Tishk International University
Engineering Faculty
Petroleum and Mining Engineering Department

Petroleum Reservoir Engineering II

Lecture 6: Fundamentals of Reservoir Fluid Flow (II)

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Reservoir Characteristics

- Reservoir Fluid Types according to Compressibility:

Homework (Part I): Explain the reason why the minus sign (-) is present in equation (1) and disappeared in equation (2)

Reservoir Characteristics

- Reservoir Fluid Types according to Compressibility:
- Slightly Compressible Fluids:

Homework (Part II): Derive the equation of isothermal compressibility coefficient for density and pressure relationship (equation 2)

Reservoir Characteristics

- Types of Flow Regimes
  - Steady-state Flow
  - Unsteady-state Flow
  - Pseudosteady-state Flow

Reservoir Characteristics

- Reservoir Fluid Types according to Compressibility
  - Incompressible Fluids
  - Slightly Compressible Fluids
  - Compressible Fluids

Reservoir Characteristics

- Types of Reservoir Geometries
  - Radial Flow
  - Linear Flow
  - Spherical and Hemispherical Flow

Reservoir Characteristics

- Number of flowing fluids in the reservoir
  - Single-phase flow (oil, water, or gas)
  - Two-phase flow (oil-water, oil-gas, or gas-water)
  - Three phase flow (oil, water, and gas)

Content:

- Fluid Flow Equations
- Darcy’s Law
- Darcy’s Law Assumptions
- Derivation of Darcy’s Law for steady-state:
- **Linear Flow of Incompressible Fluids**
- Linear Flow of Slightly Compressible Fluids
- Linear Flow of Compressible Fluids
- **Radial Flow of Incompressible Fluids**
- Radial Flow of Slightly Compressible Fluids
- Radial Flow of Compressible Fluids

Fluid Flow Equations

- The fluid flow equations that are used to describe the flow behavior in a reservoir can take many forms depending upon the combination of variables presented previously (i.e., types of flow, types of fluid, etc.).
- By combining the conservation of mass equation with the transport equation (Darcy’s equation) and various equation-of-state, the necessary flow equations can be developed.
- Since all flow equations to be considered depend on Darcy’s Law, it is important to consider this transport relationship.

Darcy’s Law

- The fundamental law of fluid motion in porous media is Darcy’s Law.
- The mathematical expression developed by Henry Darcy in 1856 states that velocity of a homogeneous fluid in a porous medium is proportional to the pressure gradient and inversely proportional to the fluid viscosity.
- For a horizontal linear system, this relationship is:

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx} --- (1)$$

Darcy's Law

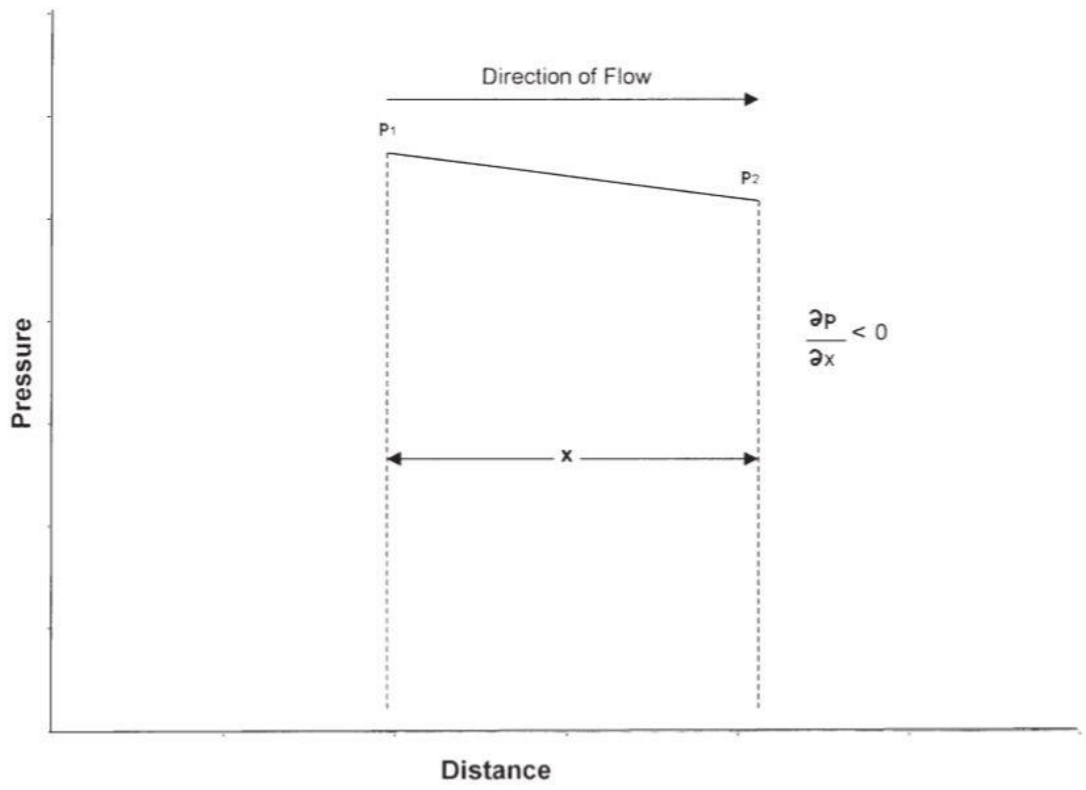
$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx} --- (1)$$

- v is the apparent velocity in centimeters per second and is equal to $q/_A$, where q is the volumetric flow rate in cubic centimeters per second and A is total cross-sectional area of the rock in square centimeters. In other words, A includes the area of the rock material as well as the area of the pore channels.
- The fluid viscosity, μ, is expressed in centipoise units, and the pressure gradient, $dp/_{dx}$, is in atmospheres per centimeter, taken in the same direction as v and q.
- The proportionality constant, k, is the permeability of the rock expressed in Darcy units.

Darcy's Law

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx} - - -(1)$$

- The negative sign in equation (1) is added because the pressure gradient is negative in the direction of flow as shown in the figure.



Pressure vs. distance in a linear flow.

Darcy's Law

- For a horizontal-radial system, the pressure gradient is positive as shown in the figure and Darcy's equation can be expressed in the following generalized radial form:

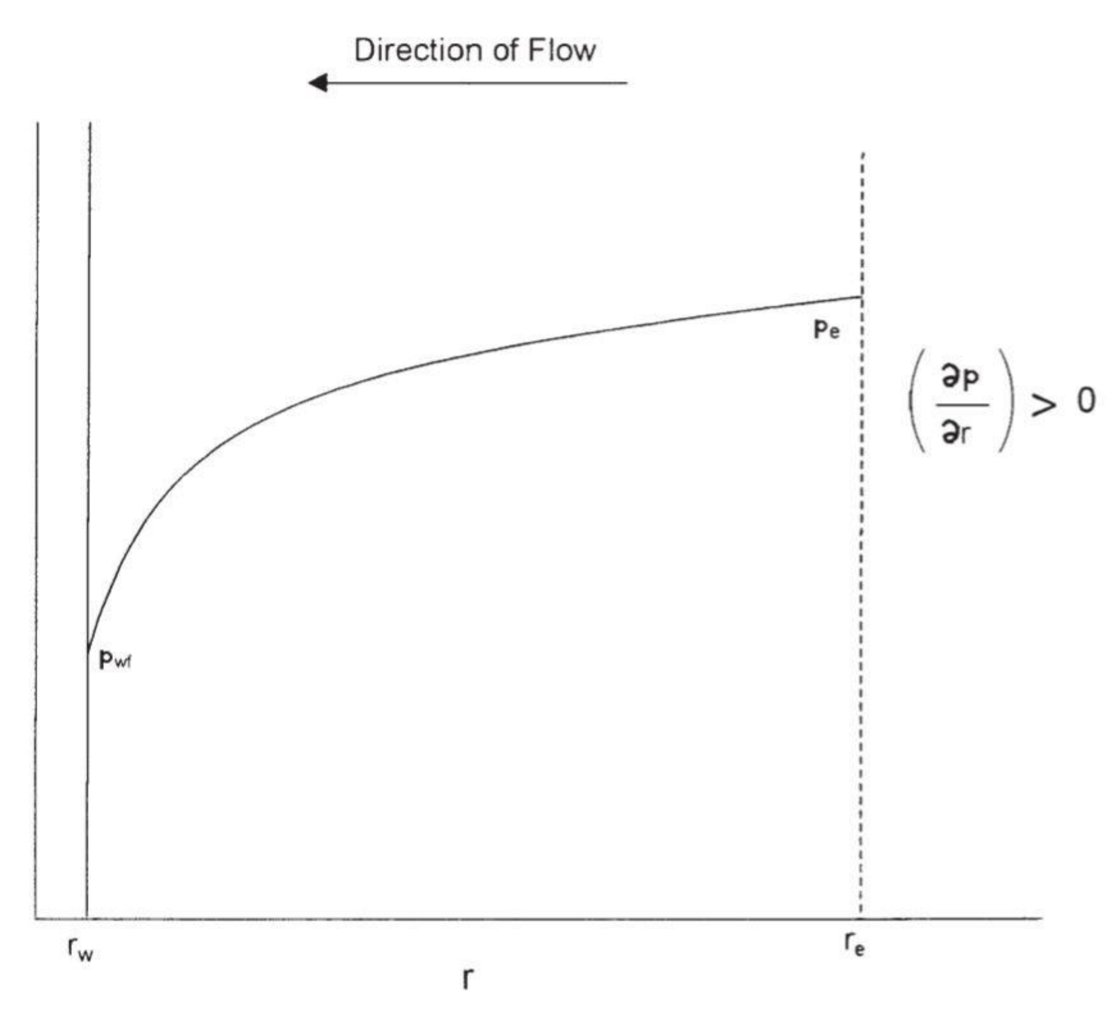
$$v = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r \;---(2)$$

Where q_r= volumetric flow rate at radius r

A_r= cross-sectional area to flow at radius r

$\left(\partial p/\partial r\right)_r$= pressure gradient at radius r

v= apparent velocity at radius r



Pressure gradient in radial flow.

Darcy’s Law

- The cross-sectional area at radius r is essentially the surface area of a cylinder. For a fully penetrated well with a net thickness of h, the cross-sectional area A_r is given by:

$$A_r = 2\pi r h \text{ --- } (3)$$

Darcy’s Law Assumptions

- Darcy’s Law applies only when the following conditions exist:

- Laminar (viscous) flow
- Steady-state flow
- Incompressible fluids
- Homogeneous formation

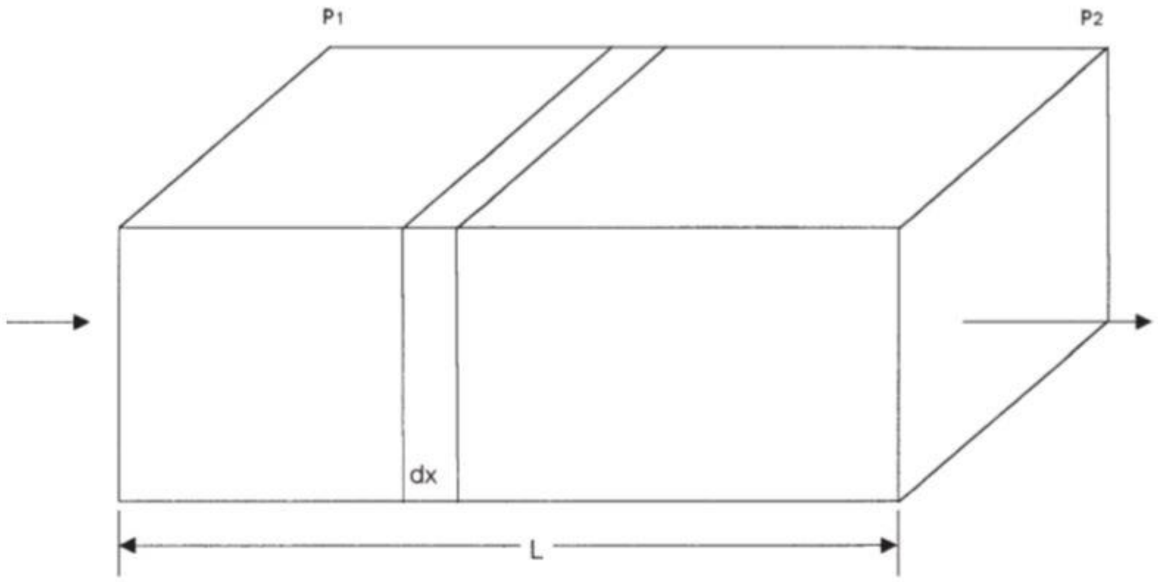
Steady-state Flow

- As defined previously, steady-state flow represents the condition that exists when the pressure throughout the reservoir does not change with time.
- The application of the steady-state flow to describe the flow behavior of several types of fluid in different reservoir geometries are presented in this lecture:

- Linear flow of incompressible fluids
- Linear flow of slightly compressible fluids
- Linear flow of compressible fluids

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**
  - In the linear system, it is assumed the flow occurs through a constant cross-sectional area A, where both ends are entirely open to flow. It is also assumed that no flow crosses the sides, top, or bottom as show in the figure.



Linear flow model.

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

- If an incompressible fluid is flowing across the element dx, then the fluid velocity v and the flow rate q are constants at all points.

- The flow behavior in this system can be expressed by the differential form of Darcy’s equation (1):

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx} --- (1)$$

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx} - - - (1)$$

- Separating the variables of equation (1) and integrating over the length of the linear system gives:

$$\frac{q}{A}\int_{0}^{L} dx = -\frac{k}{\mu}\int_{p1}^{p2} dp$$

Or:

$$q = \frac{kA\,(p1 - p2)}{\mu L}$$

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

- It is desirable to express the above relationship in customary field units, or:

$$q = \frac{0.001127\, kA\,(p1 - p2)}{\mu L} --- (2)$$

Where q= flow rate, bbl/day

k= absolute permeability, md

p= pressure, psia

μ= viscosity, cp

L= distance, ft

A= cross-sectional area, ft^2

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

Example 1: An incompressible fluid flows in a linear porous media with the following properties:

L= 2000 ft h=20′ width=300′

k= 100 md ϕ= 15% **μ**= 2cp

p1= 2000 psi p2=1990 psi

Calculate:

a. Flow rate in bbl/day

b. Apparent fluid velocity in ft/day

c. Actual fluid velocity in ft/day

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

Solution:

a. Flow rate in bbl/day:

$$q = \frac{0.001127\, kA\,(p1 - p2)}{\mu L}$$

$$A = h * w$$

$$A = 20 * 300 = 6000\, ft$$

$$q = \frac{0.001127 * 100 * 6000\ *\ (2000 - 1990)}{2 * 2000}$$

$$q = \frac{6{,}762}{4000} = 1.6905\, bbl/day$$

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

Solution:

b. Apparent fluid velocity in ft/day:

$$v = \frac{q}{A}$$

$$v = \frac{(1.6905)(5.615)}{6000} = 0.0016\ ft/day$$

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

Solution:

c. Actual fluid velocity in ft/day:

$$v = \frac{q}{\varphi\ A} = \frac{(1.6905)(5.615)}{(0.15)(6000)}$$

$$v = 0.0105\ ft/day$$

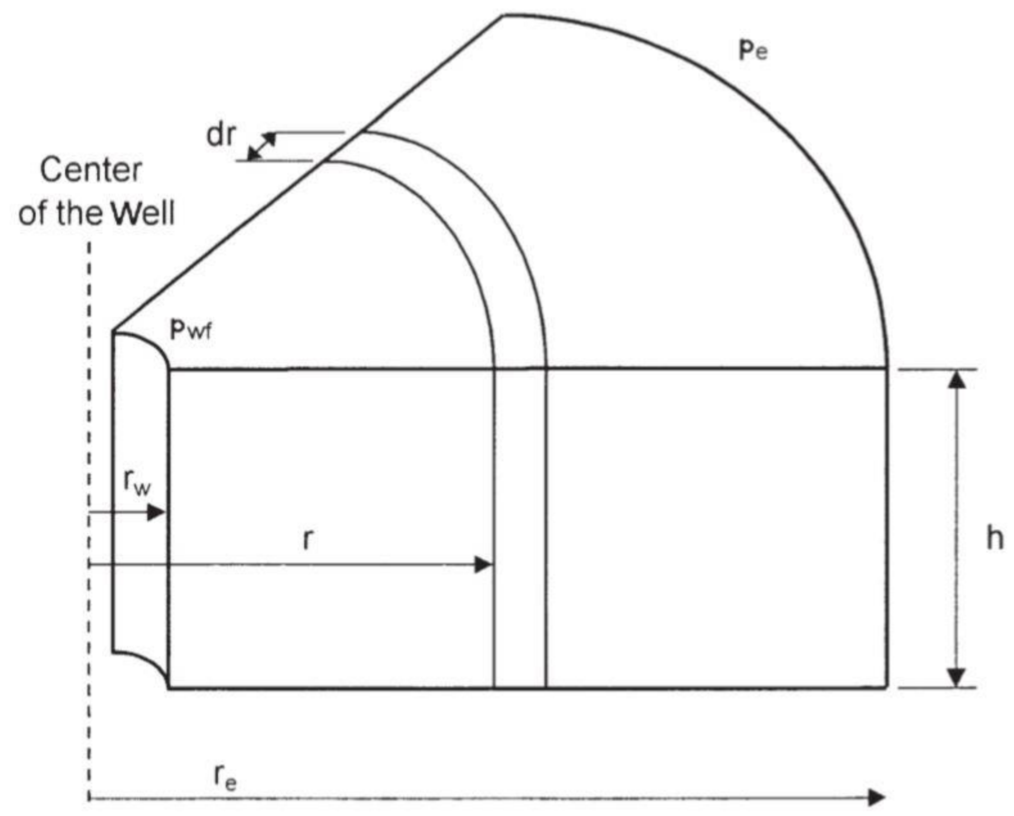
Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- In a radial flow system, all fluids move toward the producing well from all directions.
- Before flow can take place, however, a pressure differential must exist.
- Thus, if a well is to produce oil, which implies a flow of fluids through the formation to the wellbore, the pressure in the formation at the wellbore must be less than the pressure in the formation at some distance from the well.
- The pressure in the formation at the wellbore of a producing well is known as the bottom-hole flowing pressure (flowing BHP, p_{wf}).

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**
- Consider the following figure, which schematically illustrates the radial flow of an incompressible fluid toward a vertical well.
- The formation is considered to a uniform thickness h and a constant permeability k.
- Because the fluid is incompressible, the flow rate q must be constant at all radii.
- Due to the steady-state flowing condition, the pressure profile around the wellbore is maintained constant with time.



Radial flow model.

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- Let p_{wf} represent the maintained bottom-hole flowing pressure at the wellbore radius r_w and p_e denote the external pressure at the external or drainage radius.

- Darcy's equation as described by following equation can be used to determine the flow rate at any radius r:

$$v = \frac{q}{A_r} = 0.001127\frac{k}{\mu}\frac{dp}{dr} --- (1)$$

Where v= apparent fluid velocity, $bbl/day\text{-}ft^2$

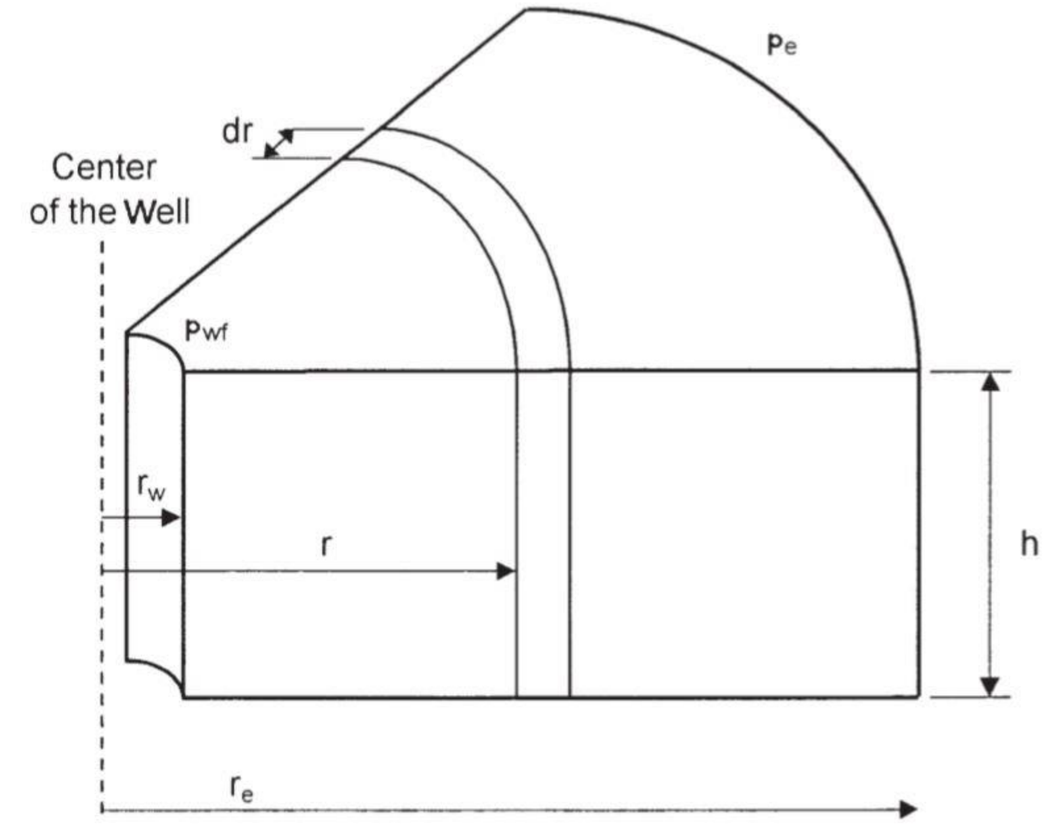
q= flow rate at radius r, bbl/day

k= permeability, md

μ= viscosity, cp

0.001127= conversion factor to express the equation in field units

A_r= cross-sectional area at radius r



Radial flow model.

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- At any point in the reservoir the cross-sectional area across which flow occurs will be the surface area of a cylinder, which is $2\pi rh$, or:

$$v = \frac{q}{A_r} = \frac{q}{2\pi rh} = 0.001127\frac{k}{\mu}\frac{dp}{dr}$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- The flow rate for a crude oil system is customarily expressed in surface units, i.e., stock-tank barrels (STB), rather than reservoir units.

- Using the symbol Q_o to represent the oil flow as expressed in STB/day, then:

$$q = B_o \, Q_o$$

Where B_o is the oil formation volume factor bbl/STB.

- The flow rate in Darcy's equation can be expressed in STB/day to give:

$$\frac{Q_o \, B_o}{2\pi r h} = 0.001127 \frac{k}{\mu_o} \frac{dp}{dr}$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- Integrating the above equation between two radii, r_1 and r_2, when the pressures are p_1 and p_2 yields:

$$\int_{r_1}^{r_2} \left(\frac{Q_o}{2\pi h}\right) \frac{dr}{r} = 0.001127 \int_{p_1}^{p_2} \left(\frac{k}{\mu_o\, B_o}\right) dp$$

- For an incompressible system in a uniform formation, above equation can be simplified to:

$$\left(\frac{Q_o}{2\pi h}\right) \int_{r_1}^{r_2} \frac{dr}{r} = 0.001127 \left(\frac{k}{\mu_o\, B_o}\right) \int_{p_1}^{p_2} dp$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- Performing the integration gives:

$$Q_o = \frac{0.00708\,k\,h\,(p_2 - p_1)}{\mu_o B_o \ln(r_2/r_1)}$$

- Frequently the two radii of interest are the wellbore radius r_w and the *external* or *drainage* radius r_e. Then:

$$Q_o = \frac{0.00708\,kh\,(p_e - p_w)}{\mu_o B_o \ln(r_e/r_w)}$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

$$Q_o = \frac{0.00708\ kh\ (p_e - p_w)}{\mu_o B_o \ln(r_e/r_w)} --- (2)$$

Where Q_o= oil flow rate, STB/day

p_e= external pressure, psi

p_{wf}= bottom-hole flowing pressure, psi

k= permeability, md

μ_o= oil viscosity, cp

B_o= oil formation volume factor, bbl/STB

h= thickness, ft

r_e= external or drainage radius, ft

r_w= wellbore radius, ft

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- The external (drainage) radius r_e is usually determined from the well spacing by equating the area of the well spacing wit that of a circle, i.e.,

$$\pi r_e^2 = 43{,}560\ A$$

Or

$$r_e = \sqrt{\frac{43{,}560\ A}{\pi}} --- (3)$$

Where A is the well spacing in acres.

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

- Equation (2) can be rearranged to solve for the pressure p at any radius r to give:

$$p = p_{wf} + \left[\frac{Q_o B_o \mu_o}{0.00708kh}\right] \ln\left(\frac{r}{rw}\right) --- (4)$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

Example 2: An oil well in the x Field is producing at a stabilized rate of 600 STB/day at a stabilized bottom-hole flowing pressure of 1,800 psi. Analysis of the pressure buildup test data indicates that the pay zone is characterized by a permeability of 120 md and a uniform thickness of 25 ft. The well drains an area of approximately 40 acres. The following additional data are available:

r_w= 0.25 ft A= 40 acres

B_o= 1.25 bbl/STB μ_o= 2.5 cp

Calculate the pressure profile (distribution) and list the pressure drop across 1 ft intervals from r_w to 1.25 ft, 4 to 5 ft, 19 to 20 ft, 99 to 100 ft, and 744 to 745 ft.

Steady-state Flow

- Radial Flow of Incompressible Fluids:

Solution:

Step 1:

$$p = p_{wf} + \left[\frac{Q_o B_o \mu_o}{0.00708kh}\right] \ln\left(\frac{r}{rw}\right)$$

$$p = 1800 + \left[\frac{(2.5)(1.25)(600)}{(0.00708)(120)(25)}\right] \ln\left(\frac{r}{0.25}\right)$$

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

Solution:

Step 2: Calculate the pressure at the designated radii:

r, ft	p, psi	Radius Interval	Pressure drop
0.25	1800		
1.25	1942	0.25–1.25	1942 – 1800 = 142 psi
4	2045		
5	2064	4–5	2064 – 2045 = 19 psi
19	2182		
20	2186	19–20	2186 – 2182 = 4 psi
99	2328		
100	2329	99–100	2329 – 2328 = 1 psi
744	2506.1		
745	2506.2	744–745	2506.2 – 2506.1 = 0.1 psi

Homework