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Lecture 6:
Reservoir Deliverability

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Introduction

Reservoir deliverability is defined as the oil or gas production rate achievable from reservoir at a given bottom-hole pressure. It is a major factor affecting well deliverability. Reservoir deliverability determines types of completion and artificial lift methods to be used. A thorough knowledge of reservoir productivity is essential for production engineers.

Reservoir deliverability depends on several factors including the following:

. Reservoir pressure

. Pay zone thickness and permeability

. Reservoir boundary type and distance

. Wellbore radius

. Reservoir fluid properties

. Near-wellbore condition

. Reservoir relative permeabilities

Reservoir deliverability can be mathematically modeled on the basis of flow regimes such as transient flow, steady state flow, and pseudo–steady state flow. An analytical relation between bottom-hole pressure and production rate can be formulated for a given flow regime. The relation is called ‘‘inflow performance relationship’’ (IPR). This chapter addresses the procedures used for establishing IPR of different types of reservoirs and well configurations.

Flow Regimes

When a vertical well is open to produce oil at production rate q, it creates a pressure funnel of radius r around the wellbore, as illustrated by the dotted line in Fig. 3.1a. In this reservoir model, the h is the reservoir thickness, k is the effective horizontal reservoir permeability to oil, mo is viscosity of oil, Bo is oil formation volume factor, rw is wellbore radius, pwf is the flowing bottom hole pressure, and p is the pressure in the reservoir at the distance r from the wellbore center line. The flow streamlines in the cylindrical region form a horizontal radial flow pattern as depicted in Fig. 3.1b.

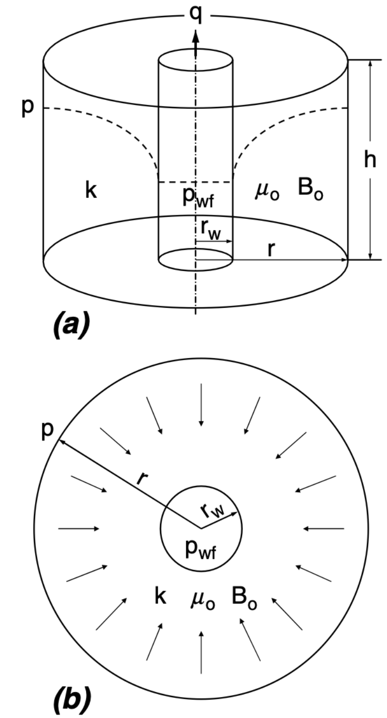

Figure 3.1 *A sketch of a radial flow reservoir model: (**a**) lateral view, (**b**) top view.*

Transient (Unsteady-State) Flow

''Transient flow'' is defined as a flow regime where/when the radius of pressure wave propagation from wellbore has not reached any boundaries of the reservoir. During transient flow, the developing pressure funnel is small relative to the reservoir size. Therefore, the reservoir acts like an infinitively large reservoir from transient pressure analysis point of view.

Assuming single-phase oil flow in the reservoir, several analytical solutions have been developed for describing the transient flow behavior. They are available from classic textbooks such as that of Dake (1978). A constant-rate solution expressed by Eq. (3.1) is frequently used in pro- duction engineering:

$$p_{wf} = p_i - \frac{162.6 q B_o \mu_o}{kh} \times \left(\log t + \log \frac{k}{\phi \mu_o c_t r_w^2} - 3.23 + 0.87S \right), \qquad (3.1)$$

where

p_{wf} = flowing bottom-hole pressure, psia
p_i = initial reservoir pressure, psia
q = oil production rate, stb/day
μ_o = viscosity of oil, cp
k = effective horizontal permeability to oil, md
h = reservoir thickness, ft
t = flow time, hour
ϕ = porosity, fraction
c_t = total compressibility, psi^{-1}
r_w = wellbore radius to the sand face, ft
S = skin factor
Log = 10-based logarithm $\log_{10}$

Transient Flow Cont.

Because oil production wells are normally operated at constant bottom-hole pressure because of constant well- head pressure imposed by constant choke size, a constant bottom-hole pressure solution is more desirable for well- inflow performance analysis. With an appropriate inner boundary condition arrangement, Earlougher (1977) developed a constant bottom-hole pressure solution, which is similar to Eq. (3.1):

$$q = \frac{kh(p_i - p_{wf})}{162.6 B_o \mu_o \left(\log t + \log \frac{k}{\phi \mu_o c_t r_w^2} - 3.23 + 0.87S \right)}, \quad \textbf{(3.2)}$$

which is used for transient well performance analysis in production engineering.

Equation (3.2) indicates that oil rate decreases with flow time. This is because the radius of the pressure funnel, over which the pressure drawdown (pi pwf) acts, increases with time, that is, the overall pressure gradient in the reservoir drops with time.

Transient Flow Cont.

For gas wells, the transient solution is

$$q_g = \frac{kh[m(p_i) - m(p_{wf})]}{1,638T\left(\log t + \log \frac{k}{\phi\mu_o c_t r_w^2} - 3.23 + 0.87S\right)} \qquad \textbf{(3.3)}$$

where qg is production rate in Mscf/d, T is temperature in 8R, and m(p) is real gas pseudo-pressure defined as

$$m(p) = \int_{pb}^{p} \frac{2p}{\mu z} dp. \qquad \textbf{(3.4)}$$

Steady-State Flow

- "Steady-state flow" is defined as a flow regime where the pressure at any point in the reservoir remains constant over time.
- This flow condition prevails when the pressure funnel shown in Fig. 3.1 has propagated to a constant- pressure boundary.
- The constant-pressure boundary can be an aquifer or a water injection well.
- A sketch of the reservoir model is shown in Fig. 3.2, where pe represents the pressure at the constant-pressure boundary.

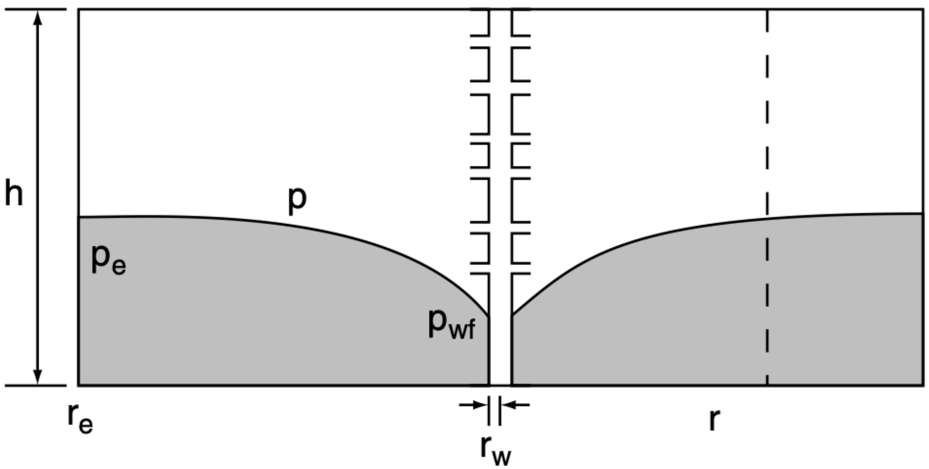

Figure 3.2 *A sketch of a reservoir with a constant-pressure boundary.*

Steady-State Flow Cont.

- Assuming single-phase flow, the following theoretical relation can be derived from Darcy's law for an oil reservoir under the steady-state flow condition due to a circular constant- pressure boundary at distance re from wellbore:

$$q = \frac{kh(p_e - p_{wf})}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} + S \right)}, \qquad \textbf{(3.5)}$$

where "ln" denotes 2.718-based natural logarithm loge.

Pseudo-Steady-State Flow

- Pseudo–steady-state'' flow is defined as a flow regime where the pressure at any point in the reservoir declines at the same constant rate over time.
- This flow condition prevails after the pressure funnel shown in Fig. 3.1 has propagated to all no-flow boundaries.
- A no-flow boundary can be a sealing fault, pinch-out of pay zone, or boundaries of drainage areas of production wells.
- A sketch of the reservoir model is shown in Fig. 3.3, where pe represents the pressure at the no-flow boundary at time t4.

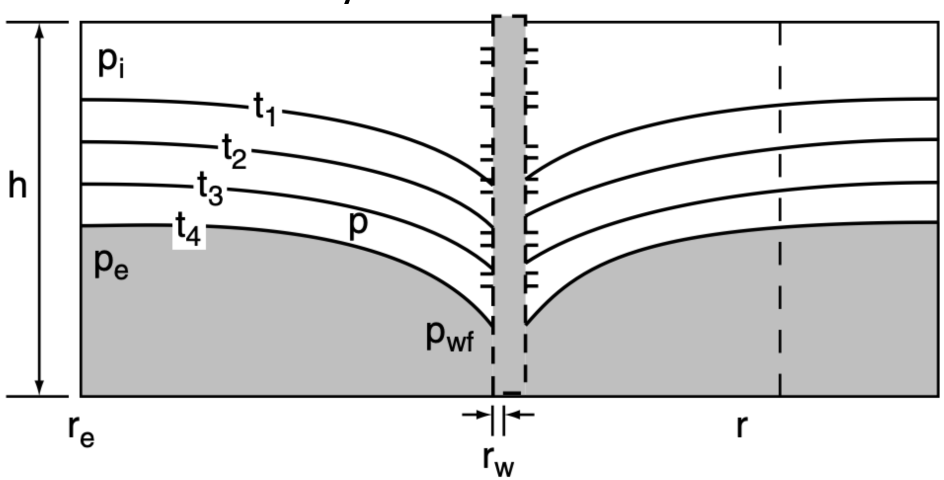

Figure 3.3 *A sketch of a reservoir with no-flow boundaries.*

Pseudo-Steady-State Flow Contd.

- Assuming single-phase flow, the following theoretical relation can be derived from Darcy's law for an oil reservoir under pseudo–steady-state flow condition due to a circular no-flow boundary at distance re from wellbore:

$$q = \frac{kh(p_e - p_{wf})}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} - \frac{1}{2} + S \right)}. \qquad \textbf{(3.6)}$$

- The flow time required for the pressure funnel to reach the circular boundary can be expressed as

$$t_{pss} = 1{,}200 \frac{\phi \mu_o c_t r_e^2}{k}. \qquad \textbf{(3.7)}$$

- Because the pe in Eq. (3.6) is not known at any given time, the following expression using the average reservoir pressure is more useful:

$$q = \frac{kh(\bar{p} - p_{wf})}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right)}, \qquad \textbf{(3.8)}$$

where p̄ is the average reservoir pressure in psia.

Pseudo-Steady-State Flow Contd.

- If the no-flow boundaries delineate a drainage area of noncircular shape, the following equation should be used for analysis of pseudo–steady-state flow:

$$q = \frac{kh(\bar{p} - p_{wf})}{141.2 B_o \mu_o \left(\frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)}, \tag{3.9}$$

where

A = drainage area, ft^2
$\gamma = 1.78$ = Euler's constant
C_A = drainage area shape factor, 31.6 for a circular boundary.

- The value of the shape factor CA can be found from Fig. 3.4.

- For a gas well located at the center of a circular drainage area, the pseudo–steady-state solution is

$$q_g = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1{,}424T\left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S + Dq_g \right)}, \tag{3.10}$$

where

D = non-Darcy flow coefficient, d/Mscf.

Pseudo-Steady-State Flow Contd.

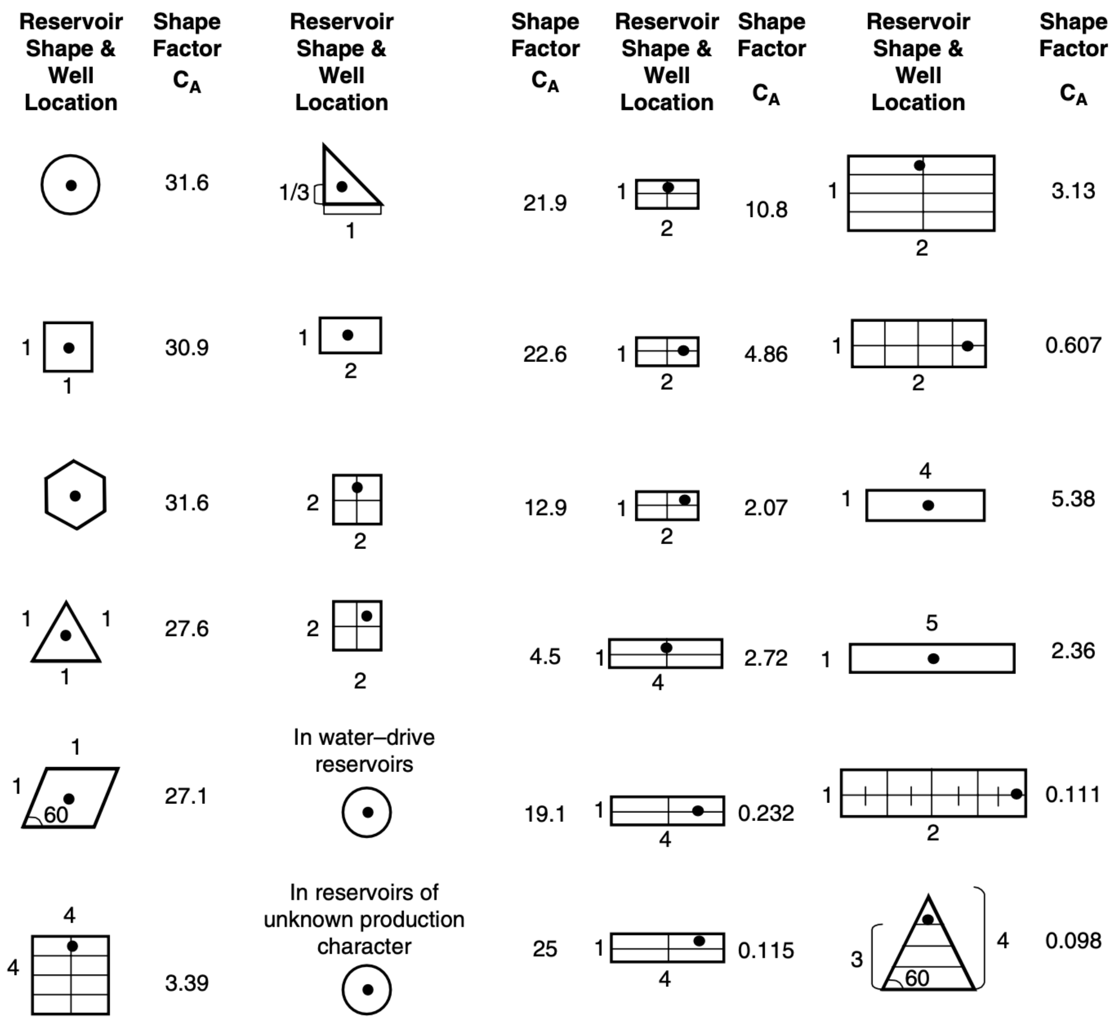

Figure 3.4 (a) Shape factors for closed drainage areas with low-aspect ratios. (b) Shape factors for closed drainage areas with high-aspect ratios (Dietz, 1965).

Inflow Performance Relationship

- IPR is used for evaluating reservoir deliverability in pro- duction engineering. The IPR curve is a graphical presentation of the relation between the flowing bottom-hole pressure and liquid production rate. A typical IPR curve is shown in Fig. 3.5. The magnitude of the slope of the IPR curve is called the ‘‘productivity index’’ (PI or J), that is,

$$J = \frac{q}{(p_e - p_{wf})}, \qquad \textbf{(3.14)}$$

where J is the productivity index. Apparently J is not a constant in the two-phase flow region.

- Well IPR curves are usually constructed using reservoir inflow models, which can be from either a theoretical basis or an empirical basis. It is essential to validate these models with test points in field applications.

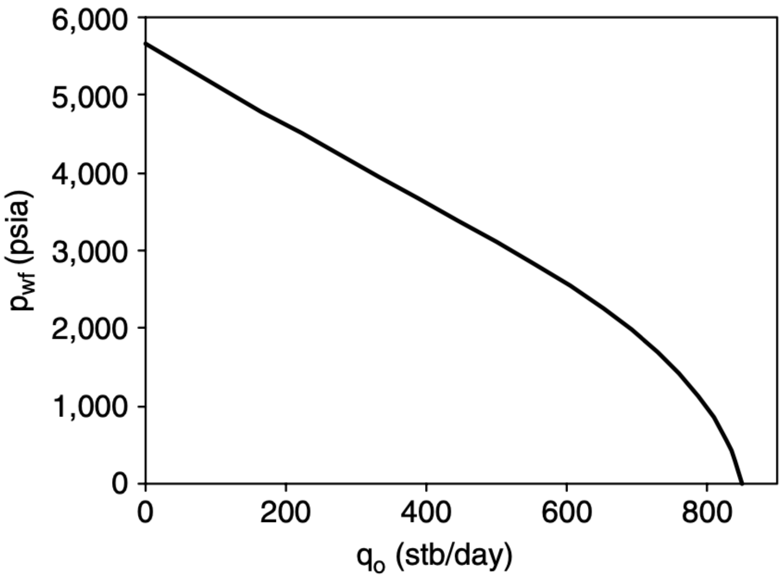

Figure 3.5 *A typical IPR curve for an oil well.*

IPR for Single (Liquid)-Phase Reservoirs

- All reservoir inflow models represented by Eqs. (3.1), (3.3), (3.7), and (3.8) were derived on the basis of the assumption of single-phase liquid flow. This assumption is valid for under- saturated oil reservoirs, or reservoir portions where the pressure is above the bubble-point pressure. These equations define the productivity index (J) for flowing bottom-hole pressures above the bubble-point pressure as follows:

$$J^* = \frac{q}{(p_i - p_{wf})} = \frac{kh}{162.6 B_o \mu_o \left(\log t + \log \frac{k}{\phi \mu_o c_t r_w^2} - 3.23 + 0.87S \right)} \quad (3.15)$$

for radial transient flow around a vertical well,

$$J^* = \frac{q}{(p_e - p_{wf})} = \frac{kh}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} + S \right)} \quad (3.16)$$

for radial steady-state flow around a vertical well,

$$J^* = \frac{q}{(\bar{p} - p_{wf})} = \frac{kh}{141.2 B_o \mu_o \left(\frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)} \quad (3.17)$$

for pseudo–steady-state flow around a vertical well, and

$$J^* = \frac{q}{(p_e - p_{wf})} = \frac{k_H h}{141.2 B \mu \left\{ \ln \left[\frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} \right] + \frac{I_{ani} h}{L} \ln \left[\frac{I_{ani} h}{r_w (I_{ani} + 1)} \right] \right\}} \quad (3.18)$$

for steady-state flow around a horizontal well.

IPR for Single (Liquid)-Phase Reservoirs

Example Problem 3.1 Construct IPR of a vertical well in an oil reservoir. Consider (1) transient flow at 1 month, (2) steady-state flow, and (3) pseudo–steady-state flow. The following data are given:

Porosity:	$\phi = 0.19$
Effective horizontal permeability:	$k = 8.2$ md
Pay zone thickness:	$h = 53$ ft
Reservoir pressure:	p_e or $\bar{p} = 5{,}651$ psia
Bubble-point pressure:	$p_b = 50$ psia
Fluid formation volume factor:,	$B_o = 1.1$
Fluid viscosity:	$\mu_o = 1.7$ cp
Total compressibility,	$c_t = 0.0000129$ psi^{-1}
Drainage area:	$A = 640$ acres
	($r_e = 2{,}980$ ft)
Wellbore radius:	$r_w = 0.328$ ft
Skin factor:	$S = 0$

Solution

1. For transient flow, calculated points are

$$J^* = \frac{kh}{162.6B\mu\left(\log t + \log\frac{k}{\phi\mu c_t r_w^2} - 3.23\right)}$$

$$= \frac{(8.2)(53)}{162.6(1.1)(1.7)\left(\log[((30)(24)] + \log\frac{(8.2)}{(0.19)(1.7)(0.0000129)(0.328)^2} - 3.23\right)}$$

$$= 0.2075 \text{ STB/d-psi}$$

Transient IPR curve is plotted in Fig. 3.6.

2. For steady state flow:

$$J^* = \frac{kh}{141.2B\mu\left(\ln\frac{r_e}{r_w} + S\right)}$$

$$= \frac{(8.2)(53)}{141.2(1.1)(1.7)\ln\left(\frac{2{,}980}{0.328}\right)}$$

$$= 0.1806 \text{ STB/d-psi}$$

Calculated points are:

p_{wf}(psi)	q_o(stb/day)
50	1,011
5,651	0

Steady state IPR curve is plotted in Fig. 3.7.

3. For pseudosteady state flow:

$$J^* = \frac{kh}{141.2B\mu\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)}$$

$$= \frac{(8.2)(53)}{141.2(1.1)(1.7)\left(\ln\frac{2{,}980}{0.328} - 0.75\right)}$$

$$= 0.1968 \text{ STB/d-psi}$$

IPR for Single (Liquid)-Phase Res

Calculated points are:

p_{wf}(psi)	q_o(stb/day)
50	1,102
5,651	0

Pseudo–steady-state IPR curve is plotted in Fig. 3.8.

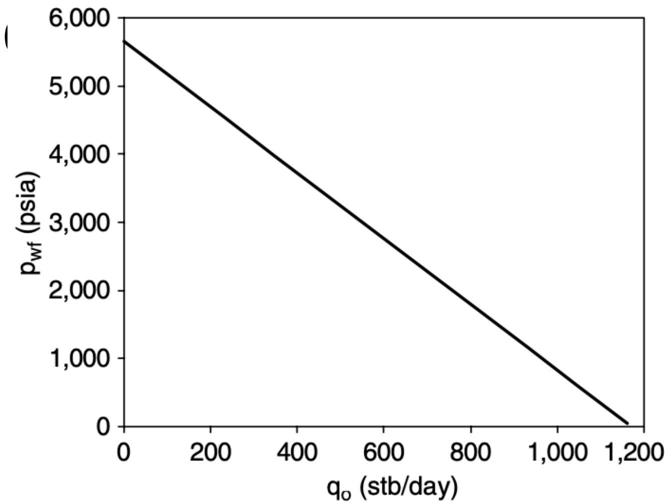

Figure 3.6 *Transient IPR curve for Example Problem 3.1.*

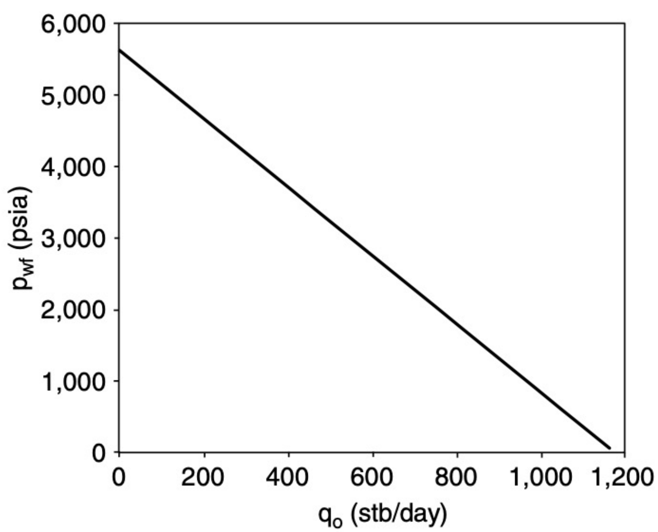

Figure 3.7 *Steady-state IPR curve for Example Problem 3.1.*

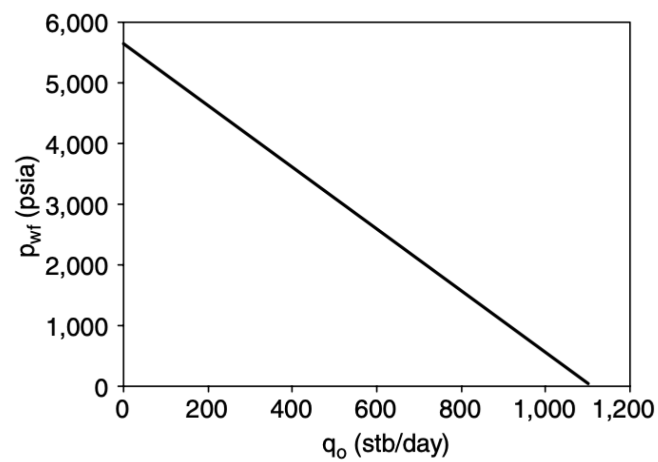

Figure 3.8 *Pseudo–steady-state IPR curve for Example Problem 3.1.*

IPR for Two Phase Reservoirs

- The linear IPR model presented in the previous section is valid for pressure values as low as bubble-point pressure. Below the bubble-point pressure, the solution gas escapes from the oil and become free gas.
- The free gas occupies some portion of pore space, which reduces flow of oil. This effect is quantified by the reduced relative permeability. Also, oil viscosity in- creases as its solution gas content drops.
- The combination of the relative permeability effect and the viscosity effect results in lower oil production rate at a given bottom-hole pressure. This makes the IPR curve deviating from the linear trend below bubble-point pressure, as shown in Fig. 3.5.
- The lower the pressure, the larger the deviation. If the reservoir pressure is below the initial bubble-point pressure, oil and gas two- phase flow exists in the whole reservoir domain and the reservoir is referred as a ‘‘two-phase reservoir.’’

IPR for Two Phase Reservoirs

- Only empirical equations are available for modelling IPR of two-phase reservoirs. These empirical equations include Vogel's (1968) equation extended by Standing (1971), the Fetkovich (1973) equation, Bandakhlia and Aziz's (1989) equation, Zhang's (1992) equation, and Retnanto and Economides' (1998) equation. Vogel's equation is still widely used in the industry. It is written as

$$q = q_{\max}\left[1 - 0.2\left(\frac{p_{wf}}{\bar{p}}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}}\right)^2\right] \quad (3.19)$$

or

$$p_{wf} = 0.125\bar{p}\left[\sqrt{81 - 80\left(\frac{q}{q_{\max}}\right)} - 1\right], \quad (3.20)$$

where qmax is an empirical constant and its value represents the maximum possible value of reservoir deliverability, or AOF. The qmax can be theoretically estimated based on reservoir pressure and productivity index above the bubble- point pressure. The pseudo–steady-state flow follows that

$$q_{\max} = \frac{J^*\bar{p}}{1.8}. \quad (3.21)$$

IPR for Two Phase Reservoirs

- Fetkovich's equation is written as

$$q = q_{\max}\left[1 - \left(\frac{p_{wf}}{\bar{p}}\right)^2\right]^n \qquad (3.22)$$

or

$$q = C(\bar{p}^2 - p_{wf}^2)^n, \qquad (3.23)$$

- where C and n are empirical constants and is related to qmax by C 1⁄4 qmax=p2n. As illustrated in Example Problem 3.5, the Fetkovich equation with two constants is more accurate than Vogel's equation IPR modelling.
- Again, Eqs. (3.19) and (3.23) are valid for average reservoir pressure p being at and below the initial bubble-point pressure. Equation (3.23) is often used for gas reservoirs.

IPR for Two Phase Reservoirs

Example Problem 3.2 Construct IPR of a vertical well in a saturated oil reservoir using Vogel's equation. The following data are given:

Porosity:	$\phi = 0.19$
Effective horizontal permeability:	$k = 8.2$ md
Pay zone thickness:	$h = 53$ ft
Reservoir pressure:	$\bar{p} = 5{,}651$ psia
Bubble point pressure:	$p_b = 5{,}651$ psia
Fluid formation volume factor:	$B_o = 1.1$
Fluid viscosity:	$\mu_o = 1.7$ cp
Total compressibility:	$c_t = 0.0000129$ psi^{-1}
Drainage area:	$A = 640$ acres
	$(r_e = 2{,}980$ ft$)$
Wellbore radius:	$r_w = 0.328$ ft
Skin factor:	$S = 0$

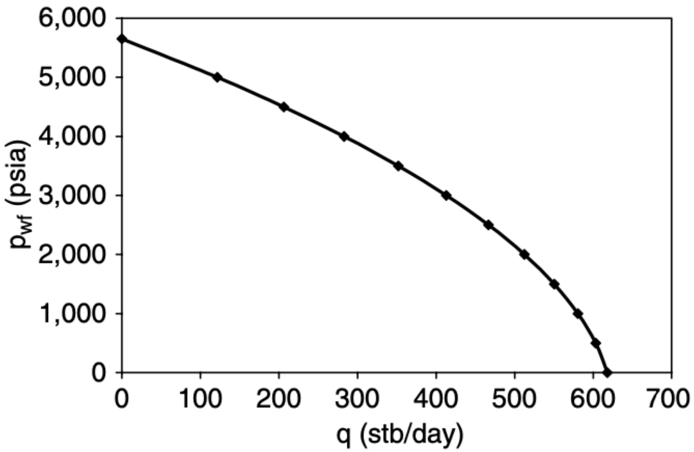

Figure 3.9 *IPR curve for Example Problem 3.2.*

Solution

$$J^* = \frac{kh}{141.2B\mu\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)}$$

$$= \frac{(8.2)(53)}{141.2(1.1)(1.7)\left(\ln\frac{2{,}980}{0.328} - 0.75\right)}$$

$$= 0.1968 \text{ STB/d-psi}$$

$$q_{max} = \frac{J^*\bar{p}}{1.8} = \frac{(0.1968)(5{,}651)}{1.8} = 618 \text{ stb/day}$$

p_{wf} (psi)	q_o (stb/day)
5,651	0
5,000	122
4,500	206
4,000	283
3,500	352
3,000	413
2,500	466
2,000	512
1,500	550
1,000	580
500	603
0	618

Calculated points by Eq. (3.19) are
The IPR curve is plotted in Fig. 3.9.

Construction of IPR Curves Using Test Points

- It has been shown in the previous section that well IPR curves can be constructed using reservoir parameters including formation permeability, fluid viscosity, drainage area, wellbore radius, and well skin factor. These parameters determine the constants (e.g., productivity index) in the IPR model. However, the values of these parameters are not always available. Thus, test points (measured values of production rate and flowing bottom-hole pressure) are frequently used for constructing IPR curves.
- Constructing IPR curves using test points involves backing calculation of the constants in the IPR models. For a single-phase (unsaturated oil) reservoir, the model constant J can be determined by

$$J^* = \frac{q_1}{(\bar{p} - p_{wf1})}, \tag{3.29}$$

where q1 is the tested production rate at tested flowing bottom-hole pressure pwf 1 .

Construction of IPR Curves Using Test Points

- For a partial two-phase reservoir, model constant J in the generalized Vogel equation must be determined based on the range of tested flowing bottom-hole pressure. If the tested flowing bottom-hole pressure is greater than bubble-point pressure, the model constant J should be determined by

$$J^* = \frac{q_1}{(\bar{p} - p_{wf1})}. \tag{3.30}$$

- If the tested flowing bottom-hole pressure is less than bubble-point pressure, the model constant J should be determined using Eq. (3.28), that is,

$$J^* = \frac{q_1}{\left((\bar{p} - p_b) + \frac{p_b}{1.8}\left[1 - 0.2\left(\frac{p_{wf1}}{p_b}\right) - 0.8\left(\frac{p_{wf1}}{p_b}\right)^2\right]\right)}. \tag{3.31}$$

Construction of IPR Curves Using Test Points

Example Problem 3.4 Construct IPR of two wells in an undersaturated oil reservoir using the generalized Vogel equation. The following data are given:

Reservoir pressure:	$\bar{p} = 5{,}000$ psia
Bubble point pressure:	$p_b = 3{,}000$ psia
Tested flowing bottom-hole pressure in Well A:	$p_{wf1} = 4{,}000$ psia
Tested production rate from Well A:	$q_1 = 300$ stb/day
Tested flowing bottom hole pressure in Well B:	$p_{wf1} = 2{,}000$ psia
Tested production rate from Well B:	$q_1 = 900$ stb/day

Construction of IPR Curves Using Test Points

Solution

Well A:

$$J^* = \frac{q_1}{(\bar{p} - p_{wf1})}$$
$$= \frac{300}{(5{,}000 - 4{,}000)}$$
$$= 0.3000 \text{ stb/day-psi}$$

Calculated points are

p_{wf} (psia)	q (stb/day)
0	1,100
500	1,072
1,000	1,022
1,500	950
2,000	856
2,500	739
3,000	600
5,000	0

The IPR curve is plotted in Fig. 3.12.

Well B:

$$J^* = \frac{q_1}{\left((\bar{p} - p_b) + \frac{p_b}{1.8}\left[1 - 0.2\left(\frac{p_{wf1}}{p_b}\right) - 0.8\left(\frac{p_{wf1}}{p_b}\right)^2\right]\right)}$$
$$= \frac{900}{\left((5{,}000 - 3{,}000) + \frac{3{,}000}{1.8}\left[1 - 0.2\left(\frac{2{,}000}{3{,}000}\right) - 0.8\left(\frac{2{,}000}{3{,}000}\right)^2\right]\right)}$$
$$= 0.3156 \text{ stb/day-psi}$$

Calculated points are

p_{wf} (psia)	q (stb/day)
0	1,157
500	1,128
1,000	1,075
1,500	999
2,000	900
2,500	777
3,000	631
5,000	0

The IPR curve is plotted in Fig. 3.13.

Construction of IPR Curves Using Test Points

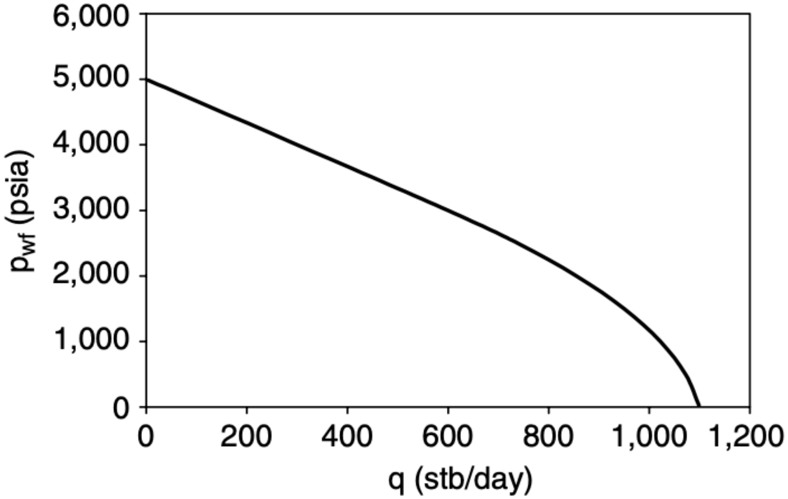

Figure 3.12 *IPR curves for Example Problem 3.4, Well A.*

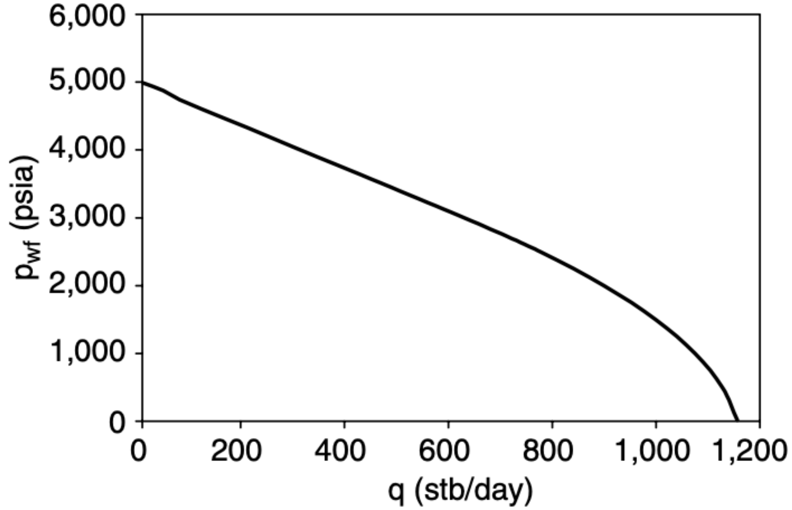

Figure 3.13 *IPR curves for Example Problem 3.4, Well B.*