Tishk International University
Engineering Faculty
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Well Testing
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Lecture 7:
Fluid Flow in Porous Media

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Content

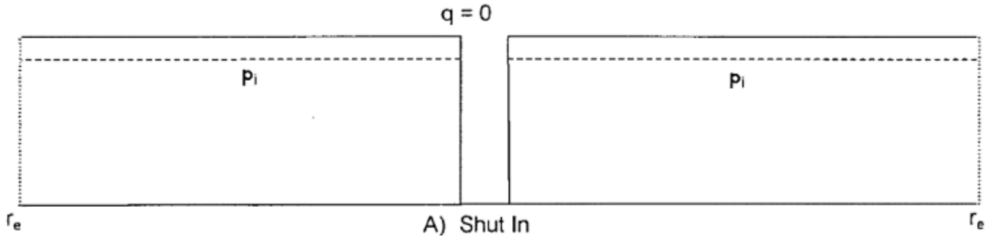
- Flow Regimes
- Unsteady (Transient) -State Flow
- Development of Radial Differential Equation
- Solution to Diffusivity Equation
- Solution to Diffusivity Equation-Transient Flow

Flow Regimes

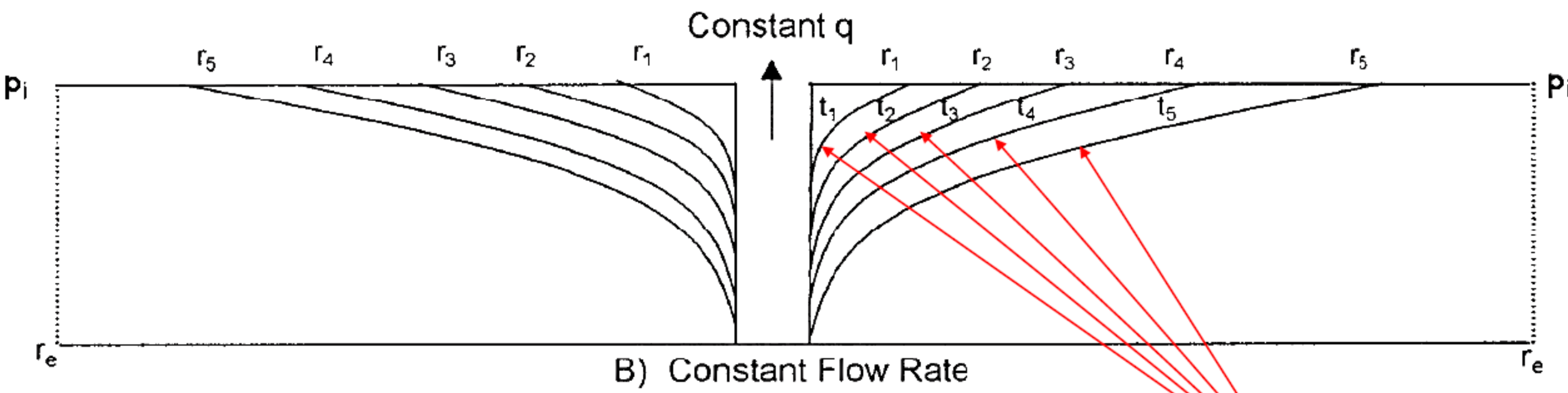
- Under the **steady-state flowing condition**, the same quantity of fluid enters the flow system as leaves it.
- In the **unsteady-state flow condition**, the flow rate into an element of volume of a porous media may not be the same as the flow rate out of that element.
- Accordingly, the fluid content of the porous medium changes with time.
- The variables in unsteady-state flow additional to those already used for steady-state flow, therefore, become:
  - Time
  - Porosity
  - Fluid viscosity
  - Total compressibility (Rock and fluid)

Unsteady (Transient) -State Flow

- If a well is centered in a homogeneous circular reservoir of radius re with a uniform pressure Pi.
- If the well is allowed to flow at a constant flow rate of q, a pressure disturbance will be created at the sand face.
- The Pwf, will drop instantaneously as the well is opened.
- The pressure disturbance will move away from the wellbore at a rate that is determined by:
  - Permeability
  - Porosity
  - Fluid viscosity
  - Rock and fluid compressibility



Unsteady (Transient) -State Flow

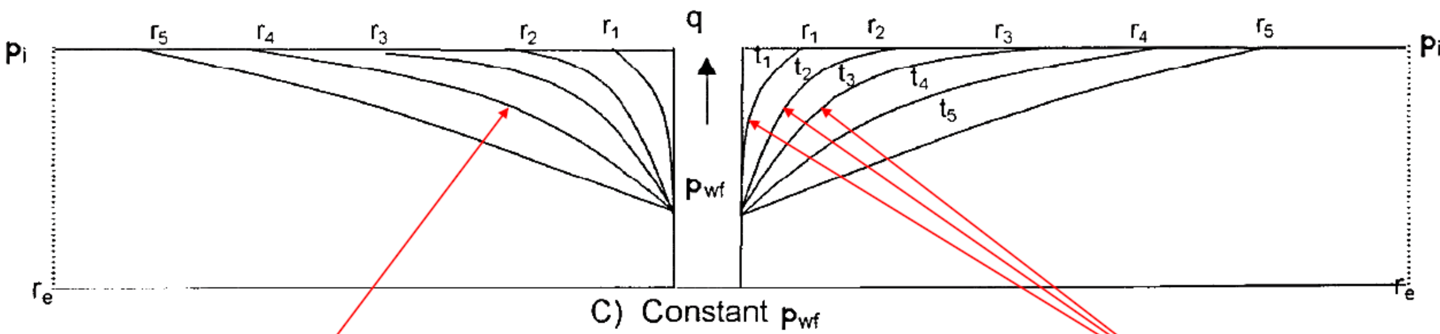


Transient flow occurs during
$t_0 < t < t_t$

Infinite acting reservoir,
$r_{inv} < r_e$

Figure-1: Transient flow

Unsteady (Transient) -State Flow



Drainage radius has been reached, $r_{inv} = r_e$

Infinite acting reservoir, $r_{inv} < r_e$

Transient flow occurs during $t_0 < t < t_4$

5

Transient (unsteady-state) flow is defined as

That time period during which the boundary has no effect on the pressure behavior in the reservoir and the reservoir will behave as its infinite in size

Development of Radial Differential Equation

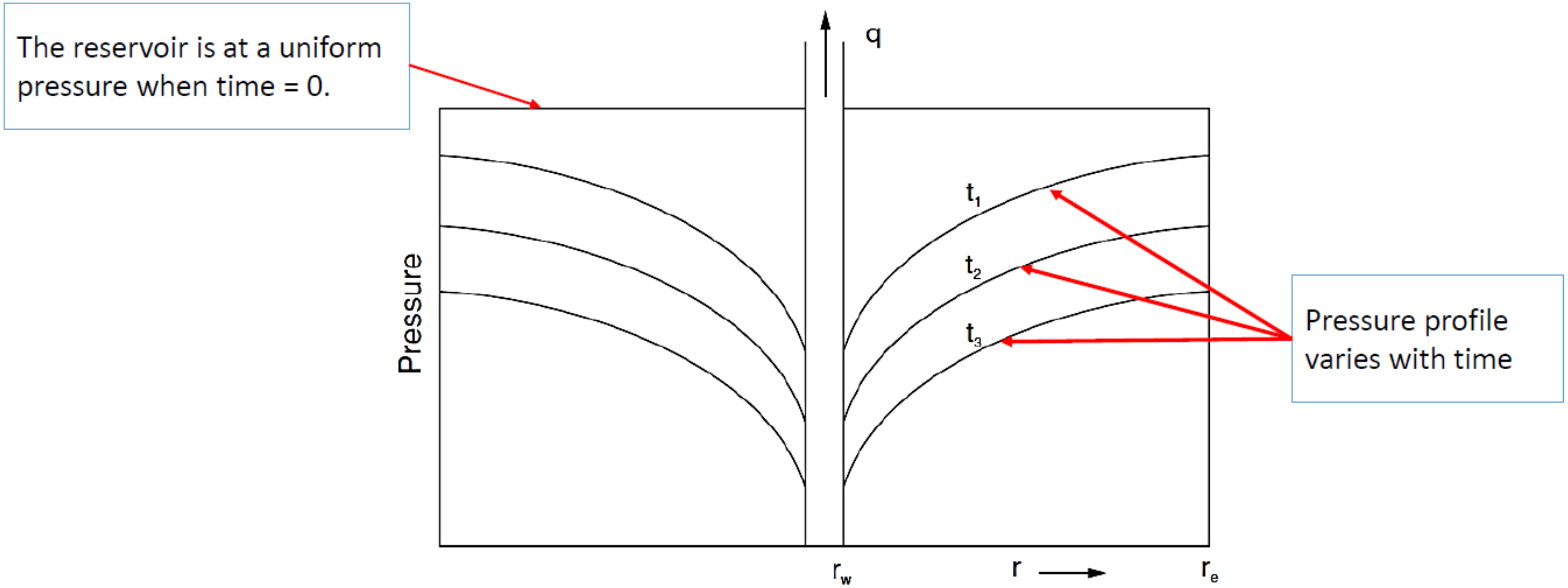


Figure-2: Transient Flow

Development of Radial Differential Equation

Ideal Reservoir Model

- Developing analysis techniques for well testing requires assumptions.

- The assumptions are introduced to combine the followings:
  - Law of mass conservation (continuity equation).
  - Darcy law.
  - Compressibility equation.
  - Initial and boundary conditions.

Development of Radial Differential Equation

Continuity Equation

- The continuity equation is essentially a material balance equation that accounts for every pound mass of fluid produced, injected, or remaining in the reservoir.

Transport Equation

- The continuity equation is combined with the equation for fluid motion (transport equation) to describe the fluid flow rate “in” and “out” of the reservoir. Basically, the transport equation is Darcy’s equation in its generalized differential form.

Compressibility Equation

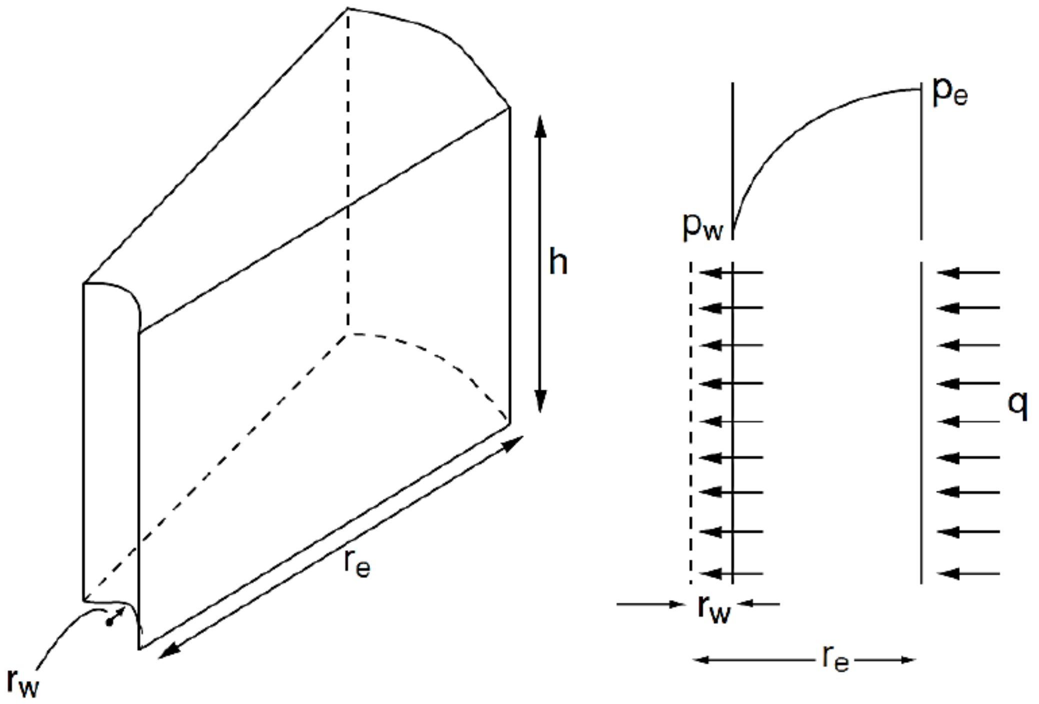
- The fluid compressibility equation (expressed in terms of density or volume) is used in formulating the unsteady-state equation with the objective of describing the changes in the fluid volume as a function of pressure.

Initial and Boundary Conditions

- There are two boundary conditions and one initial condition required to complete the formulation and the solution of the transient flow equation. The two boundary

Development of Radial Differential Equation

The two boundary conditions are:



Development of Radial Differential Equation

- According to the concept of the material-balance equation
- The rate of mass flow into and out of the element during a differential time Δt must be equal to the mass rate of accumulation during that time interval

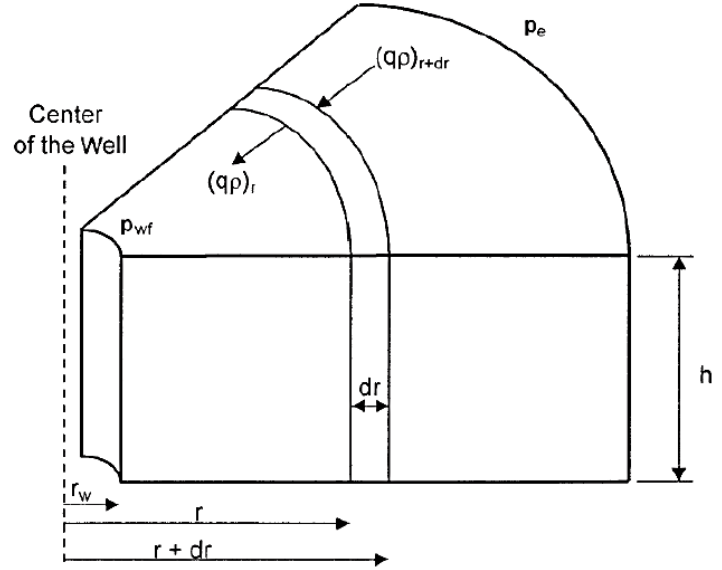


Figure-4: Radial flow

Development of Radial Differential Equation

[Mass entering volume element during Δt]-[Mass leaving volume element during Δt]

=[Rate of mass accumulation during Δt]

$$2\pi h\,(r+dr)\Delta t\,(v\rho)_{r+dr} - 2\pi hr\,\Delta t\,(v\rho)_r \qquad \text{1-1}$$

$$= (2\pi hr)dr\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$$

Dividing the above equation by (2πrh) dr Δt and simplifying gives:

$$\frac{1}{r\,dr}\,[(r+dr)\,(v\rho)_{r+dr} - r\,(v\rho)_r] = \frac{1}{\Delta t}\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$$

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\,[r(v\rho)] = \frac{\partial}{\partial t}\,(\emptyset\rho)} \qquad \text{1-2}$$

Continuity equation

Development of Radial Differential Equation

Where:

ϕ = porosity, ρ = density, lb/ft3 and v = fluid velocity, ft/day

Radial Darcy Law

$$v = (5.615)(0.001127)\,\frac{k}{\mu}\frac{\partial P}{\partial r} = 0.006328\frac{k}{\mu}\frac{\partial P}{\partial r} \qquad \text{1-3}$$

Where:

k = permeability, md

v = velocity, ft/day

Development of Radial Differential Equation

- Combining Equation 1-2 with Equation 1-3 results in:

$$\frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(r\rho)\frac{\partial P}{\partial r}\right] = \frac{\partial}{\partial t}(\emptyset\rho) \qquad 1\text{-}4$$

- Expanding the right-hand side by taking the indicated derivatives eliminates the porosity from the partial derivative term on the right-hand side:

$$\frac{\partial}{\partial t}(\emptyset\rho) = \emptyset\frac{\partial \rho}{\partial t} + \rho\frac{\partial \emptyset}{\partial t} \qquad 1\text{-}5$$

Development of Radial Differential Equation

- Since porosity is related to the formation compressibility by the following:

$$c_f = \frac{1}{\emptyset}\frac{\partial\emptyset}{\partial P} \qquad 1\text{-}6$$

- Applying the chain rule of differentiation to $\frac{\partial\emptyset}{\partial t}$

$$\frac{\partial\emptyset}{\partial t} = \frac{\partial\emptyset}{\partial P}\,\frac{\partial P}{\partial t} \qquad 1\text{-}7$$

- Substitute equation 1-6 into 1-7

$$\frac{\partial\emptyset}{\partial t} = c_f\emptyset\,\frac{\partial P}{\partial t} \qquad 1\text{-}8$$

Development of Radial Differential Equation

- Substitute equation 1-8 into 1-5

$$\frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(r\rho)\frac{\partial P}{\partial r}\right] = \emptyset\frac{\partial \rho}{\partial t} + \rho c_f \emptyset\,\frac{\partial P}{\partial t} \qquad \text{1-9}$$

General radial Partial Deferential Equation (PDE)

This equation is:

- Laminar
- Can be applied for any fluid flow (incompressible, slightly and compressible)

Development of Radial Differential Equation

In order to develop practical equations that can be used to describe the flow behavior of fluids.

- The treatments of the following systems are discussed below:
  - Radial flow of slightly compressible fluids
  - Radial flow of compressible fluids

Transient Flow Regime

- Assuming permeability and viscosity are constant

$$\frac{0.006328}{r}\frac{k}{\mu}\frac{\partial}{\partial r}\left[(r\rho)\frac{\partial P}{\partial r}\right] = \emptyset\frac{\partial \rho}{\partial t} + \rho c_f \emptyset\,\frac{\partial P}{\partial t} \qquad \text{1-10}$$

Development of Radial Differential Equation

- Expanding equation 1-10 gives:

$$0.006328\frac{k}{\mu}\left[\frac{\rho}{r}\frac{\partial P}{\partial r}+\rho\frac{\partial^2 P}{\partial r^2}+\frac{\partial P}{\partial r}\frac{\partial \rho}{\partial r}\right]=\emptyset\frac{\partial \rho}{\partial t}+\rho c_f\emptyset\ \frac{\partial P}{\partial t} \qquad \text{1-11}$$

- Using the chain rule in the above relationship yields:

$$0.006328\frac{k}{\mu}\left[\frac{\rho}{r}\frac{\partial P}{\partial r}+\rho\frac{\partial^2 P}{\partial r^2}+\frac{\partial P}{\partial r}\frac{\partial P}{\partial r}\frac{\partial \rho}{\partial P}\right]=\emptyset\frac{\partial P}{\partial t}\frac{\partial \rho}{\partial P}+\rho c_f\emptyset\ \frac{\partial P}{\partial t} \qquad \text{1-12}$$

- Dividing the above expression by the fluid density ρ gives

$$0.006328\frac{k}{\mu}\left[\frac{1}{r}\frac{\partial P}{\partial r}+\frac{\partial^2 P}{\partial r^2}+\left(\frac{\partial P}{\partial r}\right)^2\left(\frac{1}{\rho}\frac{\partial \rho}{\partial P}\right)\right]=\emptyset\frac{\partial P}{\partial t}\left(\frac{1}{\rho}\frac{\partial \rho}{\partial P}\right)+c_f\emptyset\ \frac{\partial P}{\partial t}$$

1-13

Recall that the compressibility of any fluid is related to its density by:

$$c=\frac{1}{\rho}\frac{\partial \rho}{\partial P} \qquad \text{1-14}$$

Development of Radial Differential Equation

- Combining equations 1-13 and 1-14 gives:

$$0.006328\frac{k}{\mu}\left[\frac{1}{r}\frac{\partial P}{\partial r}+\frac{\partial^2 P}{\partial r^2}+c\left(\frac{\partial P}{\partial r}\right)^2\right]=\emptyset c\frac{\partial P}{\partial t}+c_f\emptyset\frac{\partial P}{\partial t} \qquad \text{1-15}$$

- The term is considered very small and may be ignored:

$$0.006328\frac{k}{\mu}\left[\frac{1}{r}\frac{\partial P}{\partial r}+\frac{\partial^2 P}{\partial r^2}\right]=\emptyset(\underbrace{c+c_f}_{\text{Total compressibility ct}})\frac{\partial P}{\partial t} \qquad \text{1-16}$$

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\emptyset\mu c_t}{0.006328k}\frac{\partial P}{\partial t} \qquad \text{1-17}$$

Development of Radial Differential Equation

- where the time t is expressed in days.

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\emptyset \mu c_t}{0.00264\, k}\frac{\partial P}{\partial t} \qquad 1\text{-}18$$

Equation 1-18 is called the Diffusivity Equation

The equation is particularly used in analysis well testing data where the time t is commonly recorded in hours.

Where:

k = permeability, md

r = radial position, ft

p = pressure, psia

ct = total compressibility, psi−1

t = time, hrs

ϕ = porosity, fraction

μ = viscosity, cp

Development of Radial Differential Equation

- The equation can be rewritten as:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t} \qquad \text{1-19}$$

This equation is essentially designed to determine the pressure as a function of time t and position r.

Before discussing and presenting the different solutions to the diffusivity equation, it is necessary to summarize the assumptions and limitations used in developing Equation 1-19:

1. Homogeneous and isotropic porous medium
2. Uniform thickness
3. Single phase flow
4. Laminar flow
5. Rock and fluid properties independent of pressure

Total compressibility

$$c_t = S_o c_o + S_w c_w + S_g c_g$$

Total mobility

$$\lambda_t = \left(\frac{k_o}{\mu_o}\right) + \left(\frac{k_w}{\mu_w}\right) + \left(\frac{k_g}{\mu_g}\right)$$

Solution to Diffusivity Equation

- For a steady-state flow condition, the pressure at any point in the reservoir is constant and does not change with time, i.e., ∂p/∂t = 0, and therefore Equation 1-19 reduces to:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\underbrace{\cancel{\frac{\partial P}{\partial t}}}_{\text{Zero}}$$

$$\boxed{\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = 0} \qquad 1\text{-}20$$

Laplace's equation for steady-state flow

Solution to Diffusivity Equation

- Solution for the following flow regimes:
  - Steady-state
  - Pseudo-steady-state
  - Unsteady-state
- To obtain a solution to the diffusivity equation (Equation 1-19), it is necessary to specify:
  - Initial condition: states that the reservoir is at a uniform pressure pi when production begins.
  - The two boundary conditions: require that the well is producing at a constant production rate and that the reservoir behaves as if it were infinite in size

Solution to Diffusivity Equation-Transient Flow

Based on the boundary conditions imposed on (Equation 1-18), there are two generalized solutions to the diffusivity equation:

- Constant-terminal-pressure solution

  Is designed to provide the cumulative flow at any particular time for a reservoir in which the pressure at one boundary of the reservoir is held constant. This technique is frequently used in water influx calculations in gas and oil reservoirs.

- Constant-terminal-rate solution

  Solves for the pressure change throughout the radial system providing that the flow rate is held constant at one terminal end of the radial system, i.e., at the producing well

  An integral part of most transient test analysis techniques, such as with drawdown and pressure buildup analyses.

  Most of these tests involve producing the well at a constant flow rate and recording the flowing pressure as a function of time, i.e.,

  There are two commonly used forms of the constant-terminal-rate solution:

  1. The Ei-function solution
  2. The dimensionless pressure PD solution

Solution to Diffusivity Equation-Transient Flow

- The Ei-Function Solution

Assumptions:

- Infinite acting reservoir, i.e., the reservoir is infinite in size
- The well is producing at a constant flow rate
- The reservoir is at a uniform pressure, pi, when production begins
- The well, with a wellbore radius of rw, is centered in a cylindrical reservoir of radius re
- No flow across the outer boundary, i.e., at re

$$P = P_i + \left[\frac{70.6\ Q_o \mu_o B_o}{kh}\right] E_i \left[\frac{-948\ \emptyset \mu_o c_t r^2}{kt}\right] \qquad 1\text{-}21$$

line-source solution

Where:

- P = pressure at radius r from the well after t hours
- t = time, hrs
- k = permeability, md
- Qo = flow rate, STB/day

$$\frac{3.79 * 10^5\ \emptyset \mu_o c_t {r_w}^2}{k} < t < \frac{948\ \emptyset \mu_o c_t {r_e}^2}{k}$$

Solution to Diffusivity Equation-Transient Flow

Notes:

For x<0.02: $$E_i\left[\frac{-948\ \emptyset\mu_o c_t r^2}{kt}\right] = E_i(-x) = \ln(1.781x) \qquad \text{1-22}$$

For 0.02<x<10.9:

- Determine Ei(-x)-value from table 1-1(Lecture-6-Attachement)

For x>10.9:

- Ei(-x)=0

For r=rw, the logarithmic approximation will be used and equation 1- 21 will be:

$$P_{wf} = P_i + \left[\frac{70.6\ Q_o\mu_o B_o}{kh}\right]\left[\ln\left(\frac{1688\ \emptyset\mu_o c_t r_w^2}{kt}\right) - 2s\right] \qquad \text{1-23}$$

Where:

$$s = \left(\frac{k}{k_s} - 1\right)\ln\left(\frac{r_s}{r_w}\right) \qquad \text{1-24}$$

Example 1-1

Example 1-1: An oil well is producing at a constant flow rate of 300 STB/day under unsteady-state flow conditions. The reservoir has the following rock and fluid properties:

B_o = 1.25 bbl/STB	μ_o = 1.5 cp	$c_t = 12 \times 10^{-6}$ psi^{-1}
k_o = 60 md	h = 15 ft p_i = 4000 psi	ϕ = 15% r_w = 0.25 ft
p_i = 4000 psi	r_w = 0.25 ft	

1. Calculate pressure at radii of 0.25, 5, 10, 100, 1,000, 2,000, and 2,500 feet, for 1 hour. Plot the results as Pressure versus radius
2. Repeat part 1 for t = 12 hours and 24 hours. Plot the results as pressure versus radius.

Example 1-1

Solution:

$$P = P_i + \left[\frac{70.6\ Q_o \mu_o B_o}{kh}\right] E_i \left[\frac{-948\ \emptyset \mu_o c_t r^2}{kt}\right]$$

$$= 4000 + \left[\frac{70.6 * 300 * 1.5 * 1.25}{60 * 15}\right] E_i \left[\frac{-948 * 0.15 * 12 * 10^{-12} r^2}{60t}\right]$$

$$= 4000 + 44.125\ E_i \left[-42.6 * 10^{-6} \frac{r^2}{t}\right]$$

Time= 1 hour

r, ft	x	Ei(-x)	P
0.25	-2.6625E-06	-12.25907	3459.069
5	-0.001065	-6.26760548	3723.442
10	-0.00426	-4.88131111	3784.612
100	-0.426	-0.27614093	3987.815
1000	-42.6	0	4000
2000	-170.4	0	4000
2500	-266.25	0	4000

Example 1-1

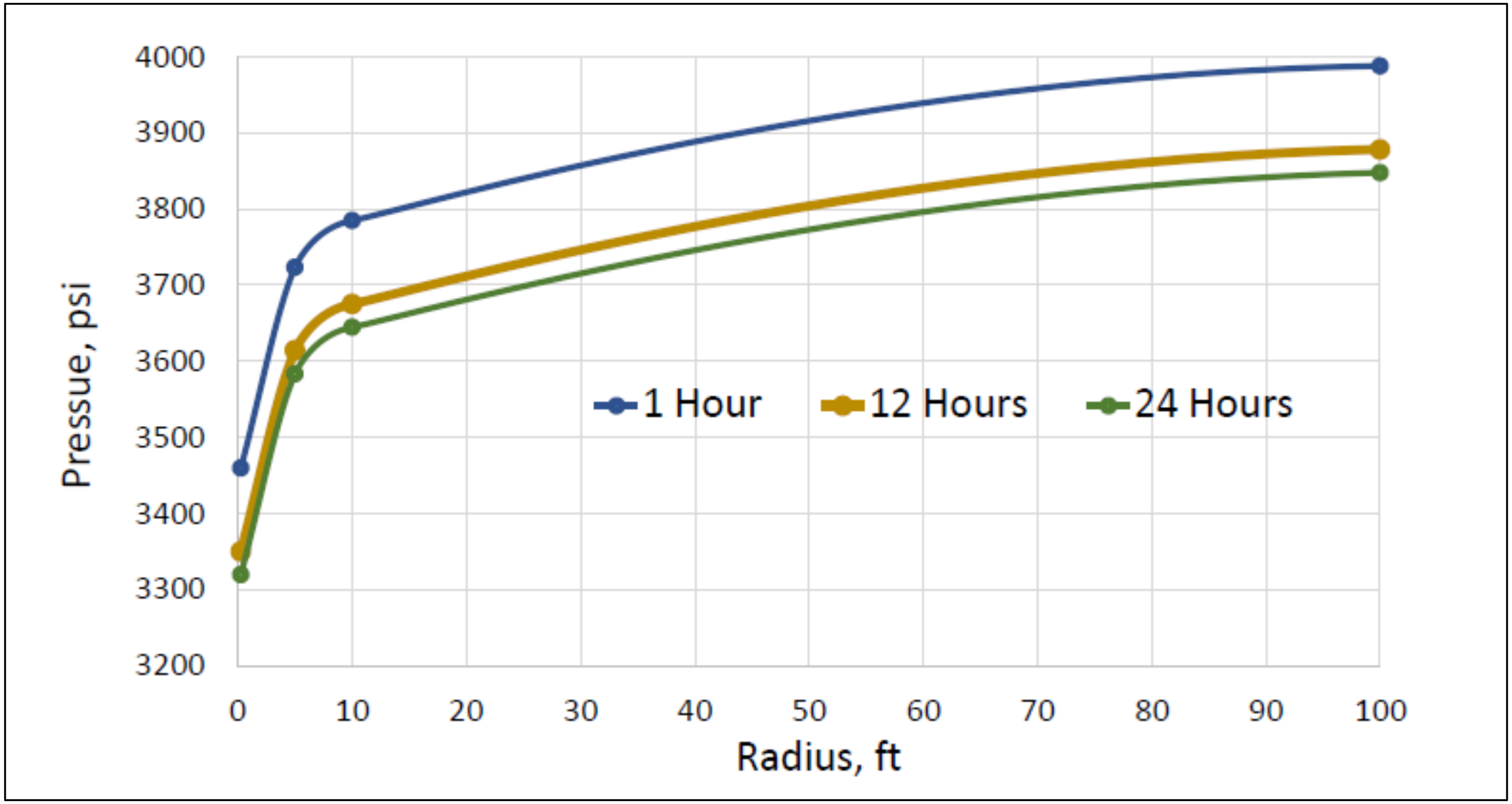
Time=12 hours

r, ft	x	Ei(-x)	P
0.25	-2.21875E-07	-14.7439767	3349.422
5	-0.00008875	-8.75251213	3613.795
10	-0.000355	-7.36621776	3674.966
100	-0.0355	-2.76104758	3878.169
1000	-3.55	0	4000
2000	-14.2	0	4000
2500	-22.1875	0	4000

Time=24 hours

r, ft	x	Ei(-x)	P
0.25	-1E-07	-15.4371	3318.837
5	-4E-05	-9.44566	3583.21
10	-0.0002	-8.05936	3644.381
100	-0.0178	-3.45419	3847.584
1000	-1.775	0	4000
2000	-7.1	0	4000
2500	-11.094	0	4000

Example 1-1



4000
3900
3800
3700
3600
3500
3400
3300
3200
Pressue, psi
0
10
20
30
40
50
60
70
80
90
100
Radius, ft
1 Hour
12 Hours
24 Hours

Example 1-2

Example 1-2: for an oil well producing at constant rate of 10 SRB/day. Blow are the description data for the well and the reservoir.

$B_o = 1.475$ bbl/STB	$\mu_o = 0.72$ cp	$c_t = 1.5 \times 10-5$ psi−1
$k_o = 0.1$ md	$h = 150$ ft	$\phi = 23\%$
$p_i = 3000$ psi	$r_w = 0.5$ ft	r_e=3000 ft

Calculate the reservoir pressure at radius of 1, 10 and 100 ft after 3 hours of production.

Example 1-2

Solution:

The Ei-function will be valid if $\frac{3.79 * 10^5\ \emptyset \mu_o c_t r_w{}^2}{k} < t < \frac{948\ \emptyset \mu_o c_t r_e{}^2}{k}$

$$\frac{3.79 * 10^5\ \emptyset \mu_o c_t r_w{}^2}{k} = \frac{3.79 * 10^5 * 0.23 * 0.72 * 1.5 * 10^{-5} * 0.5^2}{0.1}$$
$$= 2.35\ hrs$$

The equation can be used.

The end time for the reservoir to act as infinite reservoir is given by:

$$\left[\frac{948\ \emptyset \mu_o c_t r_e{}^2}{kt}\right] = \left[\frac{948 * 0.23 * 0.72 * 1.5 * 10^{-5} * 3000^2}{0.1}\right]$$
$$= 211{,}9000\ hours.$$

The reservoir will act as infinite reservoir till the time of $\mathbf{211,\ 9000}\ hrs$

Example 1-2

- The Ei-function can now be used (Equation 1-21)

$$P = P_i + \left[\frac{70.6\ Q_o\mu_o B_o}{kh}\right] E_i \left[\frac{-948\ \emptyset\mu_o c_t r^2}{kt}\right]$$

<u>At r=1 ft</u>

$$P = 3000 + \left[\frac{70.6 * 20 * 1.475 * 0.72}{0.1 * 150}\right] E_i \left[\frac{-948\ 0.23 * 0.72 * 1.5 * 10^{-5} * 1^2}{0.1 * 3}\right]$$

$$P = 3000 + 100\ \boxed{E_i[-0.007849]}$$

Use eq. 1-22

$$P = 3000 + 100 \ln[(1.781)(0.007849)] = 2573\ psi$$

Example 1-2

<u>At r=10 ft</u>

$$P = 3000 + \left[\frac{70.6 * 20 * 1.475 * 0.72}{0.1 * 150}\right] E_i \left[\frac{-948\, 0.23 * 0.72 * 1.5 * 10^{-5} * 10^2}{0.1 * 3}\right]$$

$$P = 3000 + 100\, E_i[-0.7849]$$

$$P = 3000 + 100 \ln[(1.781)(0.7849)] = 2968\, psi$$

<u>At r=100 ft</u>

$$P = 3000 + \left[\frac{70.6 * 20 * 1.475 * 0.72}{0.1 * 150}\right] E_i \left[\frac{-948\, 0.23 * 0.72 * 1.5 * 10^{-5} * 100^2}{0.1 * 3}\right]$$

$$P = 3000 + 100\, E_i[-78.49]$$

$$P = 3000 + 0 = 3000\, psi$$