Tishk International University
Engineering Faculty
Petroleum and Mining Engineering Department

Petroleum Reservoir Engineering II

Lecture 6: Fundamentals of Reservoir Fluid Flow (III)

Third Grade- Spring Semester 2021-2022

Instructor: Nabaz Ali Abdulrahman

Content:

- Derivation of Darcy's Law for steady-state:
- Linear Flow of Incompressible Fluids
- Linear Flow of Slightly Compressible Fluids
- Linear Flow of Compressible Fluids
- Radial Flow of Incompressible Fluids
- Radial Flow of Slightly Compressible Fluids
- Radial Flow of Compressible Fluids

Steady-state Flow

- **Linear Flow of Incompressible Fluids:**

$$q = \frac{0.001127\ kA\ (p1 - p2)}{\mu L} ---(1)$$

Where q= flow rate, bbl/day

k= absolute permeability, md

p= pressure, psia

μ= viscosity, cp

L= distance, ft

A= cross-sectional area, ft^2

Steady-state Flow

- **Radial Flow of Incompressible Fluids:**

$$Q_o = \frac{0.00708\ kh\ (p_e - p_w)}{\mu_o B_o \ln(r_e/r_w)} --- (2)$$

Where Q_o= oil flow rate, STB/day

p_e= external pressure, psi

p_{wf}= bottom-hole flowing pressure, psi

k= permeability, md

μ_o= oil viscosity, cp

B_o= oil formation volume factor, bbl/STB

h= thickness, ft

r_e= external or drainage radius, ft

r_w= wellbore radius, ft

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

▪ The relationship that exists between pressure and volume for slightly compressible fluids has been discussed in the previous lecture and is described by following equation:

$$V = V_{ref}\left[1 + c\left(p_{ref} - p\right)\right]$$

The above equation can be modified and written in terms of flow rate as:

$$q = q_{ref}\left[1 + c\left(p_{ref} - p\right)\right] ---(3)$$

Where q_{ref} is the flow rate at some reference pressure p_{ref}.

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

$$q = q_{ref}\left[1 + c\left(p_{ref} - p\right)\right]$$

- Substituting the above relationship in Darcy's equation gives:

$$\frac{q}{A} = \frac{q_{ref}\left[1 + c\left(p_{ref} - p\right)\right]}{A} = -0.001127\frac{k}{\mu}\frac{dp}{dx}$$

- Separating the variables and arranging:

$$\frac{q_{ref}}{A}\int_{0}^{L} dx = -0.001127\frac{k}{\mu}\int_{p1}^{p2}\left[\frac{dp}{1 + c\left(p_{ref} - p\right)}\right]$$

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

▪ Integrating gives:

$$q_{ref} = \left[\frac{0.001127\ kA}{\mu c L}\right] ln\left[\frac{1 + c(p_{ref} - p_2)}{1 + c(p_{ref} - p_1)}\right] --- (4)$$

Where q_{ref}= flow rate at a reference pressure p_{ref}, bbl/day

p_1= upstream pressure, psi

p_2= downstream pressure, psi

μ= viscosity, cp

c= average liquid compressibility, psi^{-1}

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

- Selecting the upstream pressure p_1 as the reference pressure p_{ref} and substituting in equation (4) gives the flow rate at Point 1 as:

$$q_1 = \left[\frac{0.001127\ kA}{\mu c L}\right] ln[1 + c(p_1 - p_2)] --- (5)$$

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

▪ Choosing the downstream pressure p_2 as the reference pressure and substituting in equation (4) gives:

$$q_2 = \left[\frac{0.001127\ kA}{\mu c L}\right] ln\left[\frac{1}{1 + c(p_2 - p_1)}\right] --- (6)$$

Where q_1 and q_2 are the flow rates at Points 1 and 2, respectively.

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

Example 2: A slightly compressible liquid flows in a linear porous media with the following properties:

L= 2000 ft h=20′ width=300′

k= 100 md $\phi = 15\%$ $\mu = 2cp$

$p1 = 2000$ psi $p2 = 1990$ psi

Calculate the flow rate at both ends of the linear system. The liquid has an average compressibility of $21 * 10^{-5} psi^{-1}$.

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

Solution:

- Choosing the upstream pressure as the reference pressure gives:

$$q_1 = \left[\frac{(0.001127)(100)(6000)}{(2)(21 * 10^{-5})(2000)}\right] ln[1 + (21 * 10^{-5})(2000 - 1990)]$$

$$q_1 = 1.689 \; bbl/day$$

Steady-state Flow

- **Linear Flow of Slightly Compressible Fluids:**

Solution:

- Choosing the downstream pressure, gives:

$$q_2 = \left[\frac{(0.001127)(100)(6000)}{(2)(21 * 10^{-5})(2000)}\right] ln\left[\frac{1}{1 + (21 * 10^{-5})(1990 - 2000)}\right]$$

$$q_2 = 1.692\ bbl/day$$

- The above calculations show that q_1 and q_2 are not largely different, which is due to the fact that the liquid is slightly compressible, and its volume is not a strong function of pressure.

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

- For a viscous (laminar) gas flow in a homogeneous-linear system, the real-gas equation of state can be applied to calculate the number of gas moles n at pressure p, temperature T, and volume V:

$$n = \frac{pV}{zRT} --- (7)$$

- At standard conditions, the volume occupied by the above n moles is given by:

$$V_{sc} = \frac{n z_{sc} R T_{sc}}{p_{sc}} --- (8)$$

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

- Combining the above two expressions and assuming $z_{sc} = 1$ gives:

$$\frac{pV}{zT} = \frac{p_{sc}V_{sc}}{T_{sc}}$$

- Equivalently, the above relation can be expressed in terms of the flow rate as:

$$\frac{5.615pq}{zT} = \frac{p_{sc}Q_{sc}}{T_{sc}}$$

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

▪ Rearranging the previous equation:

$$\left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{zT}{p}\right)\left(\frac{Q_{sc}}{5.615}\right) = q \text{ ---- } (9)$$

Where q= gas flow rate at pressure p in bbl/day

Q_{sc}= gas flow rate at standard conditions, scf/day

z= gas compressibility factor

T_{sc}, p_{sc}= standard temperature and pressure in °R and psia, respectively.

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

- Replacing the gas flow rate q with that Darcy's Law, i.e., equation (1) gives:

$$\frac{q}{A} = \left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{zT}{p}\right)\left(\frac{Q_{sc}}{5.615}\right)\left(\frac{1}{A}\right) = -0.001127\frac{k}{\mu}\frac{dp}{dx}$$

- The constant 0.001127 is to convert from Darcy's units to field units.

- Separating variables and arranging yields:

$$\left[\frac{q_{sc}p_{sc}T}{0.006328\, kT_{sc}A}\right]\int_0^L dx = -\int_{p1}^{p2} \frac{p}{z\mu_g}dp$$

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

- Assuming constant z and μ_g over the specified pressure, i.e., p_1 and p_2, and integrating gives:

$$Q_{sc} = \frac{0.003164 T_{sc} A\, k\, (p_1^2 - p_2^2)}{p_{sc} T L z \mu_g}$$

Where Q_{sc}= gas flow rate at standard conditions, scf/day

k= permeability, md

T= temperature, °R

μ_g= gas viscosity, cp

A= cross-sectional area, ft^2

L= total length of the linear system, ft

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

▪ Setting p_{sc}= 14.7 psi and T_{sc}= 520 °R in the above expression gives:

$$Q_{sc} = \frac{0.111924Ak\ (p_1^2 - p_2^2)}{TLz\mu_g} ---- -(10)$$

▪ The above equation is valid for applications when the pressure<2,000 psi. The gas properties must be evaluated at the average pressure $\bar{p}$ as defined below:

$$\bar{p} = \sqrt{\frac{p_1^2+p_2^2}{2}} ---- -(11)$$

Steady-state Flow

- **Linear Flow of Compressible Fluids:**

Example: A linear porous media is flowing a 0.72 specific gravity gas at 120°F. The upstream and downstream pressures are 2,100 psi and 1,894.73 psi, respectively. The cross-sectional area is constant at 4,500 ft^2. The total length is 2,500 feet with an absolute permeability of 60md. Calculate the gas flow rate in scf/day ($p_{sc} = 14.7$ psia, $T_{sc} = 520\ °R$).

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

- The relation of pressure and flow rate has been determined previously by following equation:

$$q = q_{ref}[1 + c(p_{ref} - p)]$$

- If above equation is substituted into the radial form of Darcy's Law:

$$v = \frac{q}{A_r} = \frac{q}{2\pi rh} = 0.001127\frac{k}{\mu}\frac{dp}{dr}$$

- The following is obtained:

$$\frac{q}{A_r} = \frac{q_{ref}[1 + c(p_{ref} - p)]}{2\pi rh} = 0.001127\frac{k}{\mu}\frac{dp}{dr}$$

Where q_{ref} is the flow rate at some reference pressure p_{ref}.

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

- Separating the variables in the above equation and integrating over the length of the porous medium gives:

$$\frac{q_{ref}\mu}{2\pi kh}\int_{r_w}^{r_e}\frac{dr}{r} = 0.001127\int_{p_{wf}}^{p_e}\frac{dp}{1+c(p_{ref}-p)}$$

Or

$$q_{ref} = \left[\frac{0.00708\,kh}{\mu\,c\ln\left(\frac{r_e}{r_w}\right)}\right] ln\left[\frac{1+c(p_e-p_{ref}}{1+c(p_{wf}-p_{ref})}\right]$$

Where q_{ref} is oil flow rate at a reference pressure p_{ref}.

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

- Choosing the bottom-hole flow pressure p_{wf} as the reference pressure and expressing the flow rate in STB/day gives:

$$Q_o = \left[\frac{0.00708\,kh}{\mu_o B_o c_o\,ln\left(\frac{r_e}{r_w}\right)}\right]\ln\left[1 + c_o(p_e - p_{wf})\right] --- (5)$$

Where c_o= isothermal compressibility coefficient, psi^{-1}

Q_o= oil flow rate, STB/day

k= permeability, md

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

Example: The following data are available on a well in the Red River Field:

p_e= 2506 psi p_{wf}= 1800

r_e= 745′ r_w= 0.25

B_o= 1.25 μ_o= 2.5 $c = 25 * 10^{-6}\ psi^{-1}$

k= 0.12 Darcy h= 25 ft.

Assuming a slightly compressible fluid, calculate the oil flow rate. Compare the result with that of incompressible fluid.

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

Solution:

For a slightly compressible fluid, the oil flow rate can be calculated by applying equation (5):

$$Q_o = \left[\frac{(0.00708)(120)(25)}{(2.5)(1.25)(25 * 10^{-6})\ln(745/0.25)}\right] * \ln[1 + (25 * 10^{-6})(2506 - 1800)]$$

$$Q_o = 595\, STB/day$$

Steady-state Flow

- **Radial Flow of Slightly Compressible Fluids:**

Solution:

Assuming an incompressible fluid, the flow rate can be estimated by applying Darcy's equation, i.e., equation (2):

$$Q_o = \frac{(0.00708)(120)(25)(2506 - 1800)}{(2.5)(1.25)\ln(745/0.25)}$$

$$Q_o = 600\ STB/day$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- The basic differential form of Darcy's Law for a horizontal laminar flow is valid for describing the flow of both gas and liquid systems.

- For a radial gas flow, the Darcy's equation takes the form:

$$q_{gr} = \frac{0.001127(2\pi rh)k}{\mu_g}\frac{dp}{dr} --- (6)$$

where q_{gr}= gas flow rate at radius r, bbl/day

r= radial distance, ft

h= zone thickness, ft

μ_g= gas viscosity, cp

p= pressure, psi

0.001127= conversion constant from Darcy units to field units.

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- The gas flow rate is usually expressed in scf/day. Referring to the gas flow rate at standard condition as Q_g, the gas flow rate q_{gr} under pressure and temperature can be converted to that of standard condition by applying the real gas equation-of-state to both conditions, or:

$$\frac{5.615\, q_{gr}\, p}{zTR} = \frac{Q_g\, p_{sc}}{z_{sc} R\, T_{sc}}$$

Or

$$\left(\frac{p_{sc}}{5.615\, T_{sc}}\right)\left(\frac{zT}{p}\right) Q_g = q_{gr} \quad --- (7)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

$$\left(\frac{p_{sc}}{5.615\, T_{sc}}\right)\left(\frac{zT}{p}\right) Q_g = q_{gr}$$

Where p_{sc}= standard pressure, psia

T_{sc}= standard temperature, °R

Q_g= gas flow rate, scf/day

q_{gr}= gas flow rate at radius r, bbl/day

p= pressure at radius r, psia

T= reservoir temperature, °R

z= gas compressibility factor at p and T

z_{sc}= gas compressibility factor at standard condition ≅ 1.0

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- Combining equations (6) and (7) yields:

$$\left(\frac{p_{sc}}{5.615\ T_{sc}}\right)\left(\frac{zT}{p}\right) Q_g = \frac{0.001127(2\pi rh)k}{\mu_g}\frac{dp}{dr}$$

- Assuming that T_{sc}= 520 °R and p_{sc}=14.7 psia:

$$\left(\frac{T\ Q_g}{k\ h}\right)\frac{dr}{r} = 0.703\left(\frac{2p}{\mu_g z}\right) dp \text{ --- } (8)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- Integrating equation (8) form the wellbore conditions (r_w and p_{wf}) to any point in the reservoir (r and p) gives:

$$\int_{r_w}^{r} \left(\frac{T\,Q_g}{k\,h}\right)\frac{dr}{r} = 0.703 \int_{p_{wf}}^{p} \left(\frac{2p}{\mu_g z}\right) dp --- (9)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- Imposing Darcy's Law conditions on equation (9), i.e.:

- Steady-state flow, which requires that Q_g is constant at all radii

- Homogeneous formation, which implies that k and h are constant

gives:

$$\left(\frac{TQ_g}{k\,h}\right)\ln\left(\frac{r}{r_w}\right) = 0.703 \int_{p_{wf}}^{p} \left(\frac{2p}{\mu_g z}\right) dp$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- The term $\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g z}\right)dp$ can be expanded to give:

$$\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g z}\right)dp = \int_{0}^{p}\left(\frac{2p}{\mu_g z}\right)dp - \int_{0}^{p_{wf}}\left(\frac{2p}{\mu_g z}\right)dp$$

- Combining the above relationships yields:

$$\left(\frac{TQ_g}{k\,h}\right)\ln\left(\frac{r}{r_w}\right) = 0.703\left[\int_{0}^{p}\left(\frac{2p}{\mu_g z}\right)dp - \int_{0}^{p_{wf}}\left(\frac{2p}{\mu_g z}\right)dp\right] --- (10)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- The integral $\int_0^p \left(\frac{2p}{\mu_g z}\right) dp$ is called the real gas potential or real gas pseudopressure, and it is usually represented by m(p) or ψ. Thus

$$m(p) = \psi = \int_0^p \left(\frac{2p}{\mu_g z}\right) dp \; ---(11)$$

- Equation (10) can be written in terms of the real gas potential to give:

$$\left(\frac{T Q_g}{k\ h}\right) \ln\left(\frac{r}{r_w}\right) = 0.703(\psi - \psi_w)$$

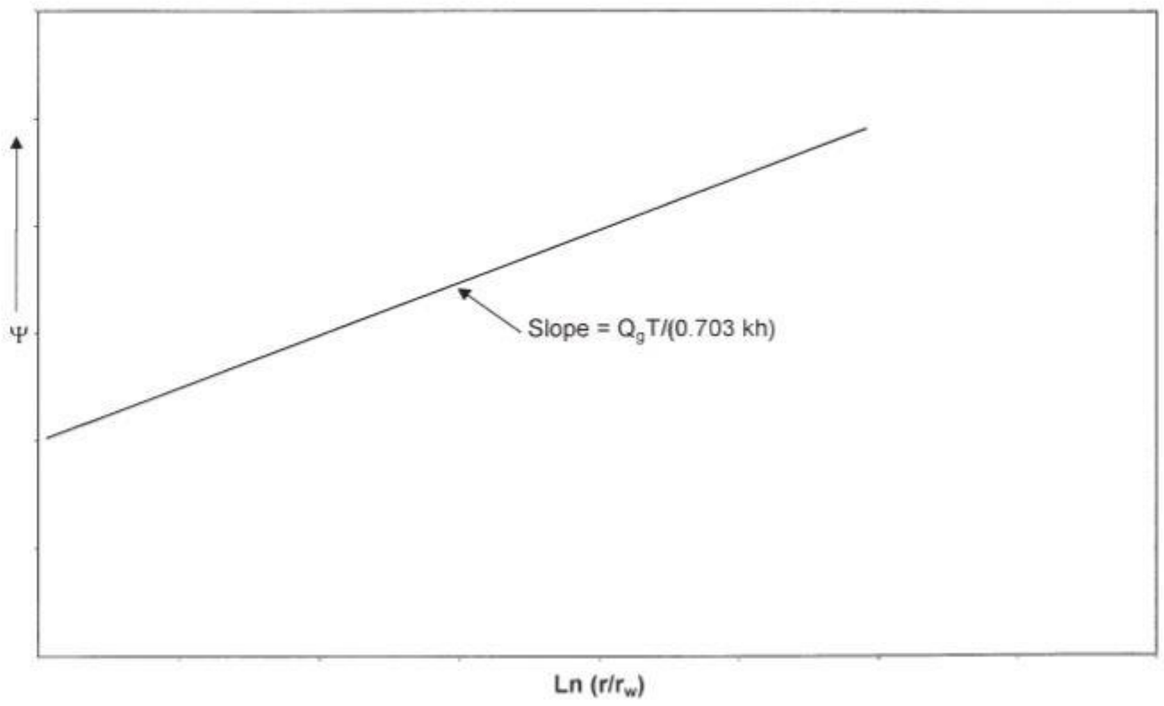
Or

$$\psi = \psi_w + \frac{Q_g T}{0.703\ kh} ln\frac{r}{r_w} \; ---(12)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

▪ Equation (12) indicates that a graph of ψ vs. $ln\, r/r_w$ yields a straight line of slope $(Q_g T/0.703\ kh)$ and intercept ψ_w as shown in the figure:



Graph of Ψ vs. ln (r/r_w).

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

- The flow rate is given exactly by:

$$Q_g = \frac{0.703\ kh(\psi - \psi_w)}{T\ ln\frac{r}{r_w}} --- (13)$$

- In the particular case when $r = r_e$, then:

$$Q_g = \frac{0.703\ kh(\psi_e - \psi_w)}{T\ ln\frac{r_e}{r_w}} --- (14)$$

Steady-state Flow

- **Radial Flow of Compressible Fluids:**

▪ The gas flow rate is commonly expressed in Mscf/day, or:

$$Q_g = \frac{kh(\psi_e - \psi_w)}{1422T \ ln\frac{r_e}{r_w}} - - -(15)$$

Where Q_g= gas flow rate, Mscf/day

ψ_e= real gas potential as evaluated from 0 to $p_e, psi^2/cp$

ψ_w= real gas potential as evaluated from 0 to $p_{wf}, psi^2/cp$

k= permeability, md

h= thickness, ft

r_e= drainage radius, ft

r_w= wellbore radius, ft