{-3} \text{ m}^2} = -26.7 \times 10^6 \text{ Pa} = -26.7 \text{ MPa}$$

Note that the sections of minimum area at A and B are not under stress, since the boom is in compression, and, therefore, *pushes* on the pins (instead of *pulling* on the pins as rod BC does).

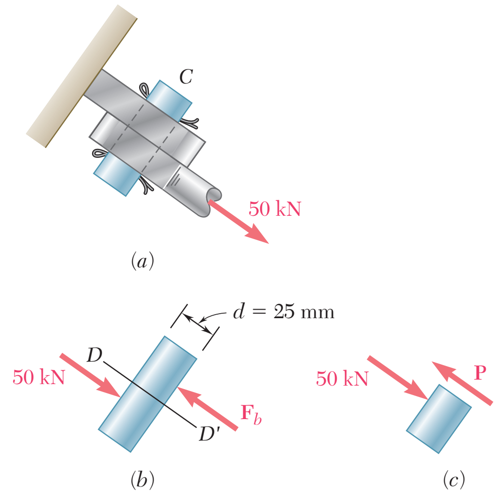


Fig. 1.23

b. Determination of the Shearing Stress in Various Connections. To determine the shearing stress in a connection such as a bolt, pin, or rivet, we first clearly show the forces exerted by the various members it connects. Thus, in the case of pin C of our example (Fig. 1.23*a*), we draw Fig. 1.23*b*, showing the 50-kN force exerted by member BC on the pin, and the equal and opposite force exerted by the bracket. Drawing now the diagram of the portion of the pin located below the plane DD' where shearing stresses occur (Fig. 1.23*c*), we conclude that the shear in that plane is $P = 50$ kN. Since the cross-sectional area of the pin is

$$A = \pi r^2 = \pi\left(\frac{25 \text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3} \text{ m})^2 = 491 \times 10^{-6} \text{ m}^2$$

we find that the average value of the shearing stress in the pin at C is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{491 \times 10^{-6} \text{ m}^2} = 102 \text{ MPa}$$

Considering now the pin at A (Fig. 1.24), we note that it is in double shear. Drawing the free-body diagrams of the pin and of the portion of pin located between the planes DD' and EE' where shearing stresses occur, we conclude that $P = 20$ kN and that

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{20 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 40.7 \text{ MPa}$$

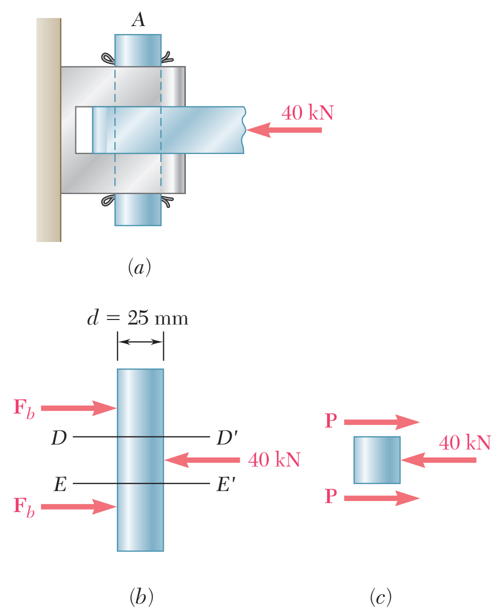


Fig. 1.24

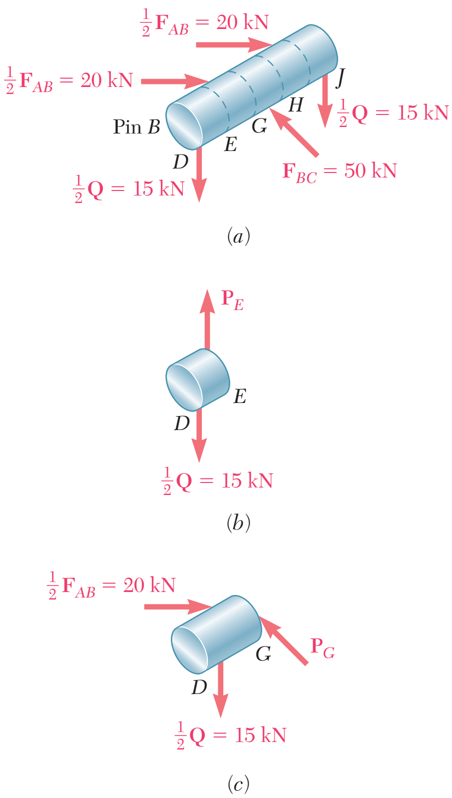


Fig. 1.25

Considering the pin at B (Fig. 1.25*a*), we note that the pin may be divided into five portions which are acted upon by forces exerted by the boom, rod, and bracket. Considering successively the portions DE (Fig. 1.25*b*) and DG (Fig. 1.25*c*), we conclude that the shear in section E is $P_E = 15$ kN, while the shear in section G is $P_G = 25$ kN. Since the loading of the pin is symmetric, we conclude that the maximum value of the shear in pin B is $P_G = 25$ kN, and that the largest shearing stresses occur in sections G and H, where

$$\tau_{\text{ave}} = \frac{P_G}{A} = \frac{25 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 50.9 \text{ MPa}$$

c. Determination of the Bearing Stresses. To determine the nominal bearing stress at A in member AB, we use formula (1.11) of Sec. 1.7. From Fig. 1.22, we have $t = 30$ mm and $d = 25$ mm. Recalling that $P = F_{AB} = 40$ kN, we have

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(30 \text{ mm})(25 \text{ mm})} = 53.3 \text{ MPa}$$

To obtain the bearing stress in the bracket at A, we use $t = 2(25 \text{ mm}) = 50$ mm and $d = 25$ mm:

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(50 \text{ mm})(25 \text{ mm})} = 32.0 \text{ MPa}$$

The bearing stresses at B in member AB, at B and C in member BC, and in the bracket at C are found in a similar way.

1.9 METHOD OF PROBLEM SOLUTION

You should approach a problem in mechanics of materials as you would approach an actual engineering situation. By drawing on your own experience and intuition, you will find it easier to understand and formulate the problem. Once the problem has been clearly stated, however, there is no place in its solution for your particular fancy. Your solution must be based on the fundamental principles of statics and on the principles you will learn in this course. Every step you take must be justified on that basis, leaving no room for your "intuition." After an answer has been obtained, it should be checked. Here again, you may call upon your common sense and personal experience. If not completely satisfied with the result obtained, you should carefully check your formulation of the problem, the validity of the methods used in its solution, and the accuracy of your computations.

The *statement* of the problem should be clear and precise. It should contain the given data and indicate what information is required. A simplified drawing showing all essential quantities involved should be included. The solution of most of the problems you will encounter will necessitate that you first determine the *reactions at supports* and *internal forces and couples*. This will require the drawing of one or several *free-body diagrams,* as was done in Sec. 1.2, from which you will write *equilibrium equations*. These equations can be solved for the unknown forces, from which the required *stresses* and *deformations* will be computed.

After the answer has been obtained, it should be *carefully checked*. Mistakes in *reasoning* can often be detected by carrying the units through your computations and checking the units obtained for the answer. For example, in the design of the rod discussed in Sec. 1.4, we found, after carrying the units through our computations, that the required diameter of the rod was expressed in millimeters, which is the correct unit for a dimension; if another unit had been found, we would have known that some mistake had been made.

Errors in *computation* will usually be found by substituting the numerical values obtained into an equation which has not yet been used and verifying that the equation is satisfied. The importance of correct computations in engineering cannot be overemphasized.

1.10 NUMERICAL ACCURACY

The accuracy of the solution of a problem depends upon two items: (1) the accuracy of the given data and (2) the accuracy of the computations performed.

The solution cannot be more accurate than the less accurate of these two items. For example, if the loading of a beam is known to be 75,000 lb with a possible error of 100 lb either way, the relative error which measures the degree of accuracy of the data is

$$\frac{100 \text{ lb}}{75{,}000 \text{ lb}} = 0.0013 = 0.13\%$$

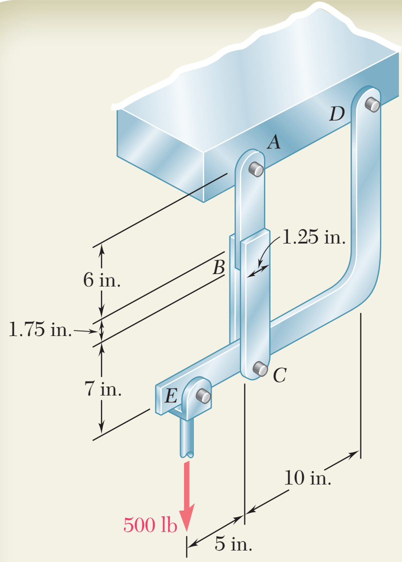
In computing the reaction at one of the beam supports, it would then be meaningless to record it as 14,322 lb. The accuracy of the solution cannot be greater than 0.13%, no matter how accurate the computations are, and the possible error in the answer may be as large as $(0.13/100)(14{,}322 \text{ lb}) \approx 20$ lb. The answer should be properly recorded as $14{,}320 \pm 20$ lb.

In engineering problems, the data are seldom known with an accuracy greater than 0.2%. It is therefore seldom justified to write the answers to such problems with an accuracy greater than 0.2%. A practical rule is to use 4 figures to record numbers beginning with a "1" and 3 figures in all other cases. Unless otherwise indicated, the data given in a problem should be assumed known with a comparable degree of accuracy. A force of 40 lb, for example, should be read 40.0 lb, and a force of 15 lb should be read 15.00 lb.

Pocket calculators and computers are widely used by practicing engineers and engineering students. The speed and accuracy of these devices facilitate the numerical computations in the solution of many problems. However, students should not record more significant figures than can be justified merely because they are easily obtained. As noted above, an accuracy greater than 0.2% is seldom necessary or meaningful in the solution of practical engineering problems.

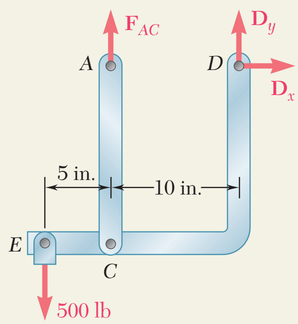
SAMPLE PROBLEM 1.1

In the hanger shown, the upper portion of link ABC is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of $\frac{3}{8}$-in. diameter while a $\frac{1}{4}$-in.-diameter pin is used at C. Determine (*a*) the shearing stress in pin A, (*b*) the shearing stress in pin C, (*c*) the largest normal stress in link ABC, (*d*) the average shearing stress on the bonded surfaces at B, (*e*) the bearing stress in the link at C.



SOLUTION

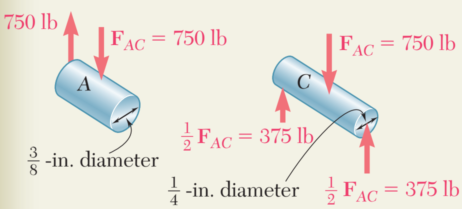
Free Body: Entire Hanger. Since the link ABC is a two-force member, the reaction at A is vertical; the reaction at D is represented by its components $\mathbf{D}_x$ and $\mathbf{D}_y$. We write



$$+\circlearrowleft \Sigma M_D = 0: \qquad (500 \text{ lb})(15 \text{ in.}) - F_{AC}(10 \text{ in.}) = 0$$

$$F_{AC} = +750 \text{ lb} \qquad F_{AC} = 750 \text{ lb} \quad \textit{tension}$$

a. Shearing Stress in Pin *A*. Since this $\frac{3}{8}$-in.-diameter pin is in single shear, we write

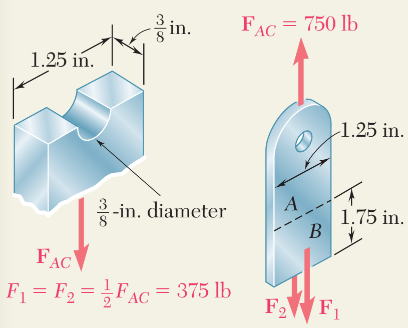


$$\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ lb}}{\frac{1}{4}\pi(0.375 \text{ in.})^2} \qquad \tau_A = 6790 \text{ psi} \blacktriangleleft$$

b. Shearing Stress in Pin *C*. Since this $\frac{1}{4}$-in.-diameter pin is in double shear, we write

$$\tau_C = \frac{\frac{1}{2}F_{AC}}{A} = \frac{375 \text{ lb}}{\frac{1}{4}\pi(0.25 \text{ in.})^2} \qquad \tau_C = 7640 \text{ psi} \blacktriangleleft$$

c. Largest Normal Stress in Link *ABC*. The largest stress is found where the area is smallest; this occurs at the cross section at A where the $\frac{3}{8}$-in. hole is located. We have

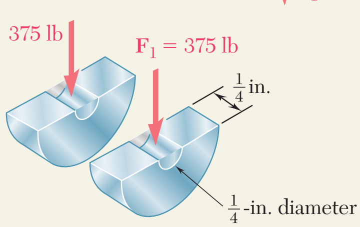


$$\sigma_A = \frac{F_{AC}}{A_{\text{net}}} = \frac{750 \text{ lb}}{(\frac{3}{8} \text{ in.})(1.25 \text{ in.} - 0.375 \text{ in.})} = \frac{750 \text{ lb}}{0.328 \text{ in}^2} \qquad \sigma_A = 2290 \text{ psi} \blacktriangleleft$$

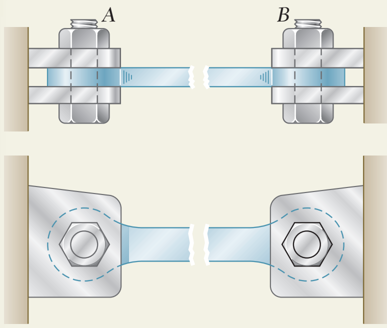
d. Average Shearing Stress at *B*. We note that bonding exists on both sides of the upper portion of the link and that the shear force on each side is $F_1 = (750 \text{ lb})/2 = 375$ lb. The average shearing stress on each surface is thus

$$\tau_B = \frac{F_1}{A} = \frac{375 \text{ lb}}{(1.25 \text{ in.})(1.75 \text{ in.})} \qquad \tau_B = 171.4 \text{ psi} \blacktriangleleft$$

e. Bearing Stress in Link at *C*. For each portion of the link, $F_1 = 375$ lb and the nominal bearing area is $(0.25 \text{ in.})(0.25 \text{ in.}) = 0.0625 \text{ in}^2$.



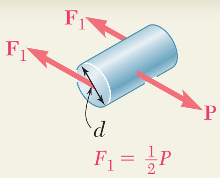
$$\sigma_b = \frac{F_1}{A} = \frac{375 \text{ lb}}{0.0625 \text{ in}^2} \qquad \sigma_b = 6000 \text{ psi} \blacktriangleleft$$



SAMPLE PROBLEM 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (*a*) the diameter d of the bolt, (*b*) the dimension b at each end of the bar, (*c*) the dimension h of the bar.

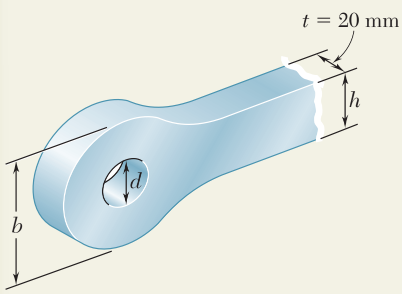
SOLUTION



a. Diameter of the Bolt. Since the bolt is in double shear, $F_1 = \frac{1}{2}P = 60$ kN.

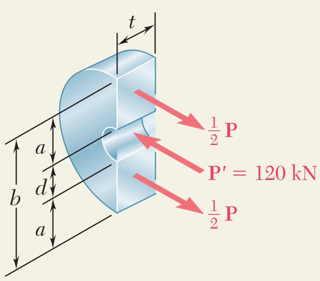
$$\tau = \frac{F_1}{A} = \frac{60 \text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad 100 \text{ MPa} = \frac{60 \text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad d = 27.6 \text{ mm}$$

We will use $d = 28$ mm ◀



At this point we check the bearing stress between the 20-mm-thick plate and the 28-mm-diameter bolt.

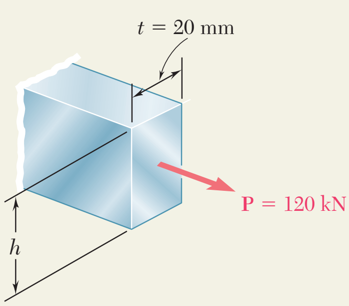
$$\tau_b = \frac{P}{td} = \frac{120 \text{ kN}}{(0.020 \text{ m})(0.028 \text{ m})} = 214 \text{ MPa} < 350 \text{ MPa} \qquad \text{OK}$$



b. Dimension *b* at Each End of the Bar. We consider one of the end portions of the bar. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, we write

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175 \text{ MPa} = \frac{60 \text{ kN}}{(0.02 \text{ m})a} \qquad a = 17.14 \text{ mm}$$

$$b = d + 2a = 28 \text{ mm} + 2(17.14 \text{ mm}) \qquad b = 62.3 \text{ mm} \blacktriangleleft$$



c. Dimension *h* of the Bar. Recalling that the thickness of the steel plate is $t = 20$ mm, we have

$$\sigma = \frac{P}{th} \qquad 175 \text{ MPa} = \frac{120 \text{ kN}}{(0.020 \text{ m})h} \qquad h = 34.3 \text{ mm}$$

We will use $h = 35$ mm ◀

PROBLEMS

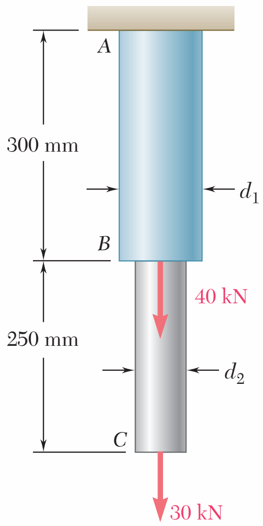


Fig. P1.1 and P1.2

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.

1.2 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* has the same magnitude as the compressive stress in rod *BC*.

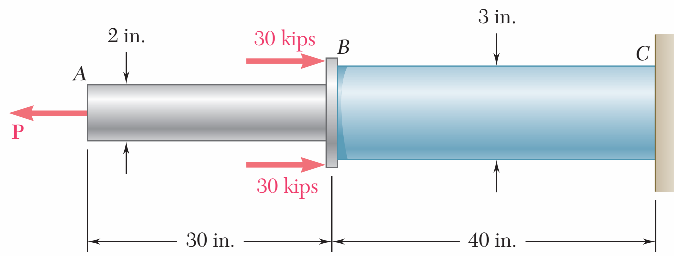


Fig. P1.3

1.4 In Prob. 1.3, knowing that $P = 40$ kips, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.5 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

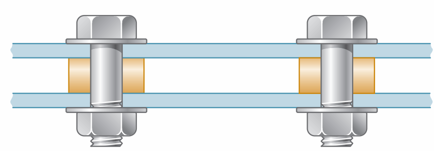


Fig. *P1.5*

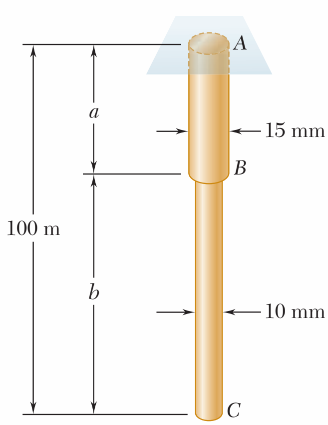


Fig. *P1.6*

1.6 Two brass rods *AB* and *BC*, each of uniform diameter, will be brazed together at *B* to form a nonuniform rod of total length 100 m which will be suspended from a support at *A* as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod *AB* for which the maximum normal stress in *ABC* is minimum, (*b*) the corresponding value of the maximum normal stress.

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

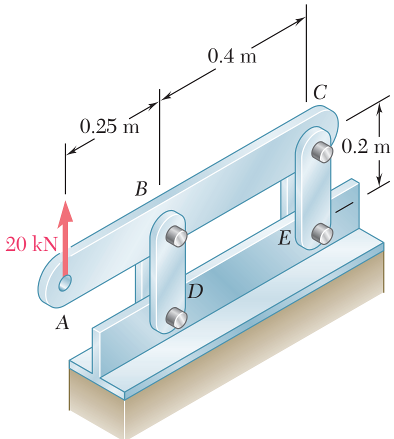


Fig. P1.7

1.8 Knowing that link *DE* is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

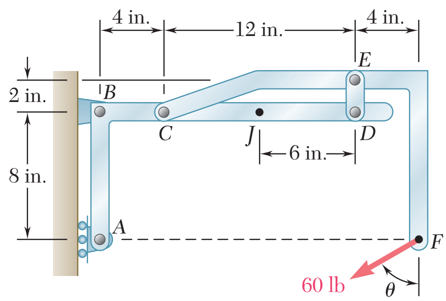


Fig. P1.8

1.9 Link *AC* has a uniform rectangular cross section $\frac{1}{16}$ in. thick and $\frac{1}{4}$ in. wide. Determine the normal stress in the central portion of the link.

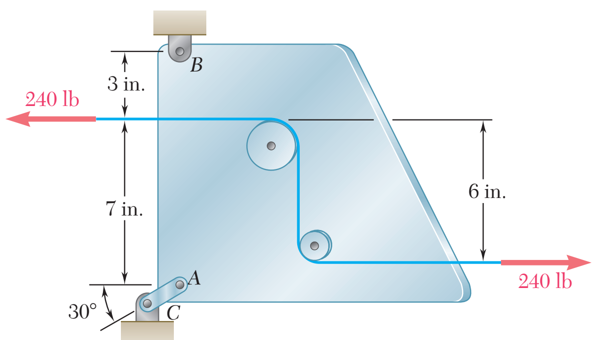


Fig. P1.9

1.10 Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod *BE* for which the normal stress in that portion is +100 MPa.

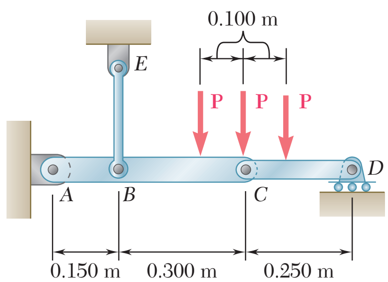


Fig. P1.10

1.11 The frame shown consists of *four* wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a $\frac{1}{2}$-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

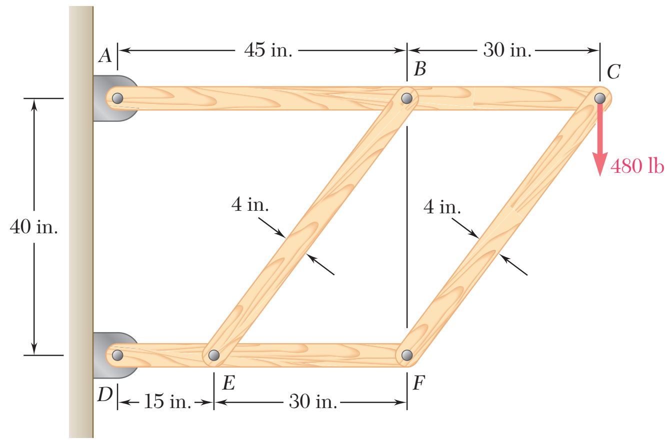


Fig. *P1.11*

1.12 For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

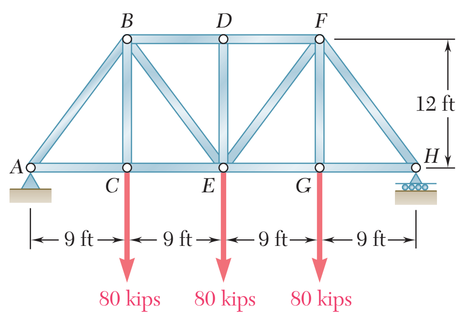


Fig. *P1.12*

1.13 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

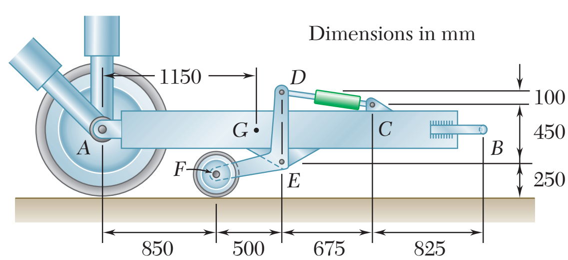


Fig. P1.13

1.14 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

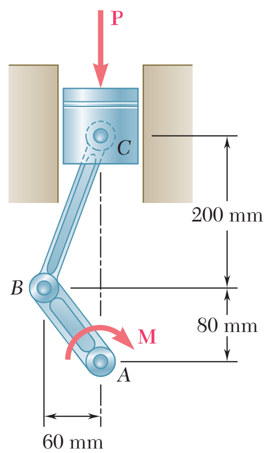


Fig. P1.14

1.15 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

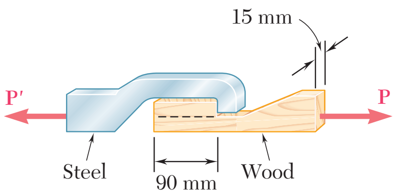


Fig. P1.15

1.16 The wooden members *A* and *B* are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 120 psi.

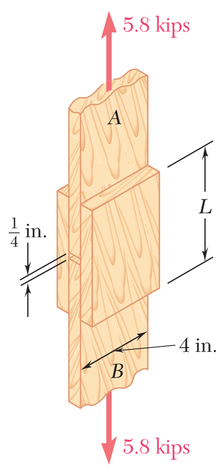


Fig. P1.16

1.17 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** that can be applied to the rod.

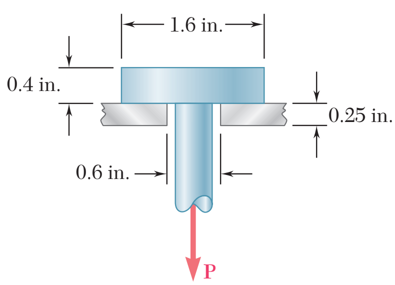


Fig. *P1.17*

1.18 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length *d* of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

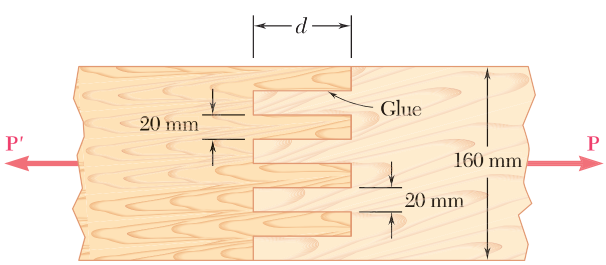


Fig. P1.18

1.19 The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

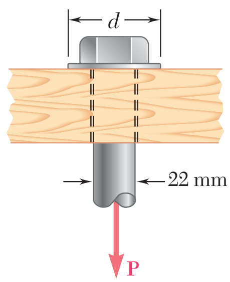


Fig. P1.19

1.20 The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

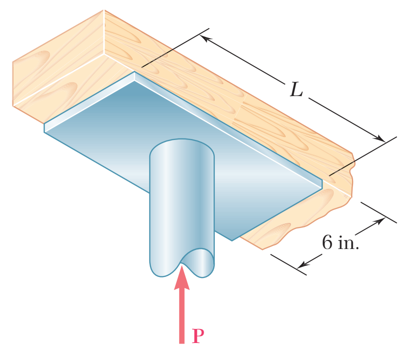


Fig. *P1.20*

1.21 An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7$ in^2 and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

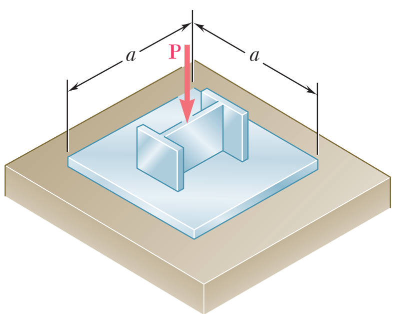


Fig. P1.21

1.22 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

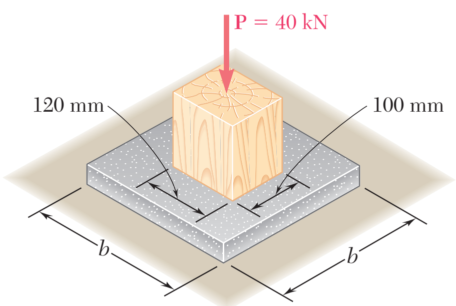


Fig. P1.22

1.23 A $\frac{5}{8}$-in.-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance b for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

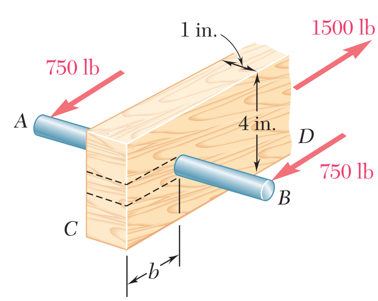


Fig. P1.23

1.24 Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

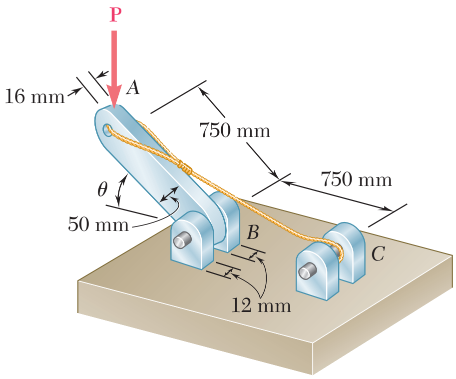


Fig. *P1.24* and *P1.25*

1.25 Determine the largest load **P** that can be applied at *A* when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at *B* must not exceed 120 MPa and that the average bearing stress in member *AB* and in the bracket at *B* must not exceed 90 MPa.

1.26 Link *AB*, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter *d* of the pins, (*b*) the average bearing stress in the link.

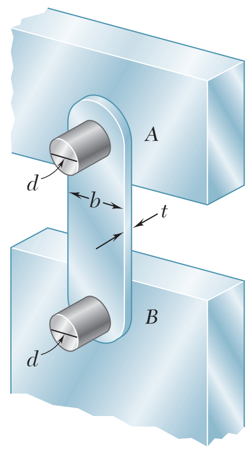


Fig. P1.26

1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50-mm uniform rectangular cross section.

1.28 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

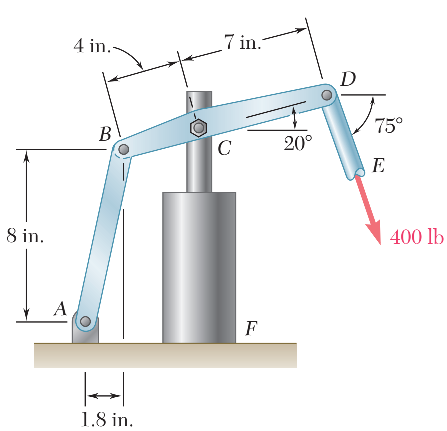


Fig. P1.28

1.11 STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

In the preceding sections, axial forces exerted on a two-force member (Fig. 1.26*a*) were found to cause normal stresses in that member (Fig. 1.26*b*), while transverse forces exerted on bolts and pins (Fig. 1.27*a*) were found to cause shearing stresses in those connections (Fig. 1.27*b*). The reason such a relation was observed between axial forces and normal stresses on one hand, and transverse forces and shearing stresses on the other, was because stresses were being determined only on planes perpendicular to the axis of the member or connection. As you will see in this section, axial forces cause both normal and shearing stresses on planes which are not perpendicular to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes which are not perpendicular to the axis of the bolt or pin.

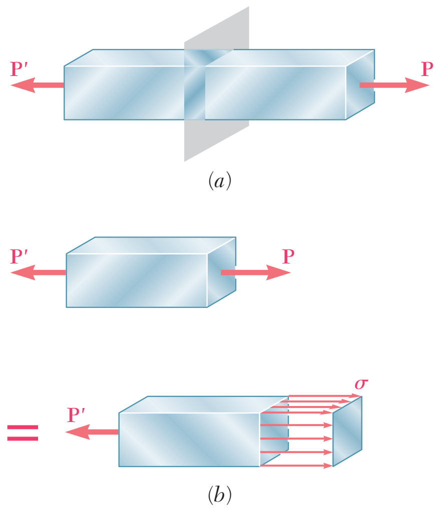


Fig. 1.26 Axial forces.

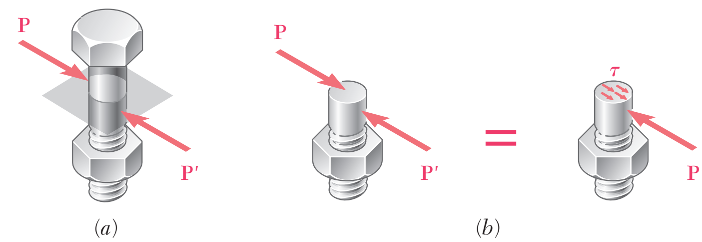


Fig. 1.27 Transverse forces.

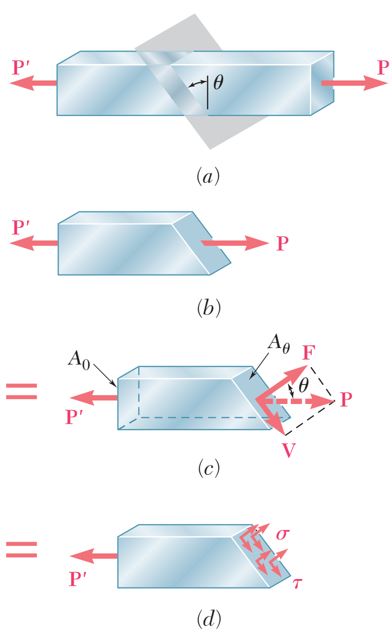


Fig. 1.28

Consider the two-force member of Fig. 1.26, which is subjected to axial forces **P** and **P**′. If we pass a section forming an angle θ with a normal plane (Fig. 1.28*a*) and draw the free-body diagram of the portion of member located to the left of that section (Fig. 1.28*b*), we find from the equilibrium conditions of the free body that the distributed forces acting on the section must be equivalent to the force **P.**

Resolving **P** into components **F** and **V,** respectively normal and tangential to the section (Fig. 1.28*c*), we have

$$F = P\cos\theta \qquad V = P\sin\theta \tag{1.12}$$

The force **F** represents the resultant of normal forces distributed over the section, and the force **V** the resultant of shearing forces (Fig. 1.28*d*). The average values of the corresponding normal and shearing stresses are obtained by dividing, respectively, F and V by the area A_θ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta} \tag{1.13}$$

Substituting for F and V from (1.12) into (1.13), and observing from Fig. 1.28*c* that $A_0 = A_\theta \cos\theta$, or $A_\theta = A_0/\cos\theta$, where A_0 denotes the area of a section perpendicular to the axis of the member, we obtain

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

or

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta \tag{1.14}$$

We note from the first of Eqs. (1.14) that the normal stress σ is maximum when $\theta = 0$, i.e., when the plane of the section is perpendicular to the axis of the member, and that it approaches zero as θ approaches 90°. We check that the value of σ when $\theta = 0$ is

$$\sigma_m = \frac{P}{A_0} \tag{1.15}$$

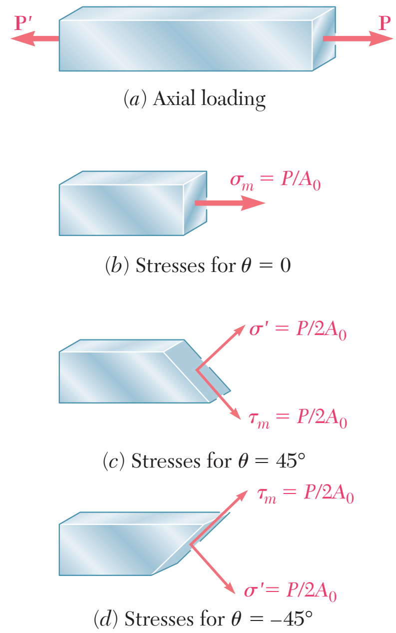
as we found earlier in Sec. 1.3. The second of Eqs. (1.14) shows that the shearing stress τ is zero for $\theta = 0$ and $\theta = 90°$, and that for $\theta = 45°$ it reaches its maximum value

$$\tau_m = \frac{P}{A_0}\sin 45°\cos 45° = \frac{P}{2A_0} \tag{1.16}$$

The first of Eqs. (1.14) indicates that, when $\theta = 45°$, the normal stress σ' is also equal to $P/2A_0$:

$$\sigma' = \frac{P}{A_0}\cos^2 45° = \frac{P}{2A_0} \tag{1.17}$$

The results obtained in Eqs. (1.15), (1.16), and (1.17) are shown graphically in Fig. 1.29. We note that the same loading may produce either a normal stress $\sigma_m = P/A_0$ and no shearing stress (Fig. 1.29*b*), or a normal and a shearing stress of the same magnitude $\sigma' = \tau_m = P/2A_0$ (Fig. 1.29 *c* and *d*), depending upon the orientation of the section.



(*a*) Axial loading
(*b*) Stresses for $\theta = 0$
(*c*) Stresses for $\theta = 45°$
(*d*) Stresses for $\theta = -45°$

Fig. 1.29

1.12 STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

The examples of the previous sections were limited to members under axial loading and connections under transverse loading. Most structural members and machine components are under more involved loading conditions.

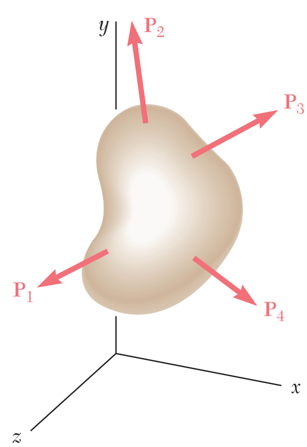


Fig. 1.30

Consider a body subjected to several loads $\mathbf{P}_1$, $\mathbf{P}_2$, etc. (Fig. 1.30). To understand the stress condition created by these loads at some point Q within the body, we shall first pass a section through Q, using a plane parallel to the yz plane. The portion of the body to the left of the section is subjected to some of the original loads, and to normal and shearing forces distributed over the section. We shall denote by $\Delta\mathbf{F}^x$ and $\Delta\mathbf{V}^x$, respectively, the normal and the shearing

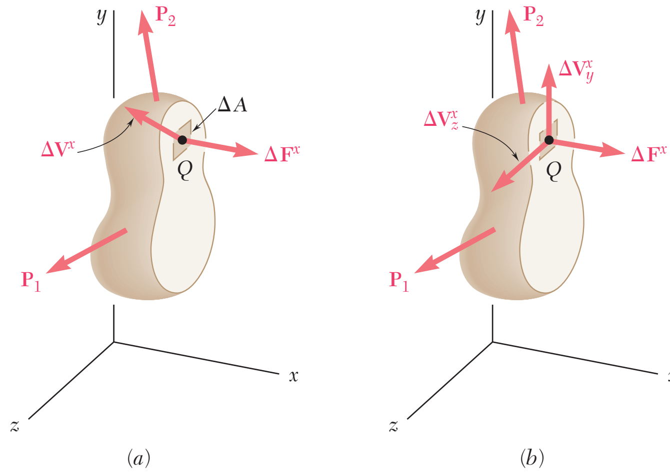


Fig. 1.31

forces acting on a small area ΔA surrounding point Q (Fig. 1.31*a*). Note that the superscript x is used to indicate that the forces $\Delta \mathbf{F}^x$ and $\Delta \mathbf{V}^x$ act on a surface perpendicular to the x axis. While the normal force $\Delta \mathbf{F}^x$ has a well-defined direction, the shearing force $\Delta \mathbf{V}^x$ may have any direction in the plane of the section. We therefore resolve $\Delta \mathbf{V}^x$ into two component forces, $\Delta \mathbf{V}^x_y$ and $\Delta \mathbf{V}^x_z$, in directions parallel to the y and z axes, respectively (Fig. 1.31*b*). Dividing now the magnitude of each force by the area ΔA, and letting ΔA approach zero, we define the three stress components shown in Fig. 1.32:

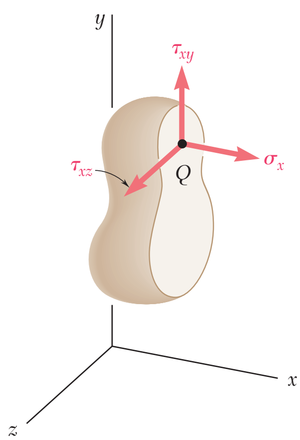


Fig. 1.32

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$
$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V^x_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V^x_z}{\Delta A} \tag{1.18}$$

We note that the first subscript in σ_x, τ_{xy}, and τ_{xz} is used to indicate that the stresses under consideration are exerted *on a surface perpendicular to the x axis.* The second subscript in τ_{xy} and τ_{xz} identifies *the direction of the component.* The normal stress σ_x is positive if the corresponding arrow points in the positive x direction, i.e., if the body is in tension, and negative otherwise. Similarly, the shearing stress components τ_{xy} and τ_{xz} are positive if the corresponding arrows point, respectively, in the positive y and z directions.

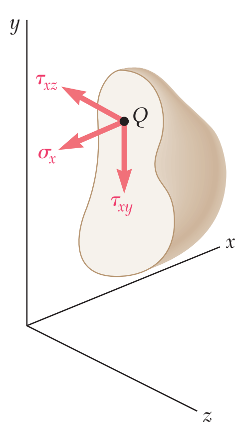


Fig. 1.33

The above analysis may also be carried out by considering the portion of body located to the right of the vertical plane through Q (Fig. 1.33). The same magnitudes, but opposite directions, are obtained for the normal and shearing forces $\Delta \mathbf{F}^x$, $\Delta \mathbf{V}^x_y$, and $\Delta \mathbf{V}^x_z$. Therefore, the same values are also obtained for the corresponding stress components, but since the section in Fig. 1.33 now faces the *negative x axis,* a positive sign for σ_x will indicate that the corresponding arrow points *in the negative x direction.* Similarly, positive signs for τ_{xy} and τ_{xz} will indicate that the corresponding arrows point, respectively, in the negative y and z directions, as shown in Fig. 1.33.

Passing a section through Q parallel to the zx plane, we define in the same manner the stress components, σ_y, τ_{yz}, and τ_{yx}. Finally, a section through Q parallel to the xy plane yields the components σ_z, τ_{zx}, and τ_{zy}.

To facilitate the visualization of the stress condition at point Q, we shall consider a small cube of side a centered at Q and the stresses exerted on each of the six faces of the cube (Fig. 1.34). The stress components shown in the figure are σ_x, σ_y, and σ_z, which represent the normal stress on faces respectively perpendicular to the x, y, and z axes, and the six shearing stress components τ_{xy}, τ_{xz}, etc. We recall that, according to the definition of the shearing stress components, τ_{xy} represents the y component of the shearing stress exerted on the face perpendicular to the x axis, while τ_{yx} represents the x component of the shearing stress exerted on the face perpendicular to the y axis. Note that only three faces of the cube are actually visible in Fig. 1.34, and that equal and opposite stress components act on the hidden faces. While the stresses acting on the faces of the cube differ slightly from the stresses at Q, the error involved is small and vanishes as side a of the cube approaches zero.

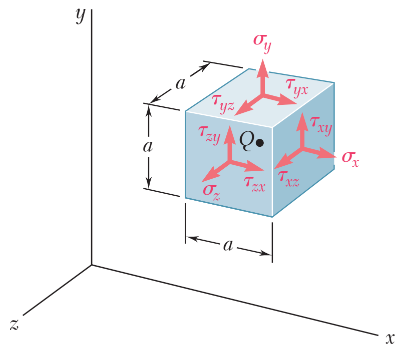


Fig. 1.34

Important relations among the shearing stress components will now be derived. Let us consider the free-body diagram of the small cube centered at point Q (Fig. 1.35). The normal and shearing forces acting on the various faces of the cube are obtained by multiplying the corresponding stress components by the area ΔA of each face. We first write the following three equilibrium equations:

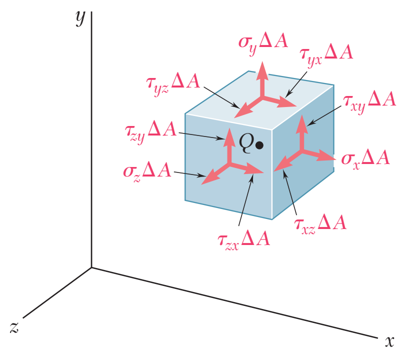


Fig. 1.35

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1.19}$$

Since forces equal and opposite to the forces actually shown in Fig. 1.35 are acting on the hidden faces of the cube, it is clear that Eqs. (1.19) are satisfied. Considering now the moments of the forces about axes x', y', and z' drawn from Q in directions respectively parallel to the x, y, and z axes, we write the three additional equations

$$\Sigma M_{x'} = 0 \qquad \Sigma M_{y'} = 0 \qquad \Sigma M_{z'} = 0 \tag{1.20}$$

Using a projection on the $x'y'$ plane (Fig. 1.36), we note that the only forces with moments about the z axis different from zero are the shearing forces. These forces form two couples, one of counterclockwise (positive) moment $(\tau_{xy}\,\Delta A)a$, the other of clockwise (negative) moment $-(\tau_{yx}\,\Delta A)a$. The last of the three Eqs. (1.20) yields, therefore,

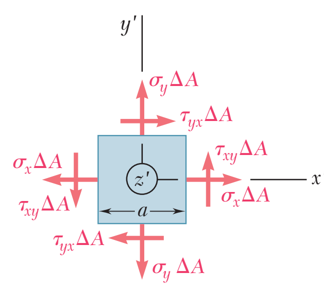


Fig. 1.36

$$+\circlearrowleft \Sigma M_z = 0: \qquad (\tau_{xy}\,\Delta A)a - (\tau_{yx}\,\Delta A)a = 0$$

from which we conclude that

$$\boxed{\tau_{xy} = \tau_{yx}} \tag{1.21}$$

The relation obtained shows that the y component of the shearing stress exerted on a face perpendicular to the x axis is equal to the x

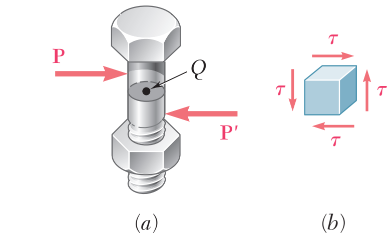


Fig. 1.37

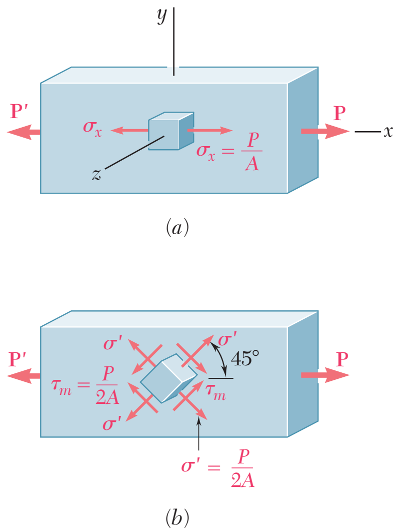


Fig. 1.38

component of the shearing stress exerted on a face perpendicular to the y axis. From the remaining two equations (1.20), we derive in a similar manner the relations

$$\tau_{yz} = \tau_{zy} \qquad \tau_{zx} = \tau_{xz} \tag{1.22}$$

We conclude from Eqs. (1.21) and (1.22) that only six stress components are required to define the condition of stress at a given point Q, instead of nine as originally assumed. These six components are σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}. We also note that, at a given point, *shear cannot take place in one plane only;* an equal shearing stress must be exerted on another plane perpendicular to the first one. For example, considering again the bolt of Fig. 1.27 and a small cube at the center Q of the bolt (Fig. 1.37*a*), we find that shearing stresses of equal magnitude must be exerted on the two horizontal faces of the cube and on the two faces that are perpendicular to the forces **P** and **P′** (Fig. 1.37*b*).

Before concluding our discussion of stress components, let us consider again the case of a member under axial loading. If we consider a small cube with faces respectively parallel to the faces of the member and recall the results obtained in Sec. 1.11, we find that the conditions of stress in the member may be described as shown in Fig. 1.38*a*; the only stresses are normal stresses σ_x exerted on the faces of the cube which are perpendicular to the x axis. However, if the small cube is rotated by 45° about the z axis so that its new orientation matches the orientation of the sections considered in Fig. 1.29*c* and *d*, we conclude that normal and shearing stresses of equal magnitude are exerted on four faces of the cube (Fig. 1.38*b*). We thus observe that the same loading condition may lead to different interpretations of the stress situation at a given point, depending upon the orientation of the element considered. More will be said about this in Chap 7.

1.13 DESIGN CONSIDERATIONS

In the preceding sections you learned to determine the stresses in rods, bolts, and pins under simple loading conditions. In later chapters you will learn to determine stresses in more complex situations. In engineering applications, however, the determination of stresses is seldom an end in itself. Rather, the knowledge of stresses is used by engineers to assist in their most important task, namely, the design of structures and machines that will safely and economically perform a specified function.

a. Determination of the Ultimate Strength of a Material. An important element to be considered by a designer is how the material that has been selected will behave under a load. For a given material, this is determined by performing specific tests on prepared samples of the material. For example, a test specimen of steel may be prepared and placed in a laboratory testing machine to be subjected to a known centric axial tensile force, as described in Sec. 2.3. As the magnitude of the force is increased, various changes in the specimen are measured, for example, changes in its length and its diameter.

Eventually the largest force which may be applied to the specimen is reached, and the specimen either breaks or begins to carry less load. This largest force is called the *ultimate load* for the test specimen and is denoted by P_U. Since the applied load is centric, we may divide the ultimate load by the original cross-sectional area of the rod to obtain the *ultimate normal stress* of the material used. This stress, also known as the *ultimate strength in tension* of the material, is

$$\sigma_U = \frac{P_U}{A} \tag{1.23}$$

Several test procedures are available to determine the *ultimate shearing stress*, or *ultimate strength in shear*, of a material. The one most commonly used involves the twisting of a circular tube (Sec. 3.5). A more direct, if less accurate, procedure consists in clamping a rectangular or round bar in a shear tool (Fig. 1.39) and applying an increasing load P until the ultimate load P_U for single shear is obtained. If the free end of the specimen rests on both of the hardened dies (Fig. 1.40), the ultimate load for double shear is obtained. In either case, the ultimate shearing stress τ_U is obtained by dividing the ultimate load by the total area over which shear has taken place. We recall that, in the case of single shear, this area is the cross-sectional area A of the specimen, while in double shear it is equal to twice the cross-sectional area.

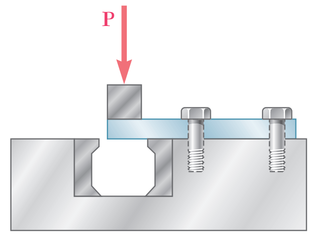


Fig. 1.39 Single shear test.

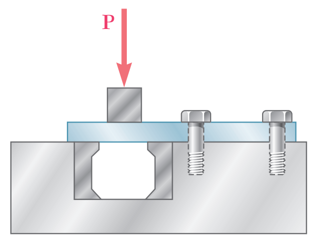


Fig. 1.40 Double shear test.

b. Allowable Load and Allowable Stress; Factor of Safety. The maximum load that a structural member or a machine component will be allowed to carry under normal conditions of utilization is considerably smaller than the ultimate load. This smaller load is referred to as the *allowable load* and, sometimes, as the *working load* or *design load*. Thus, only a fraction of the ultimate-load capacity of the member is utilized when the allowable load is applied. The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance. The ratio of the ultimate load to the allowable load is used to define the *factor of safety*.† We have

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}} \tag{1.24}$$

An alternative definition of the factor of safety is based on the use of stresses:

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate stress}}{\text{allowable stress}} \tag{1.25}$$

The two expressions given for the factor of safety in Eqs. (1.24) and (1.25) are identical when a linear relationship exists between the load and the stress. In most engineering applications, however, this relationship ceases to be linear as the load approaches its ultimate value, and the factor of safety obtained from Eq. (1.25) does not provide a

†In some fields of engineering, notably aeronautical engineering, the *margin of safety* is used in place of the factor of safety. The margin of safety is defined as the factor of safety minus one; that is, margin of safety = $F.S. - 1.00$.

true assessment of the safety of a given design. Nevertheless, the *allowable-stress method* of design, based on the use of Eq. (1.25), is widely used.

c. Selection of an Appropriate Factor of Safety. The selection of the factor of safety to be used for various applications is one of the most important engineering tasks. On the one hand, if a factor of safety is chosen too small, the possibility of failure becomes unacceptably large; on the other hand, if a factor of safety is chosen unnecessarily large, the result is an uneconomical or nonfunctional design. The choice of the factor of safety that is appropriate for a given design application requires engineering judgment based on many considerations, such as the following:

1. *Variations that may occur in the properties of the member under consideration*. The composition, strength, and dimensions of the member are all subject to small variations during manufacture. In addition, material properties may be altered and residual stresses introduced through heating or deformation that may occur during manufacture, storage, transportation, or construction.
2. *The number of loadings that may be expected during the life of the structure or machine*. For most materials the ultimate stress decreases as the number of load applications is increased. This phenomenon is known as *fatigue* and, if ignored, may result in sudden failure (see Sec. 2.7).
3. *The type of loadings that are planned for in the design, or that may occur in the future*. Very few loadings are known with complete accuracy—most design loadings are engineering estimates. In addition, future alterations or changes in usage may introduce changes in the actual loading. Larger factors of safety are also required for dynamic, cyclic, or impulsive loadings.
4. *The type of failure that may occur*. Brittle materials fail suddenly, usually with no prior indication that collapse is imminent. On the other hand, ductile materials, such as structural steel, normally undergo a substantial deformation called *yielding* before failing, thus providing a warning that overloading exists. However, most buckling or stability failures are sudden, whether the material is brittle or not. When the possibility of sudden failure exists, a larger factor of safety should be used than when failure is preceded by obvious warning signs.
5. *Uncertainty due to methods of analysis*. All design methods are based on certain simplifying assumptions which result in calculated stresses being approximations of actual stresses.
6. *Deterioration that may occur in the future because of poor maintenance or because of unpreventable natural causes*. A larger factor of safety is necessary in locations where conditions such as corrosion and decay are difficult to control or even to discover.
7. *The importance of a given member to the integrity of the whole structure*. Bracing and secondary members may in many cases be designed with a factor of safety lower than that used for primary members.

In addition to the these considerations, there is the additional consideration concerning the risk to life and property that a failure would produce. Where a failure would produce no risk to life and only minimal risk to property, the use of a smaller factor of safety can be considered. Finally, there is the practical consideration that, unless a careful design with a nonexcessive factor of safety is used, a structure or machine might not perform its design function. For example, high factors of safety may have an unacceptable effect on the weight of an aircraft.

For the majority of structural and machine applications, factors of safety are specified by design specifications or building codes written by committees of experienced engineers working with professional societies, with industries, or with federal, state, or city agencies. Examples of such design specifications and building codes are

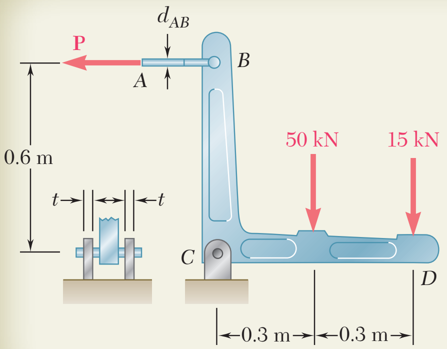
1. *Steel:* American Institute of Steel Construction, Specification for Structural Steel Buildings
2. *Concrete:* American Concrete Institute, Building Code Requirement for Structural Concrete
3. *Timber:* American Forest and Paper Association, National Design Specification for Wood Construction
4. *Highway bridges:* American Association of State Highway Officials, Standard Specifications for Highway Bridges

***d. Load and Resistance Factor Design.** As we saw previously, the allowable-stress method requires that all the uncertainties associated with the design of a structure or machine element be grouped into a single factor of safety. An alternative method of design, which is gaining acceptance chiefly among structural engineers, makes it possible through the use of three different factors to distinguish between the uncertainties associated with the structure itself and those associated with the load it is designed to support. This method, referred to as *Load and Resistance Factor Design (LRFD),* further allows the designer to distinguish between uncertainties associated with the *live load,* P_L, that is, with the load to be supported by the structure, and the *dead load,* P_D, that is, with the weight of the portion of structure contributing to the total load.

When this method of design is used, the *ultimate load,* P_U, of the structure, that is, the load at which the structure ceases to be useful, should first be determined. The proposed design is then acceptable if the following inequality is satisfied:

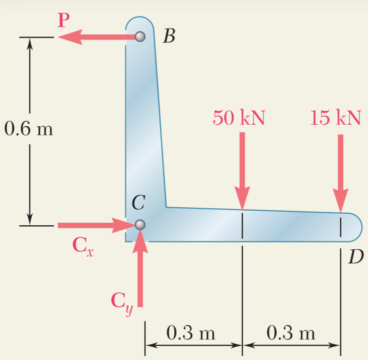
$$\gamma_D P_D + \gamma_L P_L \le \phi P_U \tag{1.26}$$

The coefficient ϕ is referred to as the *resistance factor;* it accounts for the uncertainties associated with the structure itself and will normally be less than 1. The coefficients γ_D and γ_L are referred to as the *load factors;* they account for the uncertainties associated, respectively, with the dead and live load and will normally be greater than 1, with γ_L generally larger than γ_D. While a few examples or assigned problems using LRFD are included in this chapter and in Chaps. 5 and 10, the allowable-stress method of design will be used in this text.



SAMPLE PROBLEM 1.3

Two forces are applied to the bracket *BCD* as shown. (*a*) Knowing that the control rod *AB* is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at *C* is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin *C* for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at *C* knowing that the allowable bearing stress of the steel used is 300 MPa.



SOLUTION

Free Body: Entire Bracket. The reaction at *C* is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.6 \text{ m}) - (50 \text{ kN})(0.3 \text{ m}) - (15 \text{ kN})(0.6 \text{ m}) = 0 \quad P = 40 \text{ kN}$$

$$\Sigma F_x = 0: \quad C_x = 40 \text{ k}$$

$$\Sigma F_y = 0: \quad C_y = 65 \text{ kN} \qquad C = \sqrt{C_x^2 + C_y^2} = 76.3 \text{ kN}$$

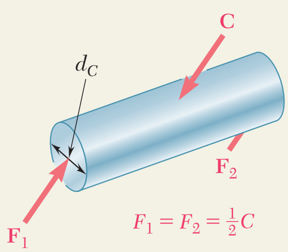
a. Control Rod *AB*. Since the factor of safety is to be 3.3, the allowable stress is

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600 \text{ MPa}}{3.3} = 181.8 \text{ MPa}$$

For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40 \text{ kN}}{181.8 \text{ MPa}} = 220 \times 10^{-6} \text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6} \text{ m}^2 \qquad d_{AB} = 16.74 \text{ mm} \quad ◀$$



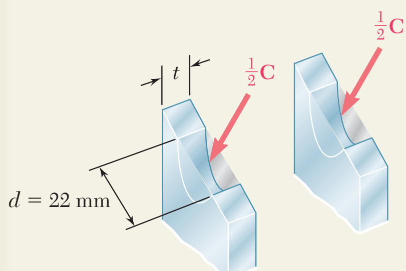
b. Shear in Pin *C*. For a factor of safety of 3.3, we have

$$\tau_{\text{all}} = \frac{\tau_U}{F.S.} = \frac{350 \text{ MPa}}{3.3} = 106.1 \text{ MPa}$$

Since the pin is in double shear, we write

$$A_{\text{req}} = \frac{C/2}{\tau_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{106.1 \text{ MPa}} = 360 \text{ mm}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_C^2 = 360 \text{ mm}^2 \qquad d_C = 21.4 \text{ mm} \qquad \text{Use: } d_C = 22 \text{ mm} \quad ◀$$

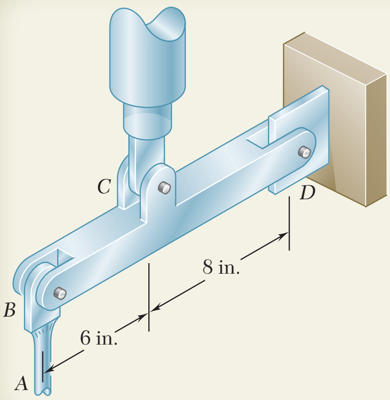


The next larger size pin available is of 22-mm diameter and should be used.

c. Bearing at *C*. Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write

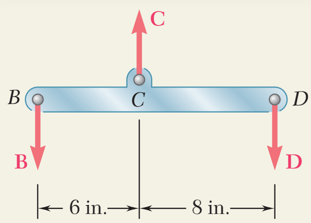
$$A_{\text{req}} = \frac{C/2}{\sigma_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{300 \text{ MPa}} = 127.2 \text{ mm}^2$$

Thus $22t = 127.2 \qquad t = 5.78$ mm $\qquad$ Use: $t = 6$ mm ◀



SAMPLE PROBLEM 1.4

The rigid beam BCD is attached by bolts to a control rod at B, to a hydraulic cylinder at C, and to a fixed support at D. The diameters of the bolts used are: $d_B = d_D = \frac{3}{8}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod AB has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C.



SOLUTION

The factor of safety with respect to failure must be 3.0 or more in each of the three bolts and in the control rod. These four independent criteria will be considered separately.

Free Body: Beam *BCD*. We first determine the force at C in terms of the force at B and in terms of the force at D.

$$+\circlearrowleft \Sigma M_D = 0: \qquad B(14 \text{ in.}) - C(8 \text{ in.}) = 0 \qquad C = 1.750B \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \qquad -D(14 \text{ in.}) + C(6 \text{ in.}) = 0 \qquad C = 2.33D \qquad (2)$$

Control Rod. For a factor of safety of 3.0 we have

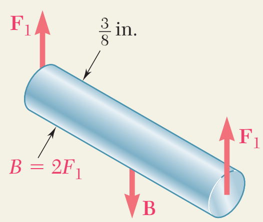
$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{3.0} = 20 \text{ ksi}$$

The allowable force in the control rod is

$$B = \sigma_{\text{all}}(A) = (20 \text{ ksi})\tfrac{1}{4}\pi(\tfrac{7}{16} \text{ in.})^2 = 3.01 \text{ kips}$$

Using Eq. (1) we find the largest permitted value of C:

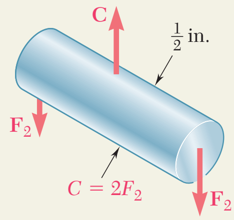
$$C = 1.750B = 1.750(3.01 \text{ kips}) \qquad C = 5.27 \text{ kips} \blacktriangleleft$$



Bolt at *B*. $\tau_{\text{all}} = \tau_U/F.S. = (40 \text{ ksi})/3 = 13.33$ ksi. Since the bolt is in double shear, the allowable magnitude of the force **B** exerted on the bolt is

$$B = 2F_1 = 2(\tau_{\text{all}} A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{3}{8} \text{ in.})^2 = 2.94 \text{ kips}$$

From Eq. (1): $\qquad C = 1.750B = 1.750(2.94 \text{ kips}) \qquad C = 5.15 \text{ kips} \blacktriangleleft$



Bolt at *D*. Since this bolt is the same as bolt B, the allowable force is $D = B = 2.94$ kips. From Eq. (2):

$$C = 2.33D = 2.33(2.94 \text{ kips}) \qquad C = 6.85 \text{ kips} \blacktriangleleft$$

Bolt at *C*. We again have $\tau_{\text{all}} = 13.33$ ksi and write

$$C = 2F_2 = 2(\tau_{\text{all}} A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{1}{2} \text{ in.})^2 \qquad C = 5.23 \text{ kips} \blacktriangleleft$$

Summary. We have found separately four maximum allowable values of the force C. In order to satisfy all these criteria we must choose the smallest value, namely: $\qquad C = 5.15 \text{ kips} \blacktriangleleft$

PROBLEMS

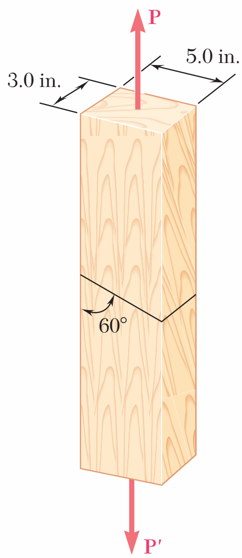


Fig. P1.29 and P1.30

1.29 The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

1.30 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

1.31 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

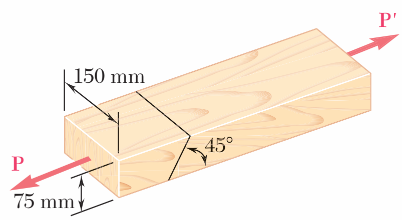


Fig. P1.31 and P1.32

1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

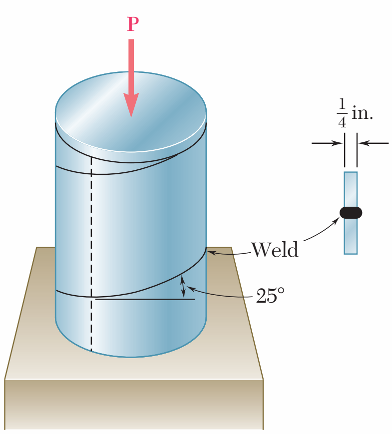


Fig. *P1.33* and *P1.34*

1.33 A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 12$ ksi and $\tau = 7.2$ ksi, determine the magnitude P of the largest axial force that can be applied to the pipe.

1.34 A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 66 kip axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

1.35 A 1060-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of that plane on which each of these maximum values occurs.

1.36 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

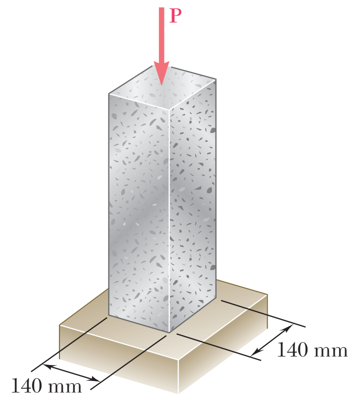


Fig. P1.35 and P1.36

1.37 Link *BC* is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What is the safety factor used if the structure shown was designed to support a 16-kN load **P**?

1.38 Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

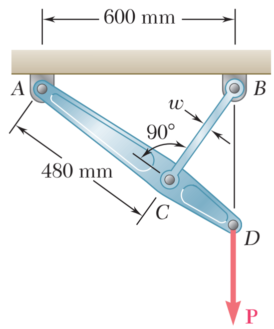


Fig. P1.37 and *P1.38*

1.39 A $\frac{3}{4}$-in.-diameter rod made of the same material as rods *AC* and *AD* in the truss shown was tested to failure and an ultimate load of 29 kips was recorded. Using a factor of safety of 3.0, determine the required diameter (*a*) of rod *AC*, (*b*) of rod *AD*.

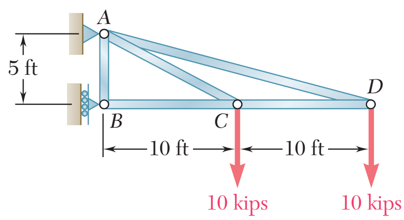


Fig. P1.39 and P1.40

1.40 In the truss shown, members *AC* and *AD* consist of rods made of the same metal alloy. Knowing that *AC* is of 1-in. diameter and that the ultimate load for that rod is 75 kips, determine (*a*) the factor of safety for *AC*, (*b*) the required diameter of *AD* if it is desired that both rods have the same factor of safety.

1.41 Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area of *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

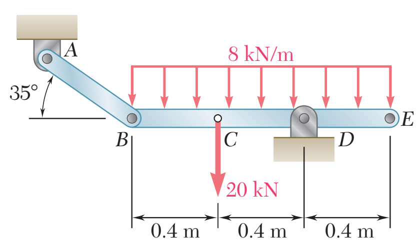


Fig. P1.41

1.42 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q.** Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa and that the ultimate strength of the aluminum used for the rod is 260 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

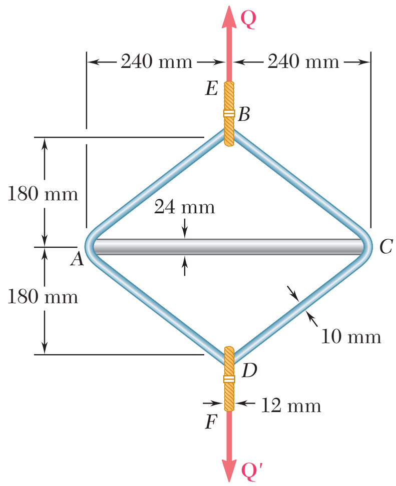


Fig. *P1.42*

1.43 Two wooden members shown, which support a 3.6-kip load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 360 psi and the clearance between the members is $\frac{1}{4}$ in. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

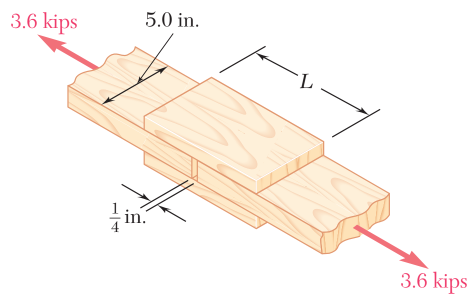


Fig. P1.43

1.44 Two plates, each $\frac{1}{8}$-in. thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surfaces is 130 psi, determine the factor of safety with respect to shear when $P = 325$ lb.

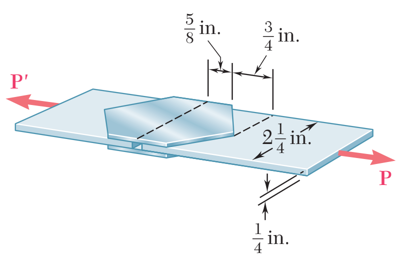


Fig. P1.44

1.45 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

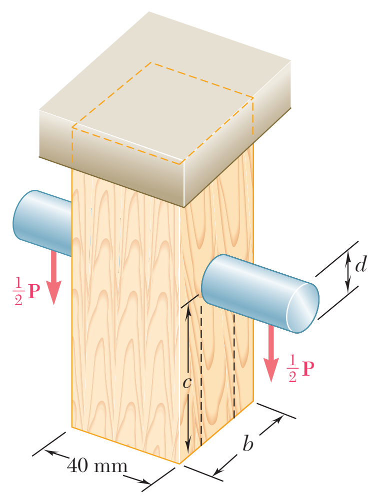


Fig. P1.45

1.46 For the support of Prob. 1.45, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of b and c if the factor of safety for the wooden member is the same as that found in part a for the pin.

1.47 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

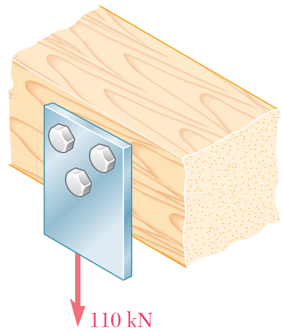


Fig. *P1.47* and P1.48

1.48 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

1.49 A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width a of the plate, (*b*) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the bottom edge of the plate.)

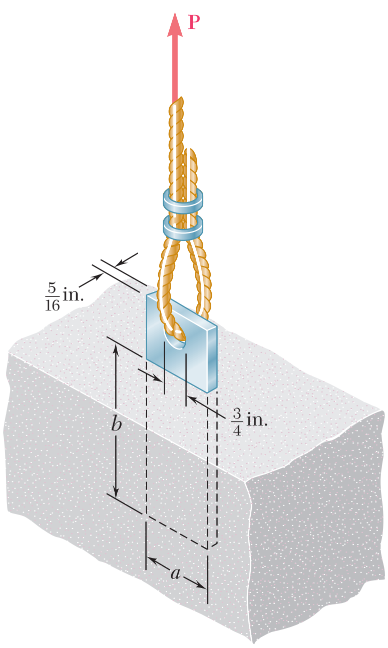


Fig. P1.49

1.50 Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

1.51 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.

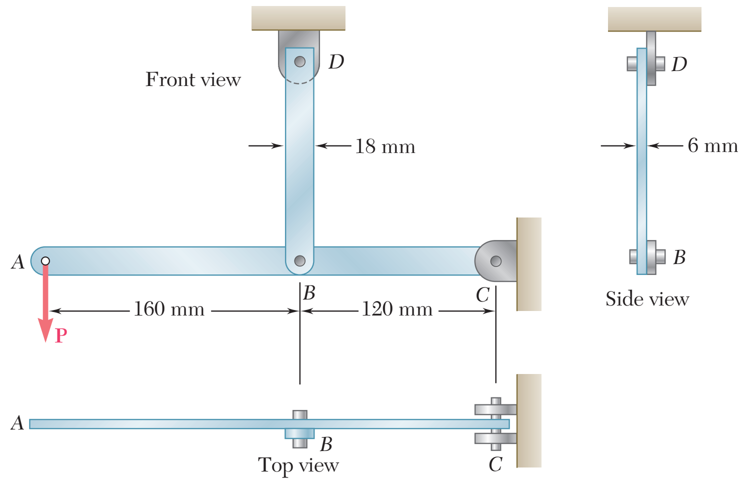


Fig. P1.51

1.52 Solve Prob. 1.51, assuming that the structure has been redesigned to use 12-mm-diameter pins at B and D and no other change has been made.

1.53 Each of the two vertical links CF connecting the two horizontal members AD and EG has a uniform rectangular cross section $\frac{1}{4}$ in. thick and 1 in. wide, and is made of a steel with an ultimate strength in tension of 60 ksi. The pins at C and F each have a $\frac{1}{2}$-in. diameter and are made of a steel with an ultimate strength in shear of 25 ksi. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

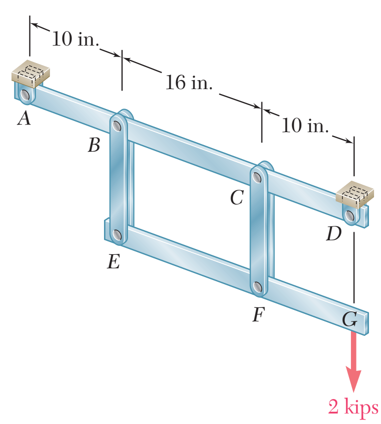


Fig. P1.53

1.54 Solve Prob. 1.53, assuming that the pins at C and F have been replaced by pins with a $\frac{3}{4}$-in. diameter.

1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

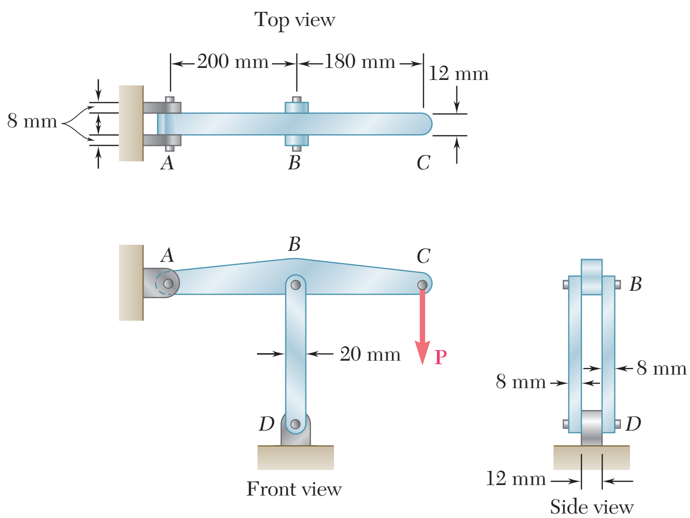


Fig. P1.55

1.56 In an alternative design for the structure of Prob. 1.55, a pin of 10-mm diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

***1.57** The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

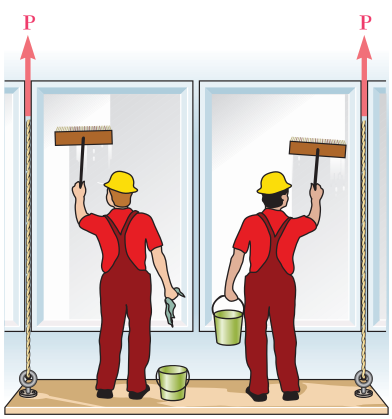


Fig. P1.57

***1.58** A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

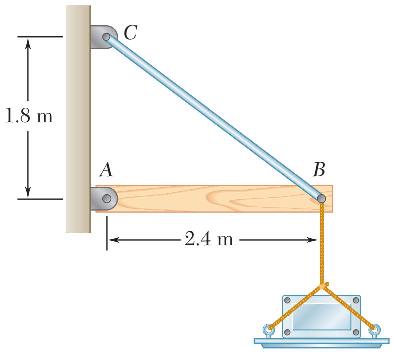


Fig. P1.58

REVIEW AND SUMMARY

This chapter was devoted to the concept of stress and to an introduction to the methods used for the analysis and design of machines and load-bearing structures.

Section 1.2 presented a short review of the methods of statics and of their application to the determination of the reactions exerted by its supports on a simple structure consisting of pin-connected members. Emphasis was placed on the use of a *free-body diagram* to obtain equilibrium equations which were solved for the unknown reactions. Free-body diagrams were also used to find the internal forces in the various members of the structure.

Axial loading. Normal stress

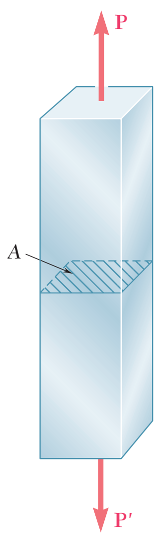


Fig. 1.41

The concept of *stress* was first introduced in Sec. 1.3 by considering a two-force member under an *axial loading*. The *normal stress* in that member was obtained by dividing the magnitude P of the load by the cross-sectional area A of the member (Fig. 1.41). We wrote

$$\sigma = \frac{P}{A} \tag{1.5}$$

Section 1.4 was devoted to a short discussion of the two principal tasks of an engineer, namely, the *analysis* and the *design* of structures and machines.

As noted in Sec. 1.5, the value of σ obtained from Eq. (1.5) represents the *average stress* over the section rather than the stress at a specific point Q of the section. Considering a small area ΔA surrounding Q and the magnitude ΔF of the force exerted on ΔA, we defined the stress at point Q as

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

In general, the value obtained for the stress σ at point Q is different from the value of the average stress given by formula (1.5) and is found to vary across the section. However, this variation is small in any section away from the points of application of the loads. In practice, therefore, the distribution of the normal stresses in an axially loaded member is assumed to be *uniform*, except in the immediate vicinity of the points of application of the loads.

However, for the distribution of stresses to be uniform in a given section, it is necessary that the line of action of the loads **P** and **P′** pass through the centroid C of the section. Such a loading is called a *centric* axial loading. In the case of an *eccentric* axial loading, the distribution of stresses is *not* uniform. Stresses in members subjected to an eccentric axial loading will be discussed in Chap 4.

Transverse forces. Shearing stress

When equal and opposite *transverse forces* **P** and **P**′ of magnitude P are applied to a member AB (Fig. 1.42), *shearing stresses* τ are created over any section located between the points of application of the two forces [Sec 1.6]. These stresses vary greatly across the section and their distribution *cannot* be assumed uniform. However, dividing the magnitude P—referred to as the *shear* in the section—by the cross-sectional area A, we defined the *average shearing stress* over the section:

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

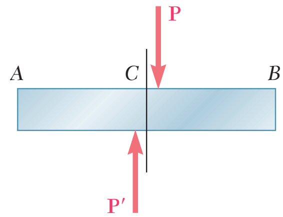


Fig. 1.42

Single and double shear

Shearing stresses are found in bolts, pins, or rivets connecting two structural members or machine components. For example, in the case of bolt CD (Fig. 1.43), which is in *single shear,* we wrote

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

while, in the case of bolts EG and HJ (Fig. 1.44), which are both in *double shear,* we had

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

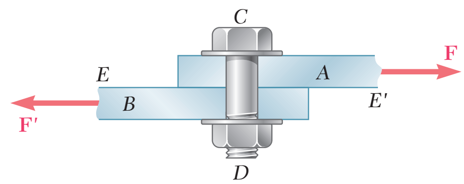


Fig. 1.43

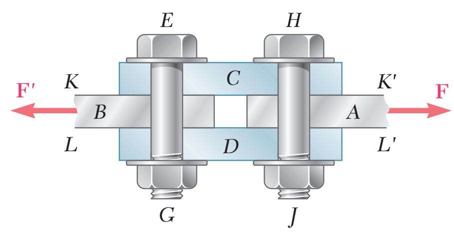


Fig. 1.44

Bearing stress

Bolts, pins, and rivets also create stresses in the members they connect, along the *bearing surface,* or surface of contact [Sec. 1.7]. The bolt CD of Fig. 1.43, for example, creates stresses on the semicylindrical surface of plate A with which it is in contact (Fig. 1.45). Since the distribution of these stresses is quite complicated, one uses in practice an average nominal value σ_b of the stress, called *bearing stress,* obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section. Denoting by t the thickness of the plate and by d the diameter of the bolt, we wrote

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

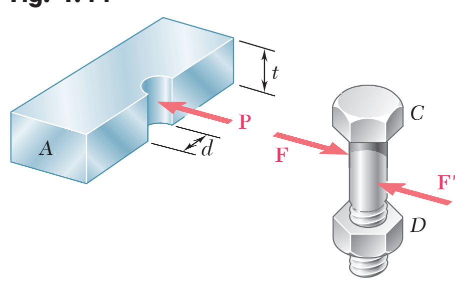


Fig. 1.45

Method of Solution

In Sec. 1.8, we applied the concept introduced in the previous sections to the analysis of a simple structure consisting of two pin-connected members supporting a given load. We determined successively the normal stresses in the two members, paying special attention to their narrowest sections, the shearing stresses in the various pins, and the bearing stress at each connection.

The method you should use in solving a problem in mechanics of materials was described in Sec. 1.9. Your solution should begin with a clear and precise *statement* of the problem. You will then draw one or several *free-body diagrams* that you will use to write *equilibrium equations*. These equations will be solved for *unknown forces,* from which the required *stresses* and *deformations* can be computed. Once the answer has been obtained, it should be *carefully checked*.

The first part of the chapter ended with a discussion of *numerical accuracy* in engineering, which stressed the fact that the accuracy of an answer can never be greater than the accuracy of the given data [Sec. 1.10].

Stresses on an oblique section

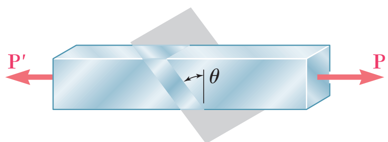


Fig. 1.46

In Sec. 1.11, we considered the stresses created on an *oblique section* in a two-force member under axial loading. We found that both *normal* and *shearing* stresses occurred in such a situation. Denoting by θ the angle formed by the section with a normal plane (Fig. 1.46) and by A_0 the area of a section perpendicular to the axis of the member, we derived the following expressions for the normal stress σ and the shearing stress τ on the oblique section:

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta \tag{1.14}$$

We observed from these formulas that the normal stress is maximum and equal to $\sigma_m = P/A_0$ for $\theta = 0$, while the shearing stress is maximum and equal to $\tau_m = P/2A_0$ for $\theta = 45°$. We also noted that $\tau = 0$ when $\theta = 0$, while $\sigma = P/2A_0$ when $\theta = 45°$.

Stress under general loading

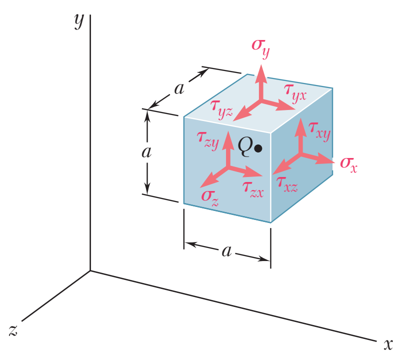


Fig. 1.47

Next, we discussed the state of stress at a point Q in a body under the most general loading condition [Sec. 1.12]. Considering a small cube centered at Q (Fig. 1.47), we denoted by σ_x the normal stress exerted on a face of the cube perpendicular to the x axis, and by τ_{xy} and τ_{xz}, respectively, the y and z components of the shearing stress exerted on the same face of the cube. Repeating this procedure for the other two faces of the cube and observing that $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, and $\tau_{zx} = \tau_{xz}$, we concluded that *six stress components* are required to define the state of stress at a given point Q, namely, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.

Section 1.13 was devoted to a discussion of the various concepts used in the design of engineering structures. The *ultimate load* of a given structural member or machine component is the load at which the member or component is expected to fail; it is computed from the *ultimate stress* or *ultimate strength* of the material used, as determined by a laboratory test on a specimen of that material. The ultimate load should be considerably larger than the *allowable load*, i.e., the load that the member or component will be allowed to carry under normal conditions. The ratio of the ultimate load to the allowable load is defined as the *factor of safety:*

Factor of safety

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}} \tag{1.24}$$

The determination of the factor of safety that should be used in the design of a given structure depends upon a number of considerations, some of which were listed in this section.

Load and Resistance Factor Design

Section 1.13 ended with the discussion of an alternative approach to design, known as *Load and Resistance Factor Design,* which allows the engineer to distinguish between the uncertainties associated with the structure and those associated with the load.

1.59 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

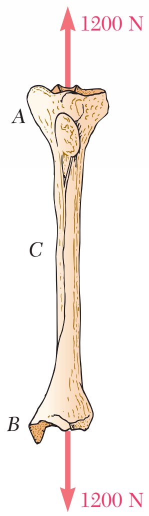


Fig. P1.59

1.60 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

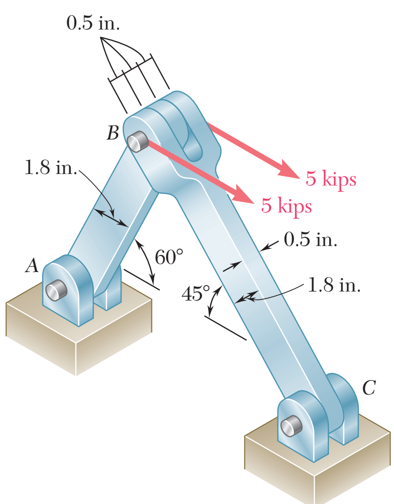


Fig. *P1.60*

1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at C, (*b*) the average bearing stress at C in member BC, (*c*) the average bearing stress at B in member BC.

1.62 In the marine crane shown, link *CD* is known to have a uniform cross section of 50×150 mm. For the loading shown, determine the normal stress in the central portion of that link.

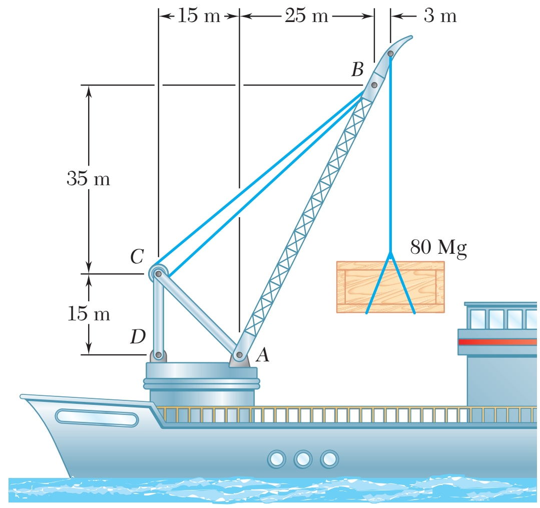


Fig. *P1.62*

1.63 Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude *P* of the axial load that will cause the joint to fail.

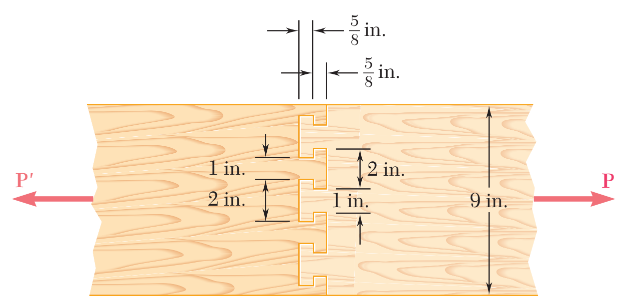


Fig. P1.63

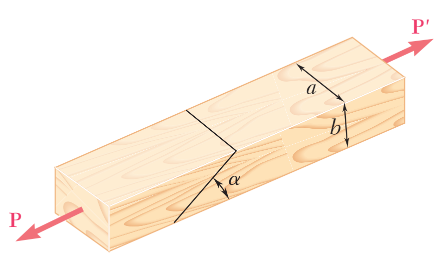


Fig. *P1.64*

1.64 Two wooden members of uniform rectangular cross section of sides $a = 100$ mm and $b = 60$ mm are joined by a simple glued joint as shown. Knowing that the ultimate stresses for the joint are $\sigma_U = 1.26$ MPa in tension and $\tau_U = 1.50$ MPa in shear and that $P = 6$ kN, determine the factor of safety for the joint when (*a*) $\alpha = 20°$, (*b*) $\alpha = 35°$, (*c*) $\alpha = 45°$. For each of these values of α, also determine whether the joint will fail in tension or in shear if *P* is increased until rupture occurs.

1.65 Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

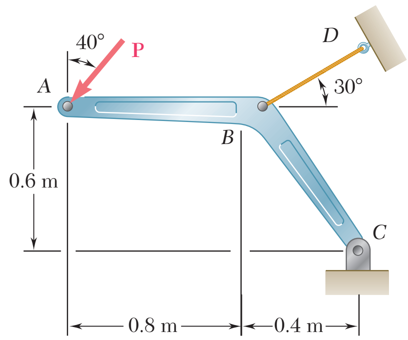


Fig. P1.65

1.66 The 2000-lb load may be moved along the beam *BD* to any position between stops at *E* and *F*. Knowing that $\sigma_{all} = 6$ ksi for the steel used in rods *AB* and *CD*, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

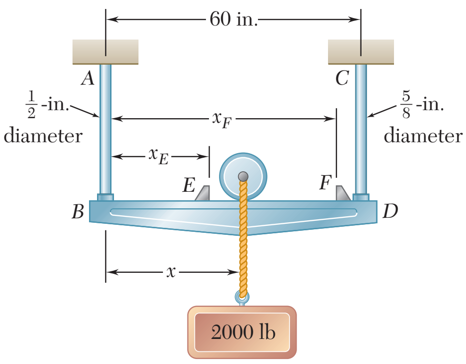


Fig. *P1.66*

1.67 Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at *C* for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at *C*, (*c*) the corresponding bearing stress in each support bracket at *C*.

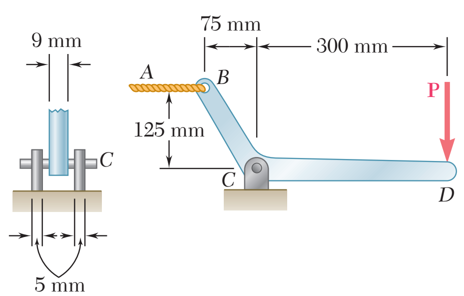


Fig. P1.67

1.68 A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

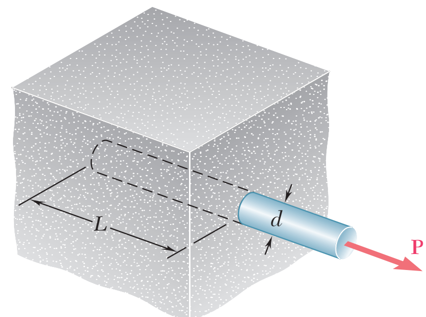


Fig. P1.68

1.69 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

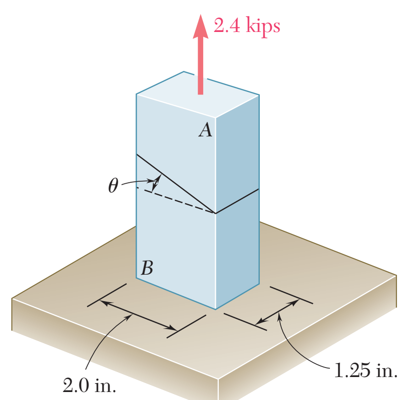


Fig. P1.69 and P1.70

1.70 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

COMPUTER PROBLEMS

The following problems are designed to be solved with a computer.

1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.2 and 1.4.

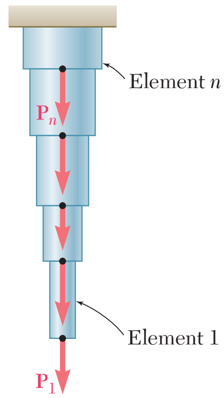


Fig. P1.C1

1.C2 A 20-kN load is applied as shown to the horizontal member ABC. Member ABC has a 10×50-mm uniform rectangular cross section and is supported by four vertical links, each of 8×36-mm uniform rectangular cross section. Each of the four pins at A, B, C, and D has the same diameter d and is in double shear. (*a*) Write a computer program to calculate for values of d from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins B and D, (2) the average normal stress in the links connecting pins C and E, (3) the average shearing stress in pin B, (4) the average shearing stress in pin C, (5) the average bearing stress at B in member ABC, (6) the average bearing stress at C in member ABC. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member ABC has been reduced from 10 to 8 mm.

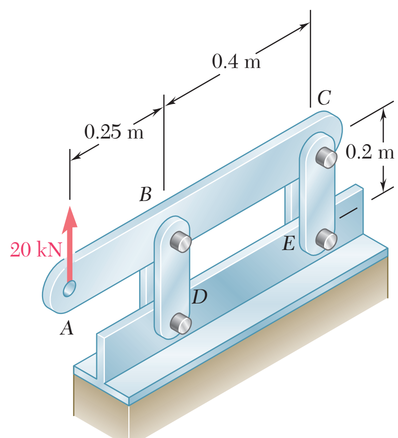


Fig. P1.C2

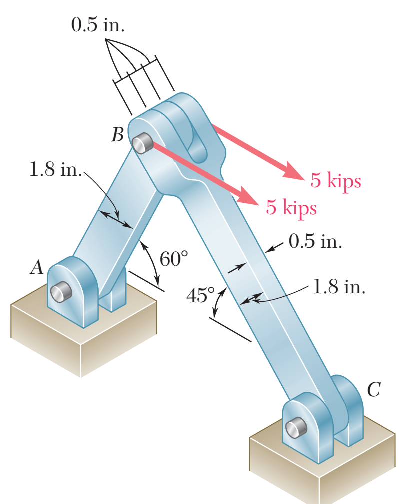


Fig. P1.C3

1.C3 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is in double shear. (*a*) Write a computer program to calculate for values of d from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (*b*) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Probs. 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (*d*) Solve part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 to 2.4 in.

1.C4 A 4-kip force **P** forming an angle α with the vertical is applied as shown to member ABC, which is supported by a pin and bracket at C and by a cable BD forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α, the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

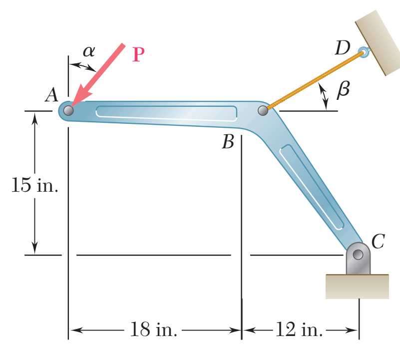


Fig. P1.C4

1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of a from 5 to 85° at 5° intervals, can calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Prob. 1.29, and that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45°$.

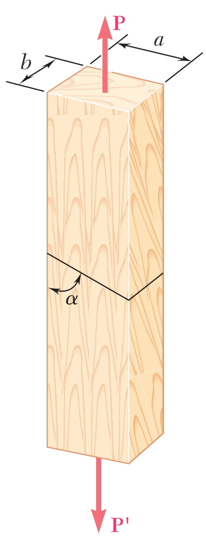


Fig. P1.C5

1.C6 Member *ABC* is supported by a pin and bracket at *A*, and by two links that are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D* (*b* and *c*). Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and *F.S.* = 3.2.

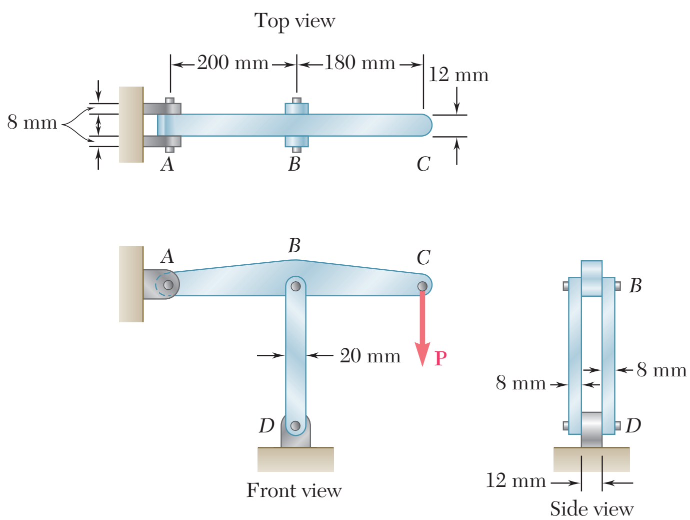


Fig. P1.C6