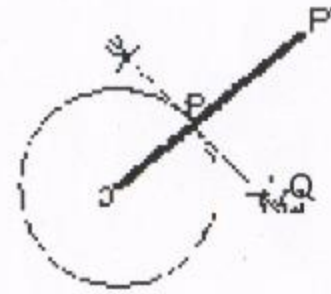


**To draw a tangent to a circle construction (Fig. a and b)**

**(a) At any point P on the circle.**

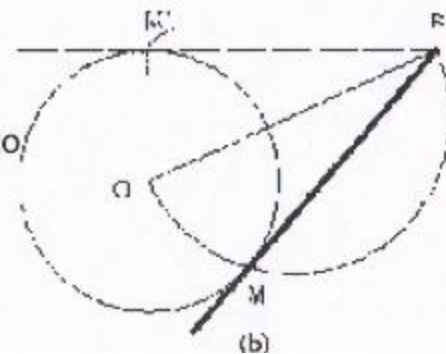
1. With O as centre, draw the given circle. P is any point on the circle at which tangent to be drawn (Fig .a)
2. Join O with P and produce it to  $p'$  so that  $OP = pp'$
3. With O and  $p'$  as centres and a length greater than OP as radius, draw arcs intersecting each other at Q.
4. Draw a line through P and Q. This line is the required tangent that will be perpendicular to OP at P.



(a)

**(b) From any point outside the circle.**

1. With O as centre, draw the given circle. P is the point outside the circle from which tangent is to be drawn to the circle (F ig. b).
2. Join O with P. With OP as diameter, draw a semi-circle intersecting the given circle at M.
3. Then, the line drawn through P and M is the required tangent.
4. If the semi-circle is drawn on the other side, it will cut the given circle at M'. Then the line through P and M' will also be a tangent to the circle from P.



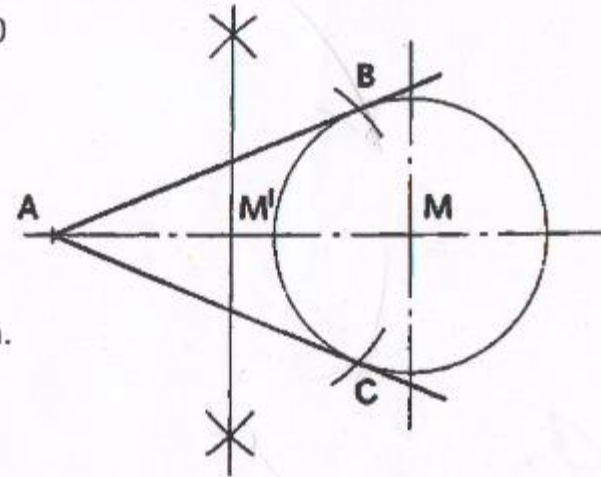
(b)

### 1- Circular Arc Connections

a-Construct the tangents to a circle having a diameter of 40 mm!.The tangents have to pass through the common intersection A. The distance AM is 55 mm. Connect A with B and A with C. AB and AC are the tangents.

Solution:

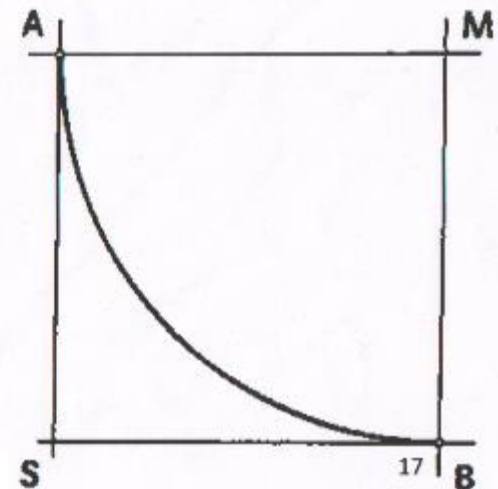
- 1-Find the centre M.
- 2-Draw circle with M as the centre and a radius of 20 mm. Find A (AM = 55 mm).
- 3-AM Bisect AM, thus obtaining M' (AM' = M'M ).
- 4-Draw arcs with M' as the centre and AM' as the radius,
- 5-thus obtaining B and C.



b-The two legs of a right angle have to be connected by a circular arc with a radius of 30 mm.

Solution:

- 1-Draw a right angle, thus obtaining S.
- 2-Draw two parallels within the angle at a distance of 30 mm in any case.
- 3-The intersection of the parallels is the centre M.
- 4-The perpendiculars (AM and BM) are the points of connection.

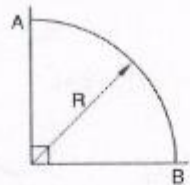


# Constructing arcs tangent to two lines

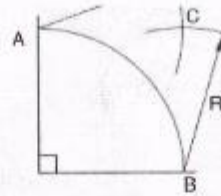
(A)



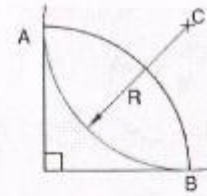
Right Angle



Step 1



Step 2

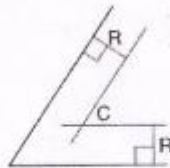


Step 3

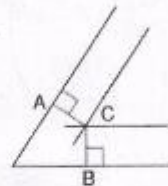
(B)



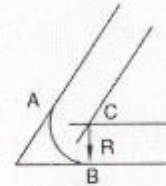
Acute Angle



Step 1



Step 2

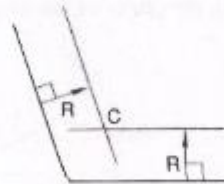


Step 3

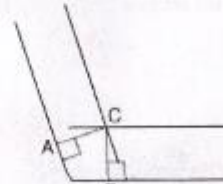
(C)



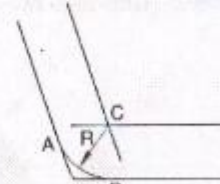
Obtuse Angle



Step 1



Step 2

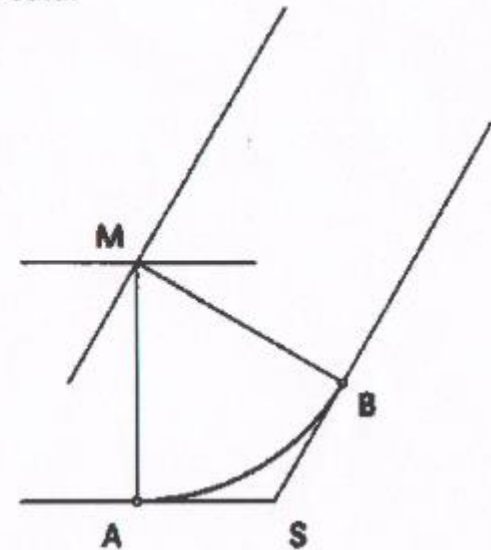


Step 3

c-The two legs of an angle of  $120^\circ$  have to be connected by a circular arc with a radius of 36 mm.

Solution:

- 1-Draw the angle ( $90^\circ + 30^\circ!$ ), thus obtaining S.
- 2-Draw two parallels at a distance of 36 mm each, thus obtaining M.
- 3-Draw circle with M as the centre and a radius of 36mm.
- 4-The perpendiculars (AM and BM) are the points of connection.



**d-Two adjacent circles have the following diameters:**

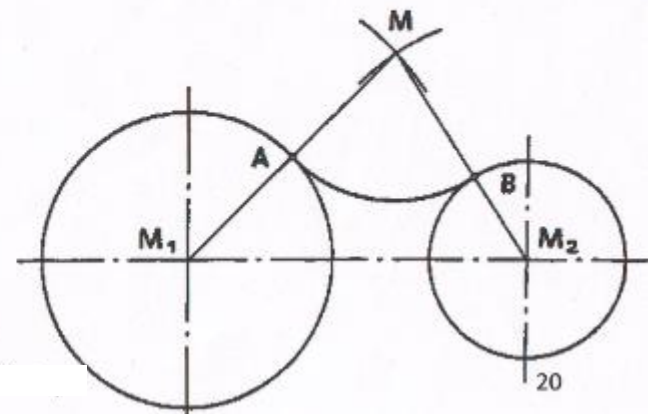
$d_1 = 50 \text{ mm}$

$d_2 = 30 \text{ mm}$

The distance between their centres is 70 mm. The two circles have to be connected with a transition radius ( $R_{tr}$ ) of 25 mm!

Solution:

- 1-Draw the two circles, thus obtaining  $M_1$  and  $M_2$ .
  - 2-Draw a circular arc with  $M_1$  as the centre and the radius of  $R_1 + R_{tr}$  ( $30 \text{ mm} + 25 \text{ mm}$ ).
  - 3-Draw a circular arc with  $M_2$  as the centre and the radius of  $R_2 + R_{tr}$  ( $15 \text{ mm} + 25 \text{ mm}$ ).
- The intersection is M.
- 4-Connect M with  $M_1$  and  $M_2$ , thus obtaining A and B.
- A and B are the points of connection.



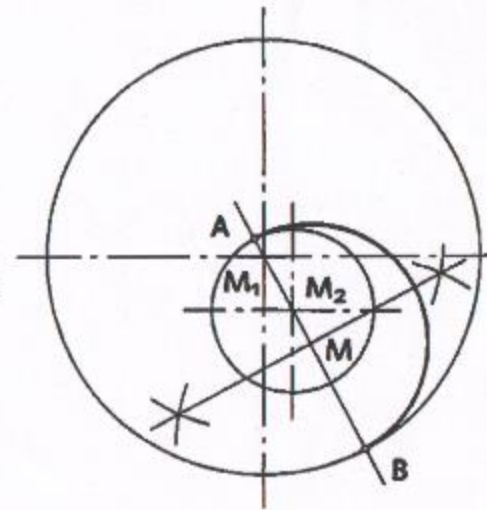
**e-Two circles, an inner circle and an outer circle,**  
have to be connected by a circular arc. The circles have the  
following diameters:  
outer circle =  $d_1 = 60$  mm, inner circle =  $d_2 = 25$  mm

Solution:

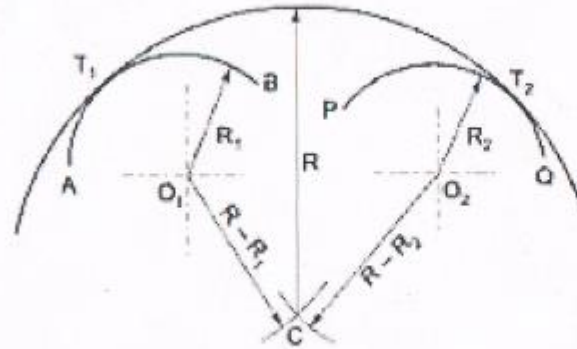
1-Draw the two circles, thus obtaining  $M_1$  and  $M_2$ .  
2- Connect the centres and extend to the circles, thus obtaining  
A and B. Bisect AB, thus obtaining M as the centre for  
the circular arc.

3-Two adjacent circles have to be connected by  
a tangent. Their diameters are:  
 $d_1 = 80$  mm  
 $d_2 = 34$  mm

The distance between the centres of the two circles is 85 mm.

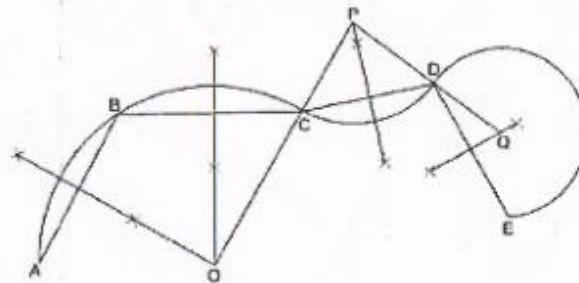


**Case :** Draw an arc of given radius  $R$  touching two given arcs of radius  $R_1$  and  $R_2$  (internally) [Fig. 1.]



1. Draw an arc with  $O_1$  as centre and radius equal to  $(R - R_1)$ .
2. With  $O_2$  as the centre and radius equal to  $(R - R_2)$  draw an arc touching the previous arc at point  $C$ .
3. Draw the required arc,  $C$  as centre and radius equal to  $R$ .

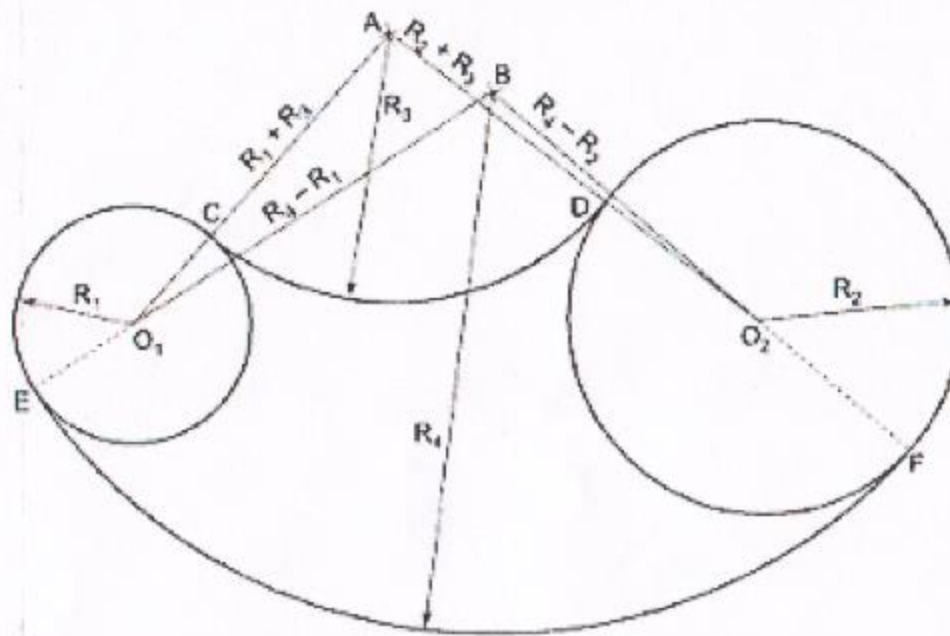
**Principle :** To draw a continuous curve of circular arcs passing through any number of given points. (Fig. 2).



**Principle :** Let  $A, B, C, D,$  and  $E$  be the location of the given points.

1. Join the points  $A$  with  $B, B$  with  $C, C$  with  $D$  and  $D$  with  $E,$  etc.
2. Draw perpendicular bisector of lines  $AB$  and  $BC$  to intersect at  $O$ .
3. Draw an arc  $ABC, O$  as centre and radius equal to  $OA$ .
4. Draw a line through the points  $O$  and  $C$ .
5. Draw the perpendicular bisector of  $CD$  to intersect  $OC$  or  $OC$  produced at  $P$ .
6. Draw an arc  $CD, P$  as the centre and radius equal to  $PC$ .
7. Draw the line through the points  $P$  and  $D$ .
8. Draw the perpendicular bisector of  $DE$  to intersect  $PD$  produced at  $Q$ .
9. Draw an arc  $DE, Q$  as centre and radius equal to  $DQ$ .

To join two circles of centres  $O_1$  and  $O_2$  with two fillet arcs of radius  $R_3$  and  $R_4$  (Fig. ).



1. Draw an arc with  $O_1$  as centre and radius  $(R_1 + R_3)$
2. With  $O_2$  as centre and  $(R_2 + R_3)$  radius draw an arc to intersect the previous arc at  $A$ , join  $O_1A$  and  $O_2A$  to intersect circles at  $C$  and  $D$ .
3. Draw the required fillet of radius  $R_3$  with  $A$  as centre and  $R_3$  radius, between  $C$  and  $D$ .
4. Draw an arc with  $O_1$  as centre and  $(R_4 - R_1)$  radius.
5. With  $O_2$  as centre and  $(R_4 - R_2)$  radius draw an arc to intersect the previous arc at  $B$ . Join  $O_1B$  and  $O_2B$  and produce to meet circles at  $E$  and  $F$ .
6. Draw the required fillet of radius  $R_4$  with  $B$  as centre and  $R_4$  radius, between  $E$  and  $F$ .

Fig. 1 Tangents to two circles

Solution:

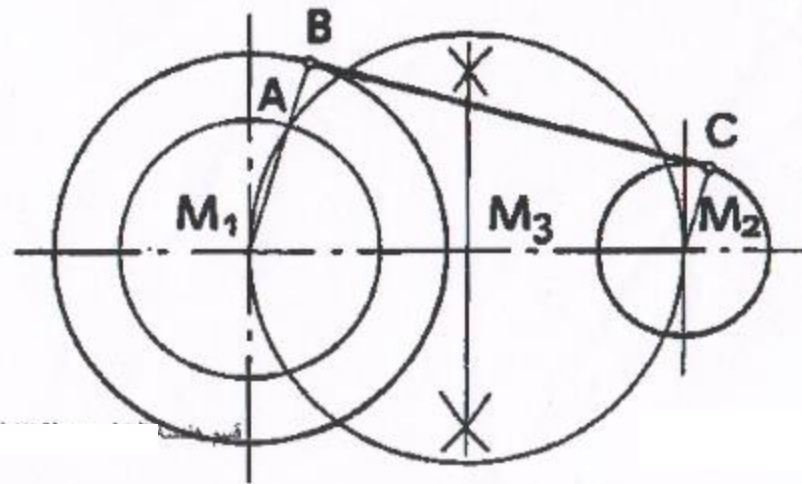
1-Draw the two circles, thus obtaining  $M_1$  and  $M_2$   
( $M_1 M_2 = 85$  mm).

2-Draw circle with the centre  $M_1$  and the radius of  $R_1 - R_2$  ( $40$  mm -  $17$  mm).

3-Bisect  $M_1 M_2$ . Thus obtaining  $M_3$ .

4-Draw a circle with  $M_3$  as the centre and  $M_1 M_3$  as the radius, thus obtaining A.

5-Extend  $M_1 A$  over A, thus obtaining B.





## 9-Ellipse

Given: Major axis AB, Minor axis CD

**Solution:** CONCENTRIC CIRCLES METHOD

1-Draw the major axis and the minor axis, thus also obtaining, A, B, C, D and M.

2-Draw a circle with M as the centre and the radius equal to the major axis.

3-Draw a circle with M as the centre and the radius equal to the minor axis.

4-Draw any number of radii from M.

5-Mark off the points on the outer circle (F, H, K, N).

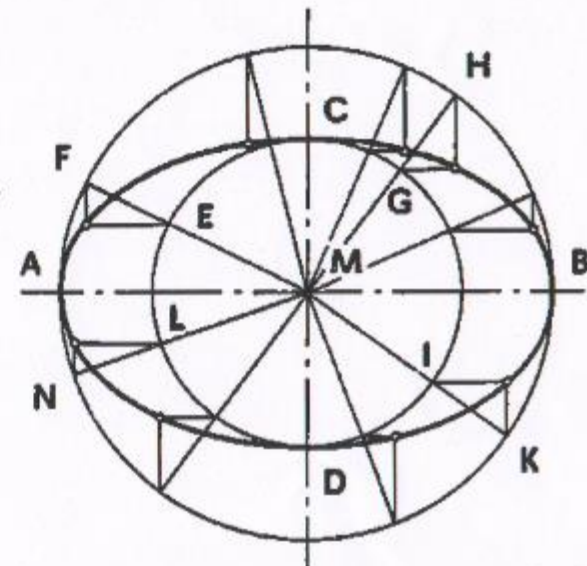
6-Mark off the points on the inner circle (E, G, I, L).

7-Draw further radii (without letter in the illustration).

8-Draw lines parallel to major axis through points provided by the intersections.

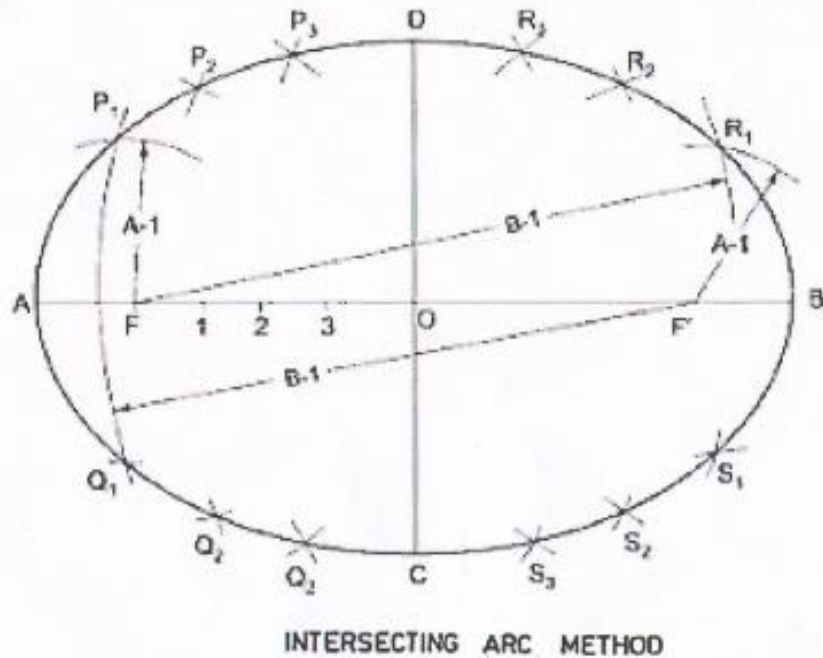
9-Draw lines parallel to the minor axis through points provided by intersections.

The intersections of the vertical and horizontal lines provide points of the ellipse.



CONCENTRIC CIRCLES METHOD

Draw an ellipse with major axis equal to 120 mm and minor axis equal to 80 mm [Arcs of Circles Method] (Fig. ).( INTERSECTING ARC METHOD)



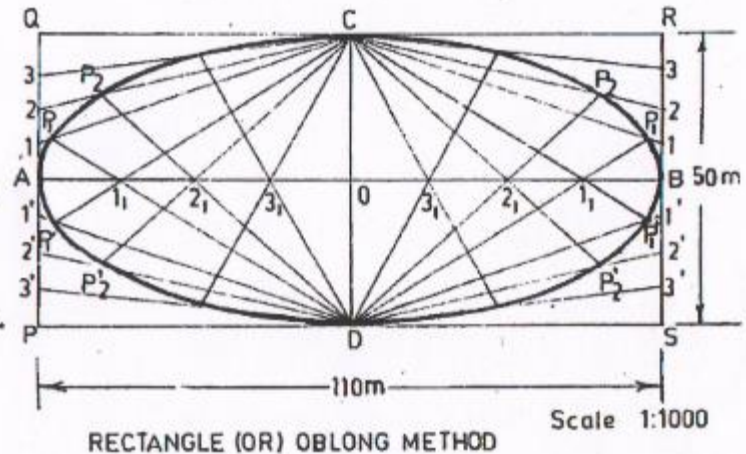
**Solution:**

1. Draw major axis  $AB = 120$  mm and minor axis  $DC = 80$  mm perpendicularly bisecting each other at  $O$ .
2. With centre  $C$  or  $D$  radius equal to  $AO$ , draw two arcs to intersect  $AB$  at  $F$  and  $F'$ .
3. Mark any number of points 1, 2, 3 ... etc. (say upto 10) at approximately equal intervals on  $AB$  in between  $F$  and  $F'$ .
4.  $F$  as centre and  $AF$  as radius draw two arcs on either side of  $AB$ .  $F'$  as centre and  $F'B$  as radius draw two arcs to cut the previous arcs at  $P_1, Q_1, R_1, S_1$ .
5. Repeat the above and mark the points  $P_2, Q_2, R_2, S_2$  and draw a smooth curve which is the required ellipse.

### Rectangle Method (or) Oblong Method

(Fig. ) A plot of ground is in the shape of a rectangle 110m x 50 m. Inscribe an elliptical lawn in it. Take a of 1 : 1000.

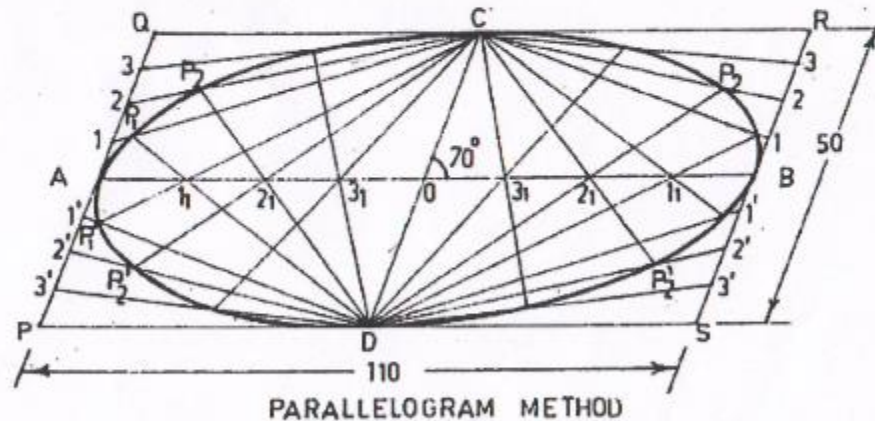
1. Draw the major axis AB = 110mm and minor axis CD = 50 mm. Both axes bisect each other at O.
2. Through A and B draw lines parallel to CD.
3. Through C and D draw lines parallel to AB and construct the rectangle PQRS. Now PS = AB and SR = CD.
4. Divide AQ and AP into any number of equal parts (say 4) and name the points as 1, 2, 3 and 1', 2', 3' respectively starting from A on AQ and AP.
5. Divide AD into same (4) number of equal parts, and name the points as 1<sub>1</sub>, 2<sub>1</sub>, 3<sub>1</sub> starting from A on AD.
6. Join 1, 2, 3 with C. Join D1<sub>1</sub> and extend it to intersect C1 at P<sub>1</sub>.
7. Similarly extend D2<sub>1</sub> and D3<sub>1</sub> to intersect C2 and C3 at P<sub>2</sub> and P<sub>3</sub> respectively. Join 1', 2', 3' with D.
8. Join C1' and extend it to intersect D1' at P<sub>1</sub>'.
9. Similarly extend C2' and C3' to intersect D2' and D3' at P<sub>2</sub>' and P<sub>3</sub>' respectively.
10. Draw a smooth curve through C, P<sub>3</sub>, P<sub>2</sub>, P<sub>1</sub>, A, P<sub>1</sub>', P<sub>2</sub>', P<sub>3</sub>', D and obtain one half (left-half) of the ellipse.
11. Repeat the above and draw the right-half of the ellipse, which is symmetrical to the left-half.



Construct an ellipse when a pair of Conjugate Diameters AB and CD are equal to 110 mm and 50 mm respectively. The angle between the conjugate diameters is  $70^\circ$ . (Fig. )

To construct the ellipse using conjugate diameters :

1. Draw a conjugate diameter AB = 110 mm and bisect it at O.
2. Angle between the conjugate diameters is  $70^\circ$ . Therefore draw another conjugate diameter CD through O such that the angle COB =  $70^\circ$ .
3. Through A and B draw lines parallel to CD. Through C and D draw lines parallel to AB and construct a parallelogram PQRS as shown.
4. Repeat the procedure given in step Nos. 4 to 11 in problem 10 and complete the construction of the ellipse inside the parallelogram PQRS.



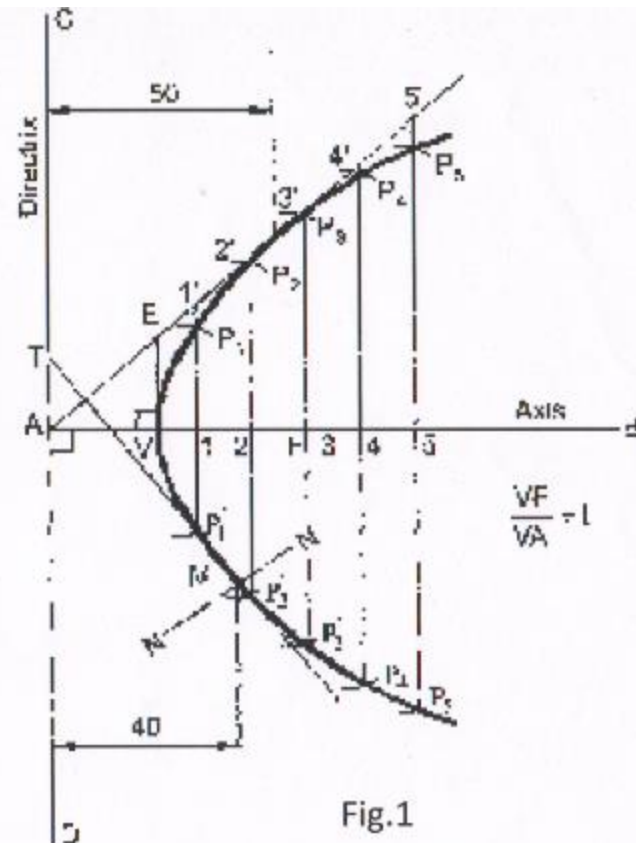
**What are Conjugate Diameters ?**

Conjugate Diameters are lines passing through the centre of ellipse and parallel to the tangents on the curve at the points of intersection of the other diameter with the ellipse.

To draw a parabola with the distance of the focus from the directrix at 50mm.

(Eccentricity method Fig.1).

1. Draw the axis AB and the directrix CD at right angles to it:
2. Mark the focus F on the axis at 50mm.
3. Locate the vertex V on AB such that  $AV = VF$
4. Draw a line VE perpendicular to AB such that  $VE = VF$
5. Join A,E and extend. Now,  $VE/vA = VF/vA = 1$ , the eccentricity.
6. Locate number of points 1,2,3, etc., to the right of V on the axis, which need not be equidistant.
7. Through the points 1,2,3, etc., draw lines perpendicular to the axis and to meet the line AE extended at 1',2',3' etc.
8. With centre F and radius 1-1' draw arcs intersecting the line 11' through I at P I and P'I.
9. Similarly, locate the points P<sub>2</sub>,p'<sub>2</sub>, P<sub>3</sub>, P'<sub>3</sub>, etc., on either side of the axis. Join the points by smooth curve, forming the required parabola.

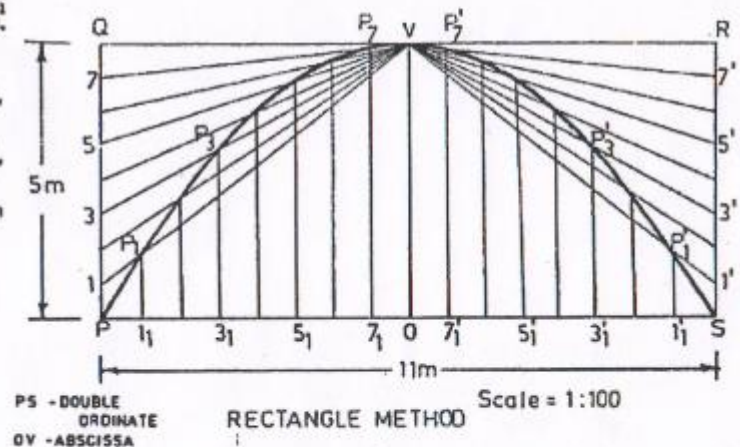


**To draw a normal and tangent through a point 40mm from the directrix.**

To draw a tangent and normal to the parabola. locate the point M which is at 40 mm from the Directrix. Then join M to F and draw a line through F, perpendicular to MF to meet the directrix at T. The line joining T and M and extended is the tangent and a line NN, through M and perpendicular to TM is the normal to the curve.

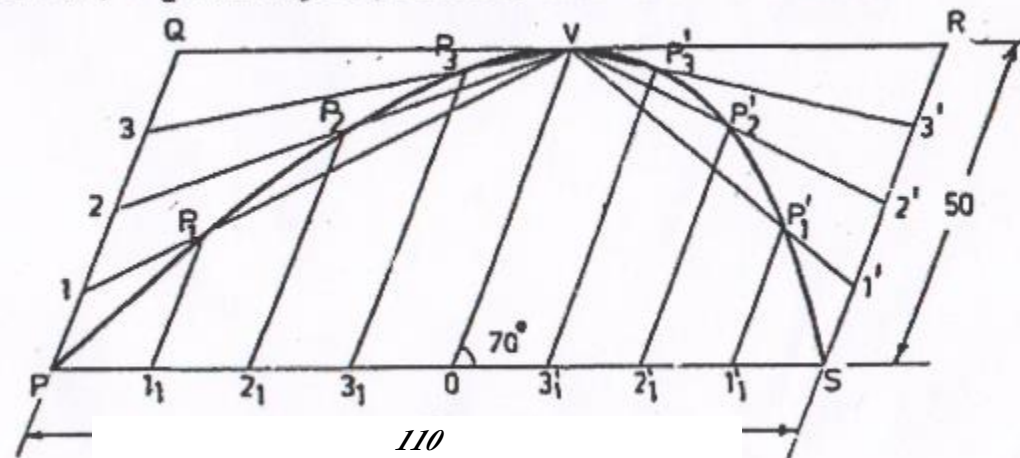
A ball thrown from the ground level reaches (Fig. ) a maximum height of 5 m and travels a horizontal distance of 11 m from the point of projection. Trace the path of the ball (parabola).

- The ball travels a horizontal distance of 11m. Take a scale of 1 : 100. Draw  $PS = 11$  cm to represent the double ordinate. Bisect  $PS$  at  $O$ .
- The ball reaches a maximum height of 5m. So from  $O$  erect vertical and mark the vertex  $V$  such that  $OV = 5$ cm. Now  $OV$  is the abscissa.
- Construct the rectangle  $PQRS$  such that  $PS$  is the double ordinate and  $PQ = RS = VO$  (abscissa).
- Divide  $PQ$  and  $RS$  into any number of (say 8) equal parts as  $1, 2, \dots, 8$  and  $1', 2', \dots, 8'$  respectively, starting from  $P$  on  $PQ$  and  $S$  on  $SR$ . Join  $1, 2, \dots, 8$  and  $1', 2', \dots, 8'$  with  $V$ .
- Divide  $PO$  and  $OS$  into 8 equal parts as  $1_1, 2_1, \dots, 8_1$  and  $1'_1, 2'_1, \dots, 8'_1$  respectively, starting from  $P$  on  $PO$  & from  $S$  on  $SO$ .
- From  $1_1$  erect vertical to meet the line  $V1$  at  $P_1$ .
- Similarly from  $2_1, \dots, 8_1$  erect verticals to meet the lines  $V2, \dots, V8$  at  $P_2, \dots, P_8$  respectively.
- Also erect verticals from  $1'_1, 2'_1, \dots, 8'_1$  to meet the lines  $V1', V2', \dots, V8'$  at  $P'_1, P'_2, \dots, P'_8$  respectively.
- Join  $P, P_1, P_2, \dots, P_7, V, P'_7, \dots, P'_1$  and  $S$  to represent the path of the ball (parabola).



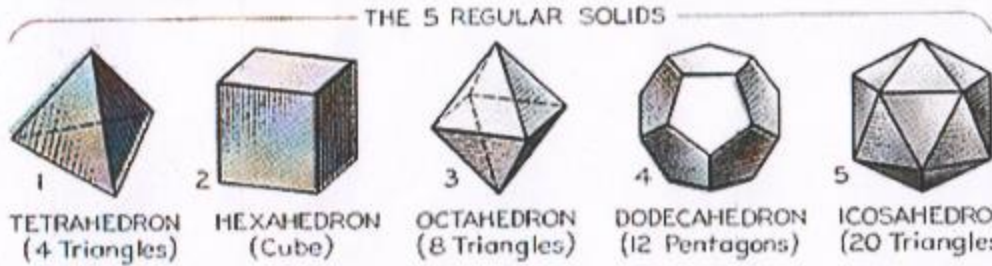
Construct a parabola within a parallelogram (Fig. ) of sides 110 mm x 50 mm. One of the included angle between the sides is  $70^\circ$ . (Parallelogram Method)

1. Construct the parallelogram PQRS (PS = 110mm and PQ = 50mm and angle QPS =  $70^\circ$ ). Bisect PS at O and draw VO parallel to PQ.
2. Divide PQ and SR into any number of (4) equal parts as 1, 2, 3 and 1', 2', 3' respectively starting from P on PQ and from S on SR. Join V1, V2 & V3. Also join V1', V2' & V3'.
3. Divide PO and OS into 4 equal parts as 1<sub>1</sub>, 2<sub>1</sub>, 3<sub>1</sub> and 1'<sub>1</sub>, 2'<sub>1</sub>, 3'<sub>1</sub> respectively starting from P on PO and from S on SO.
4. From 1<sub>1</sub> draw a line parallel to PQ to meet the line V1 at P<sub>1</sub>. Similarly obtain the points P<sub>2</sub> and P<sub>3</sub>.
5. Also from 1'<sub>1</sub>, 2'<sub>1</sub>, 3'<sub>1</sub> draw lines parallel to RS to meet the lines V1', V2' and V3' at P'<sub>1</sub>, P'<sub>2</sub> and P'<sub>3</sub> respectively and draw a smooth parabola.

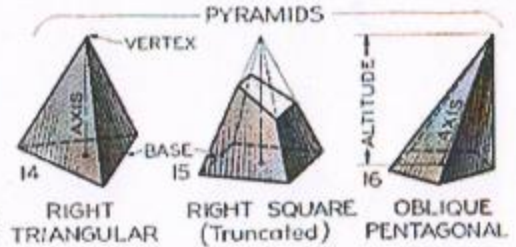


110  
PARALLELOGRAM METHOD

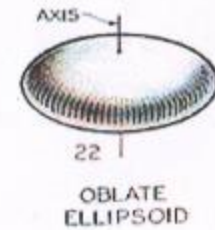
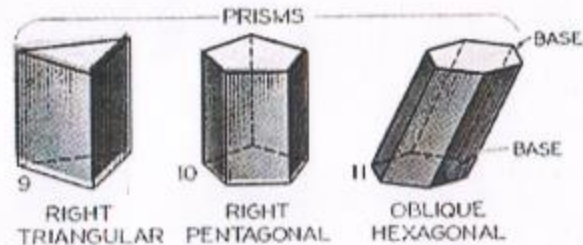
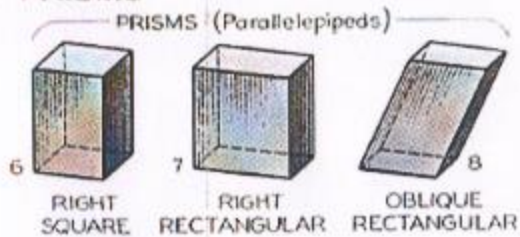
## THE 5 REGULAR SOLIDS



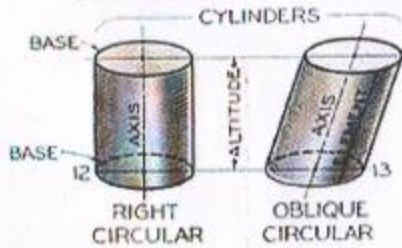
## PYRAMIDS



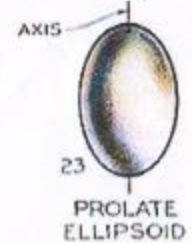
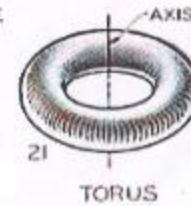
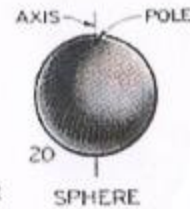
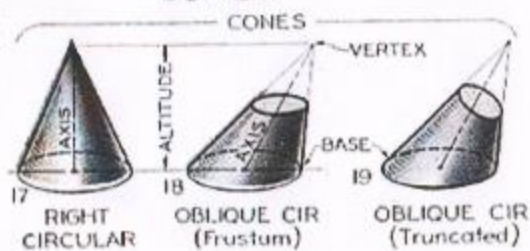
## PRISMS



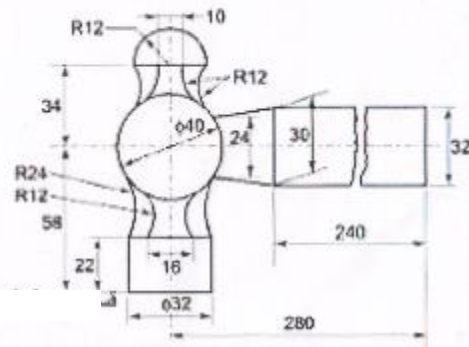
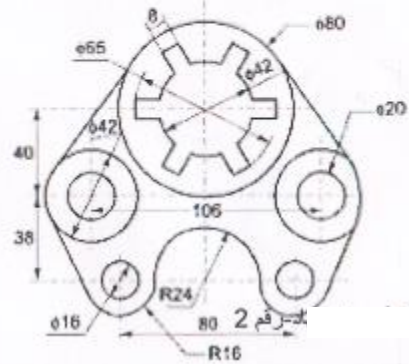
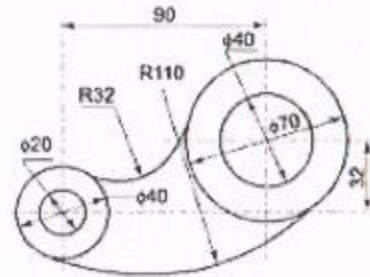
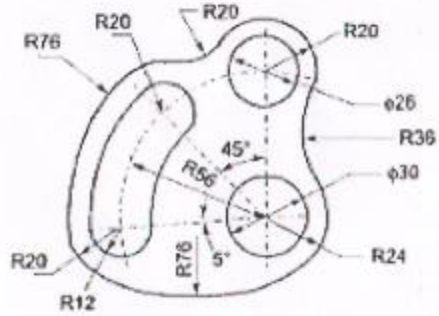
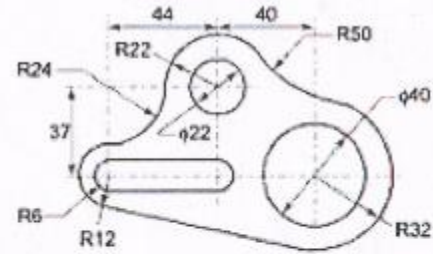
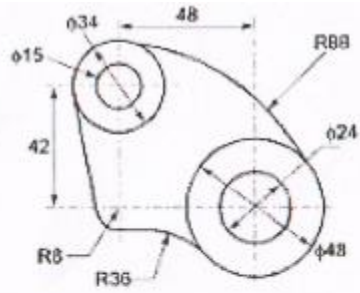
## CYLINDERS



## CONES









**Problem :** A stone is thrown from a building of 7 m high and at its highest flight it just crosses a palm tree 14 m high. Trace the path of the stone, if the distance between the building and the tree measured along the ground is 3.5 m.

### solution

1. Draw lines AB and OT, representing the building and Palm tree respectively, 3.5 m apart and above the ground level.
2. Locate C and D on the horizontal line through B such that  $CD=BC=3.5$  and complete the rectangle BDEF.
3. Inscribe the parabola in the rectangle BDEF, by rectangular method.
4. Draw the path of the stone till it reaches the ground (H) extending the principle of rectangle method.

