Lecture 5: Wellbore Performance

4th-Grade - Fall Semester 2022-2023

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Content

• Wellbore performance
• Single-phase liquid flow
• Multiphase flow in oil wells
• Liquid Holdup
• TPR Models
Wellbore performance analysis involves establishing a relationship between tubular size, wellhead and bottom hole pressure, fluid properties, and fluid production rate.

Lecture 4 described reservoir deliverability. However, the achievable oil production rate from a well is determined by wellhead pressure and the flow performance of production string, that is, tubing, casing, or both. The flow performance of production string depends on geometries of the production string and properties of fluids being produced.

Understanding wellbore flow performance is vitally important to production engineers for designing oil well equipment and optimizing well production conditions.

This lecture focuses on determination of TPR and pressure traverse along the well string. Both single-phase and multiphase fluids are considered.
Single-Phase Liquid Flow

- Single-phase liquid flow exists in an oil well only when the wellhead pressure is above the bubble-point pressure of the oil, which is usually not a reality. However, it is convenient to start from single-phase liquid for establishing the concept of fluid flow in oil wells where multiphase flow usually dominates.

- Consider a fluid flowing from point 1 to point 2 in a tubing string of length $L$ and height $\Delta z$ (Figure 5-1).

![Figure 5-1: Flow along a tubing string](image-url)
The first law of thermodynamics yields the following equation for pressure drop:

$$\Delta P = P_1 - P_2 = \frac{g}{g_c} \rho \Delta z + \frac{\rho}{2 g_c} \Delta u^2 + \frac{2 f_f \rho u^2 L}{g_c D}$$ (5.1)

The first, second, and third term in the right-hand side of the equation represent pressure drops due to changes in elevation, kinetic energy, and friction, respectively.

The Fanning friction factor ($f_F$) can be evaluated based on Reynolds number and relative roughness.

Reynolds number is defined as the ratio of inertial force to viscous force.
The Reynolds number is expressed in consistent units as:

\[ N_{Re} = \frac{Du \rho}{\mu} \quad (5.2) \]

or in U.S. field units as

\[ N_{Re} = \frac{1.48 q \rho}{d \mu} \quad (5.3) \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>fluid flow rate, bbl/day</td>
</tr>
<tr>
<td>( \rho )</td>
<td>fluid density, lbm/ft(^3)</td>
</tr>
<tr>
<td>( d )</td>
<td>tubing inner diameter, in.</td>
</tr>
<tr>
<td>( \mu )</td>
<td>fluid viscosity, cp</td>
</tr>
</tbody>
</table>
Single-Phase Liquid Flow

For laminar flow where $N_{Re} < 2000$, the fanning friction factor is inversely proportional to Reynolds number, or

$$f_f = \frac{16}{N_{Re}} \quad (5.4)$$

For turbulent flow where $N_{Re} > 2100$, the Fanning friction factor can be estimated using empirical correlations.

Chen’s correlation takes the following form:

$$\frac{1}{\sqrt{f_f}} = -4 \log \left\{ \frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{Re}} \log \left[ \frac{\varepsilon^{1.1098}}{2.8257} + \left( \frac{7.149}{N_{Re}} \right)^{0.8981} \right] \right\} \quad (5.5)$$

where the relative roughness is defined as

$$\varepsilon = \frac{\delta}{d} \quad \delta: \text{Absolute roughness of pipe wall.}$$
Single-Phase Liquid Flow

The Fanning friction factor can also be obtained based on Darcy–Wiesbach friction factor shown in Fig. 5.2.

The Darcy–Wiesbach friction factor is also referred to as the Moody friction factor \((f_M)\) in some literatures.

The relation between the Moody and the Fanning friction factor is expressed as

\[
f_F = \frac{f_M}{4}
\]

Figure 5-2: Moody friction factor diagram (After Moody, 1944)
Example Problem 5-1:

Suppose that 1000 bbl/day of 40° API, 1.2 cp oil is being produced through 2 7/8-in., 8.6-lbm/ft tubing in a well that is 15° from vertical. If the tubing wall relative roughness is 0.001, calculate the pressure drop over 1000 ft of tubing.

Solution:

Oil specific gravity:  
\[ \gamma_o = \frac{141.5}{API + 131.5} \]

\[ = \frac{141.5}{40 + 131.5} \]

\[ = 0.825 \]
Oil density:

\[ \rho = 62.5 \gamma_o \]
\[ = (62.5)(0.825) \]
\[ = 51.57 \text{ lb}_m/\text{ft}^3 \]

Elevation increase:

\[ \Delta Z = \cos(\alpha)L \]
\[ = \cos(15)(1000) \]
\[ = 966 \text{ ft} \]

The 2 7/8-in., 8.6-lb$_m$/ft tubing has an inner diameter of 2.259 in. Therefore

\[ D = \frac{2.259}{12} \]
\[ = 0.188 \text{ ft} \]
Fluid velocity can be calculated accordingly:

\[ u = \frac{4q}{\pi D^2} \]

\[ = \frac{4(5.615)(1000)}{\pi (0.188)^2 (86400)} \]

\[ = 2.34 \text{ ft/s} \]

Reynolds number:

\[ N_{Re} = \frac{1.48q\rho}{d\mu} \]

\[ = \frac{1.48(1000)(51.57)}{(2.259)(1.2)} \]

\[ = 28115 > 2100 \]

Turbulent flow
Using Reynolds number of 28115 and relative roughness of 0.001, Moody friction factor diagram gives a Moody friction factor of 0.0265.

Thus

\[ f_f = \frac{0.0265}{4} = 0.006625 \]

Chen’s correlation gives:

\[
\frac{1}{\sqrt{f_f}} = -4 \log \left\{ \frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{Re}} \log \left[ \frac{\varepsilon^{1.1098}}{2.8257} + \left( \frac{7.149}{N_{Re}} \right)^{0.8981} \right] \right\} \]

\[ = 12.3255 \]

\[ f_f = 0.006583 \]
Pressure drop calculation:

\[ \Delta P = \frac{g}{g_c} \rho \Delta z + \frac{\rho}{2g_c} \Delta u^2 + \frac{2 f_f \rho u^2 L}{g_c D} \]

\[ = \frac{32.17}{32.17} (51.57)(966) + \frac{56.57}{2(32.17)} (0)^2 + \frac{2(0.006625)(51.57)(2.34)^2(1000)}{(32.17)(0.188)} \]

\[ = 50435 \text{ lbf/ft}^2 \]

\[ = 350 \text{ psi} \]
Multiphase Liquid Flow

In addition to oil, almost all oil wells produce a certain amount of water, gas, and sometimes sand. These wells are called multiphase-oil wells.

The TPR equation for single phase flow is not valid for multiphase oil wells. To analyze TPR of multiphase oil wells rigorously, a multiphase flow model is required.

Multiphase flow is much more complicated than single phase flow because of the variation of flow regime (or flow pattern).

Fluid distribution changes greatly in different flow regimes, which significantly affects pressure gradient in the tubing.

Two-phase flow patterns in vertical pipe are Bubble, Slug, Churn and Annular flow
Liquid Holdup

In multiphase flow, the amount of the pipe occupied by a phase is often different from its proportion of the total volumetric flow rate. This is due to density difference between phases.

The density difference causes dense phase to slip down in an upward flow (i.e., the lighter phase moves faster than the denser phase).

Because of this, the in-situ volume fraction of the denser phase will be greater than the input volume fraction of the denser phase (i.e., the denser phase is “held up” in the pipe relative to the lighter phase).

Thus, liquid “holdup” is defined as

\[ y_L = \frac{V_L}{V} \]  \hspace{1cm} (5.7)

\( y_L \): liquid hold up, fraction
\( V_L \): volume of liquid phase in the pipe segment ft\(^3\)
\( V \): volume of the pipe segment, ft\(^3\)
TPR Models

Numerous TPR models have been developed for analyzing multiphase flow in vertical pipes.

TPR models for multiphase flow wells fall into two categories:

1. **Homogeneous models**
   • Homogeneous models treat multiphase as a homogeneous mixture and do not consider the effects of liquid holdup (no-slip assumption).
   • These models are less accurate and are usually calibrated with local operating conditions in field applications. The major advantage of these models comes from their mechanistic nature. They can handle gas-oil-water three-phase and gas-oil-water-sand four-phase systems. It is easy to code these mechanistic models in computer programs.

2. **Separated flow models.**
   • Separated-flow models are more realistic than the homogeneous-flow models. They are usually given in the form of empirical correlations. The effects of liquid holdup (slip) and flow regime are considered.
   • The major disadvantage of the separated flow models is that it is difficult to code them in computer programs because most correlations are presented in graphic form.
Homogeneous Models

According to Poettmann and Carpenter, the following equation can be used to calculate pressure traverse in a vertical tubing when the acceleration term is neglected:

$$\Delta p = \left( \bar{\rho} + \frac{\bar{k}}{\bar{\rho}} \right) \frac{\Delta h}{144} \quad (5.8)$$

$$\bar{k} = \frac{f_f q_o^2 M^2}{7.4137 \times 10^{10} D^5} \quad (5.9)$$

- $f_f$ = fanning friction factor for two-phase mixture
- $q_o$ = oil production rate, stb/day
- $M$ = total mass associated with 1 stb of oil
- $D$ = tubing inner diameter, ft.

Pressure traverse: Calculation of well pressure vs. depth by integrating the pressure gradient for increments of pipe length (MD)
Homogeneous Models

The average mixture density $\bar{\rho}$ can be calculated by

$$\bar{\rho} = \frac{\rho_1 + \rho_2}{2} \quad (5.10)$$

$\rho_1$ = mixture density at top of tubing segment, lb$_f$/ft$^3$
$\rho_2$ = mixture density at bottom of segment, lb$_f$/ft$^3$
Homogeneous Models

The mixture density at a given point can be calculated based on mass flow rate and volume flow rate, i.e.,

\[ \rho = \frac{M}{V_m} \]  \hspace{1cm} (5.11)

where

\[ M = 350.17(\gamma_o + WOR \: \gamma_w) + GOR \: \rho_{air} \: \gamma_g \] \hspace{1cm} (5.12)

\[ V_m = 5.615(B_o + WOR \: B_w) + (GOR - R_s)\left(\frac{14.7}{p}\right)\left(\frac{T}{520}\right)\left(\frac{z}{1.0}\right) \] \hspace{1cm} (5.13)
Homogeneous Models

If data from direct measurements are not available, solution gas-oil ratio and formation volume factor of oil can be estimated using the following correlations:

\[ R_s = \gamma_g \left[ \frac{p}{18} \times \frac{10^{0.0125API}}{10^{0.00091t}} \right]^{1.2048} \]  

(5.14)

\[ B_o = 0.971 + 0.000147 \left[ R_s \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25t \right]^{1.175} \]  

(5.15)
Homogeneous Models

For easy coding in computer programs, Guo and Ghalambor (2002) developed the following correlation to represent the chart:

\[ f_f = 10^{1.444 - 2.5 \log(D \rho v)} \]  \hspace{1cm} (5.16)

\((D \rho v)\) is the numerator of Reynolds number representing inertial force and can be formulated as:

\[ (D \rho v) = \frac{1.4737 \times 10^{-5} M q_o}{D} \]  \hspace{1cm} (5.17)
Example Problem 5-2:

For the following given data, calculate bottom hole pressure:

- Tubing head pressure: 500 psia
- Tubing head temperature: 100 °F
- Tubing inner diameter: 1.66 in.
- Tubing shoe depth (near bottom hole): 5000 ft
- Bottom hole temperature: 150 °F
- Liquid production rate: 2000 stb/day
- Water cut: 25%
- Producing GLR: 1000 scf/stb
- Oil gravity: 30 oAPI
- Water specific gravity: 1.05 (1 for water)
- Gas specific gravity: 0.65 (1 for air)
Solution:

This problem can is solved using computer program Poettmann CarpenterBHP.xls. The result is shown in Table 5-1.

Table 5-1: Result given by Poettmann-CarpenterBHP.xls for Example Problem 5-2

<table>
<thead>
<tr>
<th>Poettman-CarpenterBHP.xls</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description:</strong> This spreadsheet calculates flowing bottom hole pressure based on tubing head pressure and tubing flow performance using Poettmann-Carpenter Method.</td>
</tr>
<tr>
<td><strong>Instruction:</strong> 1) Select a unit system; 2) Update parameter values in the Input Data section; 3) Click &quot;Solution&quot; button; and 4) View result in the Solution section.</td>
</tr>
<tr>
<td>Input Data:</td>
</tr>
<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>Tubing ID:</td>
</tr>
<tr>
<td>Wellhead pressure:</td>
</tr>
<tr>
<td>Liquid production rate:</td>
</tr>
<tr>
<td>Producing gas-liquid ratio (GLR):</td>
</tr>
<tr>
<td>Water cut (WC):</td>
</tr>
<tr>
<td>Oil gravity:</td>
</tr>
<tr>
<td>Water specific gravity:</td>
</tr>
<tr>
<td>Gas specific gravity:</td>
</tr>
<tr>
<td>N₂ content in gas:</td>
</tr>
<tr>
<td>CO₂ content in gas:</td>
</tr>
<tr>
<td>H₂S content in gas:</td>
</tr>
<tr>
<td>Formation volume factor for water:</td>
</tr>
<tr>
<td>Wellhead temperature:</td>
</tr>
<tr>
<td>Tubing shoe depth:</td>
</tr>
<tr>
<td>Bottom hole temperature:</td>
</tr>
</tbody>
</table>
**Solution:**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 1</th>
<th>Unit 1</th>
<th>Value 2</th>
<th>Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil specific gravity</td>
<td>0.88</td>
<td></td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>Mass associated with 1 stb of oil</td>
<td>495.66</td>
<td>lb</td>
<td>224.54</td>
<td>kg</td>
</tr>
<tr>
<td>Solution gas ratio at wellhead</td>
<td>78.42</td>
<td>scf/stb</td>
<td>13.97</td>
<td>sm³/m³</td>
</tr>
<tr>
<td>Oil formation volume factor at wellhead</td>
<td>1.04</td>
<td>rb/stb</td>
<td>1.04</td>
<td>rm³/m³</td>
</tr>
<tr>
<td>Volume associated with 1 stb oil @ wellhead</td>
<td>45.12</td>
<td>cf</td>
<td>1.28</td>
<td>m³</td>
</tr>
<tr>
<td>Fluid density at wellhead</td>
<td>10.99</td>
<td>lb/cf</td>
<td>175.60</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Solution gas-oil ratio at bottom hole</td>
<td>301.79</td>
<td>scf/stb</td>
<td>53.75</td>
<td>sm³/m³</td>
</tr>
<tr>
<td>Oil formation volume factor at bottom hole</td>
<td>1.16</td>
<td>rb/stb</td>
<td>1.16</td>
<td>rm³/m³</td>
</tr>
<tr>
<td>Volume associated with 1 stb oil @ bottom hole</td>
<td>17.66</td>
<td>cf</td>
<td>0.50</td>
<td>m³</td>
</tr>
<tr>
<td>Fluid density at bottom hole</td>
<td>28.07</td>
<td>lb/cf</td>
<td>448.64</td>
<td>kg/m³</td>
</tr>
<tr>
<td>The average fluid density</td>
<td>19.53</td>
<td>lb/cf</td>
<td>312.12</td>
<td>kg/m³</td>
</tr>
</tbody>
</table>
| Inertial force \(Dho
\)                              | 79.21   | lb/day-ft| 117.69  | kg/day-m |
| Friction factor                                       | 0.002   |        | 0.002   |        |
| Friction term                                          | 293.12  | (lb/cf)²| 74901   | (kg/cm)²|
| Error in depth                                         | 0.00    | ft     | 0.00    | m      |
| Bottom hole pressure                                   | 1699    | psia   | 11.56   | MPa    |
Guo-Ghalambor model takes a closed (integrated) form, which makes it easy to use. Guo-Ghalambor (2005) model can be expressed as follows:

\[
144b(p - p_{hf}) + \frac{1 - 2bM}{2} \ln \left( \frac{(144p + M)^2 + N}{(144p_{hf} + M)^2 + N} \right) - \frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}} \left[ \tan^{-1}\left( \frac{144p + M}{\sqrt{N}} \right) - \tan^{-1}\left( \frac{144p_{hf} + M}{\sqrt{N}} \right) \right]
\]

\[= a(\cos \theta + d^2 e)L \]

(5.18)
where the group parameters are defined as

\[ a = \frac{0.0765 \gamma_g q_g + 350 \gamma_0 q_o + 350 \gamma_w q_w + 62.4 \gamma_s q_s}{4.07 T_{av} q_g} \]  \hspace{1cm} (5.19)

\[ b = \frac{5.615 q_o + 5.615 q_w + q_s}{4.07 T_{av} Q_g} \]  \hspace{1cm} (5.20)

\[ c = 0.00678 \frac{T_{av} q_g}{A} \]  \hspace{1cm} (5.21)
\[ d = \frac{0.00166}{A} (5.615 q_o + 5.615 q_w + q_s) \]  \hspace{1cm} (5.22)

\[ e = \frac{f_M}{2 \ g D_H} \]  \hspace{1cm} (5.23)

\[ M = \frac{cde}{\cos \theta + d^2 e} \]  \hspace{1cm} (5.24)

\[ N = \frac{c^2 e \cos \theta}{(\cos \theta + d^2 e)^2} \]  \hspace{1cm} (5.25)
$A$ = cross-sectional area of conduit, ft\(^2\)

$D_H$ = hydraulic diameter, ft

$f_M$ = Moody friction factor

$g$ = gravitational acceleration, 32.17 ft/s\(^2\)

$L$ = conduit length, ft

$p$ = pressure, psia

$p_{hf}$ = wellhead flowing pressure, psia

$q_g$ = gas production rate, scf/d

$q_o$ = oil production rate, bbl/d

$q_s$ = sand production rate, ft\(^3\)/day

$q_w$ = water production rate, bbl/d

$g_g$ = specific gravity of gas, air =1

$g_o$ = specific gravity of produced oil, fresh water =1

$g_s$ = specific gravity of produced solid, fresh water =1

$g_w$ = specific gravity of produced water, fresh water =1

$T_{av}$ = average temperature, °R
Example Problem 5-3:

For the data given below, estimate bottom hole pressure with Guo-Ghalambor method.

- Total measured depth: \(7,000\) ft
- The average inclination angle: \(20\) deg
- Tubing inner diameter: \(1.995\) in.
- Gas production rate: \(1\) MMscfd
- Gas specific gravity: \(0.7\) air=1
- Oil production rate: \(1,000\) stb/d
- Oil specific gravity: \(0.85\) \(\text{H}_2\text{O}=1\)
- Water production rate: \(300\) bbl/d
- Water specific gravity: \(1.05\) \(\text{H}_2\text{O}=1\)
- Solid production rate: \(1\) \(\text{ft}^3/\text{d}\)
- Solid specific gravity: \(2.65\) \(\text{H}_2\text{O}=1\)
- Tubing head temperature: \(100\) \(\circ\text{F}\)
- Bottom hole temperature: \(160\) \(\circ\text{F}\)
- Tubing head pressure: \(300\) psia
Solution:

This example problem is solved with the spreadsheet program Guo-GhalamborBHP.xls. The result is shown in Table 5-2.

Table 5-2: Result given by Guo-GhalamborBHP.xls for Example Problem 5-3

<table>
<thead>
<tr>
<th>Description:</th>
<th>This spreadsheet calculates flowing bottom hole pressure based on tubing head pressure and tubing flow performance using Guo-Ghalambor Method.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instruction:</td>
<td>1) Select a unit system; 2) Update parameter values in the Input Data section; 3) Click &quot;Solution&quot; button; and 4) View result in the Solution section.</td>
</tr>
<tr>
<td>Input Data</td>
<td>U.S. FieldUnits</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Total measured depth:</td>
<td>7,000 ft</td>
</tr>
<tr>
<td>Average inclination angle:</td>
<td>20 deg</td>
</tr>
<tr>
<td>Tubing I.D.:</td>
<td>1.995 in.</td>
</tr>
<tr>
<td>Gas production rate:</td>
<td>1,000,000 scfd</td>
</tr>
<tr>
<td>Gas specific gravity:</td>
<td>0.7 air=1</td>
</tr>
<tr>
<td>Oil production rate:</td>
<td>1000 stb/d</td>
</tr>
<tr>
<td>Oil specific gravity:</td>
<td>0.85 H₂O=1</td>
</tr>
<tr>
<td>Water production rate:</td>
<td>300 bbl/d</td>
</tr>
<tr>
<td>Water specific gravity:</td>
<td>1.05 H₂O=1</td>
</tr>
<tr>
<td>Solid production rate:</td>
<td>1 ft³/d</td>
</tr>
<tr>
<td>Solid specific gravity:</td>
<td>2.65 H₂O=1</td>
</tr>
<tr>
<td>Tubing head temperature:</td>
<td>100 °F</td>
</tr>
<tr>
<td>Bottom hole temperature:</td>
<td>160 °F</td>
</tr>
<tr>
<td>Tubing head pressure:</td>
<td>300 psia</td>
</tr>
<tr>
<td>Solution</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>A = 3.1243196 in²</td>
<td></td>
</tr>
<tr>
<td>D = 0.16625 ft</td>
<td></td>
</tr>
<tr>
<td>$T_{av}$ = 622 °R</td>
<td></td>
</tr>
<tr>
<td>$\cos(\theta) = 0.9397014$</td>
<td></td>
</tr>
<tr>
<td>$(D_{pv}) = 40.908853$</td>
<td></td>
</tr>
<tr>
<td>$f_{M} = 0.0415505$</td>
<td></td>
</tr>
<tr>
<td>a = 0.0001713</td>
<td></td>
</tr>
<tr>
<td>b = 2.884E-06</td>
<td></td>
</tr>
<tr>
<td>c = 1349785.1</td>
<td></td>
</tr>
<tr>
<td>d = 3.8942921</td>
<td></td>
</tr>
<tr>
<td>e = 0.0041337</td>
<td></td>
</tr>
<tr>
<td>M = 20447.044</td>
<td></td>
</tr>
<tr>
<td>N = 6.669E+09</td>
<td></td>
</tr>
<tr>
<td>Bottom hole pressure, $p_{wf} = 1,682$ psia</td>
<td></td>
</tr>
<tr>
<td>11.44 MPa</td>
<td></td>
</tr>
</tbody>
</table>