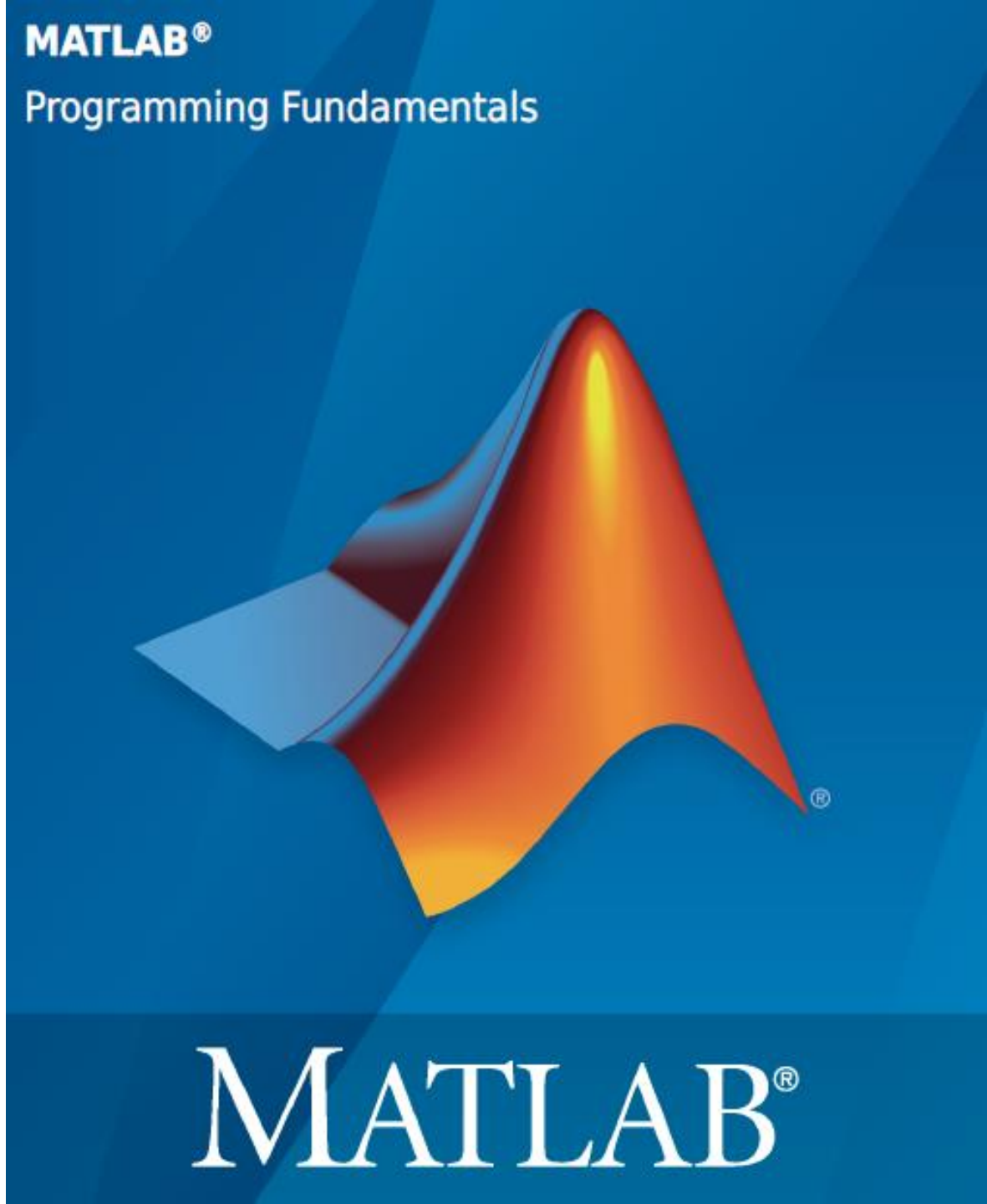
Petroleum and Mining Eng. Department

First Grade- Spring Semester

Lecture 4- Polynomial Calculation Using MATLAB

LECTURER: MS. JWAN KHALEEL M.



16. Polynomial

The polynomial is an equation that has two variables one of them called dependent and the second is independent.

Example 1:

$$y = ax^4 + b\,x^3 + c\,x^2 + dx + e$$

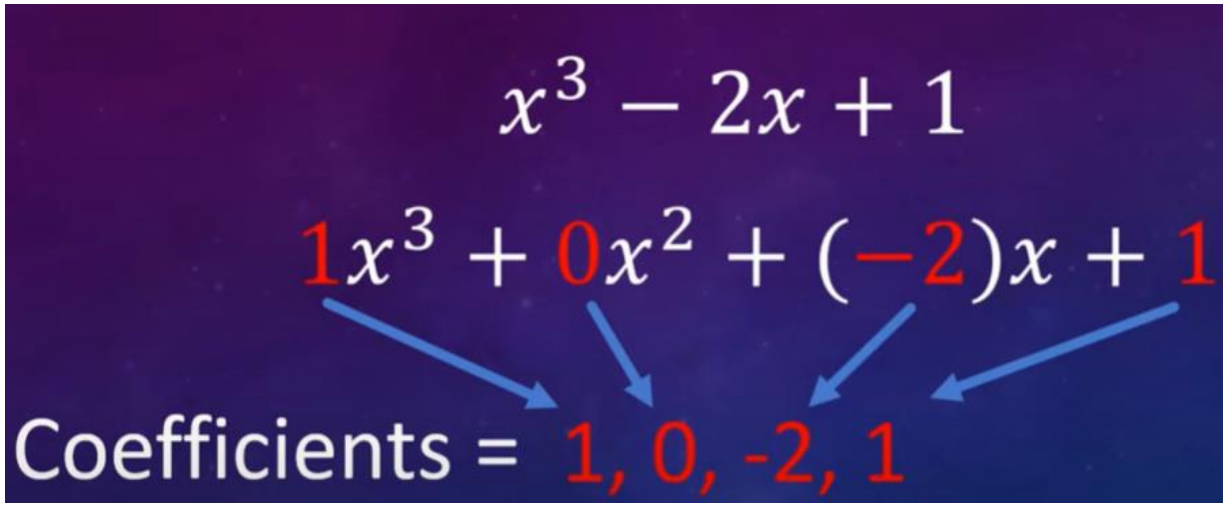
The dependent variable is (y) and the independent variables is (x)

a, b, c, d, and e are constants coefficients.

16.1 Polynomial MATLAB representation

To represent the polynomial in MATLAB for the example 1 the following syntax will be used. It's clear that we must define the values of the constants a, b, c, d, and e before they used.

```
>> a=1;b=2;c=-5;d=-2;e=9;
>> P=[a b c d]

P =

     1     2    -5    -2
```



Example 2:

Write the following equations in MATLAB form.

1. $y = x^4 - 2$
2. $y = x^4 - 12\,x^3 + 25x + 116$
3. $y = \frac{3}{5}\,x^6 + x^2 - 5x$

```
>> p=[1 0 0 0 -2]
p =
       1      0      0      0      -2

>> p=[1 -12 0 25 116]
p =
       1     -12     0      25     116

>> p=[3/5 0 0 0 1 -5 0]
p =
       0.6000    0    0      0      1.0000 -5.0000       0
```

Note: The coefficients are ordered in descending powers. $y = p_1\,x^n + p_2\,x^{n-1} + \cdots + p_n x + p_{n+1}$

The polynomial degree is equal to the length of the vector p reduced by one.

16.2 Evaluation of Polynomial Values

The (`polyval`) function can be used to evaluate the values of polynomial according to the specified points.

Syntax

```
y = polyval(p,x)
```

`y = polyval(p,x)` returns the value of a polynomial of degree `n` evaluated at `x`. The input argument p is a vector of length `n+1` whose elements are the coefficients in descending powers of the polynomial to be evaluated.

$$y = p_1 x^n + p_2 x^{n-1} + \cdots + p_n x + p_{n+1}$$

Example:

Compute the values of the following equations at x= 3, 6, and 10.

1. $y = x^4 - 2$
2. $y = x^4 - 12\,x^3 + 25x + 116$
3. $y = \frac{3}{5}\,x^6 + x^2 - 5x$

```
>> X=[3 6 10]

X =

     3     6    10

>> p=[1 0 0 0 -2]

p =

     1     0     0     0    -2

>> y=polyval(p,X)

y =

          79        1294        9998
```

```
>> X=[3 6 10]

X =

     3     6    10

>> p=[1 -12 0 25 116]

p =

     1   -12     0    25   116

>> y=polyval(p,X)

y =

         -52       -1030       -1634
```

```
>> X=[3 6 10];
>> p=[3/5 0 0 0 1 -5 0];
>> y=polyval(p,X)

y =

   1.0e+05 *

    0.0043    0.2800    6.0005
```

16.3 Roots of polynomial

The root of equation is the value of x that make the equation equal to zero.

Example1:

Find the roots of the following equations.

1. $y = x^2 - 4$
2. $y = x^2 - 5x - 6$

1. Solution:

Mathematical solution:

Set the value of y=0 then

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

1. MATLAB Solution

```
>> p=[1 0 -4]
p =
     1   0   -4
>> x=roots(p)
x =
          2.0000
         -2.0000
```

2. Mathematical solution: First solution

Set the value of y=0 then

$$x^2 - 5x - 6 = 0$$

$$(x+1)(x-6) = 0$$

$$x = -1 \; or \; x = 6$$

Second solution can be found from the following equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. MATLAB solution

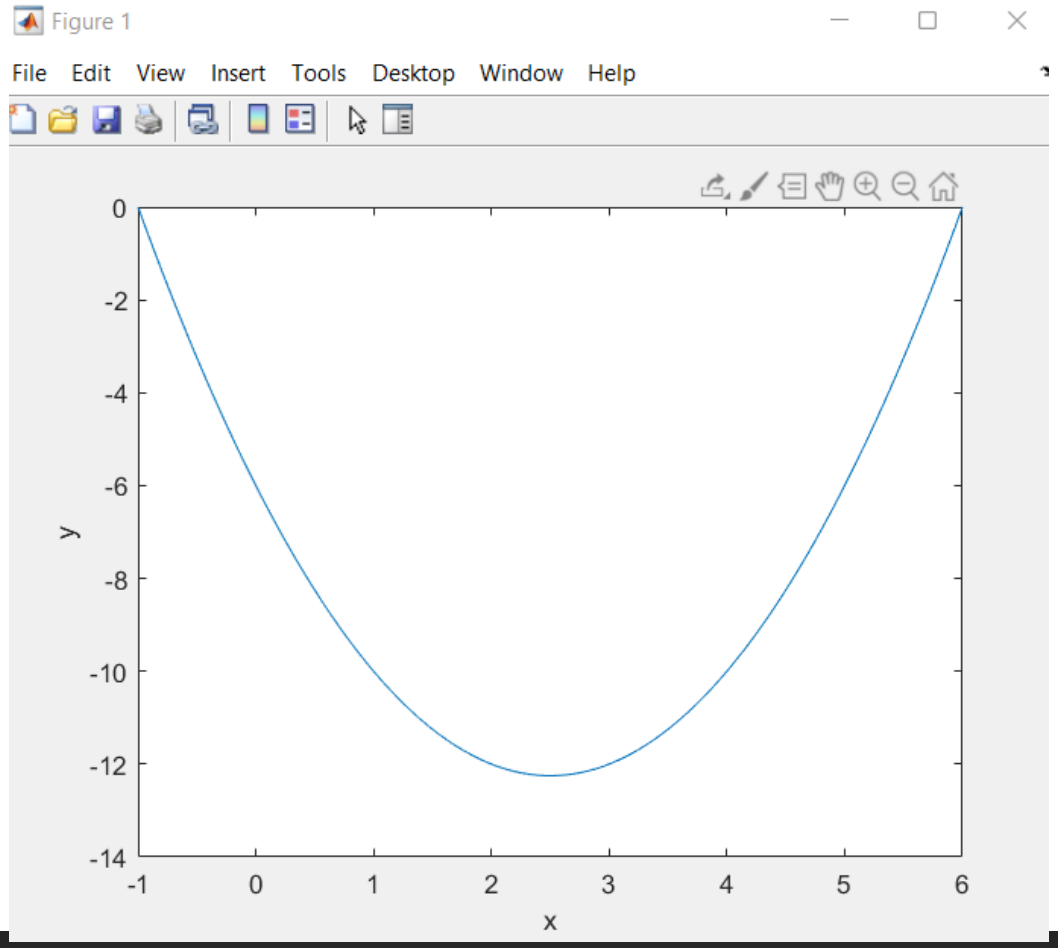
```
>> p=[1 -5 -6]

p =
     1    -5    -6

>> x=roots(p)

x =
          6
         -1
```

Example 2:

Find 100 points (x, y) that satisfy the following equation. $y = x^2 - 5x - 6$

```
>> p=[1 -5 -6]

p =

     1    -5    -6

>> r=roots(p)

r =

     6
    -1

>> x=linspace(r(1),r(2),100);
>> y=polyval(p,x);
>> plot(x,y)
```

The estimated values are shown in Figure 1.



Note: When the roots are known the polynomial coefficients can be found by using the (poly function).

Example 3:

The roots are (-1, 6) find the polynomial Solution:

```
>> r=[-1 6]
r =
     -1    6
>> p=poly(r)
p =
           1    -5    -6
```

16.4 Polynomial Operations

16.4.1 Polynomial Addition and Subtractions

The coefficient of the sum of two polynomials is the sum of the coefficients of the two polynomials. The vectors containing the coefficients must be of the same length. For example, to add

$$A(s) = s^4 - 3s^3 - s + 2$$
$$B(s) = 4\,s^3 - 2\,s^2 + 5s - 16$$

$$C(s) = A(s) + B(S) = s^4 + (4 - 3)\,s^3 - 2\,s^2 + (5 - 1)s + (2 - 16)$$
$$= s^4 + s^3 - 2\,s^2 + 4s - 14$$

MATLAB solution is

```
>> a = [1 -3 0 -1 2];
>> b = [0 4 -2 5 -16];
>> c = a + b
c =
       1       1      -2       4     -14
```

Note in the previous example the polynomial B is represented by adding the coefficients of x^4 equal zero as shown by shaded number in MATLAB code. This is to make the two vectors equal in length.

16.4 Polynomial Operations

16.4.2 Scalar Multiplication

The coefficient vector of the scalar multiple of a polynomial is simply the scalar times the coefficient vector of the polynomial. To specify

$$C(s) = 3A(s)$$

$$C(s) = 3\,s^4 - 9\,s^3 - 3s + 6$$

MATLAB solution

```
>> a = [1 -3 0 -1 2];

>> c=3*a
c =
      3     -9     0     -3     6
```

Example 1: Find the polynomial y which is the result of multiplication of the polynomial y_1 and y_2

$$y_1 = x^2 - 2$$
$$y_2 = x^2 + 3x - 1$$

Solution:

The mathematical solution is:

$$y = y_1 \times y_2$$

$$y = (x^2 - 2)(x^2 + 3x - 1)$$

$$y = x^4 + 3\,x^3 - x^2 - 2\,x^2 - 6x + 2$$

$$y = x^4 + 3\,x^3 - 3\,x^2 - 6x + 2$$

The MATLAB solution

```
>> p1=[1 0 -2]

p1 =

     1     0    -2

>> p2=[1 3 -1]

p2 =

     1     3    -1

>> p=conv(p1,p2)

p =

     1     3    -3    -6     2
```

Example 2: Write a MATLAB code to compute the velocity of an object at time (10s).

$$v_1 = t^2 + 3t - 4 \quad and \quad v_2 = t^3 + 2$$

SOLUTION:

```
pv1 =

     1     3    -4

>> pv2=[1 0 0 2]

pv2 =

     1     0     0     2

>> pv=conv(pv1,pv2)

pv =

     1     3    -4     2     6    -8

>> v=polyval(pv,10)

v =

      126252
```

The object's velocity equation can be determined by multiplying the following two velocities.

Assignments:

1. Find the roots of the polynomial $p_2(x) = x^5 - 7x^4 + 16x^2 + 25x + 52$
2. It is known that the roots of a polynomial are 1, 2, 3 & 4 . Compute the coefficients of this polynomial.
3. Let

$$p_1 = x^5 - 3x^4 + 5x^2 + 7x + 9$$

$$p_2 = 2x^6 - 8x^4 + 4x^2 + 10x + 12$$

Compute the product $p_1 \cdot p_2$ with the **conv(a,b)** function.

The End of the Lecture
Enjoy Your Day