Tishk International University
Engineering Faculty
Petroleum and Mining Engineering Department

Petroleum Reservoir Engineering II

Lecture 8: Material Balance Equation

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Oil Recovery Mechanisms

- The recovery of oil by any of the natural drive mechanisms is called **primary recovery.**
- The term refers to the production of hydrocarbons from a reservoir without the use of any process (such as fluid injection) to supplement the natural energy of the reservoir.
- In the previous lecture, an introduction of various primary recovery mechanisms and their effects on the overall performance of oil reservoirs has been given.
- The objective of this lecture is to provide the basic principles of the material balance equation and other governing relationships that can be used to predict the volumetric performance of oil reservoirs.

Primary Recovery Mechanisms

- For a proper understanding of reservoir behavior and predicting future performance, it is necessary to have knowledge of the driving mechanisms that control the behavior of fluids within reservoirs.
- The overall performance of oil reservoirs is largely determined by the nature of the energy, i.e., driving mechanism, available for moving the oil to the wellbore.
- There are basically six driving mechanisms that provide the natural energy necessary for oil recovery:

- Rock and liquid expansion drive
- Depletion drive
- Gas-cap drive
- Water drive
- Gravity drainage drive
- Combination drive

Content:

- Material Balance Equation
- Material Balance Derivation:

1. Pore Volume Occupied by the Oil Initially in Place
2. Pore Volume Occupied by the Gas in the Gas Cap
3. Pore Volume Occupied by the Remaining Oil
4. Pore Volume Occupied by the Gas Cap at Reservoir Pressure
5. Pore Volume Occupied by the Evolved Solution Gas
6. Pore Volume Occupied by the Net Water Influx
7. Change in Pore Volume due to Initial Water and Rock Expansion
8. Pore Volume Occupied by the Injection Gas and Water

- Example of Material Balance Equation Application

The Material Balance Equation

- The material balance equation (MBE) has long been recognized as one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance.

- The MBE can be used to:

- Estimate initial hydrocarbon volumes in place
- Predict future reservoir performance
- Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms.

The Material Balance Equation

- The equation is structured to simply keep inventory of all materials entering, leaving, and accumulating in the reservoir.
- In its simplest form, the equation can be written on a volumetric basis as:

Initial volume= Volume remaining + Volume removed

- Since oil, gas, and water are present in petroleum reservoirs, the material balance equation can be expressed for the total fluids or for any one of the fluids present.

The Material Balance Equation

- Before deriving the material balance, it is convenient to denote certain terms by symbols for brevity.

p_i	Initial reservoir pressure, psi
p	Volumetric average reservoir pressure
Δp	Change in reservoir pressure $= p_i - p$, psi
p_b	Bubble point pressure, psi
N	Initial (original) oil-in-place, STB
N_p	Cumulative oil produced, STB
G_p	Cumulative gas produced, scf
W_p	Cumulative water produced, bbl
R_p	Cumulative gas-oil ratio, scf/STB
GOR	Instantaneous gas-oil ratio, scf/STB
R_{si}	Initial gas solubility, scf/STB
R_s	Gas solubility, scf/STB
B_{oi}	Initial oil formation volume factor, bbl/STB
B_o	Oil formation volume factor, bbl/STB
B_{gi}	Initial gas formation volume factor, bbl/scf
B_g	Gas formation volume factor, bbl/scf
W_{inj}	Cumulative water injected, STB
G_{inj}	Cumulative gas injected, scf
W_e	Cumulative water influx, bbl
m	Ratio of initial gas-cap gas reservoir volume to initial reservoir oil volume, bbl/bbl
G	Initial gas-cap gas, scf
P.V	Pore volume, bbl
c_w	Water compressibility, psi^{-1}
c_f	Formation (rock) compressibility, psi^{-1}

The Material Balance Equation Derivation

- Several of the material balance calculations require the total pore volume (P.V) as expressed in terms of the initial oil volume N and the volume of the gas cap.
- The expression for the total pore volume can be derived by conveniently introducing the parameter m into the relationship as follows.
- Defining the ratio m as:

$$m = \frac{Initial\ volume\ of\ gas\ cap}{Volume\ of\ oil\ initially\ in\ place} = \frac{G\ B_{gi}}{N\ B_{oi}}$$

The Material Balance Equation Derivation

- Solving for the volume of the gas cap gives:

$$Initial\ volume\ of\ the\ gas\ cap = G\ B_{gi} = m\ N\ B_{oi}$$

- The total volume of the hydrocarbon system is then given by:

$$Initial\ oil\ volume + Initial\ gas\ cap\ volume = (P.V)(1 - S_{wi})$$

$$N\ B_{oi} + m\ N\ B_{oi} = (P.V)(1 - S_{wi})$$

Or

$$P.V = \frac{N\ B_{oi}(1+m)}{(1-S_{wi})} - - -(1)$$

The Material Balance Equation Derivation

$$P.V = \frac{N\,B_{oi}(1+m)}{(1-S_{wi})} ---(1)$$

Where S_{wi}= initial water saturation

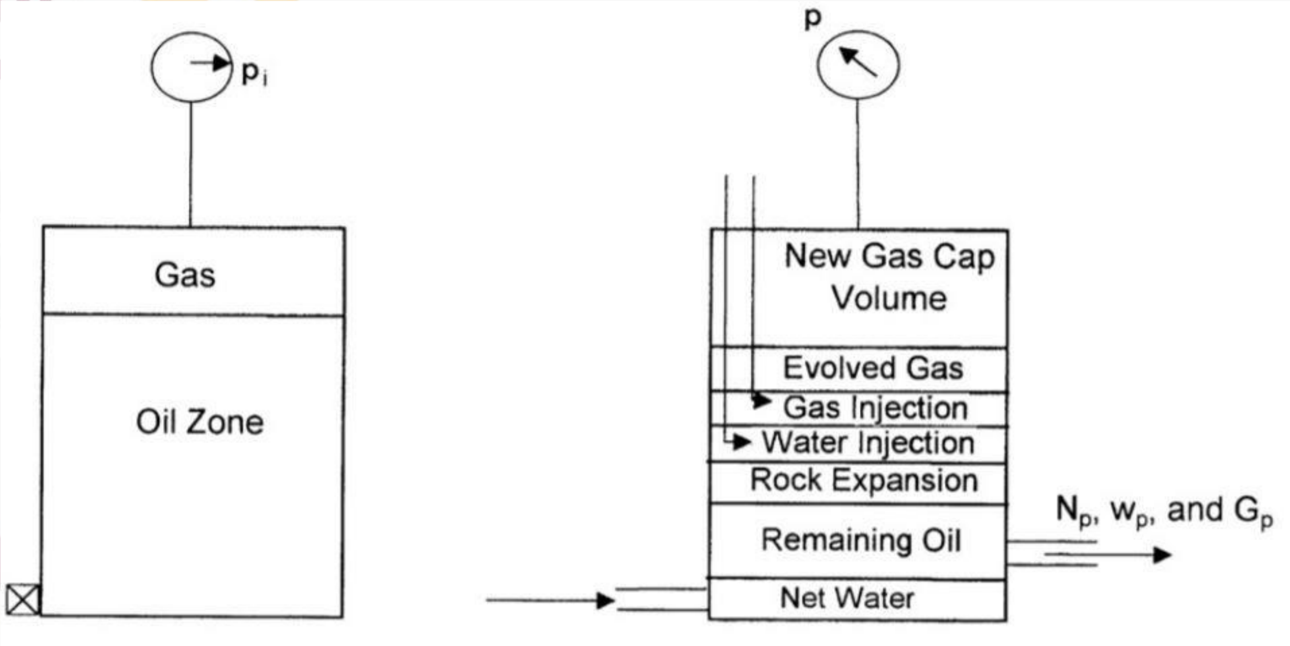
N= initial oil-in-place, STB

$P.V$= total pore volume, bbl

m= ratio of initial gas-cap gas reservoir volume to initial reservoir oil volume, bbl/bbl

The Material Balance Equation Derivation

- Treating the reservoir pore as an idealized container as illustrated below, volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.



Tank-model concept.

The Material Balance Equation Derivation

- The MBE can be written in a generalized form as follows:

Pore volume occupied by the oil initially in place at p_i + Pore volume occupied by the gas in the gas cap at p_i=

Pore volume occupied by the remaining oil at p + Pore volume occupied by the gas in the gas cap at p + Pore volume occupied by the evolved solution gas at p + Pore volume occupied by the net water influx at p + Change in pore volume due to connate-water expansion and pore volume reduction due to rock expansion + Pore volume occupied by the injected gas at p + Pore volume occupied by the injected water at p _ _ _ _ (2)

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

1. **Pore Volume Occupied by the Oil Initially in Place:**

$$Volume\ occupied\ by\ initial\ oil - in - place = N\ B_{oi} --- (3)$$

Where N= oil initially in place, STB

B_{oi}= oil formation volume factor at initial reservoir pressure p_i, bbl/STB

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

2. Pore Volume Occupied by the Gas in the Gas Cap:

$$Volume\ of\ gas\ cap = m\ N\ B_{oi} ---(4)$$

Where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

3. Pore Volume Occupied by the Remaining Oil:

$$Volume\ of\ the\ remaining\ oil = (N - N_p)\ B_o \ --- (5)$$

Where N_p= cumulative oil production, STB

B_o= oil formation volume factor at reservoir pressure p, bbl/STB

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

4. Pore Volume Occupied by the Gas Cap at Reservoir Pressure p:

- As the reservoir pressure drops to a new level p, the gas in the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$Volume\ of\ the\ gas\ cap\ at\ p = \left[\frac{m\ N\ B_{oi}}{B_{gi}}\right] B_g \quad ---(6)$$

Where B_{gi} = gas formation volume factor at initial reservoir pressure, bbl/scf

B_g = current gas formation volume factor, bbl/scf

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

5. Pore Volume Occupied by the Evolved Solution Gas:

- This volumetric term can be determined by applying the following material balance on the gas solution:

$$\begin{bmatrix} volume\ of\ the \\ eveloved \\ solution\ gas \end{bmatrix} = \begin{bmatrix} volume\ of\ gas \\ initially\ in\ solution \end{bmatrix} - \begin{bmatrix} volume\ of \\ gas\ produced \end{bmatrix} - \begin{bmatrix} volume\ of \\ gas\ remaining \\ in\ solution \end{bmatrix}$$

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

5. Pore Volume Occupied by the Evolved Solution Gas:

- This volumetric term can be determined by applying the following material balance on the gas solution:

$$\begin{bmatrix} Volume\ of\ the\ evolved \\ solution\ gas \end{bmatrix} = [N\,R_{si} - N_pR_p - (N - N_p)R_s]B_g \text{ --- }(7)$$

Where N_p= cumulative oil produced, STB

R_p= net cumulative produced gas-oil ratio, scf/STB

R_s= current gas solubility factor, scf/STB

B_g= current gas formation volume factor, bbl/scf

R_{si}= gas solubility at initial reservoir pressure, scf/STB

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

6. Pore Volume Occupied by the Net Water Influx:

$$[net\ water\ influx] = W_e - W_p B_w --- (8)$$

Where W_e= cumulative water influx, bbl

W_p= cumulative water produced, STB

B_w= water formation volume factor, bbl/STB

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

7. Change in Pore Volume Due to Initial Water and Rock Expansion:

- The compressibility coefficient c, which describes the changes in the volume (expansion) of the fluid or material with changing pressure, is given by:

$$c = \frac{-1}{V}\frac{\partial V}{\partial p}$$

Or

$$\Delta V = V \, c \, \Delta p$$

Where ΔV represents the net changes or expansion of the material as a result of changes in the pressure.

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

7. Change in Pore Volume Due to Initial Water and Rock Expansion:

- Therefore, the reduction in the pre volume due to the expansion of the connate-water in the oil zone and the gas cap is given by:

$$connate - water\ expansion = [(pore\ volume)S_{wi}]c_w\ \Delta p$$

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

7. Change in Pore Volume Due to Initial Water and Rock Expansion:

- Substituting for the pore volume (P.V) with equation (1) gives:

$$Expansion\ of\ connate\ water = \frac{N\ B_{oi}(1+m)}{1-S_{wi}} S_{wi}\ c_w\ \Delta p ---(9)$$

Where Δp= change in reservoir pressure, $(p_i - p)$

c_w= water compressibility coefficient, psi^{-1}

m= ratio of the volume of the gas-cap gas to the reservoir oil volume, bbl/bbl

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

7. Change in Pore Volume Due to Initial Water and Rock Expansion:

- Similarly, the reduction in the pore volume due to the expansion of the reservoir rock is given by:

$$Change\ in\ pore\ volume = \frac{N\ B_{oi}(1+m)}{1-S_{wi}} c_t\ \Delta p --- (10)$$

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

7. Change in Pore Volume Due to Initial Water and Rock Expansion:

- Combining the expansions of the connate-water and formation as represented by equations (9) and (10) gives:

$$Total\ changes\ in\ pore\ volume = N\ B_{oi}(1+m)\left(\frac{S_{wi}\ c_w + c_f}{1-S_{wi}}\right)\Delta p \ ---(11)$$

The Material Balance Equation Derivation

- The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

8. Pore Volume Occupied by the Injection Gas and Water:

- Assuming that G_{inj} volumes of gas and W_{inj} volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by:

$$Total\ volume = G_{inj}B_{ginj} + W_{inj}B_w \quad ---(12)$$

Where G_{inj}= cumulative gas injected, scf

B_{ginj}= injected formation volume factor, bbl/scf

W_{inj}= cumulative water injected, STB

B_w= water formation volume factor, bbl/STB

The Material Balance Equation Derivation

- Combining equations (3) through (12) with equation (2) and rearranging gives:

$$N = \frac{N_p B_o + \left(G_p - N_p R_s\right) B_g - \left(W_e - W_p B_w\right) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}}\right] \Delta p} --- (13)$$

Where N= initial oil-in-place, STB

G_p= cumulative gas produced, scf

N_p= cumulative oil produced, STB

R_{si}= gas solubility at initial pressure, scf/STB

m= ratio of gas-cap gas volume to oil volume, bbl/bbl

B_{gi}= gas formation volume factor at p_i, bbl/scf

B_{ginj}= gas formation volume factor of the injected gas, bbl/scf

The Material Balance Equation Derivation

- The cumulative gas produced G_p can be expressed in terms of the cumulative gas-oil ratio R_p and cumulative oil produced N_p by:

$$G_p = R_p N_p ---(14)$$

- Combining equation (14) with equation (13) gives:

$$N = \frac{N_p\,[B_o + (R_p - R_s)B_g] - (W_e - W_p B_w) - G_{inj}B_{ginj} - W_{inj}B_w}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p} ---(15)$$

- The above relationship is referred to as the **material balance equation** (MBE).

The Material Balance Equation Derivation

- A more convenient form of the MBE can be determined by introducing the concept of the total (two-phase) formation volume factor B_t into the equation.

- This oil PVT property is defined as:

$$B_t = B_o + (R_{si} - R_s)B_g ---(16)$$

The Material Balance Equation Derivation

- Introducing B_t into equation (15) and assuming, for the sake of simplicity, no water or gas injection gives:

$$N = \frac{N_p\ [B_t + (R_p - R_{si})B_g] - (W_e - W_p B_w)}{(B_t - B_{ti}) + mB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p} --- (17)$$

Where S_{wi}= initial water saturation

R_p= cumulative produced gas-oil ratio, scf/STB

Δp= change in the volumetric average reservoir pressure, psi

The Material Balance Equation Derivation

- In a combination-drive reservoir where all the driving mechanisms are simultaneously present, it is of practical interest to determine the relative magnitude of each of the driving mechanisms and its contribution to the production.
- Rearranging equation (17) gives:

$$\frac{N(B_t - B_{ti})}{A} + \frac{Nm\left(B_g - B_{gi}\right)/B_{gi}}{A} + \frac{W_e - W_p B_w}{A} + \frac{NB_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right](p_i - p)}{A} = 1 --- (18)$$

With the parameter A as defined by:

$$A = N_p\left[B_t + (R_p - R_{si})B_g\right] --- (19)$$

The Material Balance Equation Derivation

- Equation (18) can be abbreviated and expressed as:

$$DDI + SDI + WDI + EDI = 1.0 ---(20)$$

Where DDI= depletion-drive index

SDI= segregation (gas-cap)- drive index

WDI= water-drive index

EDI= expansion (rock and liquid)- drive index

The Material Balance Equation

Example: The big butte field is a combination-drive reservoir. The current reservoir pressure is estimated at 2,500 psi. The reservoir production data and PVT information are given below:

The following additional information is available:

Volume of bulk oil zone= 100,000 ac-ft

Volume of bulk gas zone= 20,000 ac-ft

Calculate the initial oil-in-place.

	Initial Reservoir Condition	Current Reservoir Condition
p, psi	3000	2500
B_o, bbl/STB	1.35	1.33
R_s, scf/STB	600	500
N_p, MMSTB	0	5
G_p, MMMscf		5.5
B_w, bbl/STB	1.00	1.00
W_e, MMbbl	0	3
W_p, MMbbl	0	0.2
B_g, bbl/scf	0.0011	0.0015
c_f, c_w	0	0

The Material Balance Equation

Solution:

Step1. Assuming the same porosity and connate-water for the oil and gas zones, calculate m:

$$m = \frac{20,000}{100,000} = 0.2$$

Step 2. Calculate the cumulative gas-oil ratio R_p:

$$R_p = \frac{5.5 * 10^9}{5 * 10^6} = 1100 \frac{scf}{STB}$$

Step 3. Solve for the initial oil-in-place by applying equation (15):

$$N = \frac{5*10^6[1.33+(1100-500)0.0015]-(3*10^6-0.2*10^6}{(1.35-1.33)+(600-500)0.0015+(0.2)(1.35)\left[\frac{0.0015}{0.0011}-1\right]} = 31.14\ MMSTB$$