Chapter 4: Basic Nodal Analysis

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PREVIOUS LECTURE

• KIRCHHOFF’S LAW
  – KVL
  – KCL
• VOLTAGE AND CURRENT DIVISION LAWS
WHY WE ANALYZE ELECTRIC CIRCUITS?

We learn two basic circuit analysis techniques—nodal analysis and mesh analysis—both of which allow us to investigate many different circuits with a consistent, methodical approach.

The result is a streamlined analysis, a more uniform level of complexity in our equations, fewer errors and, perhaps most importantly, a reduced occurrence of “I don’t know how to even start!”
NODAL ANALYSIS

Nodal Analysis Procedure,
1. Count the number of nodes \( N \).
2. Designate a reference node. The number of terms in your nodal equations can be minimized by selecting the node with the greatest number of branches connected to it.
3. Label the nodal voltages (there are \( N - 1 \) of them).
4. Write a KCL equation for each of the non-reference nodes. Sum the currents flowing \textit{into} a node from sources on one side of the equation. On the other side, sum the currents flowing \textit{out of} the node through resistors. Pay close attention to “-” signs.
5. Express any additional unknowns such as currents or voltages other than nodal voltages in terms of appropriate nodal voltages. This situation can occur if voltage sources or dependent sources appear in our circuit.
6. Organize the equations. Group terms according to nodal voltages.
7. Solve the system of equations for the nodal voltages (there will be \( N - 1 \) of them)
Now, for the following circuit:

- There will be 3 nodes (V₁, V₂, V\text{ ref})
- No. of equations = 3-1 = 2
**NOW**, Apply KCL to nodes 1 and 2. Do this by equating the total current leaving the node through the several resistors to the total source current entering the node. Thus,

For Node V1,

\[
\frac{V_1}{2} + \frac{V_1 - V_2}{5} = 3.1
\]

For Node V2,

\[
\frac{V_2}{1} + \frac{V_2 - V_1}{5} = -(1.4)
\]

The reference node in a schematic is implicitly defined as zero volts. However, it is important to remember that any terminal can be designated as the reference terminal. Thus, the reference node is at zero volts with respect to the other defined nodal voltages, and not necessarily with respect to *earth* ground.
EXAMPLE, write the nodal equation for node V1 & V2.

At Node V1,
\[
\frac{(V_1 - 10)}{6} + \frac{V_1}{5} + \frac{(V_1 - V_2)}{7} = 0 \quad \text{....(1)}
\]

At Node V2,
\[
\frac{(V_2 + 5)}{2} + \frac{V_2}{3} + \frac{(V_2 - V_1)}{7} = 0 \quad \text{....(2)}
\]

❖ Note: at eq. 1 (-10) because I₁ is in the reverse direction to voltage supply (10 V).
❖ Also fore equation 2 (V+5) because the current with the same direction of (5V).
EXAMPLE,

Determine the current flowing left to right through the 15 resistor of Fig.

Writing an appropriate KCL equation for node 1,

\[ 2 = \frac{V_1}{10} + \frac{V_1 - V_2}{15} \]

and for node 2,

\[ 4 = \frac{V_2}{5} + \frac{V_2 - V_1}{15} \]

Rearranging, we obtain

\[ 5v_1 - 2v_2 = 60 \quad \text{and} \quad -v_1 + 4v_2 = 60 \]

Solving, we find that \( v_1 = 20 \) V and \( v_2 = 20 \) V

so that \( v_1 - v_2 = 0 \)

In other words, \textit{zero current} is flowing through the 15 resistor in this circuit!
CLASS ACTIVITY,
For the circuit of Fig. below, write the nodal equation for $v_1$ and $v_2$. 

![Circuit Diagram]
EXAMPLE,

Determine the nodal voltages for the circuit of following Fig. as referenced to the bottom node.

Begin by writing a KCL equation for node 1:

\[-8-3 = \frac{V_1-V_2}{3} + \frac{V_1-V_3}{4}\]

or

\[0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11\]

At node 2:

\[-(-3) = \frac{V_2-V_1}{3} + \frac{V_2}{1} + \frac{V_2-V_3}{7}\]

or

\[-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3\]

And, at node 3:

\[-(-25) = \frac{V_3}{5} + \frac{V_3-V_2}{7} + \frac{V_3-V_1}{4}\]

Or simply,

\[-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25\]
The resulting three equations can be solved using a scientific calculator, software packages such as MATLAB, or more traditional “plug-and-chug” techniques such as elimination of variables, matrix methods, or Cramer’s rule. Using the latter method, we have:

\[ v_1 = \frac{\begin{vmatrix} -11 & -0.3333 & -0.2500 \\ 3 & 1.4762 & -0.1429 \\ 25 & -0.1429 & 0.5929 \end{vmatrix}}{\begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}} = \frac{1.714}{0.3167} = 5.412 \text{ V} \]

Similarly,

\[ v_2 = \frac{\begin{vmatrix} 0.5833 & -11 & -0.2500 \\ -0.3333 & 3 & -0.1429 \\ -0.2500 & 25 & 0.5929 \end{vmatrix}}{0.3167} = \frac{2.450}{0.3167} = 7.736 \text{ V} \]

and

\[ v_3 = \frac{\begin{vmatrix} 0.5833 & -0.3333 & -11 \\ -0.3333 & 1.4762 & 3 \\ -0.2500 & -0.1429 & 25 \end{vmatrix}}{0.3167} = \frac{14.67}{0.3167} = 46.32 \text{ V} \]
Verify the solution. Is it reasonable or expected?

\[
V_1 = 5.412\text{V}, \quad V_2 = 7.736\text{V}, \quad V_3 = 46.32\text{V}
\]

- **Verify** the solution:
  Substituting the nodal voltages into any of our three nodal equations is sufficient to ensure we made no computational errors.
- The resulted voltage values are “**reasonable**”?

We have a maximum possible current of \(3 + 8 + 25 = 36\) amperes anywhere in the circuit.

The largest resistor is 7\(\Omega\), so we do not expect any voltage magnitude greater than \(7 \times 36 = 252\) V.
Next Lecture

• Nodal Analysis in case of Dependent sources
• Super Node Analysis