LECTURE FOUR

THE FIRST LAW OF THERMODYNAMICS FOR NFE (closed system)

Lecture Content:

1. Definition of First law of thermodynamics
2. Energy balance and Mechanism energy transfer
3. Energy conversion efficiencies
4. Classification of thermodynamic process
5. Energy analysis of closed system

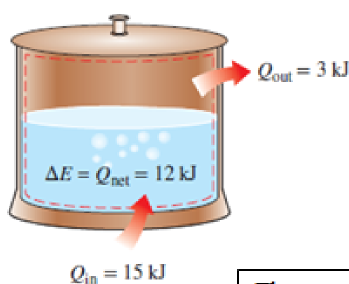
Lecture Outcomes:

At the end of this lecture, you will be able to:

- Examine the moving boundary work or PdV work
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for ideal gases.

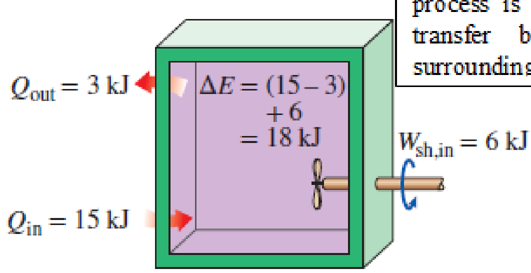
1. DEFINITION OF FIRST LAW OF THERMODYNAMICS:

The **first law of thermodynamics** states that energy can be neither created nor destroyed during a process; it can only change forms (conservation of energy).



In the absence of any work interactions, the energy change of a system is equal to the net heat transfer.

The energy change of a system during a process is equal to the network and heat transfer between the system and its surroundings.



2. ENERGY BALANCE AND MECHANISMS OF ENERGY TRANSFER, E_{IN} AND E_{OUT}:

$$(Total\ energy\ entering\ the\ system) - (Total\ energy\ leaving\ the\ system) = (change\ in\ the\ total\ energy\ of\ the\ system)$$

Or

$$E_{in} - E_{out} = \Delta E_{system}$$

Energy Change of a System, ΔE system:

$$Energy\ change = Energy\ at\ final\ state - Energy\ at\ initial\ state$$

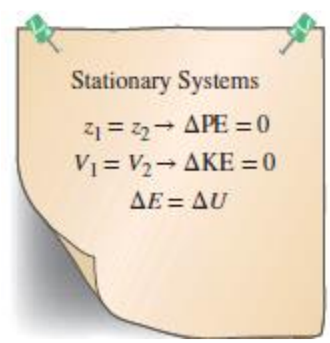
or

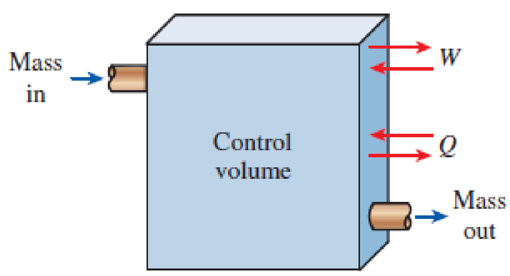
$$\Delta E_{system} = E_{final} - E_{initial} = E_2 - E_1$$

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

$$\sum Q_{1-2} - \sum W_{1-2} = \Delta E$$

$$Q_{1-2} - W_{1-2} = (PE_2 + KE_2 + U_2) - (PE_1 + KE_1 + U_1)$$
$$= \left(mgz_2 + \frac{1}{2}mV_2^{\ 2} + U_2\right) - \left(mgz_1 + \frac{1}{2}mV_1^{\ 2} + U_1\right)$$





Note:

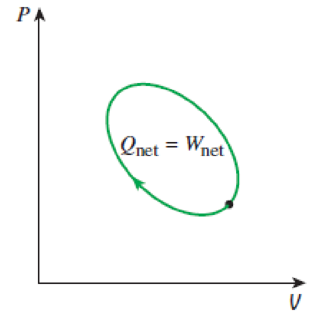
1. If isolated system so $E_2 = E_1$ (i.e $Q_{1-2} = 0, W_{1-2} = 0$)
2. **For a closed system (no mass flow):**

   $\Delta E = \Delta PE + \Delta KE + \Delta U \qquad (\Delta PE + \Delta KE = 0)$

   $\Delta E = \Delta U$

   $Q - W = \Delta U$ (NFEE)

3. **For complete cycle, the initial and final states are identical, $\Delta E = 0$, thus Q = W.** *because* $\oint Q = \oint W$

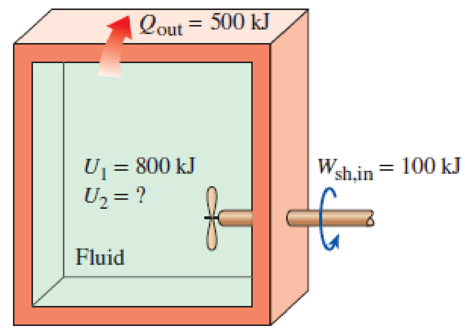


$$\sum Q_{1-2} = \sum W_{1-2}$$

$$W_{net,out} = Q_{net,in} \quad , \qquad W_{net,in} = Q_{net,out}$$

Example 1// Cooling of a Hot Fluid in a Tank

A rigid tank contains a hot fluid that is cooled while being stirred by a paddle wheel. Initially, the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat, and the paddle wheel does 100 kJ of work on the fluid. Determine the final internal energy of the fluid. Neglect the energy stored in the paddle wheel.



Solution// The tank is stationary so PE &KE=0**,** Closed system so no mass crosses the boundary

$$Q_{out} - W_{shaft} = U_2 - U_1$$

$$= -500kJ - (-100\ kJ) \ = U_2 - 800,$$

$$U_2 = 400\ kJ$$

Example 2// 3000 J of heat is added to a system and 2500 J of work is done by the system. What is the change in internal energy of the system?

Solution// Heat (Q) = +3000 Joule (add to the system)

Work (W) = +2500 Joule (done by the system)

Wanted: the change in internal energy of the system

$$\Delta U = Q - W$$

$$\Delta U = 3000 - 2500 = 500\ Joule$$

Example 3// 2000 J of heat leaves the system, and 2500 J of work is done on the system. What is the change in internal energy of the system?

Solution//

Heat (Q) =$-2000\ Jouls$ (leave the system)

Work (W) = $-3000\,Joules$ (done on the system)

$$\Delta U = Q - W \quad , \quad \Delta U = -2000 - (-3000) = 1000\,Joules$$

3. ENERGY CONVERSION EFFICIENCIES:

Efficiency is one of the most often used terms in thermodynamics, and it indicates how well an energy conversion or transfer process is accomplished.

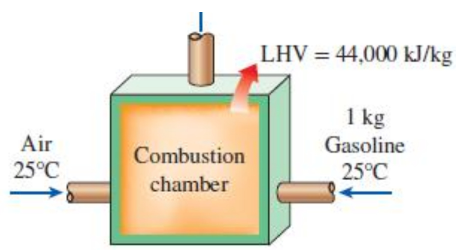
$$Efficiency = \frac{Desired\ output}{Required\ input}$$

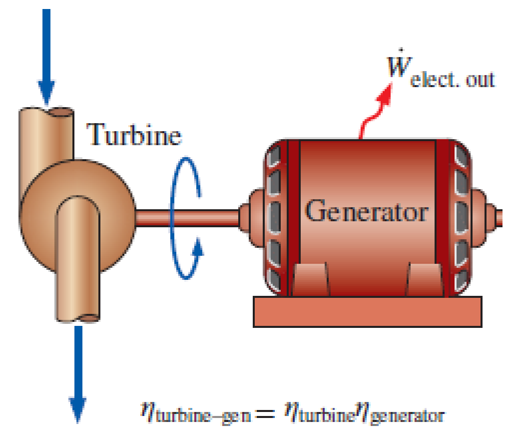
For example, Efficiency of:

- **A water heater** is defined as the ratio of the energy delivered to the house by hot water to the energy supplied to the water heater.
- **Heating value of the fuel**, which is the amount of heat released when a unit amount of fuel at room temperature is completely burned and the combustion products are cooled to the room temperature.

$$\eta_{\text{comb. equip.}} = \frac{Q_{\text{useful}}}{\text{HV}} = \frac{\text{Useful heat delivered by the combustion equipment}}{\text{Heating value of the fuel burned}}$$

$$\eta_{\text{generator}} = 0.97$$





$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = 0.75 \times 0.97 = 0.73$$

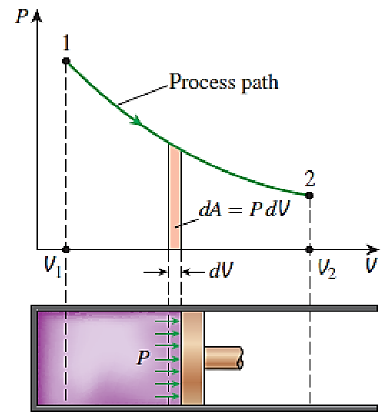
$$\text{Efficiency} = \frac{\text{Energy utilized}}{\text{Energy supplied to appliance}} = \frac{3\text{ kWh}}{5\text{ kWh}} = 0.60$$

$$\eta_{motor} = \frac{\text{Mechanical Power Output}}{\text{Electrical Power Input}} = \frac{W_{shaft,out}}{W_{electric,in}}$$

$$\eta_{Generator} = \frac{\text{Electrical Power Output}}{\text{Mechanical Power Input}} = \frac{W_{electric,out}}{W_{shaft,in}}$$

4. CLASSIFICATION OF THERMODYNAMIC PROCESSES:

1. **Closed or Non-flow processes (NFE)**. Such as filling or evacuation of vessels.
2. **Open or Flow processes (SFE)**. Such as flow through nozzles, turbines, compressors etc.



Work done for a closed or non-flow process:

As earlier explained the work for an enclosed system (piston-cylinder) happen due to moving boundary.

$$A = \int_1^2 dA = \int_1^2 PdV$$

$$W_{1-2} = F.\Delta x = P.A.\Delta x = pv_{1-2}$$

5. TYPES OF CLOSED OR NON-FLOW PROCESSES:

The heating (or cooling) and expansion (or compression) of a gas may be performed in the following ways:

1. ***Reversible non-flow processes***. These processes include
   a) Constant volume process (isometric or isochoric process).
   b) Constant pressure process (isopiestic or isobaric process).
   c) Constant temperature process (or isothermal process or hyperbolic process).
   d) Adiabatic process (or Isentropic process).
   e) Polytrophic process.

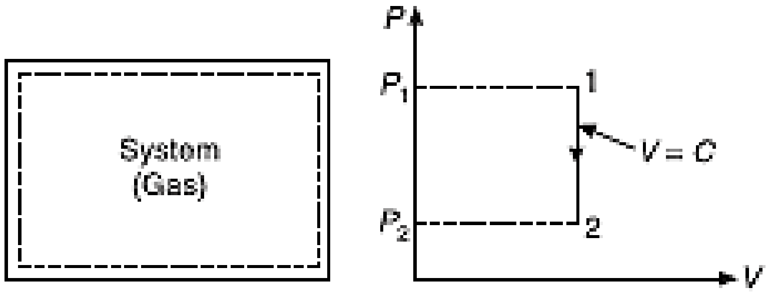
In all these processes, we shall determine the expressions for

a. Work done by the gas.
b. Change in internal energy.
c. Heat supplied or heat transfer; and
d. Change in enthalpy

2. ***Irreversible non-flow process***. The free expansion process (or unrestricted process) is an irreversible non-flow process.

We shall now discuss the above-mentioned processes, in detail, in the following:

6.1. Constant volume process (isometric or isochoric process)



$$v_1 = v_2 = v$$

$$W_{1-2} = p(v_2 - v_1) = 0$$

$$so, \qquad \Delta Q_{1-2} = \Delta U_{1-2} = U_2 - U_1 = mc_v(T_2 - T_1)$$

$$\Delta H_{1-2} = H_2 - H_1 = mc_p(T_2 - T_1)$$

Notes:

1. During expansion and heating, W_{1-2}, dU and Q_{1-2} are *positive*.
3. During compression and cooling, W_{1-2}, dU and Q_{1-2} are *negative*.

Example 4 // A constant volume chamber of 0.3 m^3 capacity contains 2 kg of gas at 5°C. Heat is transferred to the gas until the temperature is 100°C. Find the work done, the heat transferred, the changes in internal energy and enthalpy.

Take c_p = 1.97 kJ/kg K, and c_v = 1.51 kJ/kg K.

Solution.

Given:

Constant volume = 0.3 m^3

Mass of the gas, $m = 2\ kg$

Initial temperature, $T_1 = 5°C = 5 + 273 = 278\ K$

Final temperature, $T_2 = 100°C = 100 + 273 = 373\ K$

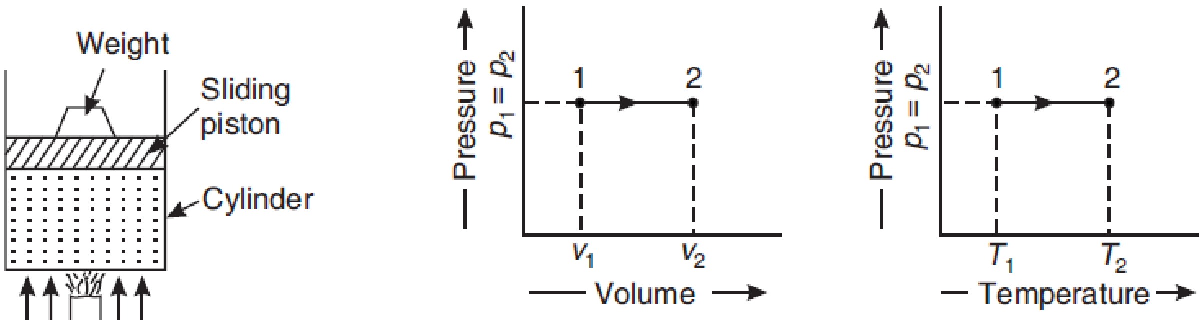
Since the volume is constant, therefore **work done is zero.**

Heat transferred, $Q_{1-2} = m\, c_v\, (T_2 - T_1) = 2 \times 1.51\,(373 - 278) = \mathbf{287\ kJ}$

Change in internal energy, $dU = m\, c_v\, (T_2 - T_1) = Q_{1-2} = \mathbf{287\ kJ}$

Change in enthalpy, $dH = m\, c_p\, (T_2 - T_1) = 2 \times 1.97\,(373 - 278) = \mathbf{374.3\ kJ}$

6.2. Constant pressure process (isobaric process):



(a) p – v diagram. (b) p – T diagram.

$$p_1 = p_2$$

- Work done by gas $\Delta W_{1-2} = p(v_2 - v_1)$

Since $p_1 v_1 = m\, R\, T_1$, and $p_2 v_2 = m\, R\, T_2$,
therefore, the above relation may be written as $\Delta W_{1-2} = mR(T_2 - T_1)$
so, $\Delta Q_{1-2} - \Delta W_{1-2} = \Delta U_{1-2}$, $Q_{1-2} - p(v_2 - v_1) = U_2 - U_1$

- change in internal energy $U_2 - U_1 = m\, c_v\, (T_2 - T_1)$

- The heat transferred or supplied:

$U_2 - U_1 = m\, c_v\, (T_2 - T_1)$and $p(v_2 - v_1) = mR(T_2 - T_1)$ therefore
SO, $Q_{1-2} = m\, c_v\, (T_2 - T_1) + mR(T_2 - T_1)$
$= m\,(T_2 - T_1)(c_v + R) = m\, c_P\, (T_2 - T_1)$ *SINCE*...($c_v + R = c_P$)

- Change in enthalpy: $H_2 - H_1 = Q_{1-2} = m\, c_P\, (T_2 - T_1)$

Example 5// One kg of air is expanded at a constant pressure of 2.5 bar from a volume of 0.3 m^3 to a volume of 0.45 m^3. Find:
1. external work done by the gas;
2. internal energy of the gas; and
3. heat transferred during the process.

Assume $R = 287\ J/kg\ K$; c_v = 0.72 kJ/kg K; and c_P = 1.005 kJ/kg K for air.

Solution.

Given: Mass of air, $m = 1\ kg$
Pressure, $p = 2.5\ bar = 250 \times 10^3\ N/m^2$
Initial volume, $v_1 = 0.3\ m^3$
Final volume, $v_2 = 0.45\ m^3$

Gas constant, $R = 287$ J/kg K
Specific heat at constant volume, $c_v = 0.72$ kJ/kg K
Specific heat at constant pressure, $c_P = 1.005$ kJ/kg K
First, let us find the initial and final temperature of the gas. Let T_1 and T_2 = Initial and final temperature of the gas respectively.
We know that $p_1 v_1 = m\,R\,T_1$

$$T_1 = \frac{p_1 v_1}{m\,R} = \frac{250 \times 10^3 \times 0.3}{1 \times 287} = 261K$$

Similarly.

$$p_2 v_2 = m\,R\,T_2$$

$$T_2 = \frac{p_2 v_2}{m\,R} = \frac{250 \times 10^3 \times 0.45}{1 \times 287} = 392K$$

1. External work done by the gas

$$W_{1\text{-}2} = p\,(v_2 - v_1) = 250 \times 103\,(0.45 - 0.3)$$
$$= 37\,500\ N.m = 37.5\ kJ$$

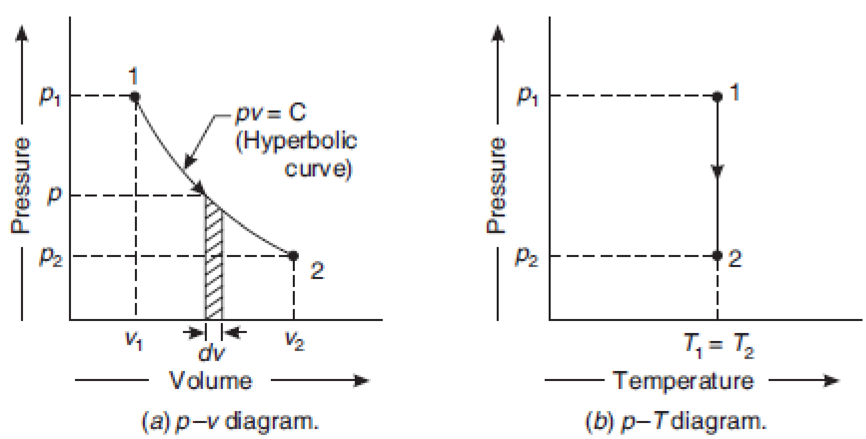
2. Internal energy of the gas

$$dU = m\,c_v(T_2 - T_1) = 1 \times 0.72\,(392 - 261) = 94.32\ kJ$$

3. Heat transferred

$$Q_{1\text{-}2} = m\,c_p(T_2 - T_1) = 1 \times 1.005\,(392 - 261) = 131.6\ kJ$$

6.3. Constant temperature process (isothermal process / hyperbolic process):



$$p_1 v_1 = p_2 v_2 = C \ \text{(NO change in internal energy } \Delta U_{1-2} = 0)$$

So, $$\Delta Q_{1-2} = \Delta W_{1-2}$$

$$\frac{v_2}{v_1} = \frac{P_1}{P_2} \quad \text{Because } P \propto \frac{1}{V}$$

- Work done by gas (T=C) (Boyle's law)

$$pv = p_1 v_1 = C\,, \quad \left(\text{During expansion}\,, p = \frac{p_1 v_1}{v}\right)$$

$$W_{1-2} = \int p dv = \int_1^2 \frac{p_1 v_1}{v}\, dv = p_1 v_1 \ln\frac{v_2}{v_1} = p_1 v_1 \ln\frac{p_1}{p_2}$$

- Change in enthalpy:

$$\int_1^2 dH = \int_1^2 dU + \int_1^2 d(pv)$$

$$H_2 - H_1 = (U_2 - U_1) + p(v_2 - v_1)$$

$$(U_2 = U_1)\,, (pv = mRT), (\,T_2 = \,T_1)$$

$$H_2 - H_1 = pv_2 - pv_1 = mRT_2 - mRT_1$$

$$= mRT_2 = mRT_1 = 0$$

So, $$H_2 = H_1$$

Example 6// 0.1 m^3 of air at 6 bars is expanded isothermally to 0.5 m^3. Calculate the final pressure and the heat supplied during the expansion process.

Solution//

Given: Initial volume, $v_1 = \ 0.1\ m^3$

Initial pressure, p_1= 6 bar = 600 × 10^3 N/m^2

Final volume, $v_2 = \ 0.5\ m^3$

Final pressure p_2= Final pressure in bar.

We know that for an isothermal process, $p_2 v_2 - p_1 v_1$

$$p_2 = \frac{p_1 v_1}{v_2} = \frac{600\ \times\ 10^3\ \times 0.1}{0.5} = 1.2\ bar$$

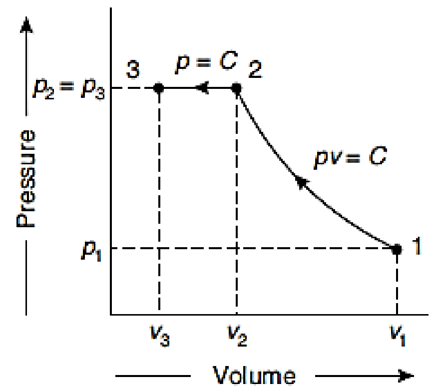
Heat supplied

We know that work done during the isothermal expansion,

$$W_{1-2} = p_1 v_1 \ln\frac{v_2}{v_1} = 600 \times 10^3 \times 0.1 \ln\frac{0.5}{0.1} = 96\,600\ N.m = 96.6\ kJ$$

We also know that heat supplied during the isothermal process is equal to the work done. Therefore, heat supplied, $Q_{1-2} = W_{1-2} = 96\,600\ N.m = 96.6\ kJ$

Example 7// Air initially at 75 kPa pressure, 1000 K temperature and occupying a volume of 0.12 m^3 is compressed isothermally until the volume is halved and subsequently it undergoes further compression at constant pressure till the volume is halved again. Sketch the process on p-v diagram and find the work transfer.



Solution.

Given: Initial pressure, $p_1 = 75 \text{ kPa} = 75 \times 10^3 \text{ N/m}^2$

Initial temperature, $T_1 = 1000 \text{ K}$

Initial volume, $v_1 = 0.12 \text{ m}^3$

Final volume, $v_2 = \frac{v_1}{2} = \frac{0.12}{2} = 0.06 \text{ m}^3$

Volume after compression at constant pressure,

$$v_3 = \frac{v_2}{2} = \frac{0.06}{2} = 0.03 \text{ m}^3$$

The *p-v* diagram of the process is shown in Fig. The curve 1–2 represents the compression of air isothermally (*i.e.* according to $pv = C$) and the horizontal line 2–3 represents compression at constant pressure (*i.e.* $p = C$)

Considering the isothermal compression 1–2, we have

$$p_1 v_1 = p_2 v_2$$

$$\therefore \quad p_2 = \frac{p_1 v_1}{v_2} = \frac{75 \times 0.12}{0.06} = 150 \text{ kPa} = 150 \times 10^3 \text{ N/m}^2$$

and workdone on the air,

$$W_{1-2} = 2.3\, p_1 v_1 \log\left(\frac{v_1}{v_2}\right) = 2.3 \times 75 \times 10^3 \times 0.12 \log\left(\frac{0.12}{0.06}\right)$$

$$= 6231 \text{ N-m or J} = 6.231 \text{ kJ}$$

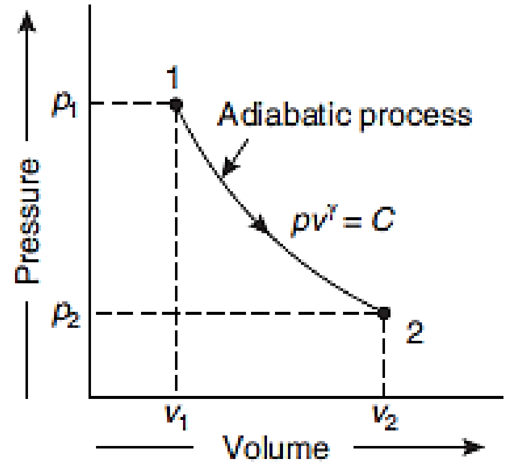
Now considering the constant pressure process 2–3, we have

Workdone on the air, $W_{1-2} = p_2 (v_2 - v_3) = 150 \times 10^3 (0.06 - 0.03)$ N-m

$= 4500$ N-m or J $= 4.5$ kJ

$\therefore$ Net work transfer, $W = W_{1-2} + W_{2-3} = 6.231 + 4.5 = 10.731$ kJ **Ans.**

6.4. Adiabatic process (or Isentropic process):



When a process is carried out in such a manner that there is no heat transfer into or out of the system (i.e., Q = 0), then the process is said to be "adiabatic". This process may be reversible or irreversible. Such a process is not really possible in practice, though it can be closely approached. This will happen when the system is thermally insulated, so no heat transfer take place during the process.

$\Delta Q_{1-2} - \Delta W_{1-2} = \Delta U_{1-2} \qquad Q = 0$

- Work done:

$\Delta W_{1-2} = -\Delta U_{1-2}$

$$W_{1-2} = \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} = \frac{mR(T_2 - T_1)}{\gamma - 1}$$

- Change in internal energy

$$\int_1^2 dU = \int_1^2 mc_v dT$$

$U_2 - U_1 = m\, c_v(T_2 - T_1)$

- Change in enthalpy

$$\int_1^2 dH = \int_1^2 dU + \int_1^2 d(pv)$$

$$H_2 - H_1 = (U_2 - U_1) + p(v_2 - v_1)$$

$U_2 - U_1 = m\, c_v(T_2 - T_1)$ and $p(v_2 - v_1) = mR(T_2 - T_1)$

$$H_2 - H_1 = m\, c_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$H_2 - H_1 = m\,(c_v + R)(T_2 - T_1) = m\, c_p(T_2 - T_1)$$

- Relation between pressure, volume, and temperature during an adiabatic change:

$$p_2 {v_2}^\gamma = p_1 v_1^\gamma$$

$$\frac{v_1}{v_2} = \left[\frac{p_2}{p_1}\right]^{\frac{1}{\gamma}}$$

From the general gas equation,

$$\frac{p_2 v_2}{T_2} = \frac{p_1 v_1}{T_1}$$

$$\frac{v_1}{v_2} = \frac{p_2 T_1}{p_1 T_2}$$

So,

$$\left[\frac{p_2}{p_1}\right]^{\frac{1}{\gamma}} = \frac{p_2 T_1}{p_1 T_2}$$

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{(p_2/p_1)^{1/\gamma}} = \left[\frac{p_2}{p_1}\right]^{1-\frac{1}{\gamma}} = \left[\frac{p_2}{p_1}\right]^{\frac{\gamma-1}{\gamma}}$$

Now let us determine the relation between the volume and temperature during an adiabatic change. We know that

$$p_2 {v_2}^{\gamma} = p_1 v_1^{\gamma}$$

Or

$$\frac{p_1}{p_2} = \left[\frac{v_2}{v_1}\right]^{\gamma}$$

and from the general gas equation,

$$\frac{p_2 v_2}{T_2} = \frac{p_1 v_1}{T_1} \quad \text{or} \; \frac{p_1}{p_2} = \frac{v_2}{v_1} \times \frac{T_1}{T_2}$$

$$\left[\frac{v_2}{v_1}\right]^{\gamma} = \left(\frac{v_2}{v_1}\right)\left(\frac{T_1}{T_2}\right)$$

$$\frac{T_1}{T_2} = \left[\frac{v_2}{v_1}\right]^{\gamma-1} \quad \text{or} \; \frac{T_2}{T_1} = \left[\frac{v_2}{v_1}\right]^{-(\gamma-1)} = \left[\frac{v_2}{v_1}\right]^{1-\gamma}$$

Example 8// 0.25 m^3 of the gas at 288 K and 100 kPa is compressed adiabatically to 700 kPa. Calculate:

1. The final temperature of the gas; and
2. The work done on the gas.

Take c_p= 1.001 kJ/kg K: c_v = 0.715 kJ/kg K for the gas.

Solution.

Given: Initial volume, $v_1 = 0.25\, m^3$

Initial temperature, $T_1 = 288\, K$

Initial pressure, $p_1 = 100\, kPa = 100\, kN/m^2$

Final pressure,$p_2 = 700\, kPa = 700\, kN/m^2$

c_p= 1.001 kJ/kg K

c_v = 0.715 kJ/kg K for the gas

1- Final temperature of gas

T_2 = Final temperature of the gas.

$$\gamma = \frac{c_p}{c_v} = \frac{1.001}{0.715} = 1.4$$

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{\frac{\gamma-1}{\gamma}} = \left[\frac{700}{100}\right]^{\frac{1.4-1}{1.4}} = (7)^{0.286} = 1.7446$$

$$T_2 = T_1 \times 1.7446 = 288 \times 1.7446 = 502.4\, K$$

2- Work done on the gas

First, let us find the final volume of the gas after compression (i.e., v_2). We know that

$$p_2 {v_2}^{\gamma} = p_1 v_1^{\gamma} \quad \text{Or} \quad \frac{p_1}{p_2} = \left[\frac{v_2}{v_1}\right]^{\gamma}$$

$$\frac{v_2}{v_1} = \left[\frac{p_1}{p_2}\right]^{\gamma} = \left[\frac{100}{700}\right]^{1.4} = (0.143)^{0.714} = 0.2494$$

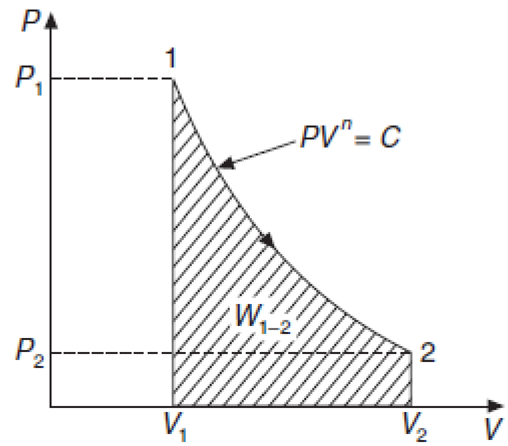
$$v_2 = 0.25 \times 0.2494 = 0.0624 m^3$$

Hence, the work done is.

$$W_{1-2} = \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} = \frac{700 \times 0.0624 - 100 \times 0.25}{1.4 - 1} = 46.7\ \text{kN.m} = 46.7\ \text{kJ}$$

6.5. polytropic process ($PV^n = C$):

Both heat and work can cross the boundary of the system T,P,V are all change from state 1 to 2.



$\Delta Q_{1-2} - \Delta W_{1-2} = \Delta U_{1-2}$

$$Q = \frac{p_2 v_2 - p_1 v_1}{1-n} + \Delta U$$

For ideal gas where $0 < n < \infty$

$$\frac{P_1}{P_2} = \left[\frac{v_2}{v_1}\right]^n, \frac{T_1}{T_2} = \left[\frac{v_2}{v_1}\right]^{n-1}, \frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{n-1}{n}}$$

Work done: $W = \frac{p_2 v_2 - p_1 v_1}{1-n} = \frac{mR(T_1 - T_2)}{n-1}$

$\Delta U_{1-2} = mc_v(T_2 - T_1)$

Heat transfer or supplied $Q = \frac{\gamma - n}{\gamma - 1} \times$ work done during polytropic process

Example 9// 0.016 m^3 of gas at constant pressure of 2055 kN/m^2 expands to a pressure of 215 kN/m^2 by following the law $pv^{1.35} = C$. Determine the work done by the gas during expansion process.

Solution: Given: Initial volume, $v_1 = 0.015\ m3$

Initial pressure, $p_1 = 2055\ kN/m2$, Final pressure,$p_2 = 215\ kN/m2$

Polytropic index, $n = 1.35$

First of all, let us find the final volume of the gas after expansion (i.e., V_2). We know that

$$p_2 {v_2}^n = p_1 {v_1}^n \quad or \quad \frac{p_1}{p_2} = \left[\frac{v_2}{v_1}\right]^n$$

$$\frac{v_2}{v_1} = \left[\frac{p_1}{p_2}\right]^{\frac{1}{n}} = \left[\frac{2055}{215}\right]^{\frac{1}{1.35}} = (9.56)^{0.74} = 5.315$$

$$v_2 = v_1 \times 5.315 = 0.016 \times 5.315 = 0.085\ m^3$$

$W = \frac{p_2 v_2 - p_1 v_1}{1-n} = \frac{215 \times 0.085 - 2055 \times 0.016}{1-1.35} = 41.74$ kN-m = 41.74 kJ **Ans.**

Example 10// 0.2 kg of air at 1.5 bar and 27°C is compressed to 15 bars according to $PV^{1.25}$ = constant. Determine: 1. Initial and final parameters of the air; 2. Work done on or by the air; and 3. Heat flow to or from the air.

Solution.

Given: Mass of air, $m = 0.2\ kg$

Initial pressure, $p_1 = 1.5\ bar = 150 \times 10^3\ N/m^2$

Initial temperature, $T_1 = 27°C = 27 + 273 = 300\ K$

Final pressure, $p_2 = 15\ bar$

Polytropic index, $n = 1.25$

4. Initial and final parameters of the air

First of all, let us find the final temperature of the air (i.e., T_2).

$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{n-1}{n}} = \left[\frac{15}{150 \times 10^3}\right]^{\frac{1.25-1}{1.25}} = (10)^{0.2} = 1.585$$

$$T_2 = T_1 \times 1.585 = 300 \times 1.585 = 475.5\ K$$

$$T_2 = 475.5\ K - 273 = 202.5°C$$

Let v_1 and v_2 be the initial and final volume of the air respectively. Using the relation, taking gas constant (R) for air = 287 J/kg K, we have

$$p_1 v_1 = m\,RT_1$$

$$v_1 = \frac{m\,RT_1}{p_1} = \frac{0.2 \times 287 \times 300}{150 \times 10^3} = 0.1148 m^3$$

We know that

$$p_2 {v_2}^n = p_1 {v_1}^n \quad or \quad \frac{p_1}{p_2} = \left[\frac{v_2}{v_1}\right]^n$$

$$\frac{v_2}{v_1} = \left[\frac{p_1}{p_2}\right]^{\frac{1}{n}} = \left[\frac{1.5}{15}\right]^{\frac{1}{1.25}} = (0.1)^{0.8} = 0.1585$$

$$v_2 = v_1 \times 0.1585 = 0.1148 \times 0.1585 = 0.0182 m^3$$

5. work done

$$W_{1-2} = \frac{mR(T_1 - T_2)}{n-1} = \frac{0.2 \times 287(300 - 475.5)}{1.25 - 1} = -40295\,J = -40.295\,kJ$$

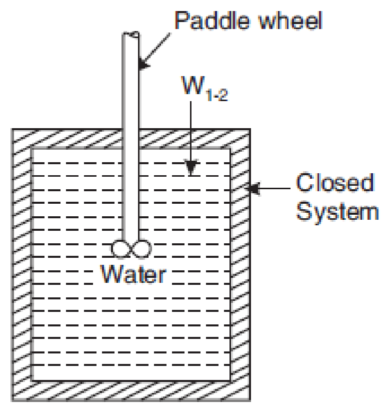
6. Heat flow to or from the air

First of all, let us find the change in internal energy (dU) by assuming the value of c_v for air as 0.712 kJ/kg K. We know that

$$dU = m\,c_v\,(T_2 - T_1) = 0.2 \times 0.712\,(475.5 - 300) = 254\,kJ$$

Heat flow, $Q_{1-2} = dU + W_{1-2} = 25 - 40.295 = -\ 15.295\ kJ\ Ans.$

ASSIGNMENTS



1. A closed system consists of water contained in a cylinder and being stirred by a paddle wheel. During the process, 35 kJ/h of work was imparted to the system and the internal energy is increased to 145 kJ from an initial value of 120 kJ during one hour of stirring. Sketch the process with the help of neat diagram of the arrangement and determine the heat transfer. Is the temperature of the system rising or falling?
2. 0.14 m^3 of air at 100°C and at a pressure of 1.5 bar is compressed at constant pressure until the volume is 0.112 m^3. Find: 1. Final temperature of the air; 2. Mass of the air compressed; 3. Work done in compressing the air; 4. Heat given out by the air; and 5. Specific heat at constant volume. Take R = 287 J/kg K; and c_p = 1 kJ/kg K.
3. The values of specific heats at constant pressure and constant volume for an ideal gas are 0.984 kJ/kg K and 0.728 kJ/kg K respectively. Find the values of the characteristic gas constant (R) and ratio of specific heat (γ) for the gas.

4. 0.4 kg of air at a pressure of 1 bar and a temperature of 20°C is compressed isothermally until its volume is 0.08 m^3. Determine the work done on the air during this compression process. Take R = 287 J/kg K.
5. One kg of a gas expands reversibly and adiabatically. Its temperature during the process falls from 240°C to 115°C while its volume is doubled. The gas does 90 kJ of work in this process. Find: 1. The value of c_p and c_v; and 2. Molecular mass of the gas.

Review:

Process	*p-v-T relationship*	*Workdone* (W_{1-2})	*Change in internal energy* $(dU = U_2 - U_1)$	*Heat Supplied* $(Q_{1-2}) = W_{1-2} + dU$	*Change in enthalpy* $(dH = H_2 - H_1)$
(*a*) Constant volume (or isochoric) process	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	0	$m\, c_v (T_2 - T_1)$	$m\, c_v (T_2 - T_1)$	$m\, c_p (T_2 - T_1)$
(*b*) Constant pressure (or isobaric) process	$\frac{v_1}{T_1} = \frac{v_2}{T_2}$	$p\,(v_2 - v_1)$ or $mR\,(T_2 - T_1)$	$m\, c_v (T_2 - T_1)$	$m\, c_p (T_2 - T_1)$	$m\, c_p (T_2 - T_1)$
(*c*) Hyperbolic or Constant temperature or Isothermal) process	$p_1 v_1 = p_2 v_2$	$p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$ or $mRT \log_e \left(\frac{v_2}{v_1}\right)$	0	$p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$ or $mR\,T \log_e \left(\frac{v_2}{v_1}\right)$	0
(*d*) Adiabatic (or Isentropic) process	$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$	$\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$ or $\frac{mR(T_1 - T_2)}{\gamma - 1}$	$m\, c_v (T_2 - T_1)$	0	$m\, c_p (T_2 - T_1)$
(*e*) Polytropic (pv^n = Constant) process	$p_1 v_1^{n} = p_2 v_2^{n}$	$\frac{p_1 v_1 - p_2 v_2}{n - 1}$ or $\frac{mR(T_1 - T_2)}{n - 1}$	$m\, c_v (T_2 - T_1)$	$\frac{p_1 v_1 - p_2 v_2}{n - 1} + mc_v (T_2 - T_1)$ or $\frac{\gamma - n}{\gamma - 1} \times \frac{mR\,(T_1 - T_2)}{n - 1}$	$m\, c_p (T_2 - T_1)$

EXTRA EXAMPLES

1. An ideal gas of mass 0.25 kg has a pressure of 3 bar, a temperature of 80°C and a volume of 0.07 m3. The gas undergoes an *irreversible adiabatic process to a final pressure of 3 bar and a final volume of 0.1 m3, during which the workdone on the gas is 25 kJ. Evaluate cp and cv of the gas.

Solution.

Given: Mass of the gas, $m = 0.25\ kg$

Initial pressure, $p_1 = 3\ bar = 300 \times 10^3\ N/m^2$

Initial temperature, $T_1 = 80°C = 80 + 273 = 353\ K$

Initial volume, $v_1 = 0.07\ m^3$

Final pressure,$p_2 = 3\ bar = 300 \times 10^3\ N/m^2$

Final volume, $v_2 = 0.1\ m^3$

Work done on the gas, $W1-2 = 25$ kJ

Let R = Characteristic gas constant, and T_2 = Final temperature of the gas.

We know that $p_1\, v_1 = m\, RT_1$

$$R = \frac{p_1\, v_1}{mT_1} = \frac{300 \times 10^3 \times 0.07}{0.25 \times 353} = 238\ \text{J/kgK}$$

$$p_2\, v_2 = m\, RT_2$$

$$R = \frac{p_2\, v_2}{mT_2} = \frac{300 \times 10^3 \times 0.07}{0.25 \times 353} = 504\ \text{J/kgK}$$

Since the gas undergoes an irreversible adiabatic process, therefore there is no heat transfer during the process, i.e.

$$Q_{1-2} = 0$$

We also know that heat transfer,

$$Q_{1-2} = dU + W_{1-2} = m\, c_V(T_2 - T_1) + W_{1-2}$$

$$0 = 0.25\, c_V\, (504 - 353) - 25 = 37.75\, c_V - 25$$

(W1–2 is taken –ve, because work is done on the gas)

$$c_V = \frac{25}{37.75} = 0.662\frac{\text{kJ}}{\text{kgK}}$$

and

$$c_p - c_V = R = 238\ \text{J / kg K} = 0.238\ \text{kJ / kg K}$$

$$c_p = R + c_V = 0.238 + 0.662 = 0.9\ \text{kJ / kg}$$

2. In a closed system during a process, a volume changes from 4 to 8 m3. The process takes place according to the relation P = V^3 + 10/V, where P is in bar and V is in m^3. Calculate the work done.

Solution:

$$W = \text{Work done} = \int_{V_1}^{V_2} PdV = \int 10^5\left(V^3 + \frac{10}{V}\right)dV = 10^5\left[\frac{V^4}{4} + 10\ln V\right]$$

$$= 10^5\left[\frac{(V_2^4 - V_1^4)}{4} + 10\ln\frac{V_2}{V_1}\right]_4^8 = 10^5\left[\frac{(8^4 - 4^4)}{4} + 10\ln\frac{8}{4}\right] = \mathbf{96693\ kJ}$$

3. Air is compressed in an air compressor from 1 bar to 6 bar according to the relation $PV^{1.3}$ = C. Calculate

(a) The work done during the compression processes if the entrance and exit velocities are assumed to be the same,

(b) The shaft work of the compressor. Take specific volume of air = 0.9 m^3/kg (V_1).

Solution:

(a) Work done during the compression process (W.D.)

$$\text{W.D.} = \int_{V_1}^{V_2} PdV; \quad PV^{1.3} = C$$

This is a polytropic process with index n = 1.3.

$$\therefore \quad \text{W.D.} = \frac{(P_1V_1 - P_2V_2)}{(n-1)}$$

$$\frac{V_2}{V_1} = \left[\frac{P_1}{P_2}\right]^{1/1.3}$$

$$\therefore \quad V_2 = 0.9\left[\frac{1}{6}\right]^{1/1.3} = 0.2268\ \frac{\text{m}^3}{\text{kg}}$$

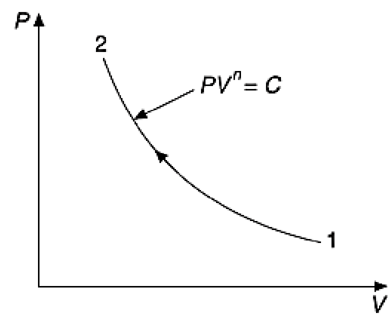


Figure E2.32

$$\text{W.D.} = \frac{10^5\,[1\times 0.9 - 6\times 2268]}{0.3} = \mathbf{-1.536\times 10^5\ J}$$ Work is done on the system

References:

1. Thermodynamic an engineering approach, by Y.A. Çengel, M. A. Boles, & M. Kanoğlu

2.Basic Thermodynamics by B.K. Venkanna

3. Engineering Thermodynamics, by Er. S.K. Gupta