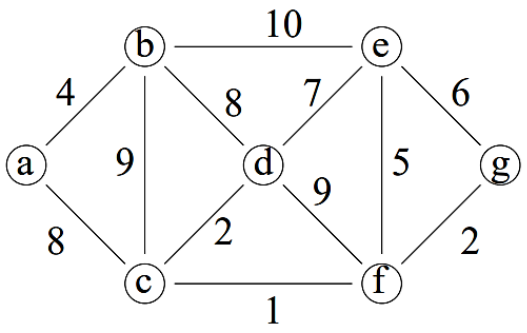
Final Exam Topics:

1. Set Theory (cardinality, relationships, operations, identities)
2. Cartesian Products
3. Propositional Logic & Logic Circuits
4. Truth Tables
5. Logic Laws (proofs)
6. Translating propositions and Negations
7. Proofs (direct, by case, contradiction)
8. Relations (properties, digraphs, databases, matrices)
9. Cryptography (Caesar cipher, shift cipher, affine cipher, substitution cipher, Vigenère cipher)
10. Algorithms (searching, sorting, optimization-greedy algorithms)
11. Graph Theory (terminology, hand-shaking theorem, graphs, applications)
12. Trees (properties, Dijkstra's, MST, Prim's, Kruskal's Algorithms)

Note: Revise and re-solve all Lecture Notes & Examples, Homework, Quizzes and Question Banks.

1. Find the minimum Spanning Tree using Prim's Algorithm, and give the weight of the spanning tree.



2. Using the keyword "mercedes", encode the plaintext: "Complaining never makes anything better" using the substitution cipher.

a	b	c	d	e	f	g	h	I	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z

3. How many edges are there if you have 16 vertices, each degree of 7?

4. What is the power set of a set $A = \{a, b, c, d\}$ and set $B = \{1, 2, 3\}$, and their cardinality?

Note: This Question Bank may not include some topics.

5. Consider the relation P on the set $A = \{0,1,2,3,4,5\}$ defined by:
$P = \{((0,0),(1,1),(2,2),(3,3),(4,4),(5,5),(1,2),(1,3),(2,1),(3,1),(4,5),(5,4)\}$
a) Draw the digraph of the relation P.
b) Determine properties.

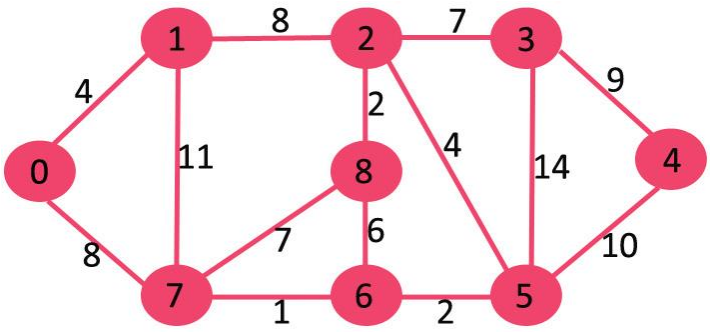
6. List of numbers is given: {3, 5, 1, 2, 7, 12, 65, 33, 23, 14, 55, 13}. Using Linear Search, find a $\boldsymbol{x = 14}$. (explain)

7. List of numbers is given: {3, 5, 1, 2, 7, 12, 65, 33, 23, 14, 55, 13}. Using Binary Search, find a $\boldsymbol{x = 12}$. (show every step)

8. List of numbers is given: {3, 5, 1, 2, 7, 12, 65, 33, 23, 14, 55, 13}. Using Bubble Sort, show every pass.

9. List of numbers is given: {3, 5, 1, 2, 7, 12, 65, 33, 23, 14, 55, 13}. Using Insertion Sort, show every pass.

10. Find the shortest paths from vertex A to every other vertex using Dijkstra's Algorithm.

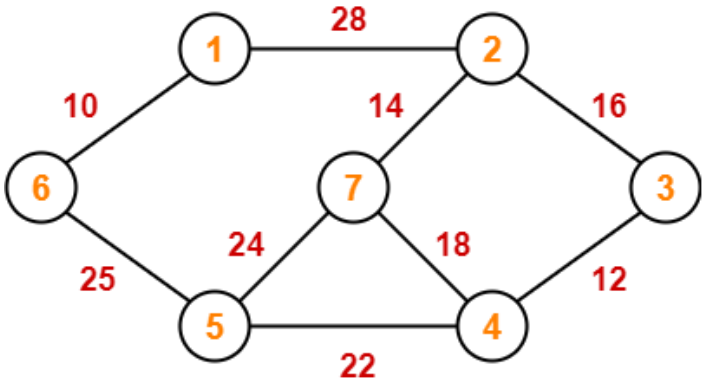


11. Encrypt the message "**Stop wasting time**" using shift cipher $k = 10$ and decrypt the message "**Lphwxcvidc**" with $k = 15$.

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

Note: This Question Bank may not include some topics.

12. Find a Minimum Spanning Tree using the Kruskal's Algorithm. Then, give the weight of the spanning tree.



13. Encrypt the message **BLESSINGS** with keyword *try* using Vigenère cipher and decrypt the message "**UBMHMDONASZOF**" with keyword *mouse*.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

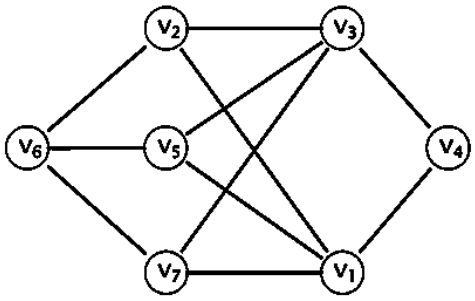
14. Encrypt "**DSSUHFLDWLRQ**" and decrypt "**FKRFRODWHV DUH VZHHW**" using Caesar Cipher. (Use the table from a Question 11)

Note: This Question Bank may not include some topics.

15. A graph contains 21 edges, 3 vertices of degree 4 and all other vertices of degree 2. Find total number of vertices. (use the Handshaking theorem)

16. Make change for 0.81$ using Greedy Algorithm (Optimization Algorithms).

17. Is a graph below bipartite? (use a graph coloring technique)



18. Using greedy algorithm for scheduling find the optimal choice for participating in Talks:
Talk 1 starts at 8 a.m. and ends at 12 noon,
Talk 2 starts at 9 a.m. and ends at 10 a.m.,
Talk 3 starts at 11 a.m. and ends at 12 noon.
Talk 4 starts at 10 am and ends at 1 pm
Talk 5 starts at 11 am and ends at 12.30 noon

19. Construct a combinational circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg q \wedge r \wedge \neg p)$ from input bits p, q, and r.

20. What results when the projection $P_{2,5}$ is applied to the 5-tuples (2, 3, 0, 4,5), (Jane Doe, 234111001, Geography, 3.14, California), and (a1, a2, a3, a4, a5)?

21. Find the truth set of the predicate, where the domain is $\mathbb{Z}$. $P(x)$: $\sqrt{x+2} \leq 7$

22. Suppose Adam, Ben, Chris, David and Eric are training for tasks at work. Adam and Chris are training for task 1, Ben, Chris and Eric are training for Task 2, David is training for task 3, Chris and Eric are training for task 4 and Eric is training for task 5. Create a graph to model this, then determine if a matching is possible.

Note: This Question Bank may not include some topics.

23. Suppose someone starts a chain letter. Each person is asked to send the letter to five other people. Everyone does it. How many people have seen the letter, including the original sender, if no one receives more than one letter, and the chain ends after 200 people read it but did not send it on. How many people sent out the letter?

24. From the code below, explain the function of **temp.**

```
int main()
{
 int numb[7];
 int i, j;

 for(i=0;i<=6;i++)
 {
  cout << "Please enter number: ";
  cin >> numb[i];
 }

 for(i=0;i<=5;i++)
 {
  for(j=i+1;j<=6;j++)
  {
   int temp;

   if(numb[i] > numb[j])
   {
    temp = numb[i];
    numb[i] = numb[j];
    numb[j] = temp;
   }
  }
 }

 for(i=0;i<=6;i++)
 {
  cout << endl <<  numb[i] << endl;
 }
```

25. Where do we use “Graph Theory” in real life?

26. Where do we use “Relations” in computer science?

27. What is the difference between Prim’s and Kruskal’s algorithms?

28. What is an incident in graph theory?

29. What is a degree in graph theory?

30. Show “adjacent to” term for directed graphs?

31. What types of graphs we have?

Note: This Question Bank may not include some topics.

32. Simplify/Prove statements using Logic Laws:

a) $(p \vee q) \longrightarrow (q \rightarrow q) \equiv T$

b) $p \rightarrow q \quad \Leftrightarrow \neg q \rightarrow \neg p$

c) $(p \rightarrow r) \wedge (q \rightarrow r) \Leftrightarrow (p \vee q) \rightarrow r$

d) $\neg\big(p \vee (\neg p \wedge q)\big) \equiv \neg p \wedge \neg q$

33. Use all conditional types to translate and formulate given sentence: "If system works well, students can see their marks".

34. Negate the following:

a) $\forall x(x^2 > x)$

b) $\exists x(x^2 = 2)$

c) $\forall x(P(x) \rightarrow Q(x))$

d) $\forall x \exists y \forall z T\ (x, y, z)$

35. Construct a truth table for each of these compound propositions.

a) *$(p \oplus q) \wedge (p \oplus \neg q)$*

b) *$\neg p \rightarrow (q \leftrightarrow$ p)*

c) *$(p \rightarrow q) \vee (\neg p \oplus (p \wedge \neg p))$*

d) *$(\neg p \leftrightarrow \neg q) \uparrow (p \leftrightarrow q)$*

36. Let U=letters of the alphabet, A=letters={a, b, c, d, e, f, g}, B={mathematics}. Find:

a) $A \cap B$

b) $\bar{A} - B$

c) $\overline{B - A}$

37. $U = \{x | x \in R \ \wedge -14 \leq x \leq 14\}, A = [-5,\ 12], C = (-1, 14)$, Find:

a) $A \cap B$

b) $\bar{A} - B$

c) $\overline{B - A}$

38. Determine the truth values of these expressions.

(a) $\emptyset \in \emptyset$ **(b)** $1 \subseteq \{1\}$ **(c)** $\emptyset \in \{\emptyset\}$ **(d)** $\emptyset \subseteq \{\emptyset\}$

e) $(-1{,}5) \subseteq [-1{,}6)$ **(f)** $\{1\} \subseteq \{1, \{1{,}2\}\}$ **(g)** $\{1\} \subseteq \{\{1\}, \{1{,}2\}\}$ (h) $[9, 12] \subseteq (9, 12]$

39. Evaluate:

$(\boldsymbol{a})\ P(\{\emptyset\})$ $(\boldsymbol{b})\ P(P(\{a, b\}))$

$(\boldsymbol{c})\ P(P(P(P(\emptyset))))$ $(d)\ P(\emptyset)$

Note: This Question Bank may not include some topics.

40. Prove:

a) Prove that product of three consecutive odd numbers is odd. (direct)
b) Prove that $(n+10)^2-(n+5)^2$ is always multiple of 5. (direct)
c) If $5|2a$ for $a\in\mathbb{Z}$, then $5|a$. (direct)
d) If $n\in\mathbb{Z}$, n^2+3n+4 is even (by cases)
e) $-5 \le |x+2|-|x-3| \le 5$ (by cases)
f) If $x\ or\ y$ are odd, check if xy is odd. (by cases)
g) Show that $\sqrt{2}$ is irrational. (by contradiction)
h) Prove that the sum of the squares of two odd integers is always even.
i) $(A-B)\cap(B-A)=\emptyset$ by contradiction.
j) Prove that the sum of even number and odd number is odd.

Note: This Question Bank may not include some topics.