Tishk International University
Mechatronics Engineering Department
Fluid Mechanics
Week 9 : 1/12 /2021

Differential manometer and Continuity Equation

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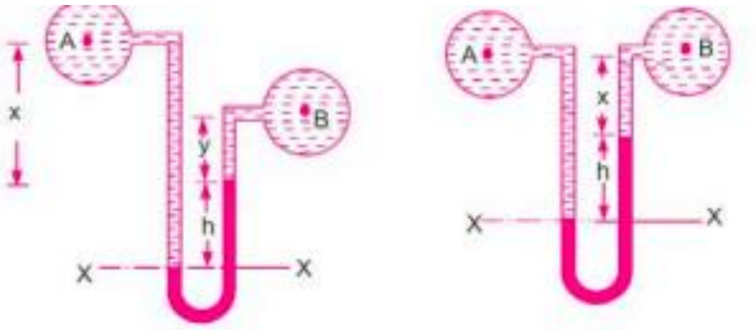
▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1 U-tube Differential Manometer. Fig. 2.18 shows the differential manometers of U-tube type.



(a)Two pipes at different levels (b) A and B are at the same level

Fig. 2.18 *U-tube differential manometers.*

In Fig. 2.18 (*a*), the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \qquad \text{...(2.12)}$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

In Fig. 2.18 (*b*), the two points *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \qquad \text{...(2.13)}$$

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.*

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5 \quad \therefore \quad \rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9 \quad \therefore \quad \rho_2 = 900$

Pressure at A, $p_A = 1 \text{ kgf/cm}^2 = 1 \times 10^4 \text{ kgf/m}^2$

$= 10^4 \times 9.81 \text{ N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8 \text{ kgf/cm}^2$

$= 1.8 \times 10^4 \text{ kgf/m}^2$

$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

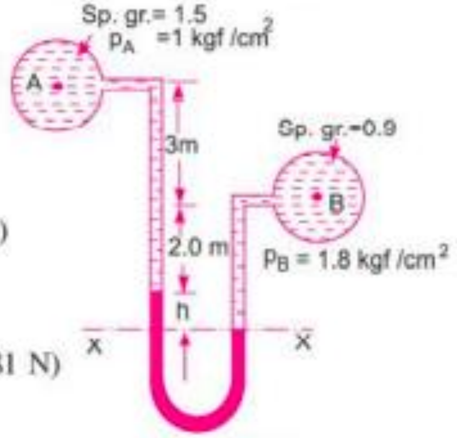


Fig. 2.19

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1 \text{ cm}}. \text{ Ans.}$$

2.7.2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

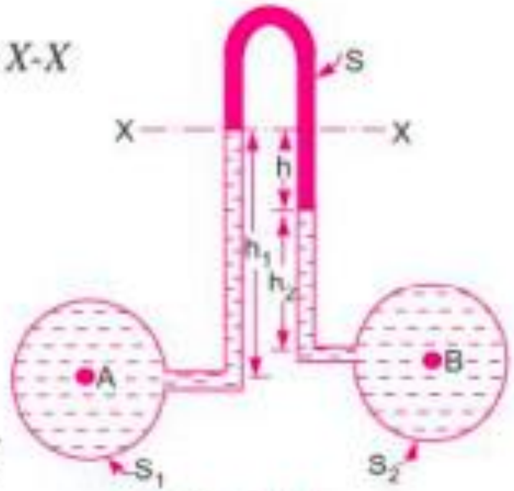


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18 *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Solution. Given :

Pressure head at $A = \dfrac{p_A}{\rho g} = 2$ m of water

$\therefore$ $p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb $= p_A - \rho_1 \times g \times h_1$

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$

or $p_B = \mathbf{1.8599 \text{ N/cm}^2}$. **Ans.**

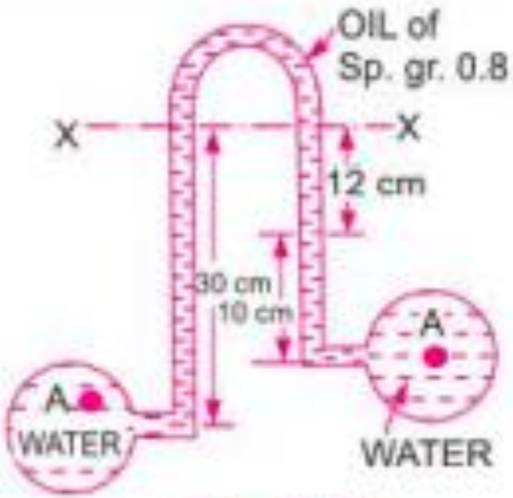


Fig. 2.22

▶ 5.5 CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in Fig. 5.1.

Let V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding values at section, 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad ...(5.2)$$

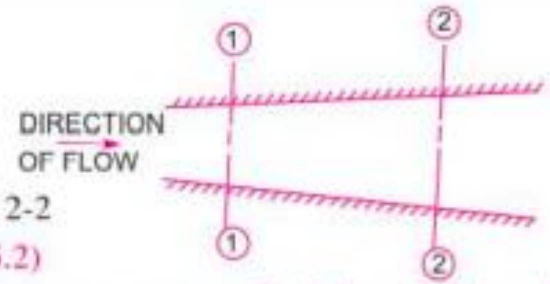


Fig. 5.1 *Fluid flowing through a pipe.*

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation**. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2 \qquad ...(5.3)$$

Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

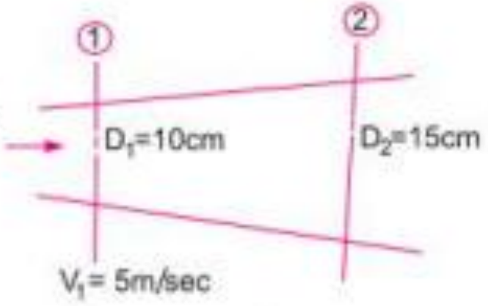


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $$Q = A_1 \times V_1$$

$$= 0.007854 \times 5 = \mathbf{0.03927 \text{ m}^3/\text{s}}. \textbf{ Ans.}$$

Using equation (5.3), we have $A_1V_1 = A_2V_2$

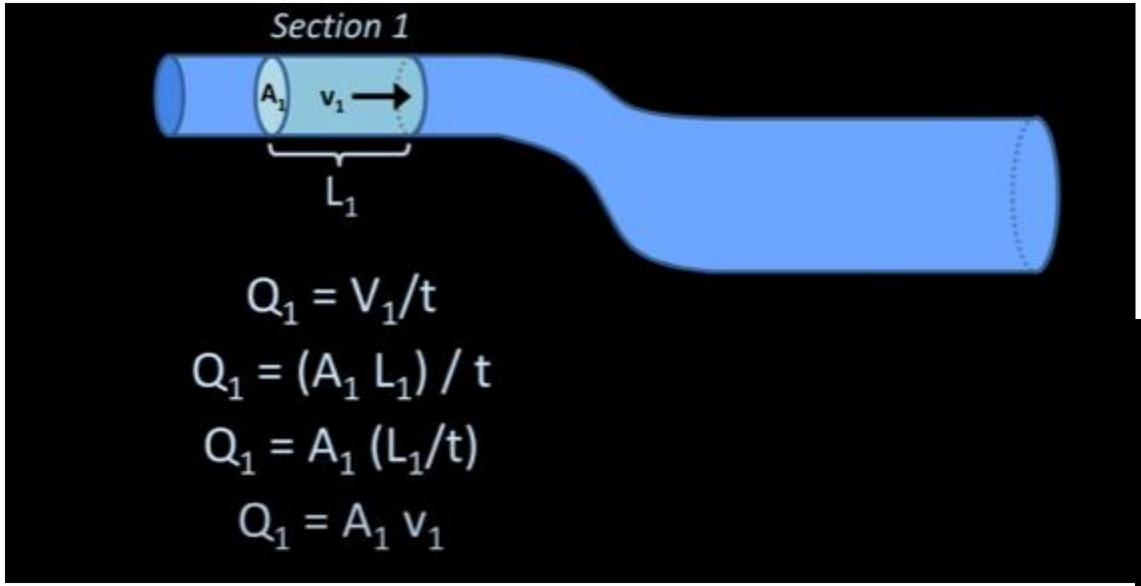
(*ii*) ∴ $$V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22 \text{ m/s}}. \textbf{ Ans.}$$

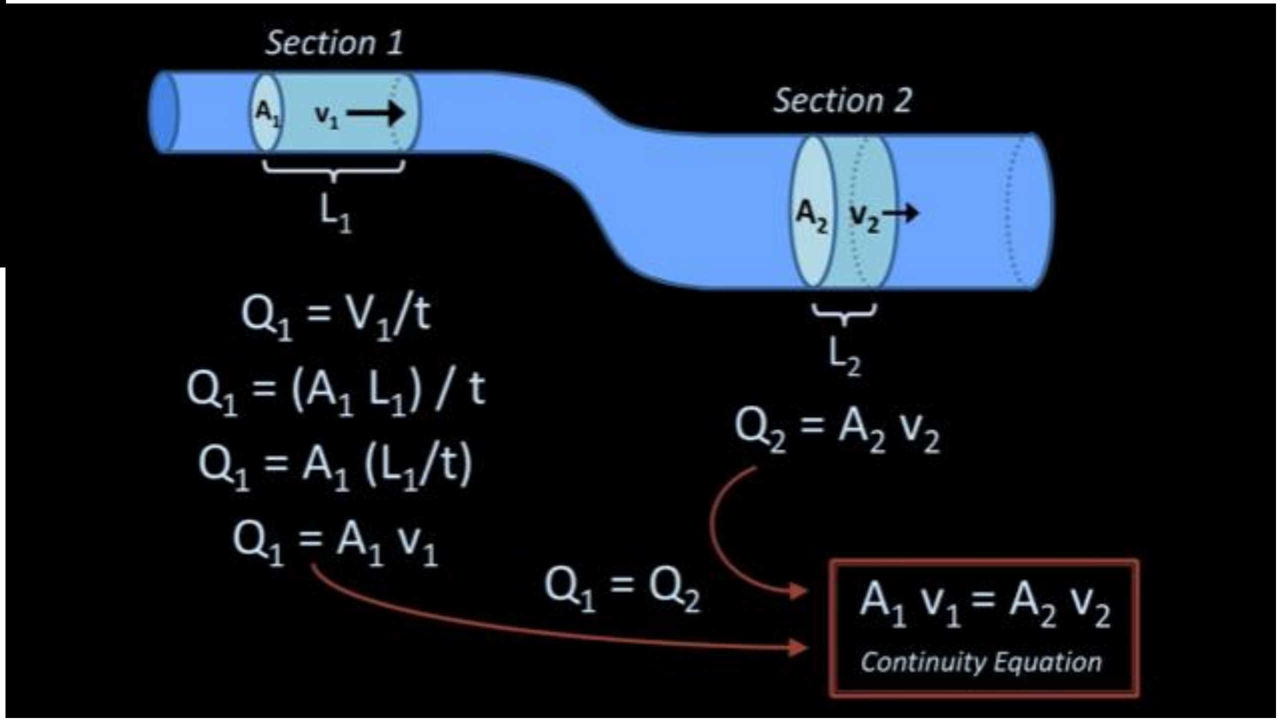
Continuity equation

To define fluid flow, and understand how conservation of flow can help one determine fluid velocity or the cross sectional area of fluid conduits.

Fluid flow is the volume of fluid that moves past a certain point per unit time.

$$Q = V / t$$


Section 1
A_1
v_1
L_1
$Q_1 = V_1/t$
$Q_1 = (A_1 L_1) / t$
$Q_1 = A_1 (L_1/t)$
$Q_1 = A_1 v_1$


Section 1
A_1
v_1
L_1
Section 2
A_2
v_2
L_2
$Q_1 = V_1/t$
$Q_1 = (A_1 L_1) / t$
$Q_1 = A_1 (L_1/t)$
$Q_1 = A_1 v_1$
$Q_2 = A_2 v_2$
$Q_1 = Q_2$
$A_1 v_1 = A_2 v_2$
Continuity Equation

Example

The showing fire hydrant that connected to the fire hose and turn it on the radius of the outlet of the hydrant is 2cm and water passing through it at 4m/s and the radius of the nozzle is 1cm what would be the velocity of the nozzle at the exit of nozzle.

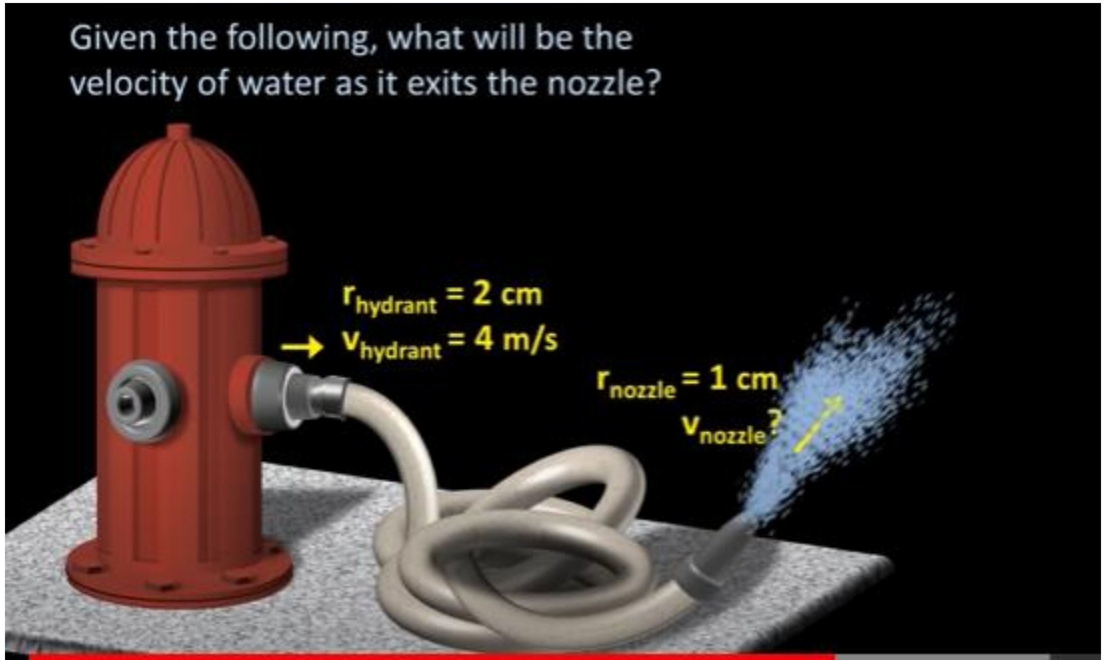
$$A_1 v_1 = A_2 v_2$$

$$(\pi r^2_1)(v_1) = (\pi r^2_2)(v_2)$$

$$\not\pi (2\ \text{cm})^2 (4\ \text{m/s}) = \not\pi (1\ \text{cm})^2 (v_2)$$

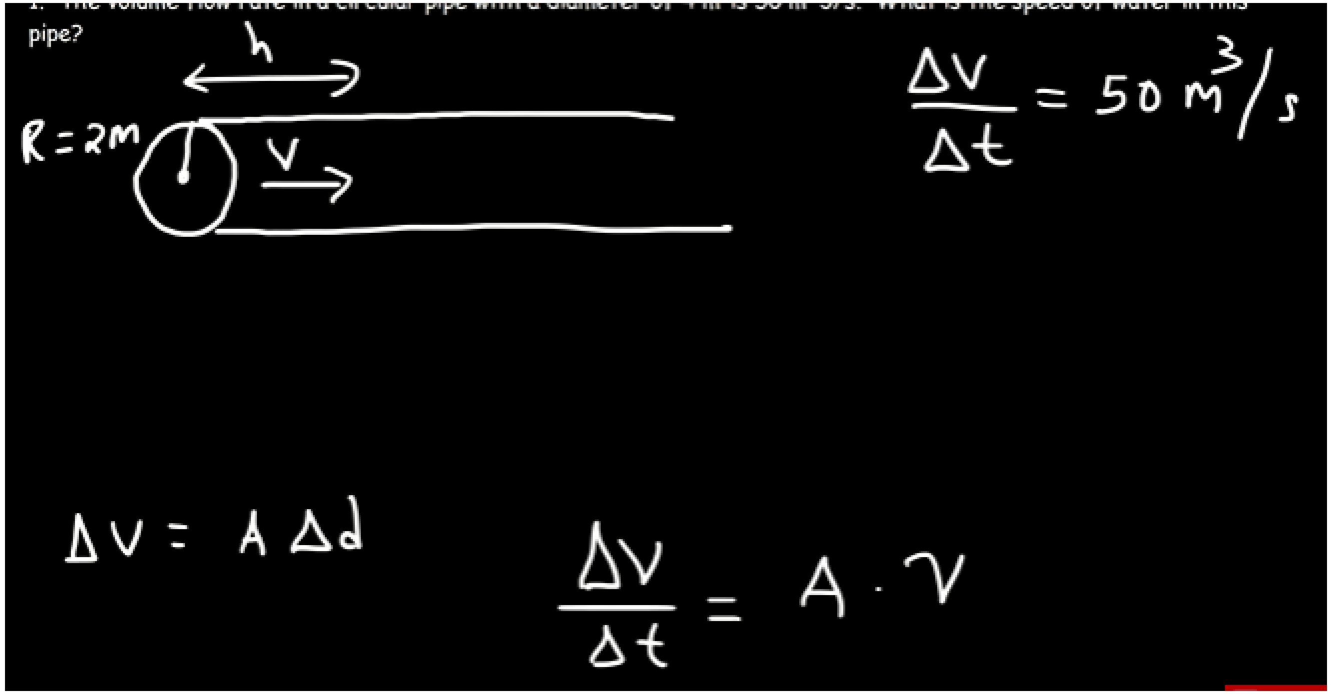
$$\frac{(4)(4)}{(1)} = v_2$$

$$16\ \text{m/s} = v_2$$



Example

1. The volume flow rate in a circular pipe with a diameter of 4 m is 50 m^3/s. What is the speed of water in this pipe?



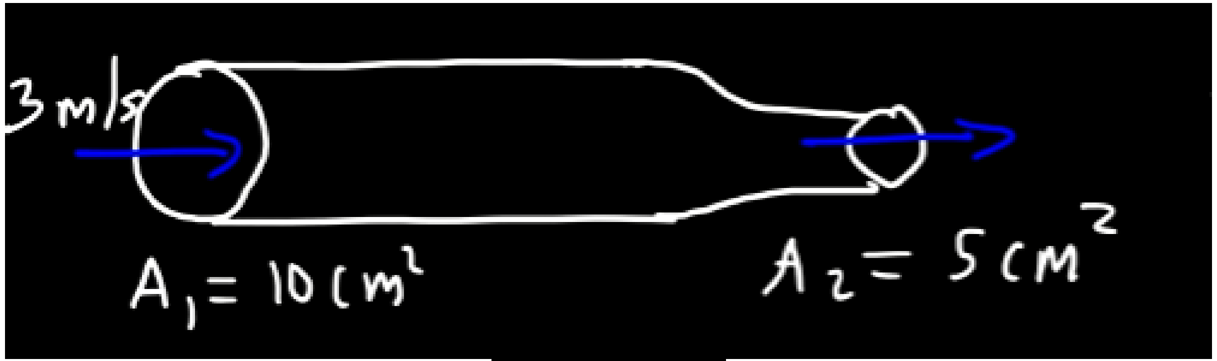
$$\left(\frac{50\,m^3}{s}\right) = \pi (2)^2 v$$

$$50 = 4\pi v$$

$$v = 3.98\ m/s$$

Example

2. Water flows through a pipe with a cross-sectional area of 10 cm^2 at 3 m/s. What is the flow speed in the pipe if the cross-sectional area is reduced to 5 cm^2?



$A\downarrow \; v\uparrow$

$\rho = \frac{m}{V}$

$m = \rho V$

$\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t}$

$\frac{\rho A \Delta d}{\Delta t}$

$$\frac{\Delta m}{\Delta t} = \boxed{\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2}$$

$$A_1 V_1 = A_2 V_2$$

$$A_1 V_1 = A_2 V_2$$

$$(10\,cm^2)(3\,m/s) = (5\,cm^2)V_2$$

$$V_2 = 6\,m/s$$