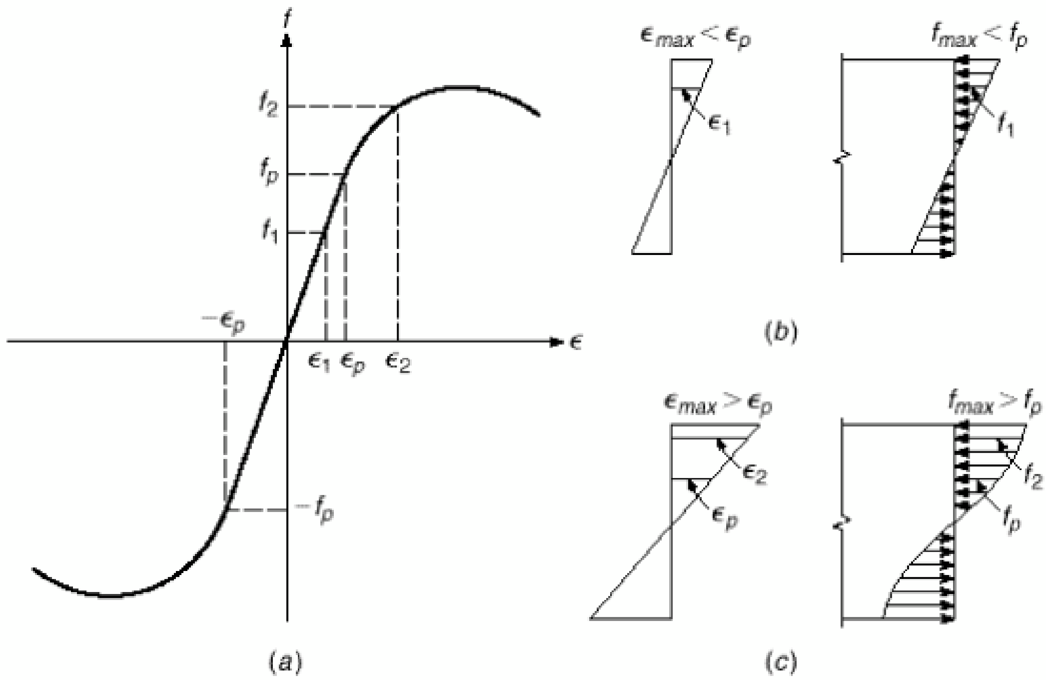
2 Flexural Analysis and Design of RC Beams

Fundamental assumptions

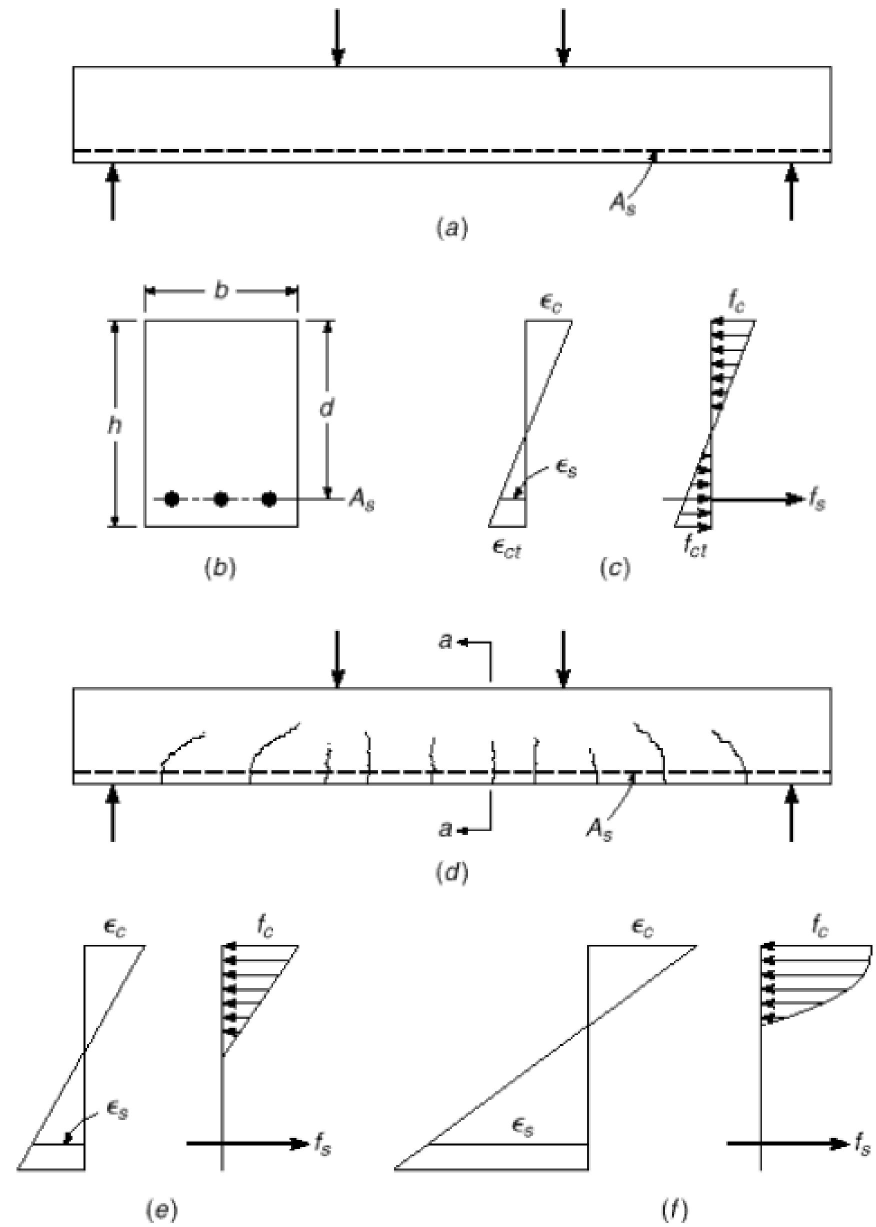
1. A cross section that was plane before loading remains plane under load.
2. The bending stress f at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material. (See Fig.)



3. Distribution of shear stresses over the depth of the section depends on the shape of the cross section and the stress-strain diagram.
4. At any point in the beam there are inclined stresses of tension and compression, the largest of which form angle 90° with each other. The inclined stresses of tension and compression at any point form an angle of 45° with the horizontal.
5. When the stresses in the outer fibers are smaller than the proportional limit f_p, the beam behaves elastically, as shown in fig. b. In this case the following pertains:

a) The NA passes thru the c.g. of the cross section.
b) The intensity of bending stress increases directly with the distance from NA: $f = M.y / I$, $f_{max} = M.c / I = M / S$, $(S = I / c)$
c) The shear stress at any point is given by $v = V.Q / (I.b)$
d) The intensity of shear stress varies as a parabola being zero at outer fibers and max at NA.

RC Beam Behavior



1st stage (fig. c): At low loads, all stresses are of small magnitude and are proportional to strains.
2nd stage (fig. e): When the load is increased, the tensile strength of concrete is reached; tension cracks develop; concrete does not transmit any tensile stresses. The steel resists the entire tension. If concrete compressive stresses do not exceed ≈ $0.5 f'_c$, stresses and strains continue to be proportional (linear stress distribution)
3rd stage (fig. f): When the load is further increased, stresses and strains are no longer proportional; the distribution of concrete stresses on the compression side is of the same shape as concrete stress-strain curve.

<u>Failure can be caused in one of two ways:</u>

A) When moderate amounts of reinforcement are employed, the steel will reach its <u>yield</u> point; the reinforcements stretches a large amount; the tension cracks widen and propagate upward; significant deflection of the beam;
When this happens, the strains in compression zone of concrete increase to ensue <u>crushing</u> (*secondary compression failure*) at a load only slightly greater than that which cause the steel to yield.
Such **<u>yield failure</u>** is gradual and is preceded by visible signs of distress: cracks and deflection.

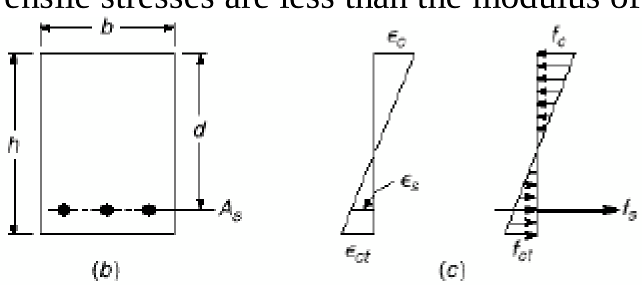
B) **When large amounts of reinforcement are employed,** the compressive strength of concrete is exhausted before the steel starts yielding. It has been observed that beams fail in compression when the concrete strains reach values of about 0.003 to 0.004.
Such **<u>compression failure</u>** is sudden; explosive, and occurs without warning.

It is good practice to dimension beams that they will fail by yielding of the steel (A) rather than by crushing of concrete (B).

<u>Analysis of Stresses and Strength in the Different Stages</u>

a) <u>Stresses Elastic and Section Uncracked (figure c)</u>

Tensile stresses are less than the modulus of rupture f_r.

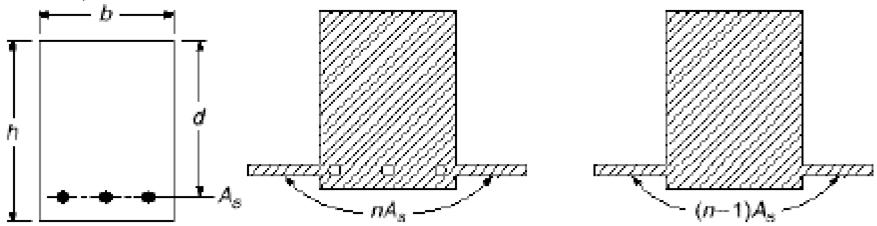


At the level of reinforcement:

$$\varepsilon_s = \varepsilon_c \; ;$$
$$f_s / E_s = f_c / E_c \; ;$$
$$f_s = (E_s / E_c) \; f_c$$
$$f_s = n \; f_c$$

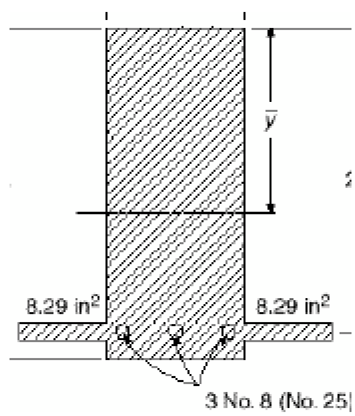
Where $n = E_s / E_c$ is known as the *modular ratio*.

It means that the stress in steel (***$f_s = n\ f_c$***) is ***n*** times that of the concrete. The analysis shall depend on the "transformed section". In this fictitious section, the actual area of the reinforcement is replaced with an equivalent concrete area equal to ***nA_s***, located at the level of steel. (Figure below)



Example 1

A rectangular beam has the dimensions b = 250 mm, h = 650 mm, and d = 600 mm and is reinforced with 3 No. 25 bars so that A_s = 1530 mm^2. The concrete cylinder strength f'_c is 28 MPa, and the tensile strength in bending (modulus of rupture) f_r is 3.27 MPa. The yield point of the steel f_y is 420 MPa. Determine the stresses caused by a bending moment M = 61 kN-m.



Solution

$E_c = 4700\ \sqrt{f'_c}$ = 24870 MPa

$n = E_s / E_c$ = 200000 / 24870 = 8

Add an area $(n - 1)A_s$ = 7 × 1530

= 10710 mm^2 {5355 mm^2 (8.29 in^2) as shown}

$y^{-} = \sum A.y / \sum A$ = 342 mm from top (check this)

I = 6,481,000,000 mm^4 (check this)

Compression stress at top $f_c = M.y^{-} / I$

= 61,000,000 × 342 / 6,481,000,000 = 3.22 MPa

Tension stress at bottom f_{ct} = 61,000,000 × 308 / 6,481,000,000 = 2.90 MPa

Since 2.90 MPa < f_r = 3.27 MPa (given), no tensile cracks will form, and calculation by uncracked transformed section is justified.

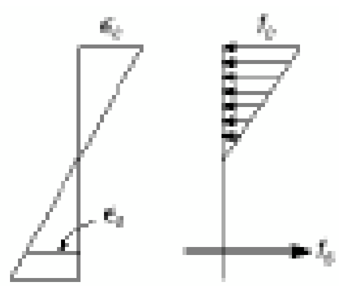
Steel stress $f_s = n\, M.y / I$

$= 8\ (61{,}000{,}000 \times 258 / 6{,}481{,}000{,}000) = 19.43$ MPa

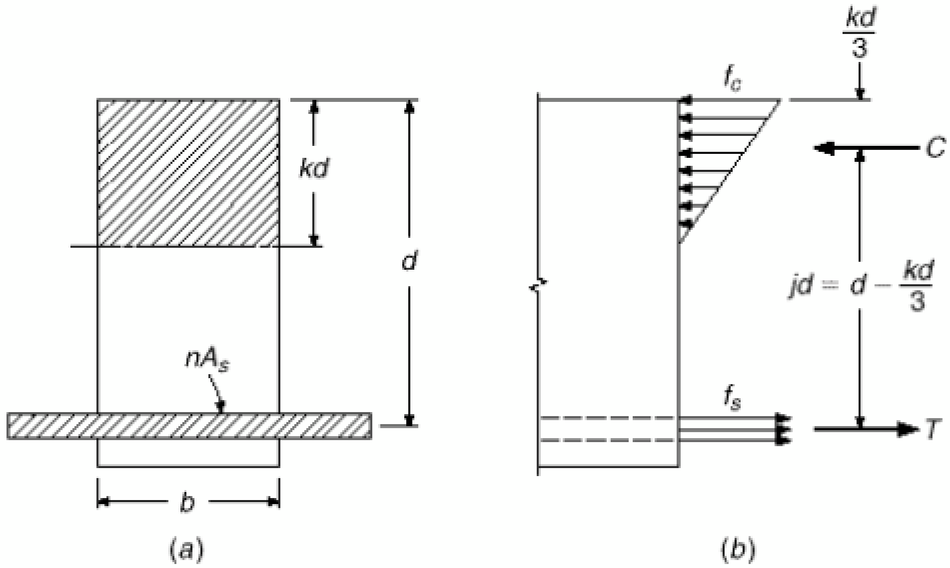
It is seen that at this stage the actual stresses are quite small compared with the available strengths of steel and concrete.

b) Stresses Elastic and Section Cracked (figure e)

When the tensile stress f_{ct} exceeds the modulus of rupture f_r , cracks form. If the compressive stress f_c is less than ≈ 0.5 f'_c and the steel stress has not reached the yield point ($f_s < f_y$) , both materials continue to behave elastically.
This situation occurs in structures under normal service conditions and loads. This situation with regard to strain and stress distribution is that shown in figure e:



The fact is that all of the concrete that is stressed in tension is assumed cracked, and therefore effectively absent. (Figure below: cracked transformed section)



To determine the location of the N.A. (*kd* from top), the moment of the tension area about the axis is set equal to the moment of the compression area:

$$b(kd)^2/2 - nA_s(d - kd) = 0 \qquad \text{.......(1)}$$

Then determine the moment of inertia.

Alternatively,

$$C = \frac{f_c}{2} bkd \qquad \text{and} \qquad T = A_s f_s \tag{3.6}$$

The requirement that these two forces be equal numerically has been taken care of by the manner in which the location of the neutral axis has been determined.

Equilibrium requires that the couple constituted by the two forces C and T be equal numerically to the external bending moment M. Hence, taking moments about C gives

$$M = Tjd = A_s f_s jd \tag{3.7}$$

where jd is the internal lever arm between C and T. From Eq. (3.7), the steel stress is

$$f_s = \frac{M}{A_s jd} \tag{3.8}$$

Conversely, taking moments about T gives

$$M = Cjd = \frac{f_c}{2} bkdjd = \frac{f_c}{2} kjbd^2 \tag{3.9}$$

from which the concrete stress is

$$f_c = \frac{2M}{kjbd^2} \tag{3.10}$$

In using Eqs. (3.6) through (3.10), it is convenient to have equations by which k and j may be found directly, to establish the neutral axis distance kd and the internal lever arm jd. First defining the *reinforcement ratio*

$$\boldsymbol{\rho = A_s / bd}$$

then substituting $\boldsymbol{A_s = \rho\, bd}$ into equation (1), solving for $\boldsymbol{k}$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

then $jd = d - kd/3,$

$$j = 1 - k/3$$

Values of ***k*** and ***j*** are tabulated (Nilson Table A6):

Example 2

The beam of Example 1 is subjected to a bending moment M = 122 kN-m (rather than 61 kN-m). Calculate the relevant properties and stresses.

Solution

Check the section is cracked:
Tension stress at bottom f_{ct} = 122,000,000 × 308 / 6,481,000,000 = 5.8 MPa

Since 5.8 MPa > f_r = 3.27 MPa (given), tensile cracks will form, and calculation must adapt the cracked transformed section.

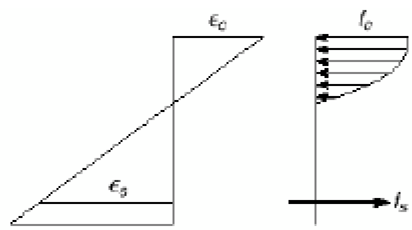
Equation 1, $b(kd)^2/2 - nA_s(d - kd) = 0$, with b = 250 mm, n = 8, and A_s = 1530 mm^2 inserted, gives
kd = 198 mm (distance to N.A.)
k = 198/600 = 0.33
$j = 1 - k/3$ = 0.89
$f_s = M / A_s\, jd$ = 122,000,000/[1530 × 0.89] = <u>153.6 MPa</u>
$f_c = 2M / kjbd^2$ = 2×122,000,000/[0.33×0.89×250×600^2] = <u>9.58 MPa</u>

Notes: (compared with Example 1; doubling M)

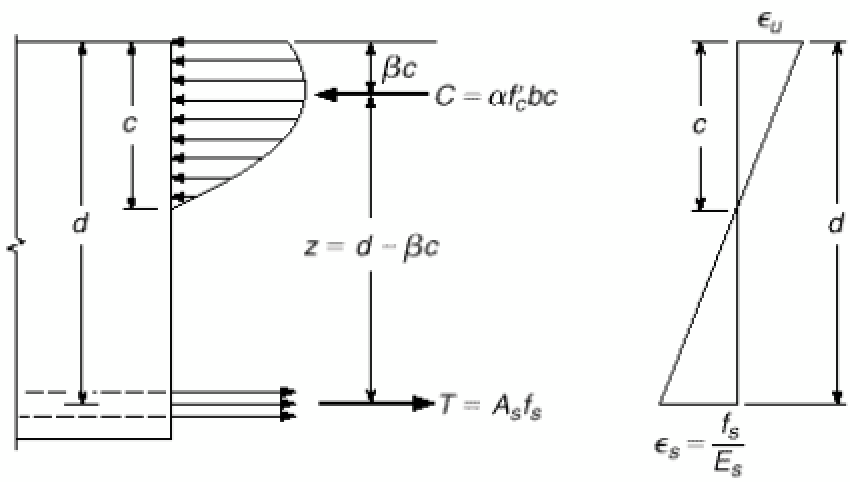
1. N.A. has moved upward: changed from 342 to 198 mm.
2. The steel stress changed from 19.43 to 153.6 MPa (about 8 times).
3. The concrete compressive stress has increased from 3.22 to 9.58 MPa (about 3 times).
4. The moment of inertia of cracked section (2,625,000,000 mm^4 check this!) is less than that of uncracked section (6,481,000,000 mm^4). This affects the magnitude of the deflection.

c) Flexural Strength (figure f)

At high loads, close to failure, the distribution of stresses and strains is that of fig. f:



Stress and strain distributions at ultimate load are assumed as shown in fig. below:



For failure mode A, two criteria are implied

- $f_s = f_y$
- The concrete crushes when the maximum strain reaches ε_u **= 0.003**.

It is necessary to know, for a given distance c of N.A.,

1. The total resultant compression force C in the concrete.
2. Its vertical location, i.e., its distance from the outer compression fiber.

In rectangular beams, area in compression is bc, and $C = f_{av}bc$

Let $\alpha = f_{av} / f'_c$ then $C = \alpha f'_c bc$
The location of C is at βc from top.

Knowing **α** and **β** will define the compressive stresses.

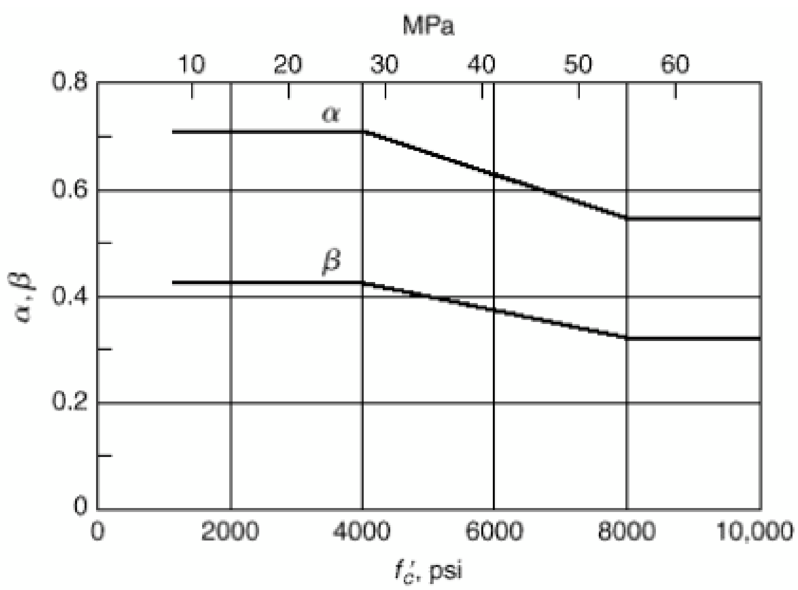
If **α** and **β** are known, then equilibrium requires that
$C = T$ or $\alpha f'_c bc = A_s f_s$
Also
$M = Tz = A_s f_s (d - \beta c)$
Or $M = \alpha f'_c bc(d - \beta c)$

Set $f_s = f_y$ then $c = A_s f_y / \alpha f'_c$
Using $A_s = \rho bd$, then $c = \rho f_y d / \alpha f'_c$

Substitute, M_n is then obtained

$$M_n = \rho f_y bd^2(1 - \beta \rho f_y/ \alpha f'_c)$$

From extensive experimental work, the values of α and β have shown to be as in the figure below (for $f'_c \leq 28$ MPa, $\alpha = 0.72$ and $\beta = 0.425$)



Now, the nominal moment equation becomes:

$$M_n = \rho f_y bd^2(1 - 0.59 \rho f_y/ f'_c)$$

Balanced reinforcement ratio ρ_b

The balanced reinforcement ratio, ρ_b represents that amount of reinforcement necessary for the beam to fail by crushing of the concrete at the same load that causes the steel to yield.

Hooke's law: $f_s = \varepsilon_s E_s$

From strain distribution (see fig.), similar triangles give

$$f_s = \varepsilon_u E_s(d - c)/c$$

Setting $f_s = f_y$, and substituting ε_y for f_y / E_s, the value of c defining the unique position of the N.A. corresponding to simultaneous crushing of the concrete and initiation of yielding in the steel,

$$c = d.\ \varepsilon_u/(\ \varepsilon_u + \varepsilon_y)$$

Substituting c in equation $C = T$ or $\alpha f'_c\, bc = A_s f_s$ with $A_s f_s = \rho b d f_y$, the ρ_b is obtained

$$\rho_b = (\alpha f'_c / f_y)[\ \varepsilon_u/(\ \varepsilon_u + \varepsilon_y)]$$

Example 3

Determine the nominal moment M_n at which the beam of Examples 1 and 2 will fail.

Solution

$\rho = A_s / bd$ = 1530 / (250×600) = 0.0102 (*always write ρ with 4 digits*)

check

$\rho_b = (\alpha f'_c / f_y)[\ \varepsilon_u/(\ \varepsilon_u + \varepsilon_y)] = 0.0282$ (α = 0.72)

Since $\rho < \rho_b$, the beam will fail in tension by yielding of the steel, its nominal moment is

$$M_n = \rho f_y bd^2(1 - 0.59\, \rho f_y / f'_c)$$
$$= 0.0102 \times 420 \times 250 \times 600^2 (1 - 0.59 \times 0.0102 \times 420 / 28)$$
$$= 350{,}800{,}000 \text{ N-mm} = 350.8 \text{ kN-m}$$

At this M_n, the distance to neutral axis is

$c = \rho f_y d / \alpha f'_c$

= 0.0102 × 420 × 600 / (0.72 × 28) = 127.5 mm

Summary

	Ex 1:Uncracked	**Ex 2:Cracked**	**Ex 3:Ultimate**
NA from top,mm	342	198	127.5
f_c / f_s (MPa/MPa)	3.22 / 19.43	9.58 / 153.6	28 / 420
M , kN-m	61	122	350.8

The differences between various stages (as the load is increased) are

1. The migration of the N.A. toward the compression edge.
2. The increase in steel stress.
3. The increase in concrete compressive stress.

Design of Tension-Reinforced Rectangular Beams

To provide sufficient strength to RC structures:

1. The nominal strength is modified by a ***strength reduction factor*** **φ**, less than unity, to obtain the *design strength*.
2. The *required strength* is found by applying ***load factors*** **γ**, greater than unity, to loads actually expected (*service loads*).

Thus, RC members are proportioned such that $M_u \le \varphi M_n$; $V_u \le \varphi V_n$; $P_u \le \varphi P_n$

where subscripts ***n*** denote the nominal strengths in flexure, shear, and axial load respectively, and ***u*** denote the factored load moment, shear and axial load.

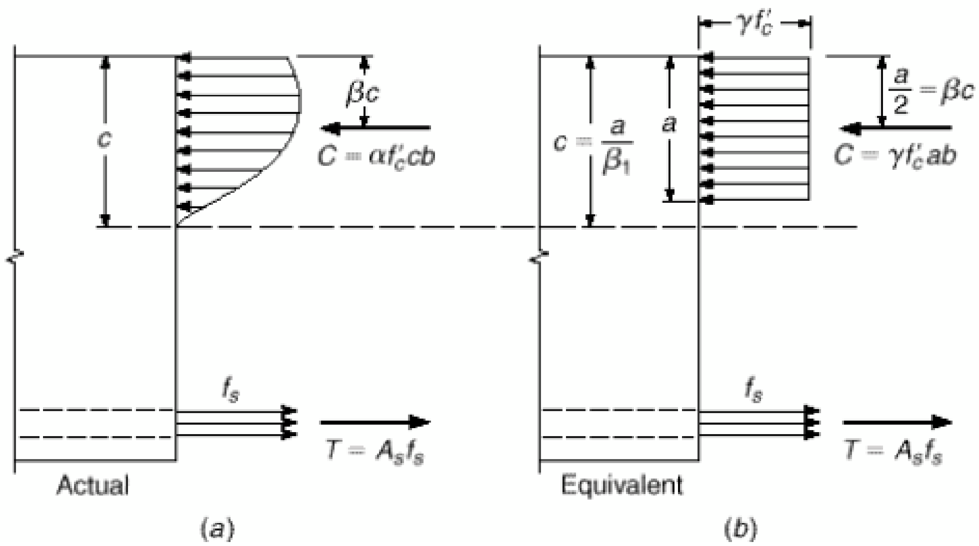
See page 4, chapter 1 of the lecture notes.

a. Equivalent Rectangular Stress Distribution

It was noted that the actual shape of the concrete compressive stress distribution varies considerably. The magnitude ***C*** and location ***βc*** of the resultant of the concrete compressive stresses are obtained from experiments and expressed in the two parameters ***α*** and ***β***.

For simplicity, the actual stress distribution is replaced by an equivalent one of simple rectangular outline. See next figure.

The conditions are that the magnitude of ***C*** and its location must be the same in the equivalent rectangular as in the actual stress distribution.



$C = \alpha f'_c bc = \gamma f'_c ab$ from which $\gamma = \alpha c / a$

With $a = \beta_1 c$, this gives $\gamma = \alpha / \beta_1$

The force C is located at the same distance: $\beta_1 = 2\beta$.

$\gamma = \alpha / \beta_1 = \alpha / 2\beta$ is seen independent of f'_c and can be taken as 0.85 throughout (e.g. 0.72/(2×0.425) = 0.85):

The force ***C***: $C = 0.85 f'_c\, ab$

The distance ***a***: $a = \beta_1 c$

$\beta_1 = 0.85$ ………………………… for $f'_c \le 28$ **MPa**

$\beta_1 = 0.85 - 0.05\,(f'_c - 28) / 7$ …………… for $f'_c > 28$ **MPa**

$$0.65 \le \beta_1 \le 0.85$$

b. <u>Balanced Strain Condition</u>

From strain diagram $c = d.\ \varepsilon_u / (\varepsilon_u + \varepsilon_y)$

Equilibrium C = T ; $0.85\ \beta_1 f'_c\ bc = \rho_b b d f_y$

$$\rho_b = 0.85\ \beta_1 (f'_c / f_y)[\ \varepsilon_u / (\varepsilon_u + \varepsilon_y)]$$

c. <u>Under-reinforced Beams</u>

To ensure that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete, this can be done, <u>theoretically</u> by requiring

$$\rho < \rho_b$$

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons:

1. To get significant yielding before failure.
2. Material properties are never known exactly.
3. Strain-hardening of the steel may lead to concrete compression failure.
4. Actual steel area provided will always be equal to or larger than required.
5. Lower ρ increases deflection and thus provides warning prior to failure.

d. <u>ACI Code Provisions for Under-reinforced Beams</u>

ACI Code defines the safe limits of maximum reinforcement by two forms both are based on the ***net tensile strain*** ε_t of the reinforcement farthest from the compression face of the concrete at depth d_t

1. The minimum tensile reinforcement strain allowed at nominal strength:

$\varepsilon_t = \varepsilon_u . (d_t - c) / c$

$\rho = 0.85\, \beta_1 (f'_c / f_y)(d_t / d)[\varepsilon_u / (\varepsilon_u + \varepsilon_t)]$

conservatively

$\rho = 0.85\, \beta_1 (f'_c / f_y)[\varepsilon_u / (\varepsilon_u + \varepsilon_t)]$

To ensure under-reinforced behavior, ACI Code 10.3.5 establishes a minimum net tensile strain ε_t at nominal of 0.004 for members subjected to axial loads less that $0.10 f'_c A_g$, where is the gross area of the cross section. Substituting in ρ equation:

$$\rho = 0.85\, \beta_1 (f'_c / f_y)[\varepsilon_u / (\varepsilon_u + 0.004)]$$

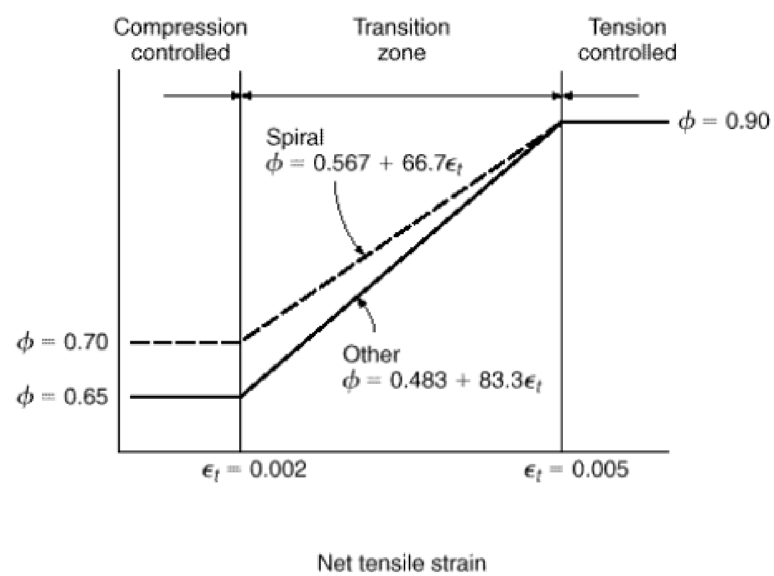
2. Allowing ***strength reduction factors*** that depend on the tensile strain at nominal strength. The Code defines:

a. ***Tension-controlled member***: The one with a net tensile strain $\varepsilon_t \geq 0.005$. the corresponding strength reduction factor $\varphi = 0.9$.

b. ***Compression-controlled member:*** The one with a net tensile strain $\varepsilon_t \leq 0.002$. The corresponding strength reduction factor $\varphi = 0.65$. For spirally-reinforced members $\varphi = 0.75$

For ε_t between 0.002 and 0.005, φ varies linearly, and ACI Code allows linear interpolation of φ based on ε_t. see Figure below:

FIGURE 3.9
Variation of strength reduction factor with net tensile strain.

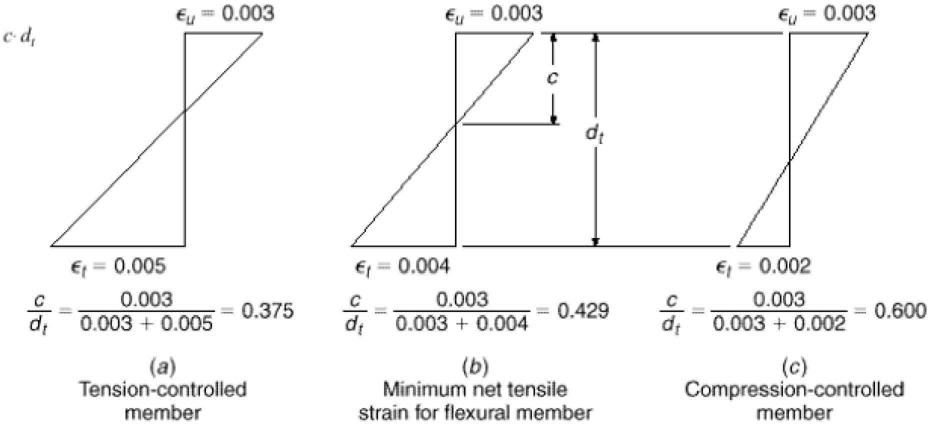


The maximum reinforcement ration for a tension-controlled beam is: (recommended for flexural members)

$$\rho_{0.005} = 0.85\,\beta_1\,(f'_c/\,f_y)[\varepsilon_u/(\varepsilon_u + 0.005)]$$

The depth of equivalent rectangular stress block a:
Since $c = a/\beta_1$, it is more convenient to compute c/d_t rather than ρ or net ε_t, see Figure. Maximum value of $c/d_t = 0.375$ for $\varepsilon_t \geq 0.005$

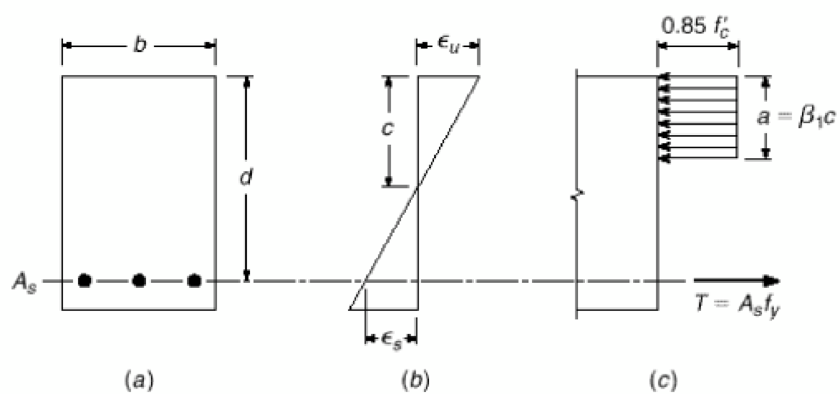
FIGURE 3.10
Net tensile strain and c/d_t ratios.



The nominal flexural strength is given by (see figure below)

$$M_n = A_s f_y (d - a/2), \quad a = A_s f_y / 0.85\, f'_c\, b$$

FIGURE 3.11
Singly reinforced rectangular beam.



Example 4
Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 3. Recall $\boldsymbol{b}$ = 250 mm, $\boldsymbol{d}$ = 600 mm, $\boldsymbol{A_s}$ = 1530 mm2, $\boldsymbol{f'_c}$ = 28 MPa, $\boldsymbol{f_y}$ = 420 MPa.

Solution

$\beta_1 = 0.85$ ($f'_c = 28$ MPa)

$\rho_{0.005} = 0.85\,\beta_1 (f'_c / f_y)[\varepsilon_u /(\varepsilon_u + 0.005)]$

$= 0.85\times0.85\,(28/420)[0.003 /(0.003 + 0.005)] = 0.0181$

Actual $\rho = 1530 /(250\times600) = 0.0102$

Since $\rho < \rho_{0.005}$, the member will fail by yielding of steel.

Alternatively, recall c = 127.5 mm,

$c/d_t = 127.5/600 = 0.213 < 0.375$, the member will fail by yielding of steel

$a = A_s f_y / 0.85 f'_c b = 1530\times420/(0.85\times28\times250) = 108$ mm

$M_n = A_s f_y(d - a/2) = 1530\times420(600 - 108/2) = 350.9\times10^6$ Nmm = 350.9kNm

Moment equation can be re written (as derived previously) as follows:

$$M_n = \rho f_y bd^2(1 - 0.59\,\rho\, f_y/ f'_c)$$

$= 0.0102\times420\times250\times600^2\,(1 - 0.59\times0.0102\times420/28)[10^{-6}] = 350.8$ kNm

This equation may be simplified further for everyday design as follows

$$M_n = Rbd^2$$

In which

$$R = \rho f_y\,(1 - 0.59\,\rho\, f_y/ f'_c) \qquad \text{(MPa)}$$

The values of the *flexural resistance factor* R are tabulated in Appendix A5 (Nilson)

In accordance with safety the safety provisions of the ACI Code, the *nominal flexural strength* M_n is reduced by imposing the strength reduction factor φ to obtain the *design strength* φM_n

$$\varphi M_n = \varphi A_s f_y(d - a/2)$$

Or, alternatively, $\varphi M_n = \varphi\,\rho f_y\, bd^2(1 - 0.59\,\rho\, f_y/ f'_c)$

Or $\varphi M_n = \varphi R\, bd^2$

Example 4 (*continued*): Since $\rho<\rho_{0.005}$ (or $c/d_t < 0.375$), then $\varepsilon_t > 0.005$. Therefore, $\varphi = 0.9$ and design capacity is $\varphi M_n = 0.9\times350.9 = 315.8$ kNm

e. Minimum Reinforcement Ratio

In very lightly reinforced beams, if the flexural strength < the moment that produce cracking, the beam will fail immediately and without warning upon formation of the first flexural crack.

To ensure against this type of failure, a ***lower limit*** can be established for the reinforcement.
According to ACI Code 10.5, at any section where tensile reinforcement is required by analysis, the area A_s provided must not be less than

$$A_{s,min} = \rho_{min}\, b_w d$$

$$\rho_{min} = 0.25\ \sqrt{f'_c} / f_y \geq 1.4 / f_y$$

f. Examples of Rectangular Beams

Example 5 (*Analysis problem*)
A rectangular beam has width **300 mm** and effective depth **440 mm**. it is reinforced with **4 No.29** (#9) bars in one row. If $f_y = 420$ **MPa** and $f'_c = 28$ **MPa**, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to ACI Code?
Solution:
Area of 4 No.29 bars = 4 × 645 = 2580 mm^2 (Table A2, Nilson)
$a = A_s f_y / 0.85 f'_c\, b$ = 2580×420/(0.85×28×300) = 151.8 mm
$c = a / \beta_1$ = 151.8 / 0.85 = 178.5 mm
c / d_t = 178.5 / 440 = 0.406 (between 0.429 and 0.375)
i.e. (ε_t between 0.004 and 0.005)
Thus, the beam is under-reinforced
or, $\rho = A_s/bd$ = 2580 / (300 × 440) = 0.0195 which just exceeds
$\rho_{0.005} = 0.85\, \beta_1 (f'_c / f_y)[\varepsilon_u /(\varepsilon_u + 0.005)]$
= 0.85×0.85 (28/420)[0.003 /(0.003 + 0.005)] = 0.0181
Since $\varepsilon_t = \varepsilon_u(d - c)/c$ = 0.003(440 – 178.5)/178.5 = 0.00439
Using interpolation $\varphi = 0.85$
$\varphi M_n = \varphi A_s f_y (d - a/2)$= 0.85 ×2580×420(440 – 151.8/2)
= 0.85 × 394.5×10^6 Nmm = 335.3 kNm
Check $\rho_{max} = \rho_{0.004}$ = 0.0206, and $\rho_{min} = 0.25\sqrt{28} / 420 \geq 1.4 / 420$ = 0.0033. Thus $\rho_{min} < \rho = 0.0195 < \rho_{max}$ is satisfactory.

Example 6 (*Design problem*)
Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of **4.5 m** that is to carry a computed dead load of **20 kN/m** and a service live load of **31 kN/m**. Material strengths are $f'_c = 28$ **MPa** and $f_y = 420$ **MPa**.

Solution:

Factored load $w_u = 1.2\ D + 1.6\ L$

$= 1.2 \times 20 + 1.6 \times 31 = 73.6$ kN/m

$M_u = w_u l^2 / 8 = 73.6 \times 4.5^2 / 8 = 186.3$ kNm

To minimize section dimensions, it is desirable to select the maximum permissible reinforcement ratio:

$\rho_{0.005} = 0.85\ \beta_1 (f'_c / f_y)[\varepsilon_u /(\varepsilon_u + 0.005)]$

$= 0.85 \times 0.85\ (28/420)[0.003 /(0.003 + 0.005)] = 0.0181$

$\varphi = 0.9$ ($\varepsilon_t = 0.005$)

$M_u = \varphi\ M_n$

$186.3 \times 10^6 = 0.9 \times 0.0181 \times 420\ bd^2\ (1 - 0.59 \times 0.0181 \times 420/28)$

$bd^2 = 32{,}420{,}000$ mm^2

Say b = 250 mm, d = 360 mm, then

A_s, *required* $= 0.0181 \times 250 \times 360 = 1630$ mm^2

USE 2 No.32 (1638 mm^2)

Total depth of section $h = d_t + d_b/2 + d_b$ (*stirrup*) **+ concrete cover**

$= 360 + 16 + 12 + 40 = 428$ mm

Round – up to the nearest 25 mm: h = 450 mm.

Note:

1. *The effective depth will be increased:* $d = 450 - 40 - 12 - 16 = 382$ *mm. Improved economy may be possible by refining the steel area based on the actual, larger* **d***.*
2. *Infinite number of solutions is possible depending upon the reinforcement ratio selected.*

Example 7: (*Design problem: section dimensions are given, A_s is required*)

Find the steel area required to resist a moment M_u of **150 kNm** using a concrete section having b = **250 mm**, d = **435 mm**, and h = **500 mm**. f'_c = **28 MPa** and f_y = **420 MPa.**

Solution:

Assume $a = 100$ mm

$\varphi M_n = \varphi A_s f_y (d - a/2)$

$A_s = \varphi M_n / \varphi f_y (d - a/2) = 150 \times 10^6 / 0.9 \times 420\ (435 - 100/2) = 1031$ mm^2

Check $a = A_s f_y / 0.85 f'_c\ b = 1031 \times 420 / 0.85 \times 28 \times 250 = 72.8$ mm

Next assume $a = 70$ mm and recalculate A_s:

$A_s = 150 \times 10^6 / 0.9 \times 420\ (435 - 70/2) = 992$ mm^2

No further iteration is required. ***USE $A_s = 992$ mm^2 (2 No.25 bars A_s = 1080 mm^2)***

Check $\rho = 0.0091 < \rho_{0.005}$***, then*** $\varphi = 0.9$ ***OK***