Tishk International University
Engineering Faculty
Petroleum and Mining Department

Petroleum Production Engineering I

Lecture 5: Wellbore Performance

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Content

- Wellbore performance
- Single-phase liquid flow
- Multiphase flow in oil wells
- Liquid Holdup
- TPR Models

Wellbore performance

Wellbore performance analysis involves establishing a relationship between tubular size, wellhead and bottom hole pressure, fluid properties, and fluid production rate.

Lecture 4 described reservoir deliverability. However, the **achievable oil production rate** from a well is determined by wellhead pressure and the flow performance of production string, that is, tubing, casing, or both. The flow performance of production string depends on geometries of the production string and properties of fluids being produced.

Understanding wellbore flow performance is vitally important to production engineers for **designing oil well equipment** and **optimizing well production** conditions.

This lecture focuses on determination of **TPR and pressure traverse** along the well string. Both **single-phase** and **multiphase fluids** are considered.

Single-Phase Liquid Flow

- Single-phase liquid flow exists in an oil well only when the wellhead pressure is above the bubble-point pressure of the oil, which is usually **not a reality**. However, it is convenient to start from single-phase liquid for establishing the concept of fluid flow in oil wells where multiphase flow usually dominates.

- Consider a fluid flowing from point 1 to point 2 in a tubing string of length *L* and height Δz (Figure 5-1).

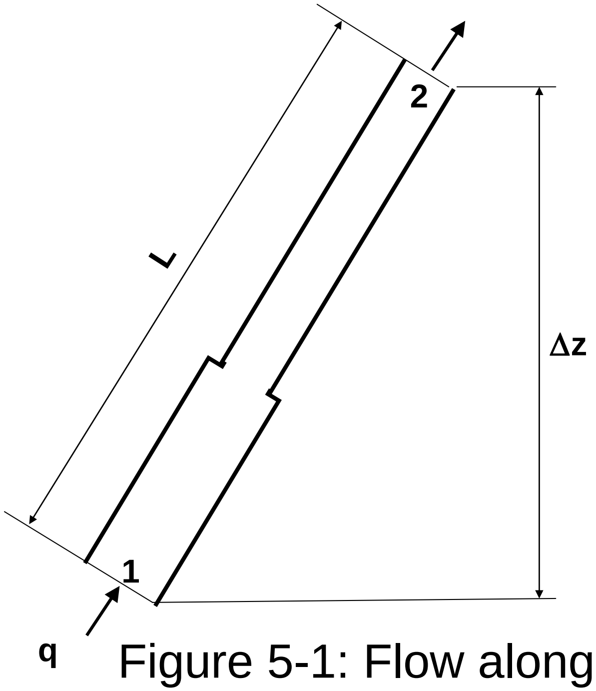


Figure 5-1: Flow along a tubing string

Single-Phase Liquid Flow

The first law of thermodynamics yields the following equation for pressure drop:

$$\Delta P = P_1 - P_2 = \frac{g}{g_c}\rho\Delta z + \frac{\rho}{2g_c}\Delta u^2 + \frac{2f_f\rho u^2 L}{g_c D} \qquad \textbf{(5.1)}$$

The first, second, and third term in the right-hand side of the equation represent pressure drops due to changes in **elevation**, **kinetic energy**, and **friction**, respectively.

The Fanning friction factor (f_F) can be evaluated based on Reynolds number and relative roughness.

Reynolds number is defined as the ratio of inertial force to viscous force.

ΔP = pressure drop, lb_f/ft^2
P_1 = pressure at point 1, lb_f/ft^2
P_2 = pressure at point 2, lb_f/ft^2
g = gravitational acceleration, 32.17 ft/s^2
g_c = unit conversion factor, 32.17 $\text{lb}_m\text{-ft}/\text{lb}_f\text{-s}^2$
ρ = fluid density lb_m/ft^3
Δz = elevation increase, ft
u = fluid velocity, ft/s
f_F = Fanning friction factor
L = tubing length, ft
D = tubing inner diameter, ft

Single-Phase Liquid Flow

The Reynolds number is expressed in consistent units as:

$$N_{\mathrm{Re}} = \frac{Du\rho}{\mu} \qquad \textbf{(5.2)}$$

or in U.S. field units as

$$N_{\mathrm{Re}} = \frac{1.48q\rho}{d\mu} \qquad \textbf{(5.3)}$$

N_{Re} = Reynolds number
q = fluid flow rate, bbl/day
ρ = fluid density $\mathrm{lb_m/ft^3}$
d = tubing inner diameter, in.
μ = fluid viscosity, cp

Single-Phase Liquid Flow

For **laminar flow** where $N_{\text{Re}} < 2000$, the fanning friction factor is inversely proportional to Reynolds number, or

$$f_f = \frac{16}{N_{\text{Re}}} \quad \textbf{(5.4)}$$

For **turbulent flow** where NRe > 2,100, the Fanning friction factor can be estimated using empirical correlations.

Chen's correlation takes the following form:

$$\frac{1}{\sqrt{f_f}} = -4\log\left\{\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{\text{Re}}}\log\left[\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{\text{Re}}}\right)^{0.8981}\right]\right\} \quad \textbf{(5.5)}$$

where the relative roughness is defined as

$$\varepsilon = \frac{\delta}{d}$$

δ :Absolute roughness of pipe wall.

Single-Phase Liquid Flow

The Fanning friction factor can also be obtained based on Darcy–Wiesbach friction factor shown in Fig. 5.2.

The Darcy–Wiesbach friction factor is also referred to as the Moody friction factor (f_M) in some literatures.

The relation between the Moody and the Fanning friction factor is expressed as

$$f_F = \frac{f_M}{4} \qquad \textbf{(5.6)}$$

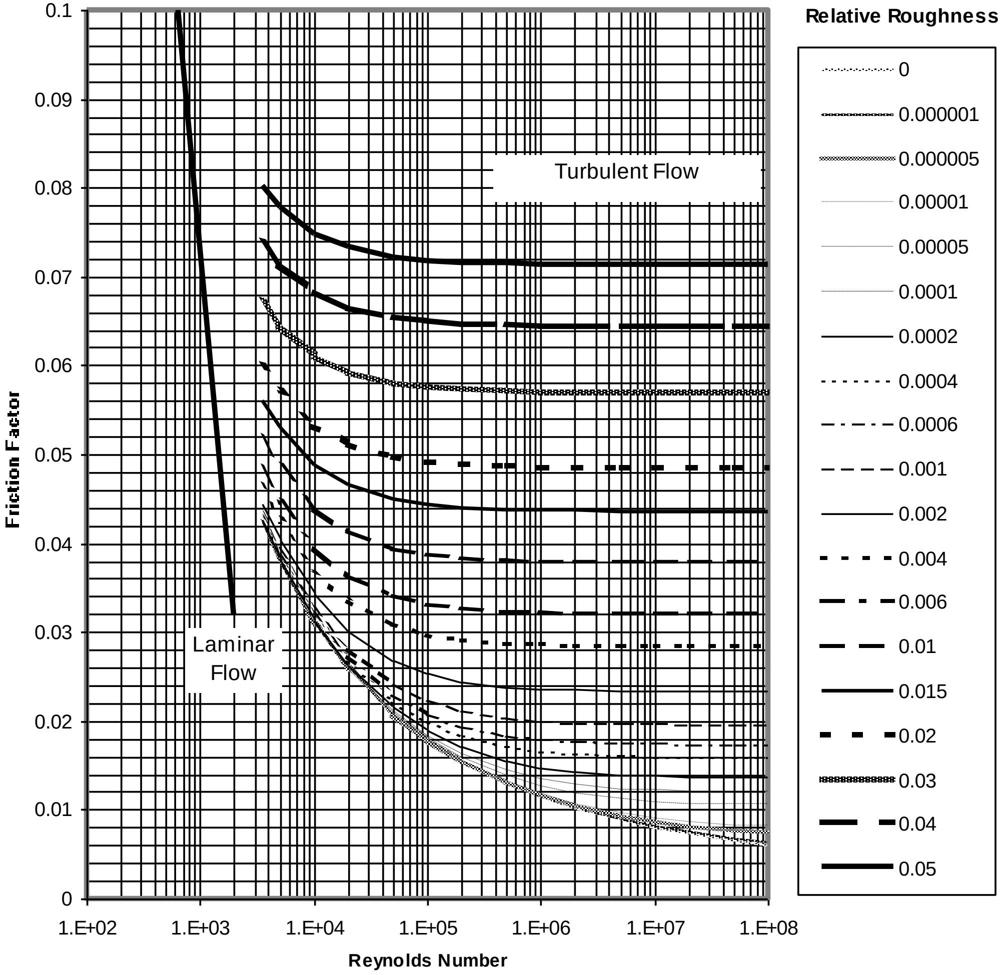


Figure 5-2: Moody friction factor diagram (After Moody, 1944)

Example Problem 5-1:

Suppose that 1000 bbl/day of 40º API, 1.2 cp oil is being produced through 2 $^{7/8}$-in., 8.6-lb$_m$/ft tubing in a well that is 15º from vertical. If the tubing wall relative roughness is 0.001, **calculate the pressure drop over 1000 ft of tubing.**

Solution:

Oil specific gravity:

$$\gamma_o = \frac{141.5}{{}^o API + 131.5}$$

$$= \frac{141.5}{40 + 131.5}$$

$$= 0.825$$

Oil density:

$$\rho = 62.5\gamma_o$$

$$= (62.5)(0.825)$$

$$= 51.57\ \text{lb}_m/\text{ft}^3$$

Elevation increase:

$$\Delta Z = \cos(\alpha)L$$

$$= \cos(15)(1000)$$

$$= 966\ \text{ft}$$

The 2 $^{7/8}$-in., 8.6-lb_m/ft tubing has an inner diameter of 2.259 in. Therefore

$$D = \frac{2.259}{12}$$

$$= 0.188\ \text{ft}$$

Fluid velocity can be calculated accordingly:

$$u = \frac{4q}{\pi D^2}$$

$$= \frac{4(5.615)(1000)}{\pi (0.188)^2 (86400)}$$

$$= 2.34 \text{ ft/s}$$

Reynolds number:

$$N_{\mathrm{Re}} = \frac{1.48 q\rho}{d\mu}$$

$$= \frac{1.48(1000)(51.57)}{(2.259)(1.2)}$$

$$= 28115 > 2100$$

Turbulent flow

Using Reynolds number of 28115 and relative roughness of 0.001, Moody friction factor diagram gives a Moody friction factor of 0.0265.

Thus

$$f_f = \frac{0.0265}{4} \qquad = 0.006625$$

Chen’s correlation gives:

$$\frac{1}{\sqrt{f_f}} = -4\log\left\{\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{\mathrm{Re}}}\log\left[\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{\mathrm{Re}}}\right)^{0.8981}\right]\right\}$$

$$= 12.3255$$

$$f_f = 0.006583$$

Pressure drop calculation:

$$\Delta P = \frac{g}{g_c}\rho\Delta z + \frac{\rho}{2g_c}\Delta u^2 + \frac{2f_f\rho u^2 L}{g_c D}$$

$$= \frac{32.17}{32.17}(51.57)(966) + \frac{56.57}{2(32.17)}(0)^2 + \frac{2(0.006625)(51.57)(2.34)^2(1000)}{(32.17)(0.188)}$$

$= 50435 \text{ lbf/ft}^2$

$= 350$ psi

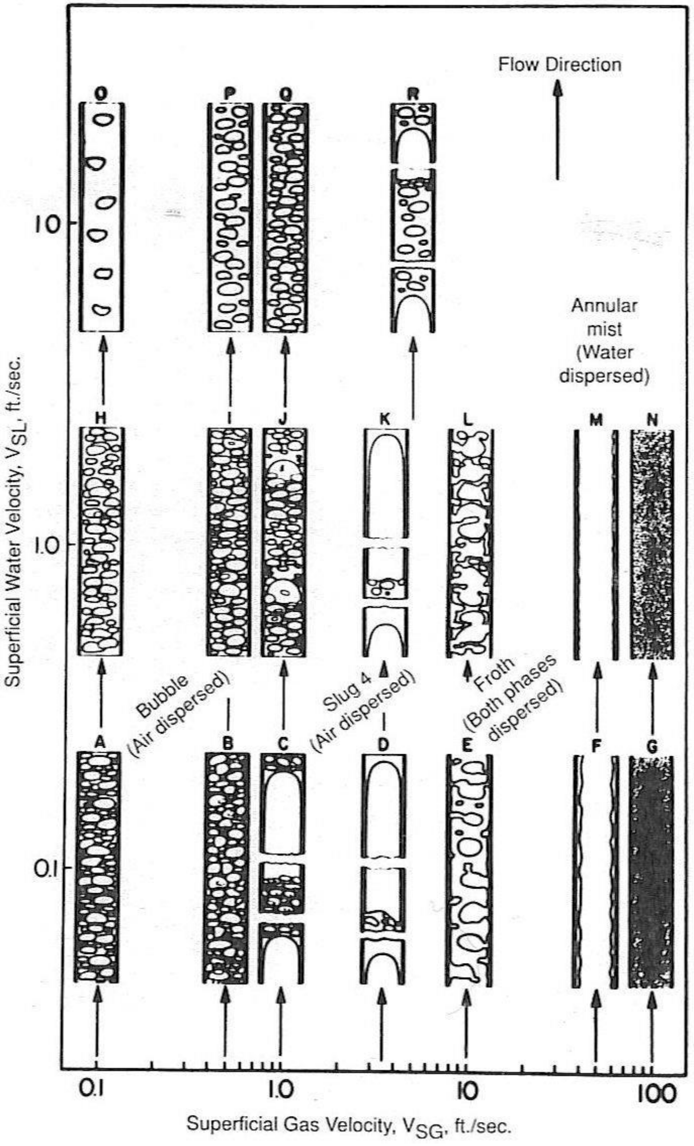
Multiphase Liquid Flow

In addition to oil, almost all oil wells produce a certain amount of water, gas, and sometimes sand. These wells are called multiphase-oil wells.

The TPR equation for single phase flow is not valid for multiphase oil wells. To analyze TPR of multiphase oil wells rigorously, a multiphase flow model is required.

Multiphase flow is much more complicated than single phase flow because of the variation of flow regime (or flow pattern).

Fluid distribution changes greatly in different flow regimes, which significantly affects pressure gradient in the tubing.
Two-phase flow patterns in vertical pipe are
Bubble, Slug, Churn and Annular flow



Flow Regimes

Liquid Holdup

In multiphase flow, the amount of the pipe **occupied by a phase** is often different from its **proportion of the total volumetric flow rate**. This is due to density difference between phases.

The density difference causes **dense phase to slip down** in an upward flow (i.e., the lighter phase moves faster than the denser phase).

Because of this, the **in-situ volume fraction of the denser phase** will be greater than the **input volume fraction of the denser phase** (i.e., the denser phase is ‘‘held up’’ in the pipe relative to the lighter phase).

Thus, liquid ‘‘holdup’’ is defined as

$$y_L = \frac{V_L}{V} \quad \textbf{(5.7)}$$

$y_{L:}$ liquid hold up, fraction

$V_{L:}$ volume of liquid phase in the pipe segment ft^3

V: volume of the pipe segment, ft^3

TPR Models

Numerous TPR models have been developed for analyzing multiphase flow in vertical pipes.

TPR models for multiphase flow wells fall into two categories:

1. **Homogeneous models**
    - Homogeneous models treat multiphase as a homogeneous mixture and do not consider the effects of liquid holdup (no-slip assumption).

    - These models are less accurate and are usually calibrated with local operating conditions in field applications. The major advantage of these models comes from their mechanistic nature. They can handle gas-oil water three-phase and gas-oil-water-sand four-phase systems. It is easy to code these mechanistic models in computer programs.

2. Separated flow models.

- Separated-flow models are more realistic than the homogeneous-flow models. They are usually given in the form of empirical correlations. The effects of liquid holdup (slip) and flow regime are considered.

- The major disadvantage of the separated flow models is that it is difficult to code them in computer programs because most correlations are presented in graphic form.

Homogeneous Models

According to Poettmann and Carpenter, the following equation can be used to calculate pressure traverse in a vertical tubing when the acceleration term is neglected:

$$\Delta p = \left(\bar{\rho} + \frac{\bar{k}}{\bar{\rho}} \right) \frac{\Delta h}{144} \qquad \textbf{(5.8)}$$

$$\bar{k} = \frac{f_f q_o{}^2 M^2}{7.4137 \times 10^{10} D^5} \qquad \textbf{(5.9)}$$

f_f = fanning friction factor for two-phase mixture
q_o = oil production rate, stb/day
M = total mass associated with 1 stb of oil
D = tubing inner diameter, ft.

Pressure traverse: Calculation of well pressure vs. depth by integrating the pressure gradient for increments of pipe length (MD)

Homogeneous Models

The average mixture density $\bar{\rho}$ can be calculated by

$$\bar{\rho} = \frac{\rho_1 + \rho_2}{2} \quad \textbf{(5.10)}$$

ρ_1 = mixture density at top of tubing segment, lb_f/ft^3

ρ_2 = mixture density at bottom of segment, lb_f/ft^3

Homogeneous Models

The mixture density at a given point can be calculated based on mass flow rate and volume flow rate, i.e.,

$$\rho = \frac{M}{V_m} \qquad \textbf{(5.11)}$$

where

$$M = 350.17(\gamma_o + WOR\ \gamma_w) + GOR\ \rho_{air}\gamma_g \qquad \textbf{(5.12)}$$

$$V_m = 5.615(B_o + WOR\ B_w) + (GOR - R_s)\left(\tfrac{14.7}{p}\right)\left(\tfrac{T}{520}\right)\left(\tfrac{z}{1.0}\right) \qquad \textbf{(5.13)}$$

Homogeneous Models

If data from direct measurements are not available, **solution gas-oil ratio** and **formation volume factor** of oil can be estimated using the following correlations:

$$R_s = \gamma_g \left[\frac{p}{18} \frac{10^{0.0125API}}{10^{0.00091t}} \right]^{1.2048} \qquad \textbf{(5.14)}$$

$$B_o = 0.971 + 0.000147 \left[R_s \left(\frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25t \right]^{1.175} \qquad \textbf{(5.15)}$$

Homogeneous Models

For easy coding in computer programs, Guo and Ghalambor (2002) developed the following correlation to represent the chart:

$$f_f = 10^{1.444-2.5\log(D\rho v)} \tag{5.16}$$

$(D\rho v)$ is the numerator of Reynolds number representing inertial force and can be formulated as:

$$(D\rho v) = \frac{1.4737\times 10^{-5} M q_o}{D} \tag{5.17}$$

Example Problem 5-2:

For the following given data, **calculate bottom hole pressure:**

Tubing head pressure:		500	psia
Tubing head temperature:	100	°F	
Tubing inner diameter:	1.66	in.	
Tubing shoe depth (near bottom hole):	5000	ft	
Bottom hole temperature:	150	°F	
Liquid production rate:	2000	stb/day	
Water cut:		25	%
Producing GLR:	1000	scf/stb	
Oil gravity:		30	oAPI
Water specific gravity:	1.05	1 for water	
Gas specific gravity:	0.65	1 for air	

Solution:

This problem can is solved using computer program Poettmann CarpenterBHP.xls. The result is shown in Table 5-1.

Table 5-1: Result given by Poettmann-CarpenterBHP.xls for Example Problem 5-2

Poettman-CarpenterBHP.xls
Description: This spreadsheet calculates flowing bottom hole pressure based on tubing head pressure and tubing flow performance using Poettmann-Carpenter Method.
Instruction: 1) Select a unit system; 2) Update parameter values in the Input Data section; 3) Click "Solution" button; and 4) View result in the Solution section.

Input Data:		***US Field Units***		***SI Units***
Tubing ID:	**1.66**	in		
Wellhead pressure:	**500**	psia		
Liquid production rate:	**2000**	stb/d		
Producing gas-liquid ratio (GLR):	**1000**	scf/stb		
Water cut (WC):	**25**	%		
Oil gravity:	**30**	°API		
Water specific gravity:	**1.05**	fresh water =1		
Gas specific gravity:	**0.65**	1 for air		
N_2 content in gas:	**0**	mole fraction		
CO_2 content in gas:	**0**	mole fraction		
H_2S content in gas:	**0**	mole fraction		
Formation volume factor for water:	**1.2**	rb/stb		
Wellhead temperature:	**100**	°F		
Tubing shoe depth:	**5000**	ft		
Bottom hole temperature:	**150**	°F		

Solution:					
	Oil specific gravity =	0.88	fresh water =1	0.88	fresh water =1
	Mass associated with 1 stb of oil =	495.66	lb	224.54	kg
	Solution gas ratio at wellhead =	78.42	scf/stb	13.97	sm^3/m^3
	Oil formation volume factor at wellhead =	1.04	rb/stb	1.04	rm^3/m^3
	Volume associated with 1 stb oil @ wellhead =	45.12	cf	1.28	m^3
	Fluid density at wellhead =	10.99	lb/cf	175.60	kg/m^3
	Solution gas-oil ratio at bottom hole =	301.79	scf/stb	53.75	sm^3/m^3
	Oil formation volume factor at bottom hole =	1.16	rb/stb	1.16	rm^3/m^3
	Volume associated with 1 stb oil @ bottom hole =	17.66	cf	0.50	m^3
	Fluid density at bottom hole =	28.07	lb/cf	448.64	kg/m^3
	The average fluid density =	19.53	lb/cf	312.12	kg/m^3
	Inertial force ($D\rho v$) =	79.21	lb/day-ft	117.69	kg/day-m
	Friction factor =	0.002		0.002	
	Friction term =	293.12	$(lb/cf)^2$	74901	$(kg/cm)^2$
	Error in depth =	0.00	ft	0.00	m
	Bottom hole pressure =	1699	psia	11.56	MPa

Guo-Ghalambor model takes a closed (integrated) form, which makes it easy to use.

Guo-Ghalambor (2005) model can be expressed as follows:

$$144b\left(p-p_{hf}\right)+\frac{1-2bM}{2}\ln\left|\frac{\left(144p+M\right)^{2}+N}{\left(144p_{hf}+M\right)^{2}+N}\right| -\frac{M+\frac{b}{c}N-bM^{2}}{\sqrt{N}}\left[\tan^{-1}\left(\frac{144p+M}{\sqrt{N}}\right)-\tan^{-1}\left(\frac{144p_{hf}+M}{\sqrt{N}}\right)\right]$$

$$=a\left(\cos\theta+d^{2}e\right)L \qquad \textbf{(5.18)}$$

where the group parameters are defined as

$$a = \frac{0.0765\gamma_g q_g + 350\gamma_o q_o + 350\gamma_w q_w + 62.4\gamma_s q_s}{4.07 T_{av} q_g} \quad \textbf{(5.19)}$$

$$b = \frac{5.615 q_o + 5.615 q_w + q_s}{4.07 T_{av} Q_g} \quad \textbf{(5.20)}$$

$$c = 0.00678 \frac{T_{av} q_g}{A} \quad \textbf{(5.21)}$$

$$d = \frac{0.00166}{A}\left(5.615q_o + 5.615q_w + q_s\right) \qquad (5.22)$$

$$e = \frac{f_M}{2gD_H} \qquad (5.23)$$

$$M = \frac{cde}{\cos\theta + d^2 e} \qquad (5.24)$$

$$N = \frac{c^2 e \cos\theta}{\left(\cos\theta + d^2 e\right)^2} \qquad (5.25)$$

A = cross-sectional area of conduit, ft2
D_H = hydraulic diameter, ft
f_M = Moody friction factor
g = gravitational acceleration, 32.17 ft/s2
L = conduit length, ft
p = pressure, psia
p_{hf} = wellhead flowing pressure, psia
q_g = gas production rate, scf/d
q_o = oil production rate, bbl/d
q_s = sand production rate, ft3/day
q_w = water production rate, bbl/d
g_g = specific gravity of gas, air =1
g_o = specific gravity of produced oil, fresh water =1
g_s = specific gravity of produced solid, fresh water =1
g_w = specific gravity of produced water, fresh water =1
T_{av} = average temperature, °R

Example Problem 5-3:

For the data given below, estimate bottom hole pressure with Guo-Ghalambor method.

Total measured depth:	7,000	ft
The average inclination angle:	20	deg
Tubing inner diameter	1.995	in.
Gas production rate:	1	MMscfd
Gas specific gravity:	0.7	air=1
Oil production rate:	1,000	stb/d
Oil specific gravity:	0.85	$H_2O=1$
Water production rate:	300	bbl/d
Water specific gravity:	1.05	$H_2O=1$
Solid production rate:	1	ft^3/d
Solid specific gravity:	2.65	$H_2O=1$
Tubing head temperature:	100	^oF
Bottom hole temperature:	160	^oF
Tubing head pressure:	300	psia

Solution:

This example problem is solved with the spreadsheet program Guo-GhalamborBHP.xls. The result is shown in Table 5-2.

Table 5-2: Result given by Guo-GhalamborBHP.xls for Example Problem 5-3

Guo-GhalamborBHP.xls
Description: This spreadsheet calculates flowing bottom hole pressure based on tubing head pressure and tubing flow performance using Guo-Ghalambor Method.
Instruction: 1) Select a unit system; 2) Update parameter values in the Input Data section; 3) Click "Solution" button; and 4) View result in the Solution section.

		U.S. FieldUnits		**SI Units**	
Input Data					
	Total measured depth:	7,000	ft		
	Average inclination angle:	20	deg		
	Tubing I.D.:	1.995	in.		
	Gas production rate:	1,000,000	scfd		
	Gas specific gravity:	0.7	air=1		
	Oil production rate:	1000	stb/d		
	Oil specific gravity:	0.85	$H_2O=1$		
	Water production rate:	300	bbl/d		
	Water specific gravity:	1.05	$H_2O=1$		
	Solid production rate:	1	ft^3/d		
	Solid specific gravity:	2.65	$H_2O=1$		
	Tubing head temperature:	100	°F		
	Bottom hole temperature:	160	°F		
	Tubing head pressure:	300	psia		

Solution					
	A =	3.1243196	in^2		
	D =	0.16625	ft		
	T_{av} =	622	^oR		
	$\cos(\theta)$ =	0.9397014			
	$(D\rho v)$ =	40.908853			
	f_M=	0.0415505			
	a =	0.0001713			
	b =	2.884E-06			
	c =	1349785.1			
	d =	3.8942921			
	e =	0.0041337			
	M =	20447.044			
	N =	6.669E+09			
	Bottom hole pressure, p_{wf} =	**1,682**	psia	11.44	MPa