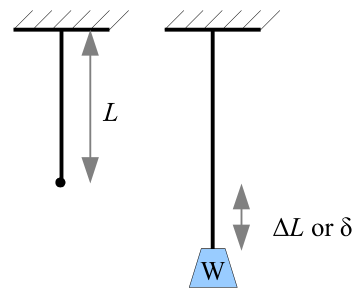
Chapter 2: Stress and Strain

Normal Stress and Strain



The words "stress" and "strain" are used interchangeably in popular culture in a psychological sense: "I'm feeling stressed" or "I'm under a lot of strain." In engineering, these words have specific, technical meanings. If you tie a steel wire to a hook in the ceiling and hang a weight on the lower end, the wire will stretch. Divide the change in length by the original length, and you have the strain in the wire. Divide the weight hanging from the wire by the wire's cross sectional area, and you have the tensile stress in the wire. Stress and strain are ratios.

The symbol for tensile stress is σ, the lower case Greek letter sigma. If the weight is 25 lb. and the cross-sectional area of the wire is 0.002 in.^{2}, then the stress in the wire is $\sigma=\dfrac{W}{A}=\dfrac{25\text{ lb.}}{0.002\text{ in.}^2}=12{,}700\dfrac{\text{lb.}}{\text{in.}^2}=12{,}700\text{ psi}$.

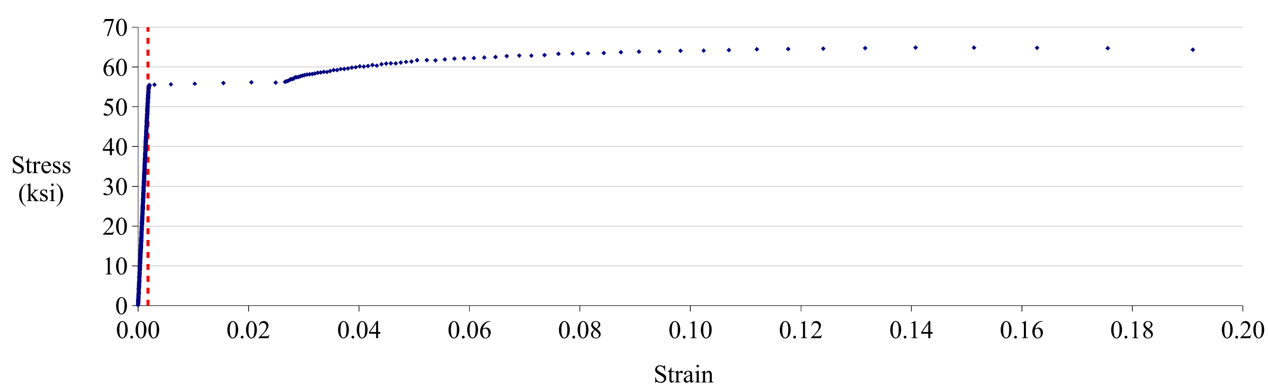
The symbol for strain is ε, the lower case Greek letter epsilon. If the original length of the wire $L=40\text{ in.}$ and the change in length $\Delta L=0.017\text{ in.}$ (also written $\delta=0.017\text{ in.}$), then strain $\varepsilon=\dfrac{\Delta L}{L}=\dfrac{\delta}{L}=\dfrac{0.017\text{ in.}}{40\text{ in.}}=0.000425$. This is a small number, so sometimes the strain number is multiplied by 100 and and reported as a percent: $0.000425=0.0425\%$. You may also see strain reported in microstrain: $0.000425\times10^6=425$ microstrain. Strain is usually reported as a percent for highly elastic materials like rubber.

Example #1

A 6 inch long copper wire is stretched to a total length of 6.05 inches. What is the strain?

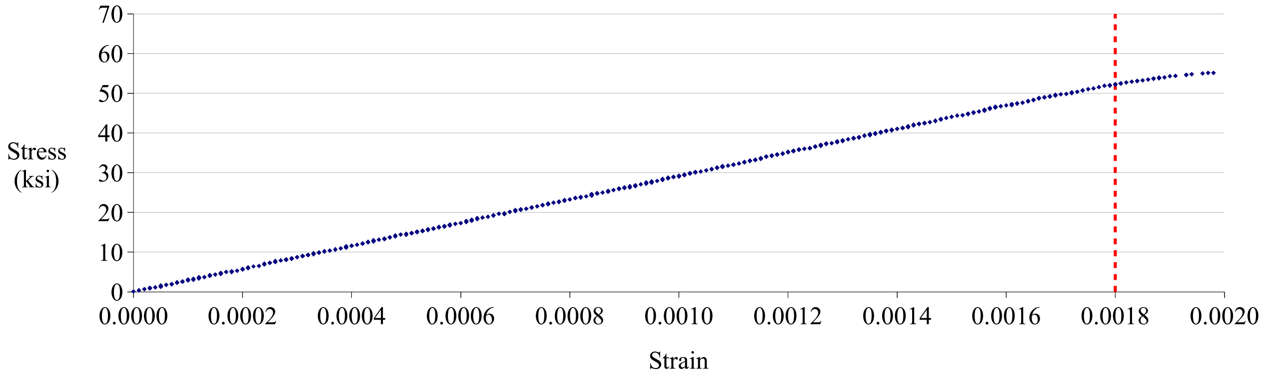
Solution The change in anything is the final dimension minus the initial dimension. Here, the change in length is the final length minus the initial length: $\Delta L=L_f-L_o=6.05\text{ in.}-6.0\text{ in.}=0.05\text{ in.}$. Strain is $\varepsilon=\dfrac{\Delta L}{L}=\dfrac{0.05\text{ in.}}{6.0\text{ in.}}=0.0083$.

If we hang a bucket from the wire and gradually fill the bucket with water, the weight will gradually increase along with the stress and the strain in the wire, until finally the wire breaks. We can plot the stress vs. strain on an x-y scatter graph, and the result will look like this:



This graph shows the stress-strain behavior of a low-carbon sheet steel specimen. Stress is in units of ksi, or kips per square inch, where 1 kip = 10^3 lb. (1 kilopound). The points at the left end of the curve (left of the red dashed line) are so close together that they are smeared into a line. This straight part of the stress-strain curve is the elastic portion of the curve. If you fill the bucket with only enough water to stretch the wire in the elastic zone, then the wire will return to its original length when you empty the bucket.

We can change the range of the strain axis from 0.0-0.2 to 0.000-0.002, to show the elastic data only:



This graph shows the leftmost 1% of the previous graph. The dashed red line is in the same position on both graphs. Now the individual data points are visible, and the curve is almost perfectly straight up to a strain of about 0.0018. The straight line has a slope, called Young's Modulus,[2] or Elastic Modulus, *E*. The slope of a straight line is the rise over run, so within this elastic zone, $E=\frac{\sigma}{\varepsilon}$. Since strain is unitless, Young's modulus has the same units as stress. Young's modulus is a mechanical property of the material being tested: $30\times10^6\,\text{psi}$ or 207 GPa for steels, $10\times10^6\,\text{psi}$ or 70 GPa for aluminum alloys. See **Appendix B** for materials properties of other materials.

Example #2

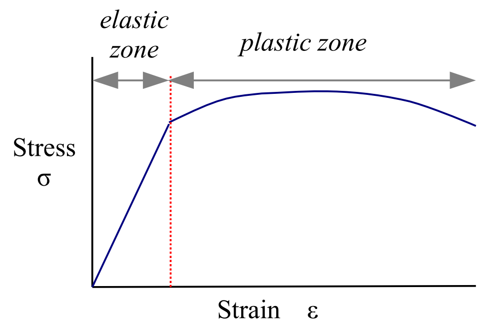
What tensile stress is required to produce a strain of 8×10^{-5} in aluminum? Report the answer in MPa.

Solution Aluminum has a Young's modulus of E = 70 GPa. Rewrite $E=\frac{\sigma}{\varepsilon}$, solving for stress:

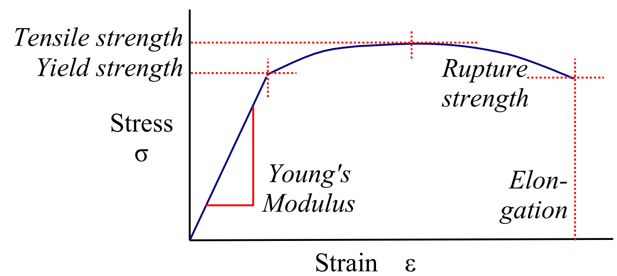
$$\sigma=E\,\varepsilon=\frac{8\times10^{-5}\cdot70\,\text{GPa}}{}\left|\frac{10^3\,\text{MPa}}{\text{GPa}}\right.=5.6\,\text{MPa}$$

2 Named for Thomas Young, an English physics professor, who defined it in 1807.

This cartoon of a stress-strain curve illustrates the *elastic* and *plastic* zones. If you hang a light weight to the wire hanging from the ceiling, the wire stretches elastically; remove the weight and the wire returns to its original length. Apply a heavier weight to the wire, and the wire will stretch beyond the elastic limit and begins to plastically[3] deform, which means it stretches permanently. Remove the weight and the wire will be a little longer (and a little skinnier) than it was originally. Hang a sufficiently heavy weight, and the wire will break.



Two stress values are important in engineering design. The yield strength, σ_{YS}, is the limit of elastic deformation; beyond this point, the material "yields," or permanently deforms. The ultimate tensile strength, σ_{UTS} (also called tensile strength, σ_{TS}) is the highest stress value on the stress-strain curve. The rupture strength is the stress at final fracture; this value is not particularly useful, because once the tensile strength is exceeded, the metal will break soon after. Young's modulus, *E*, is the slope of the stress-strain curve before the test specimen starts to yield. The strain when the test specimen breaks is also called the elongation.

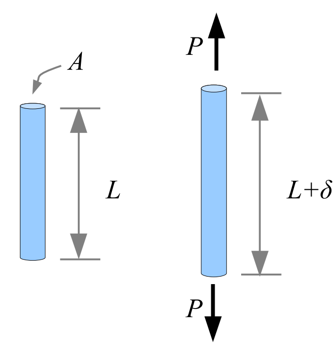


Many manufacturing operations on metals are performed at stress levels between the yield strength and the tensile strength. Bending a steel wire into a paperclip, deep-drawing sheet metal to make an aluminum can, or rolling steel into wide-flange structural beams are three processes that permanently deform the metal, so $\sigma_{YS} < \sigma_{Applied}$. During each forming operation, the metal must not be stressed beyond its tensile strength, otherwise it would break, so $\sigma_{YS} < \sigma_{Applied} < \sigma_{UTS}$. Manufacturers need to know the values of yield and tensile strength in order to stay within these limits.

After they are sold or installed, most manufactured products and civil engineering structures are used below the yield strength, in the elastic zone.[4] In this *Strength of Materials* course, almost all of the problems are elastic, so there is a linear relationship between stress and strain.

Take an aluminum rod of length *L*, cross-sectional area *A*, and pull on it with a load *P*. The rod will lengthen an amount δ. We can calculate δ in three separate equations, or we can use algebra to find a simple equation to calculate δ directly. Young's modulus is defined as $E = \frac{\sigma}{\varepsilon}$. Substitute the definition of stress, $\sigma = \frac{P}{A}$, and $E = \frac{\sigma}{\varepsilon} = \frac{P}{A \cdot \varepsilon}$.

Substitute the definition of strain, $\varepsilon = \frac{\delta}{L}$, and $E = \frac{P}{A \cdot \varepsilon} = \frac{PL}{A\delta}$. Rewrite this equation to solve for deflection: $\delta = \frac{PL}{AE}$. Now we have a direct equation for calculating the change in length of the rod.



3 Here, the word *plastic* is used in its 17th century sense "capable of being deformed" rather than the 20th century definition "polymer."

4 One exception is the crumple zones in a car. During an auto accident, the hood and other sheet metal components yield, preventing damage to the driver and passengers. Another exception is a shear pin in a snow blower. If a chunk of ice jams the blades, the shear pin exceeds its ultimate strength and breaks, protecting the drivetrain by working as a mechanical fuse.

Example #3

A 6 foot long aluminum rod has a cross-sectional area of 0.08 in.². How much does the rod stretch under an axial tensile load of 400 lb.? Report the answer in inches.

Solution Aluminum has a Young's modulus of $E = 10\times10^6\ \text{psi}$.

Deflection $\delta = \dfrac{PL}{AE} = \dfrac{400\,\text{lb.}\ \ 6\,\text{ft.}}{0.08\,\text{in.}^2}\,\left.\dfrac{\text{in.}^2}{10\times10^6\,\text{lb.}}\right|\dfrac{12\,\text{in.}}{\text{ft.}} = 0.036\,\text{in.}$

Note Young's modulus is in units of psi, but when you write it in an equation, split up the lb. and the in.² between numerator and denominator to avoid unit confusion.

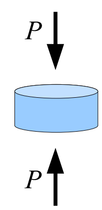
Sign Convention

A load that pulls is called a *tensile* load. If the load pushes, we call it a *compressive* load. The equations are the same: compressive stress $\sigma = P/A$, compressive strain $\varepsilon = \delta/L$, and compressive deflection $\delta = PL/AE$. We need a way to differentiate between compression and tension, so we use a sign convention. Tensile loads and stresses are positive; compressive loads and stresses are negative. Increases in length are positive; decreases in length are negative.

Example #4

A 70 kN compressive load is applied to a 5 cm diameter, 3 cm tall, steel cylinder. Calculate stress, strain, and deflection.

Solution The load is –70 kN, so the stress is . $\sigma = \dfrac{P}{A} = \dfrac{4P}{\pi d^2} = \left.\dfrac{4(-70\,\text{kN})}{\pi(5\,\text{cm})^2}\right|\left.\dfrac{\text{MPa}\ \ \text{m}^2}{10^3\,\text{kN}}\right|\dfrac{(100\,\text{cm})^2}{\text{m}^2} = -35.6\,\text{MPa}$



The negative sign tells us the stress is compressive.

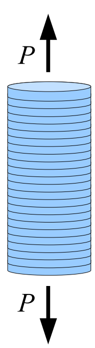
Young's modulus $E = \dfrac{\sigma}{\varepsilon}$. Rewrite the equation to solve for strain: $\varepsilon = \dfrac{\sigma}{E} = \left.\dfrac{-35.6\,\text{MPa}}{207\,\text{GPa}}\right|\dfrac{\text{GPa}}{10^3\,\text{MPa}} = -0.000172$

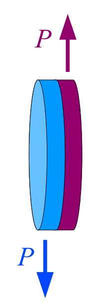
Strain is defined as $\varepsilon = \dfrac{\delta}{L}$. Rewrite to solve for deflection: $\delta = \varepsilon L = \left.-0.000172\cdot 3\,\text{cm}\right|\dfrac{10\,\text{mm}}{\text{cm}} = -0.0052\,\text{mm}$.

The negative signs tell us that the cylinder is shrinking along the direction of the load.

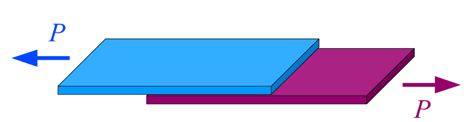
Shear Stress and Strain

The stress in the previous examples is called "normal stress" because the stress acts on an area that is normal, or perpendicular, to the direction of the applied load. Imagine a tall stack of coins glued together on their faces. If you pull on the ends of the stack, the glue will experience a stress that is normal (perpendicular) to the face of each coin. If the glue is thick and tacky, maybe it will tend to stretch, and you can see the coins gradually pull apart along the direction of the applied load. If the load is 100 lb. and the face area of each coin is 1 in.², then the normal stress is 100 psi.



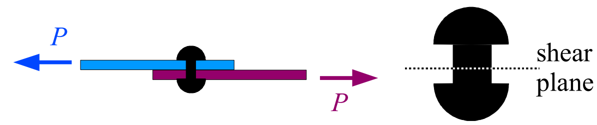


Next, imagine taking two coins that are glued together on their faces, and try to slide them apart. Now the stress is acting parallel to the glue instead of perpendicular to it. This stress is called *shear stress*, symbolized by the lower case Greek letter tau, τ. The units are the same as for normal stress because shear stress is also force divided by area. If the load is 25 lb. and the face area of each coin is 1 in.², then the shear stress is 25 psi.



Sheet metal joints are often manufactured this way, with adhesive bonding two lapped sheets to form a lap joint. The load is parallel to the area under stress (the adhesive in the shear plane between the two lapped panels). Joints can be designed to put the adhesive in either tension or in shear; typically, the shear strength of an adhesive is not the same as the strength in tension. For example, cyanoacrylate adhesive ("superglue") is stronger in shear than in tension. An adhesive lap joint will fail when the shear strength of the adhesive is exceeded.

If the sheet metal is held together with rivets instead of glue, then each rivet is loaded in shear across its cross-section. The shear plane passes through the rivet where the two sheets meet. In a bolted joint, use a bolt with a smooth shank instead of a bolt that is threaded along its entire length. This way, the shear plane can pass through the smooth shank, which has a larger cross-sectional area than the root of a thread, and therefore can handle a higher applied load. Later in the book, we will see that the thread root also acts as a stress concentration site; yet another reason for keeping threads out of shear planes.



One way to produce holes in sheet metal is by punching them out with a punch and die set. The punch shears the sheet metal, so we can use shear stress calculations to figure out the stress in the sheet metal. The sheared area is perimeter of the shape that is punched times the thickness of the sheet metal *t*. The shear stress is the punch force divided by the sheared surface: $\tau = \frac{P}{A}$.

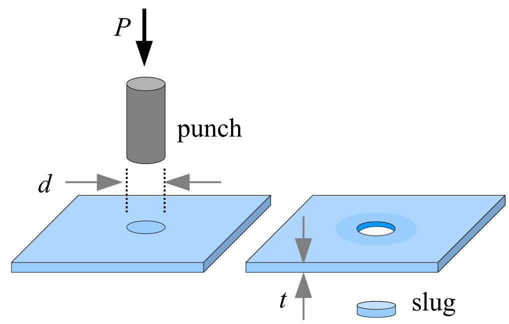
Example #5

A 3 mm thick aluminum sheet is cut with a 4 cm diameter round punch. If the punch exerts a force of 6 kN, what is the shear stress in the sheet? Report the answer in MPa.

Solution The punch will create a round slug, where the cut edge is around the circumference of the slug. Think of the cut edge as the wall of a cylinder with a height of 3 mm and a diameter of 4 cm. The area equals the circumference of the circle times the thickness of the sheet metal: $A = \pi\, dt$.

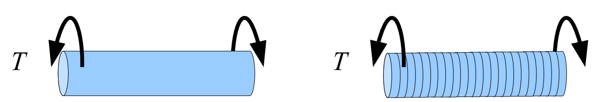
Shear stress

$$\tau = \frac{P}{A} = \frac{P}{\pi\, dt} = \frac{6\,\text{kN}}{\pi \cdot 4\,\text{cm} \cdot 3\,\text{mm}} \left| \frac{\text{MPa}\ \ \text{m}^2}{10^3\,\text{kN}} \right| \frac{100\,\text{cm}}{\text{m}} \left| \frac{10^3\,\text{mm}}{\text{m}} \right. = 15.9\,\text{MPa}$$

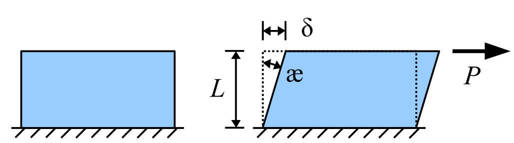


A process engineer in a stamping plant will rewrite this equation to solve for *P* in order to find out whether a press is capable of punching out blanks of a given size in a sheet metal of known shear strength.

Shear stress controls the design of torsion members. Think of a round shaft as a series of disks glued together on their faces. If you twist the shaft with a torque *T*, the glue will be loaded in shear because the load is parallel to the face of each disk.



Consider a rectangular block loaded in shear. The block will distort as a parallelogram, so the top edge moves an amount δ. Divide the distortion by length *L* perpendicular to the distortion, and you have the shear strain, $\gamma = \frac{\delta}{L}$. Like normal strain, shear strain is unitless.



Consider the angle formed between the initial and loaded positions of the block. From trigonometry, we know that $\tan\phi = \frac{\delta}{L}$ The amount of strain in the cartoon is exaggerated. For metals, concrete, wood, and most polymers, angle æ is so small that $\tan\phi \approx \phi$ if we measure the angle in radians, therefore $\phi \approx \gamma = \frac{\delta}{L}$.

Key Equations

Chapter 2: Stress and Strain

Normal stress in a tensile or compressive member is the load divided by the cross-sectional area: $\sigma = \frac{P}{A}$

Normal strain is the change in length parallel to the load divided by initial length: $\varepsilon = \frac{\Delta L}{L} = \frac{\delta}{L}$

Young's modulus is the ratio of stress over strain within the elastic zone of the stress-strain diagram: $E = \frac{\sigma}{\varepsilon}$

The change in length of a tensile or compressive member is derived from the three previous equations: $\delta = \frac{PL}{AE}$

Shear stress is the load divided by the area parallel to the load: $\tau = \frac{P}{A}$

Shear strain is the deformation parallel to the load divided by initial length perpendicular to the load: $\gamma = \frac{\delta}{L}$.