EXP. (6)

"Determination of Tension on a String with Conical Pendulum"

Purpose:

This experiment is performed to determine the tension force on a string in a conical pendulum, and understanding centripetal force.

Safety First

During the experiment, do not rotate the bob fast.

Equipment's:

1. Rope
2. Mass hanger
3. Circular path
4. Stop-watch
5. Ruler
6. Calculator
7. Stand and holders

Pre-Lab Questions:

1. How does the tension on the rope change with rotation speed?
2. When can the tension be minimum? What is its value?
3. Why does the tension stay constant in the fixed radius?

Introduction and Theory:

When an object is exposed to make a circular motion, there must be a force to keep the object in the circle towards the center called centripetal force. For a car taking the curve, the centripetal force is friction. For a stone rotated with a rope, the centripetal force is the tension on the rope. Sometimes the centripetal force is the combination of two or more forces.

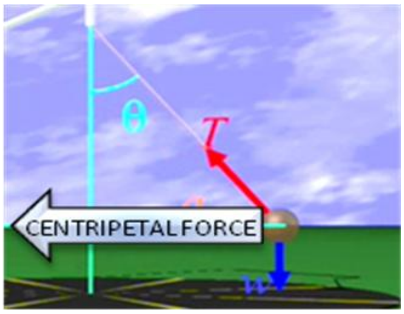


Figure 1: Expression for centripetal force.

Suppose that an object, mass m, is attached to the end of a light inextensible string whose other end is attached to a rigid beam. Suppose, further, that the object is given an initial horizontal velocity such that it executes a horizontal circular orbit of radius r with angular velocity ω. See Fig. 1. Let h be the vertical distance between the beam and the plane of the circular orbit, and let θ be the angle subtended by the string with the downward vertical.

The object is subject to two forces: the gravitational force mg which acts vertically downwards and the tension force T which acts upwards along the string. The tension force can be resolved into a component $T\ \cos\theta$ which acts vertically upwards, and a component $T\,sin\theta$ which acts towards the center of the circle. Force balance in the vertical direction yields

$$T\cos\theta = mg \qquad (1)$$

In other words, the vertical component of the tension force balances the weight of the object.
Since the object is executing a circular orbit, radius r, with angular velocity ω (The rate of change of angular displacement and is a vector quantity which specifies the angular speed or rotational speed of an object and the axis about which the object is rotating.), it experiences a centripetal acceleration $\omega^2 r$. Hence, it is subject to a centripetal force $m\omega^2 r$ (Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path. This force is called the centripetal force which means "center seeking" force. The force has the magnitude)

$$F_{cetripital} = m\frac{v^2}{r} \qquad (2)$$

This force is provided by the component of the string tension which acts towards the center of the circle. In other words,

$$T\sin\theta = mw^2 r \qquad (3)$$

Taking the ratio of equation (1) & (3)

$$\tan\theta = \frac{w^2 r}{g} \qquad (4)$$

However, by simple trigonometry, $$\tan\theta = \frac{r}{h} \qquad (5)$$

Hence, we find $$w = \sqrt{\frac{g}{h}} \qquad (6)$$

Note that if (ℓ) is the length of the string then $h = \ell\cos\theta$ (theta). It follows that

$$w = \sqrt{\frac{g}{\ell\cos\theta}} \qquad (7)$$

For instance, if the length of the string is $\ell = 0.2\,m$ and the conical angle is $\theta = 30^o$ then the angular velocity of rotation is given by

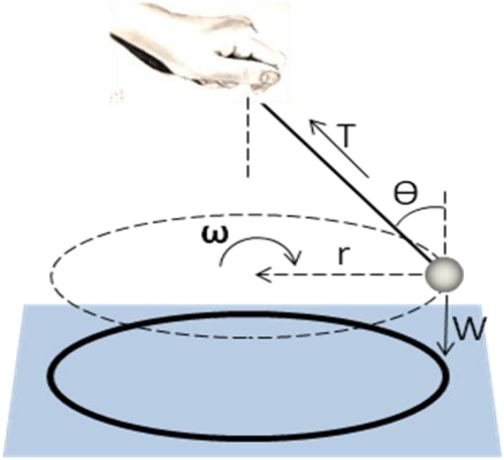
$$w = \sqrt{\frac{9.81}{0.2\times\cos 30^\circ}} = 7.526\ rad\ /s \qquad (8)$$

This translates to a rotation frequency in cycles per second of

$$f = \frac{w}{2\pi} = 1.20\,Hz\,. \qquad (9)$$

Experimental Procedure:

The apparatus consists of a rotating mass suspended by a rope. Students measure the period of rotation and the average speed of the mass in uniform circular motion. Then they determine the static force that keeps the mass in a defined circular path. In order to get constant speed, the mass must move <u>in the indicated circle</u> otherwise the force cannot be constant due to the change in radius.



Free Body Diagram

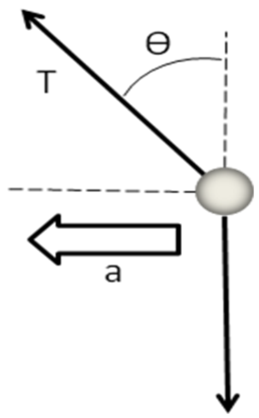


Figure 2: Uniform circular motion.

1. Hold the rope connected to the mass high and then rotate just above the indicated circle. For each circle, the mass should revolve 20 times.
2. (a partner) Measure the time taken for 20 cycles using stopwatch and record on the data table.
3. Calculate the average period and frequency using stopwatch.
4. Calculate the average linear (v) and angular (ω) velocity.
5. Calculate the centripetal force.
6. Calculate the angle using the radius and the length of the rope.
7. Calculate the tension (empirical) on the rope using trigonometric functions.
8. Calculate the tension (theoretical) using the weight and trigonometric functions.
9. Record the data on the table.
10. Repeat for other circles.
11. Write your comment.

Data collection and Calculations:

Show the results of the experiment.

m_{bob} = ………… kg

r_1 = …………m

r_2 = …………m

r_3 = …………m

r_n	**Time ,t , for 20 cycles (s)**	**T, period, t/20 (s)**	***f* (Hz)**	**$\omega = 2\pi f$ (rad/s)**	**$V = 2\pi fr$ (m/s)**	**$F_c = m\,\omega^2\,r$ (N)**	**$T = F_c/\sin\theta$ * (N)**
r_1							
r_2							
r_3							

$$* F_c = T_x \sin\theta$$
$$W = T_x \cos\theta$$