Tishk International University
Engineering Faculty
Petroleum and Mining Engineering Department
Well Testing
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Lecture 8:
Pressure Transient Analysis

Fourth Grade - Spring Semester 2022-2023

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Well Testing Operation:

- Drill Stem Testing (DST) – Drilling
- Surface Well Testing (SWT) – Before/during production

Operation Stages:

- Build up (BU) – Well Shut in
- Drawdown (DD) – Well Flowing

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Well Testing Data

- Pressure vs Time
- Flowrate vs Time
- Oil and Gas Sample Collection

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Pressure Transient Analysis

- Permeability - Skin - Productivity Index – Other Reservoir Parameters

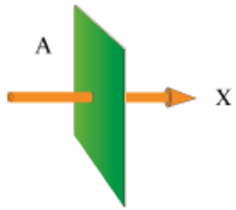
Pressure Transient Analysis - PTA

- Theories
    - Darcy's Law
    - Diffusivity Equation
    - IARF
    - The Line Source Solution
    - Wellbore storage
    - Skin
    - Superposition
    - Physical Meaning of Diffusion

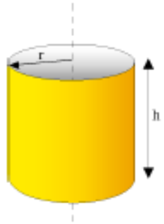
- PTA
    - Introduction
    - Drawdown response and MDH Plot
    - Build-up response and Horner Plot
    - Bourdet Derivate
    - Bourdet Derivate Flow Regimes and Models

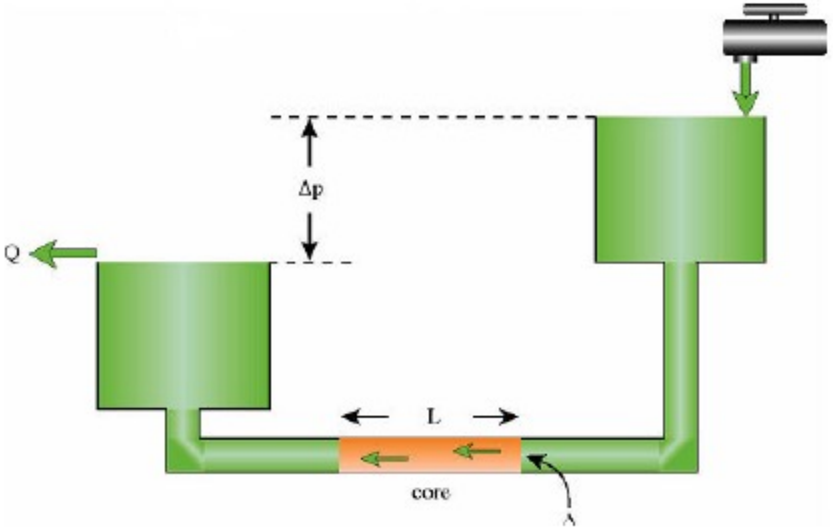
Theories - Darcy's Law

Linear flow: $$\frac{\partial p}{\partial x} = -887.2 \frac{q_x \mu}{k_x A}$$



Radial Flow: $$r\frac{\partial p}{\partial r} = 141.2 \frac{q \mu}{kh}$$





$$\frac{\Delta p}{L} = -887.2 \frac{Q \mu}{kA}$$

Theories - Diffusivity Equation

Fluid flow in porous media is governed by the diffusivity equation.

Diffusivity Equation explains the effect of pressure on reservoir properties. Since pressure is changing versus time, the equation can also explain how reservoir properties are changing with time.

- **General :** $$\frac{\partial p}{\partial t} = 0.0002637 \frac{k}{\Phi \mu c_t} \nabla^2 p$$
- **Radial Flow :** $$\frac{\partial p}{\partial t} = 0.0002637 \frac{k}{\Phi \mu c_t} \frac{1}{r} \left[\frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) \right]$$
- **Linear flow :** $$\frac{\partial p}{\partial t} = 0.0002637 \frac{k}{\Phi \mu c_t} \frac{\partial^2 p}{\partial x^2}$$

Darcy's Law

+

Conservation of mass

+

Equation of state

=

Diffusion Equation

Theories - Diffusivity Equation

Assumptions made in Diffusivity Equation

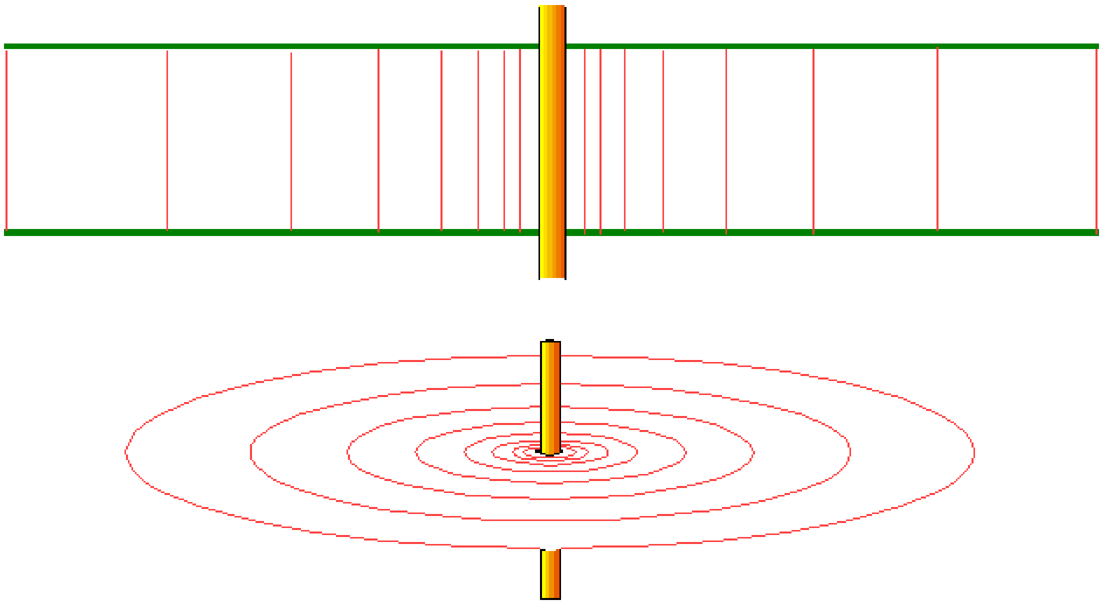
1. Homogeneous reservoir
2. No gravity effects
3. Darcy’s law valid
4. Single phase Flow
5. Slightly compressible fluid
6. Constant viscosity
7. Small pressure gradient

Theories - IARF

Infinite-Acting Radial Flow

Fluid flows towards the wellbore equally from all directions – the pressure drop expands radially. The upper and lower bed boundaries are parallel and clearly defined, the reservoir rock between them is homogeneous, and the wellbore is perpendicular to the bed boundaries:

The initial radial flow (IARF) regime is called infinite-acting because until the first boundary is reached, the flow pattern and corresponding pressure drop at the wellbore are exactly as would be obtained if the reservoir were truly infinite.



Theories - The Line Source Solution

- The line source solution in a homogeneous infinite reservoir:
  - The diffusion in a homogeneous infinite reservoir, starting at initial uniform pressure Pi, produced by a vertical line source well.
  - The solution at any point and time, for a Line Source well producing a homogeneous infinite reservoir, is given by the following equation

$$p(r,t) = p_i - \frac{70.6 qB\mu}{kh}\left[-E_i\left(-\frac{948.1\Phi\mu c_t r^2}{kt}\right)\right]$$

Zero radius – No wellbore storage – No skin

Line source assumptions:

Diffusion equation : $\frac{\partial p}{\partial t} = 0.0002637\frac{k}{\Phi\mu c_t}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]$

Initial condition : $p(t=0,r) = p_i$

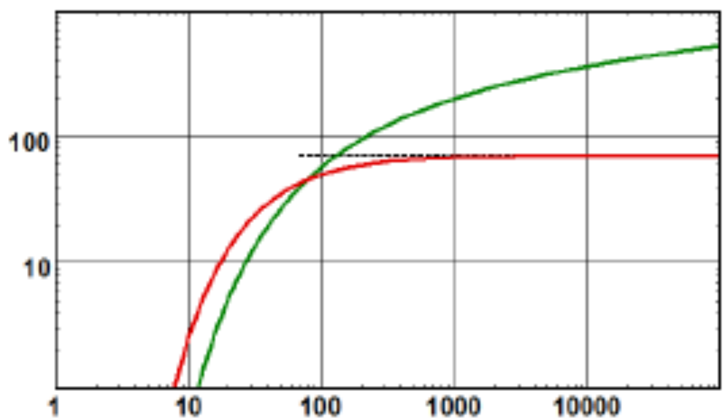
Zero Radius : $\lim\left[r\frac{\partial p}{\partial r}\right]_{r\to 0,t} = 141.2\frac{qB\mu}{kh}$

Infinite reservoir : $\lim_{r\to\infty}\left[p(r,t)\right] = p_i$

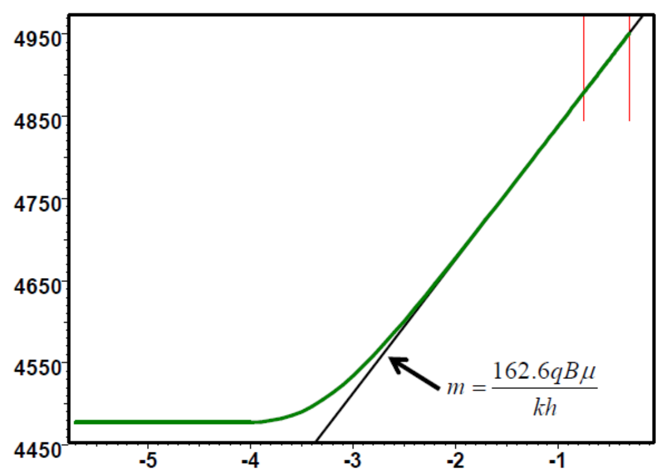
Theories - The Line Source Solution

Semilog approximation when : $t \geq \frac{379200\Phi\mu c_t r_w^2}{k}$

$$p(t) \approx p_i - \frac{162.6q\mu}{kh}\left[\log(t) + \log\left(\frac{k}{\Phi\mu c_t r_w^2}\right) - 3.228\right]$$



Line source loglog plot



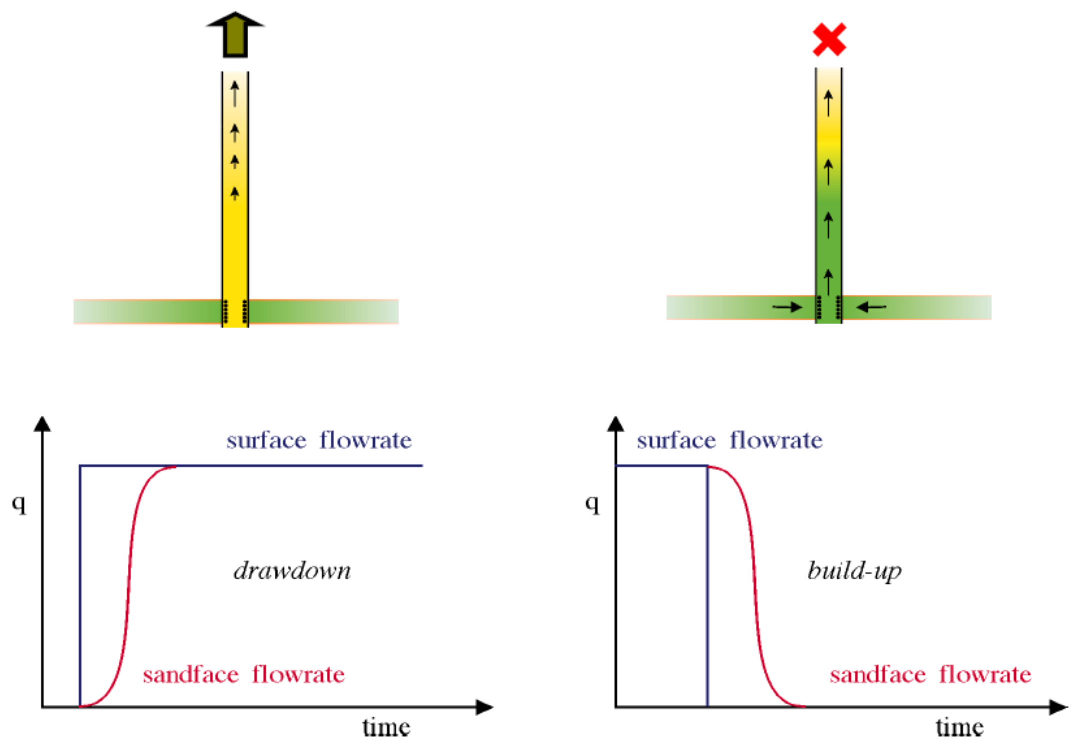
Line Source semilog plot

Knowing the slope m, we can find K

X-axis is a linear scale of log(Δt)

Theories - Wellbore storage

- Wellbore storage prevents the sandface flowrate from instantaneously following the surface
- flowrate. Initially, flow at surface is due only to decompression of fluid in the wellbore.
- Eventually, decompression effects become negligible and the downhole flowrate approaches the surface rate
- The reverse happens during a build-up, as for a while 'the bottom of the well does not know what the top is doing', and the reservoir continues to flow into the well after it has been shut in. This is known as afterflow and is also called wellbore storage.



Theories - Wellbore storage

Fluid filled wellbore : $$C = V_w c_w$$

Well with a liquid level : $$C = 144 \frac{V_{unity}}{\rho(g/g_c)}$$

Constant storage sandface flow rate : $$q_{sf} = qB + 24C \frac{\partial p_{wf}}{\partial t}$$

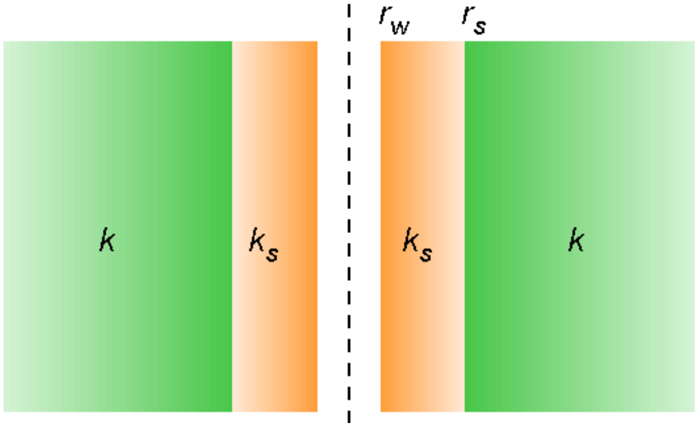
Well condition with wellbore storage: $$\left[r \frac{\partial p}{\partial r} \right]_{r_w, t} = \frac{141.2 \left[qB + 24C \frac{\partial p}{\partial t} \right] \mu}{kh}$$

Early time pressure response : $$\Delta p = \frac{qB}{24C} \Delta t$$

Theories - Skin

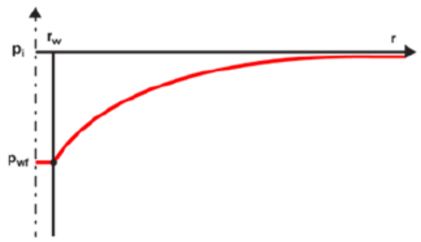
- If after drilling, completion, cementing and perforating, the overall pressure drop during production into the wellbore is identical to that for the ideal case, of undamaged wellbore in an open hole completion, the well is said to have a zero skin. Often the reservoir near the wellbore has been invaded by (typically water-based) drilling fluid, and has undergone changes in permeability, absolute and/or relative to the reservoir fluid.

  - Pressure loss (Positive Skin)
    - Invasion during drilling
    - Non-ideal perforations
    - Limited entry
    - Turbulent flow
    - Fluid block
  - Pressure Gain (Negative Skin)
    - Stimulation

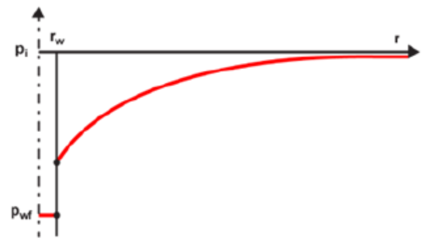


Theories - Skin

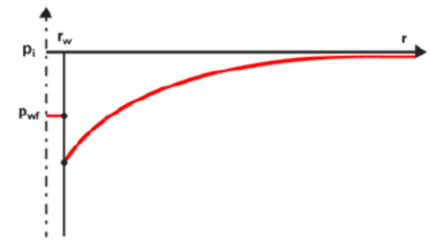
Pressure loss or gain : $$\Delta p_{Skin} = \frac{141.2 qB\mu}{kh} \cdot S$$



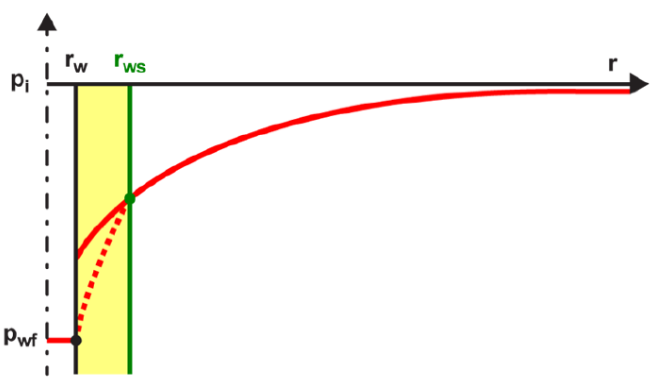
Non damaged skin = 0

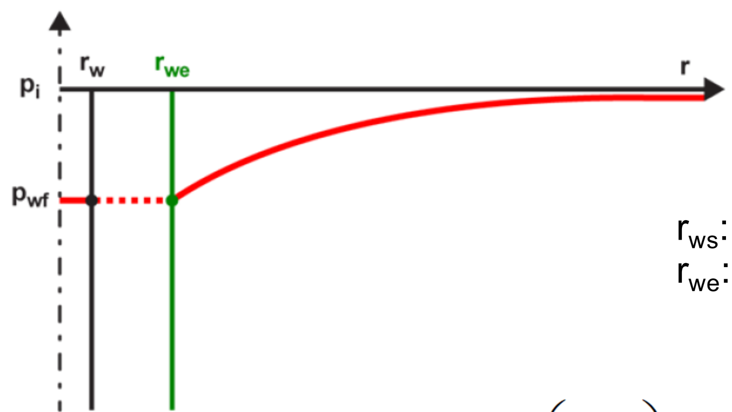


Damaged Skin > 0



Stimulated Skin < 0





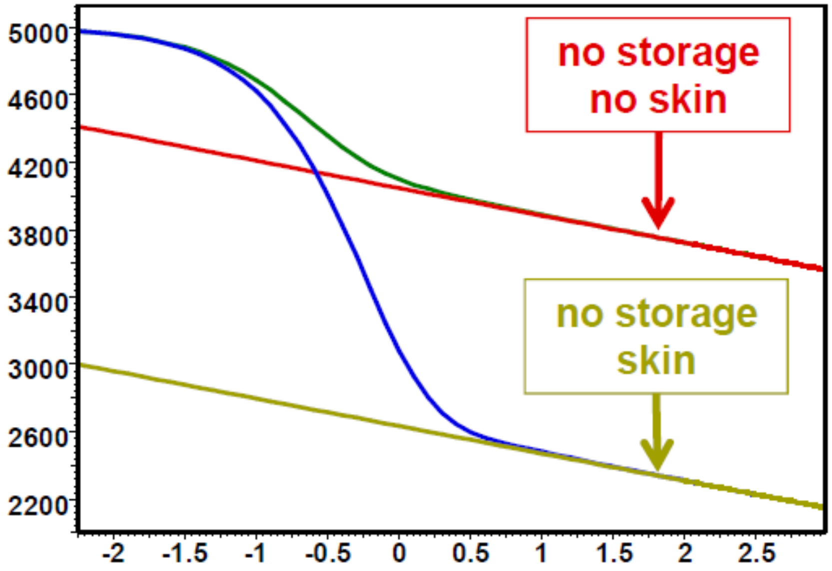
r_{ws}: Skin damaged zone radius
r_{we}: Equivalent wellbore radius

$$S = \left(\frac{k}{k_S} - 1 \right) \ln \left(\frac{r_S}{r_w} \right)$$

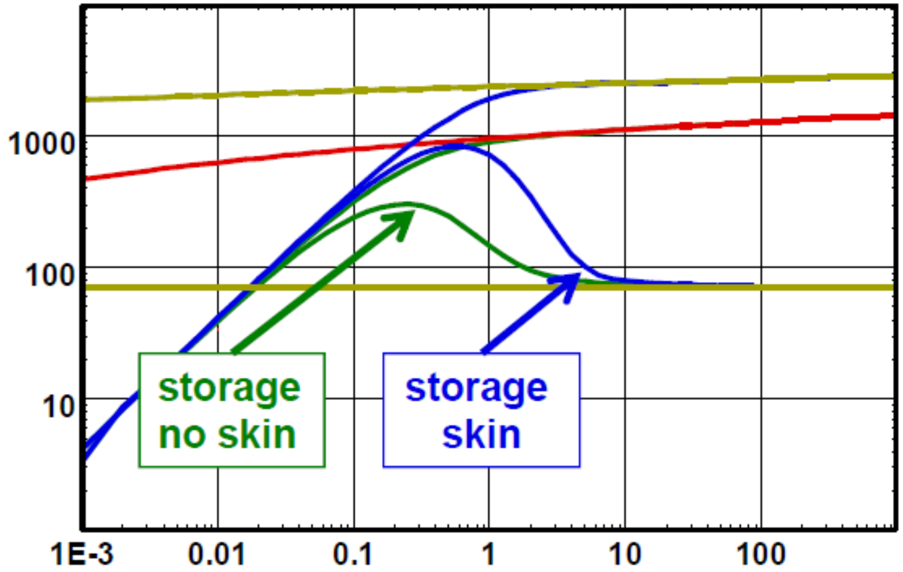
$$S = -\ln \left(\frac{r_{we}}{r_w} \right)$$

$$r_{we} = r_w e^{-S}$$

Theories – Wellbore Storage & Skin



Finite radius solution, semilog scale

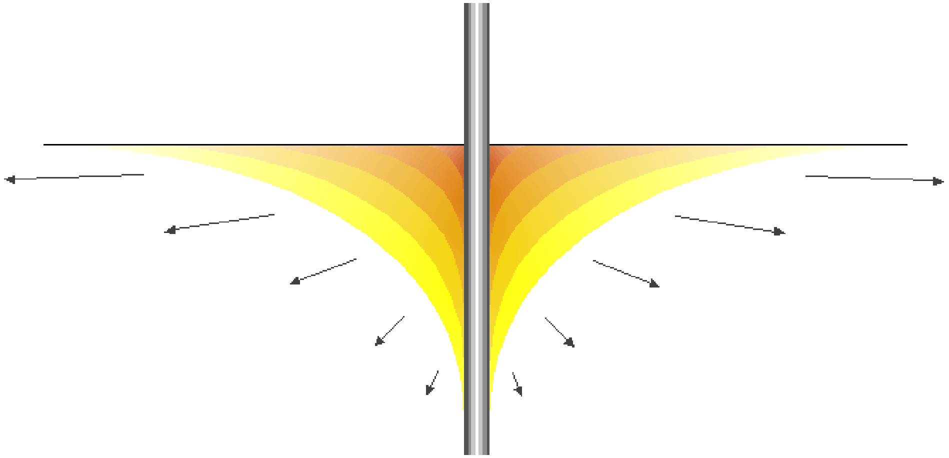


Finite radius solution, loglog scale

Theories - Superposition

- If we start producing a well, from a reservoir initially at a uniform pressure pi, we will induce a distortion of the pressure profile at the wellbore, the slope of which is given by Darcy's Law. The 'bending' of the pressure profile in the case of a drawdown is described as concave, and the diffusivity equation will describe how quickly this distortion will evolve within the reservoir:

- Throughout the production phase the profile will be concave, the pressure dropping everywhere, and it will be most concave close to the well. The concavity around the well will reduce in time, as more and more of the fluid is produced from further into the reservoir.

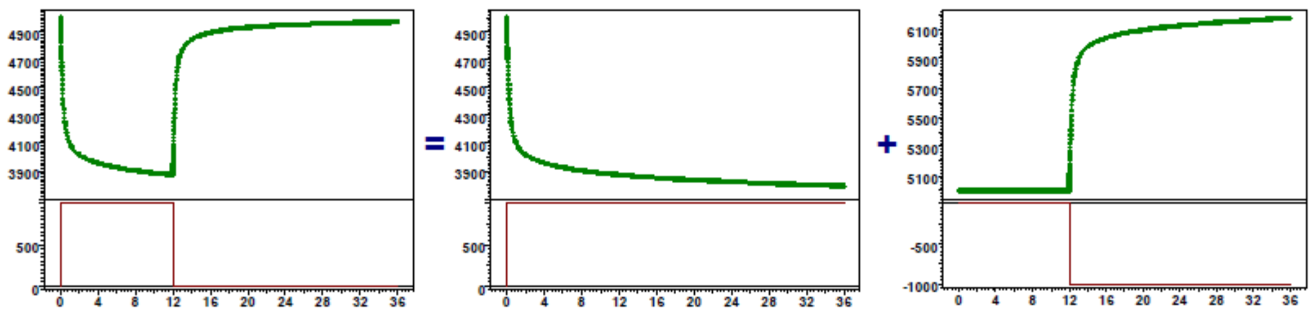


Theories - Superposition

Superposition:

Response to ‘’n’’ disturbances (rate changes) = To the sum of the responses to each disturbance (rate change)

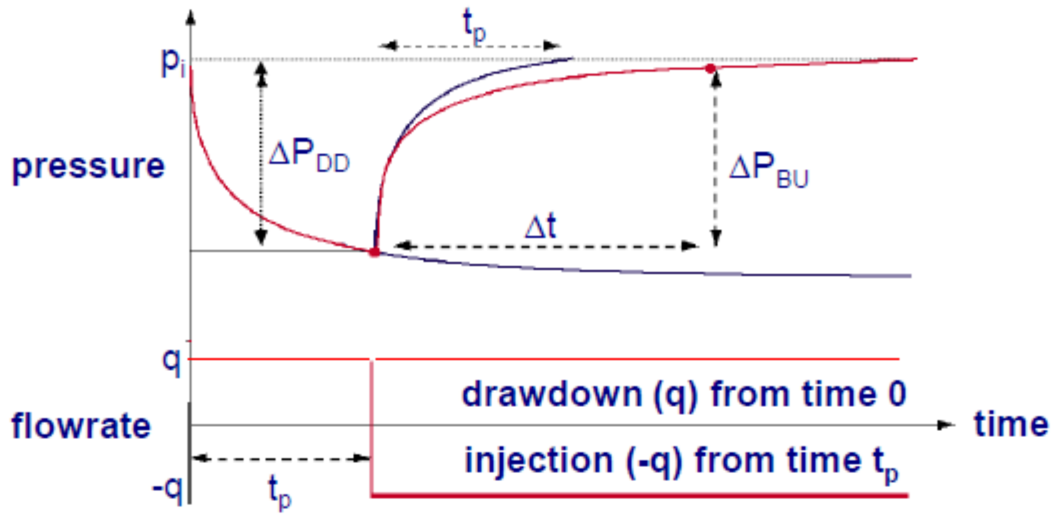
Superposition single buildup following single production



12h @ 1000 b/d + 24h @ 0 b/d	=	36h @ 1,000 b/d	+	12h @ 0 b/d + 24h @ -1,000 b/d

Theories - Superposition

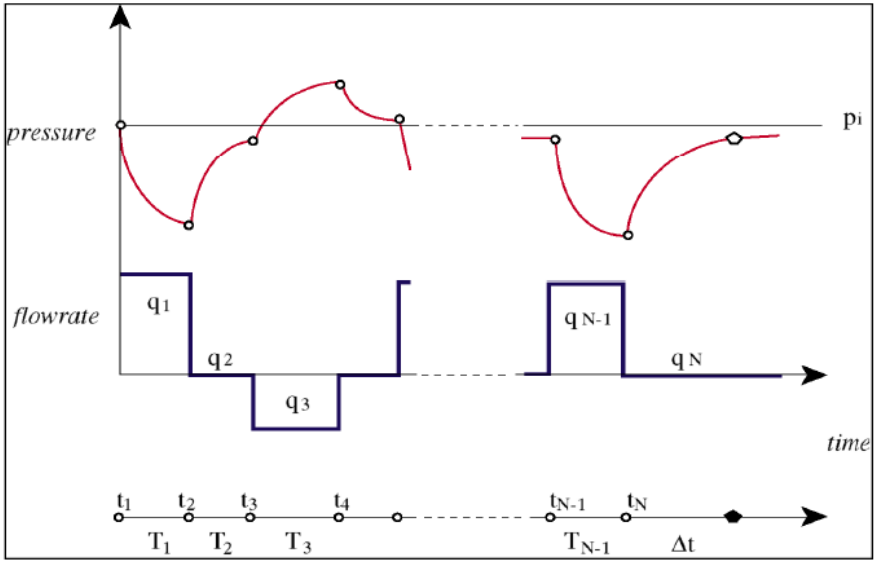
Buildup Superposition :



$$p_{BU}(\Delta t) = p_i - p_{Dd}(t_p + \Delta t) + p_{Dd}(\Delta t)$$

$$\Delta p_{BU}(\Delta t) = \Delta p_{DD}\left(t_p\right) + \Delta p_{DD}\left(\Delta t\right) - \Delta p_{DD}\left(t_p + \Delta t\right)$$

Theories - Superposition



Drawdown:

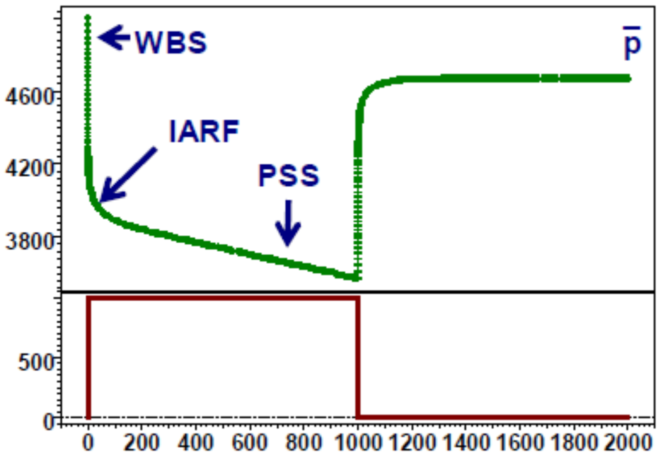
$$p(t) = p_i - \sum_{i=1}^{n} (q_i - q_{i-1}) \Delta p_{unit}(t - t_i)$$

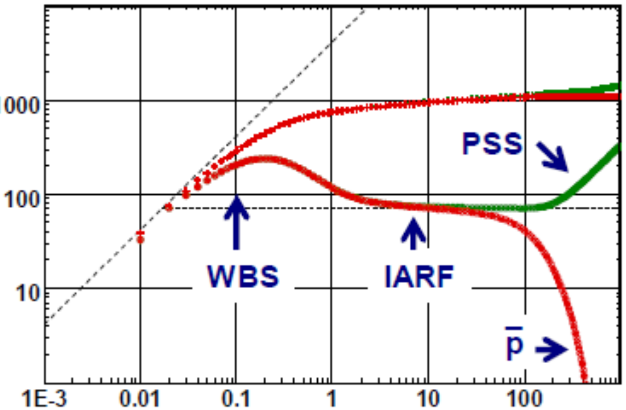
Build up:

$$\Delta p_{BU}(\Delta t) = \sum_{i=1}^{n-1} (q_i - q_{i-1}) \Delta p_{unit}(t_n - t_i) - \sum_{i=1}^{n} (q_i - q_{i-1}) \Delta p_{unit}(t_n + \Delta t - t_i)$$

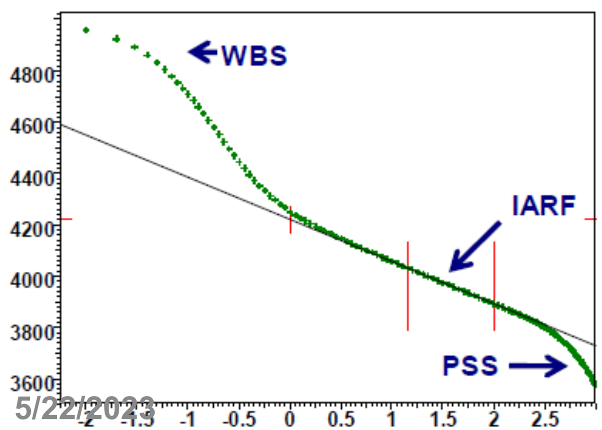
Theories - Physical Meaning of Diffusion

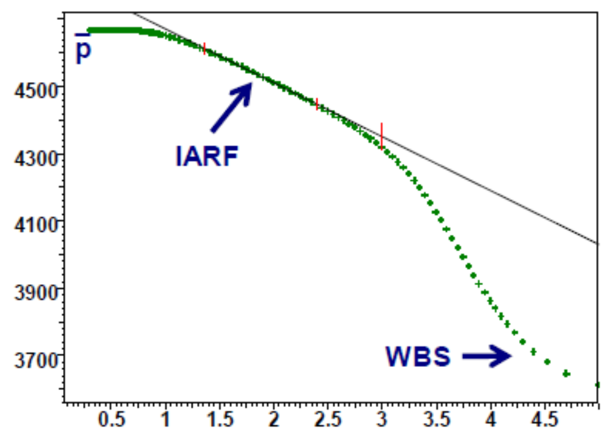
Test Design of the reference case: history and loglog plot





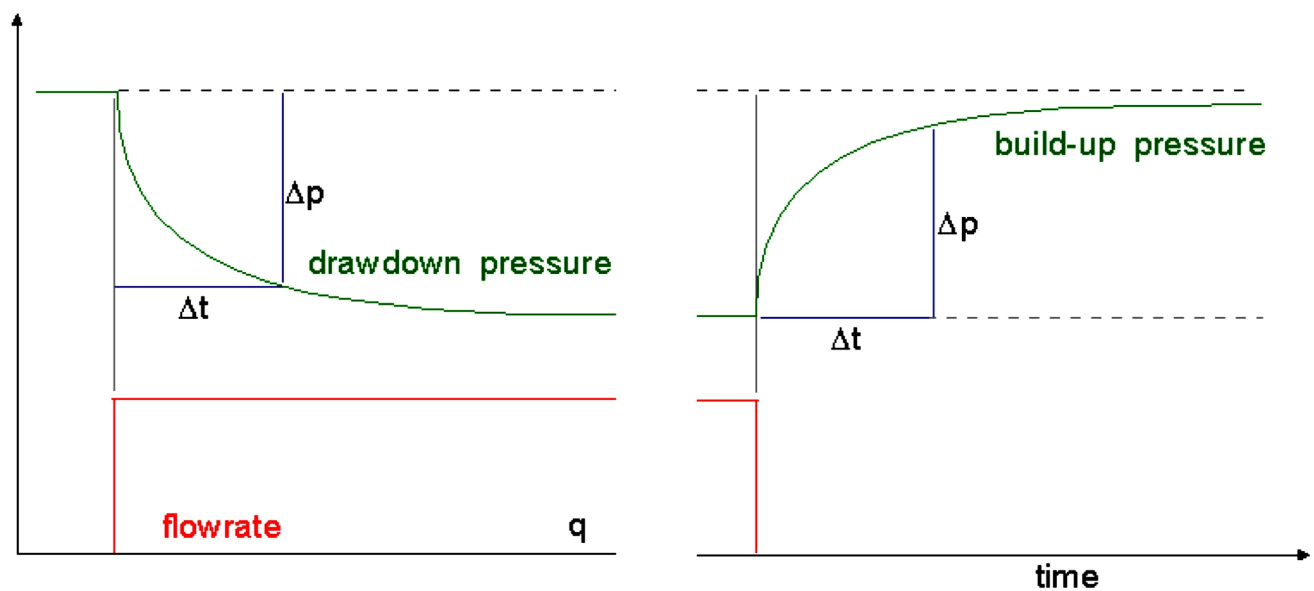
Semilog plot (production) and Horner plot (shut-in)





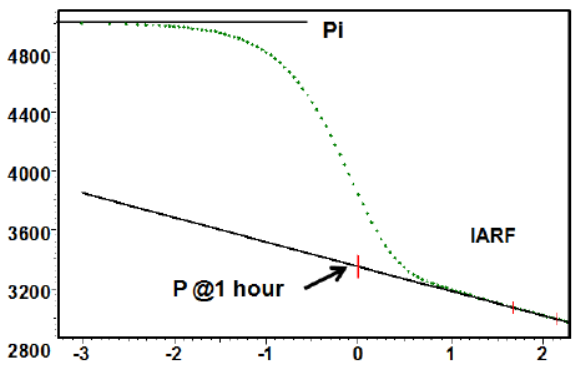
PTA - Introduction

- The pressure response during a transient well test is a function of both the well and reservoir characteristics and the flowrate history. In interpretation terms, the actual pressure and time are unimportant, with analysis performed in terms of pressure change Δp versus elapsed time, Δt
- The linear or Cartesian plot of pressure versus time, as shown above, is of limited value in well testing, but does have specialized uses, as will be seen later. Well test interpretation is predominantly carried out using semi-log and log-log techniques.

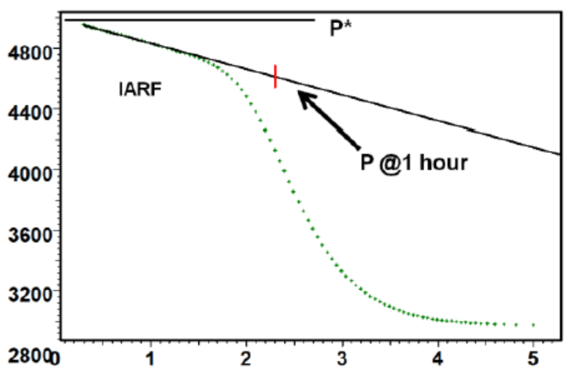


PTA - Introduction

- The main flow regime of interest is the infinite acting Radial Flow (IARF), which occurs after well effects have faded and before boundaries are detected. IARF may not always be seen. IARF provides an average permeability around the well, the well productivity (skin).
- When the well is shut in we also get an estimate of reservoir static pressure (P* or Pi). The first PTA methods were specialized plots (MDH, Horner) introduced in the 1950s to identify and quantify IARF.



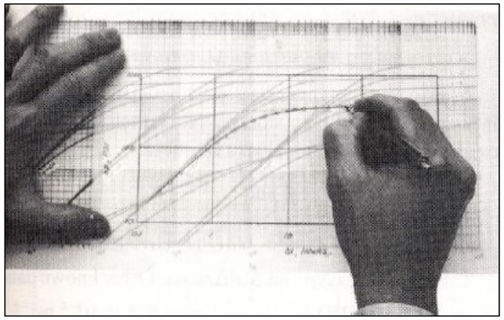
MDH plot



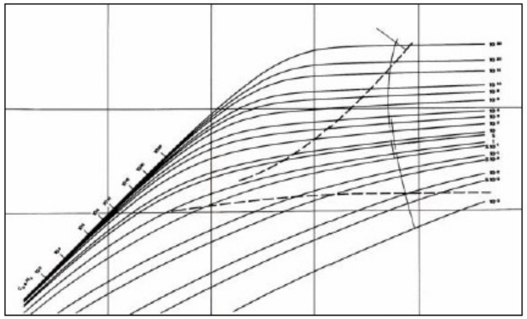
Horner plot

PTA - Introduction

- In the 1970s loglog type-curve matching techniques were developed to complement straight lines. One would plot pressure response on a loglog scale on tracing paper and slide it over pre-printed type-curves until a match is obtained.
- The choice of the type-curve and the relative position of the data (the match point) provided physical results. These methods were of poor resolutions until the Bourdet derivate was introduced.



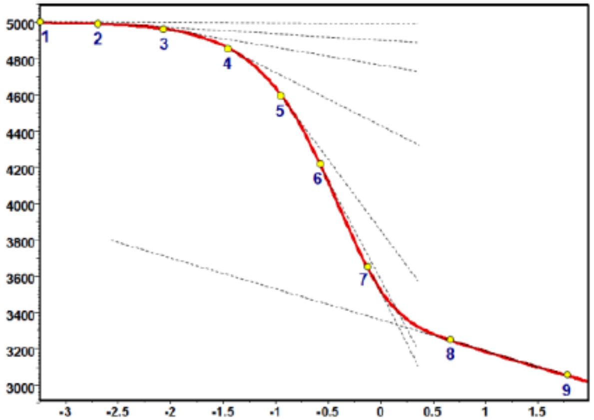
Manual Drawdown type curve matching



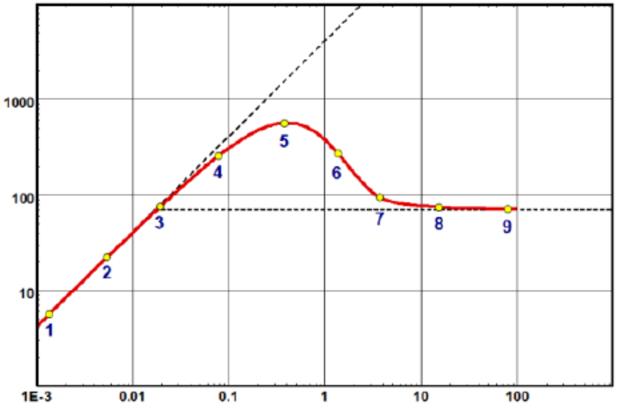
Drawdown Type Curve

PTA - Introduction

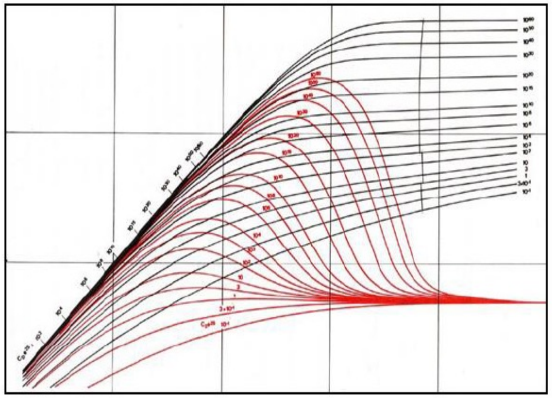
- In 1983, the Bourdet derivates, i.e., the slope of the semi-log plot displayed on the loglog plot, increased the resolution and reliability of a new generation of type-curves



Superposition Plot



Derivate Plot

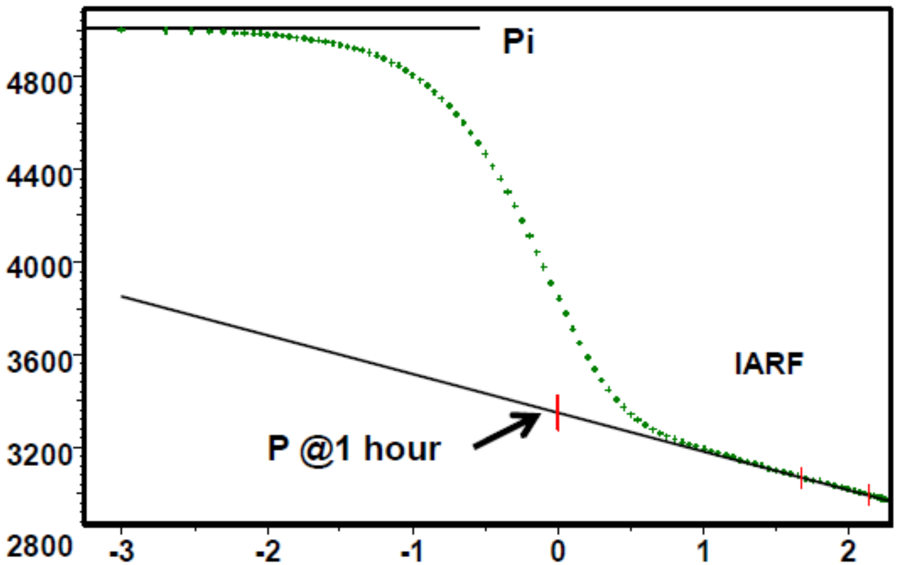


Bourdet derivate type curve

PTA - Drawdown response and MDH Plot

MDH plot is a graph of the pressure or the pressure change as a function of the logarithm of time

$$\left(p_i - p_{wf}\right) = \frac{162.6\, qB\mu}{kh}\left[\log t + \log \frac{k}{\Phi \mu c_t r_w^2} + 0.8686S - 3.2275\right]$$



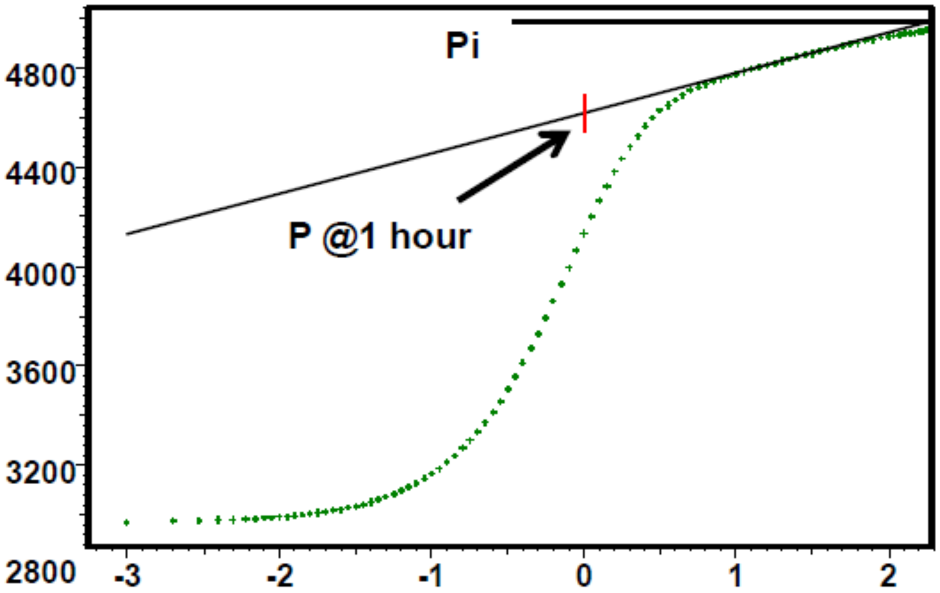
$$m = \frac{162.6 qB\mu}{kh}$$

$$kh = \frac{162.6 qB\mu}{m}$$

$$S = 1.151\left[\frac{p_i - p_{1hr}}{m} - \log \frac{k}{\Phi \mu c_t r_w^2} + 3.227\right]$$

Drawdown MDH plot

PTA – Build up response and MDH Plot



Build up MDH plot

$$m = \frac{162.6 q B \mu}{kh}$$

$$kh = \frac{162.6 q B \mu}{m}$$

$$S = 1.151\left[\frac{p_i - p_{1hr}}{m} - \log\frac{k}{\Phi \mu c_t r_w^{2}} + 3.227\right]$$

MDH Analysis Limits: First constant rate production period

PTA - Build-up response and Horner Plot

- To use semi-log analysis for any flow period other than the first drawdown, it is necessary to consider superposition effects.

Drawdown: effect of q during t_p+Δt:

$$p_i - p_{wf} = \frac{162.6 qB\mu}{kh}\left[\log(t_p + \Delta t) + \log\frac{k}{\Phi \mu c_t r_w^2} - 3.22 + .87S\right]$$

Shut-in: Effect of -q during Δt:

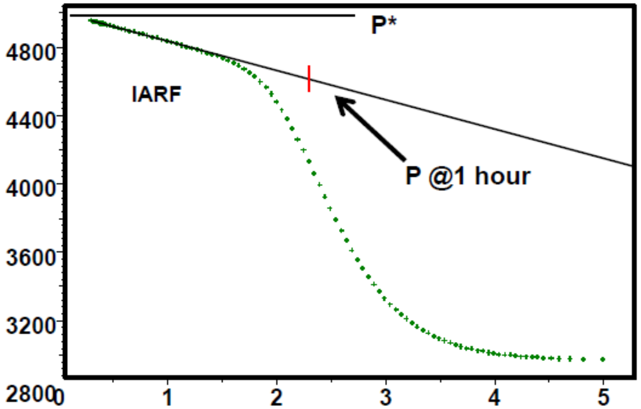
$$p_{wf} - p_{ws} = \frac{162.6(-q)B\mu}{kh}\left[\log\Delta t + \log\frac{k}{\Phi \mu c_t r_w^2} - 3.22 + .87S\right]$$

Final result:

$$p_{ws} = p_i - \frac{162.6 qB\mu}{kh}\log\frac{(t_p + \Delta t)}{\Delta t}$$

PTA - Build-up response and Horner Plot

- To use semi-log analysis for any flow period other than the first drawdown, it is necessary to consider superposition effects.



Horner plot

$$p_{ws} = p_i - \frac{162.6 q B \mu}{kh} \log \frac{(t_p + \Delta t)}{\Delta t}$$

$$m = \frac{162.6 q B \mu}{kh}$$

$$kh = \frac{162.6 q B \mu}{m}$$

The Horner method is the solution of the superposition of one rate change

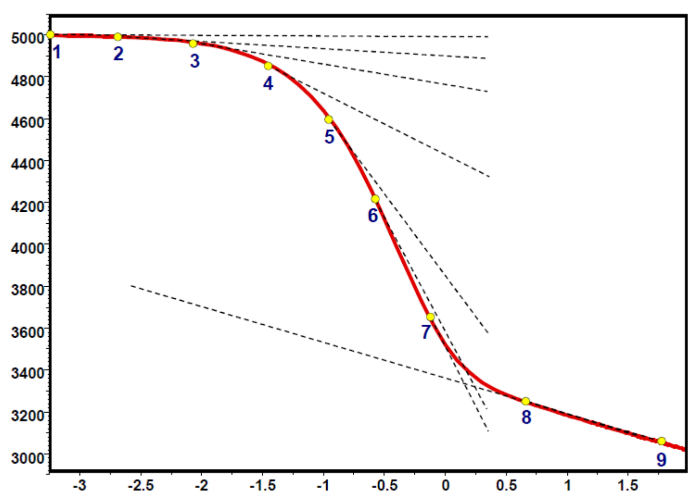
$$t_{pe} = \frac{cumulative\ \ production}{last\ \ rate}$$

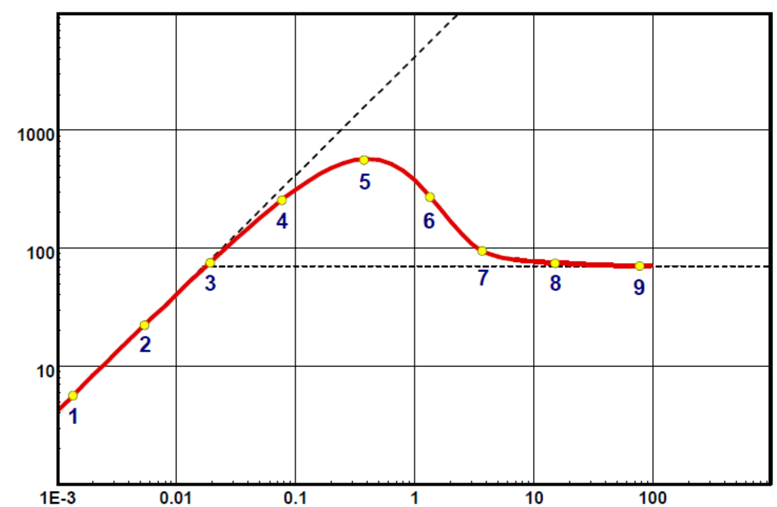
$$S = 1.151 \left[\frac{p_{1hr} - p_{wf}}{m} + \log\left(\frac{t_p + 1}{t_p}\right) - \log \frac{k}{\Phi \mu c_t r_w^2} + 3.227 \right]$$

$p^* = p_i$ only when reservoir infinite

PTA - Bourdet Derivate

- Bourdet derivative is the slope of the semi-log plot displayed on the loglog plot. It is the slope of this semi-log plot when the time scale is the neutral log. It must be multiplied by ln(10)=2.31 when the decimal logarithm is used in the semi-log plot. The semi-log is not ‘any' semi-log plot (Horner, MDH, etc). The reference logarithmic time scale must be the superposition time.





PTA - Bourdet Derivate

First drawdown : $\Delta p' = \frac{d\Delta p}{d(\ln \Delta t)} = \Delta t \frac{d\Delta p}{d\Delta t}$

any other period : $\Delta p' = \frac{d\Delta p}{d\sup(\Delta t)}$

Superposition time function

Wellbore storage : $\Delta p = C\Delta t$

Approximation : $\sup(\Delta t) \approx \ln(\Delta t)$

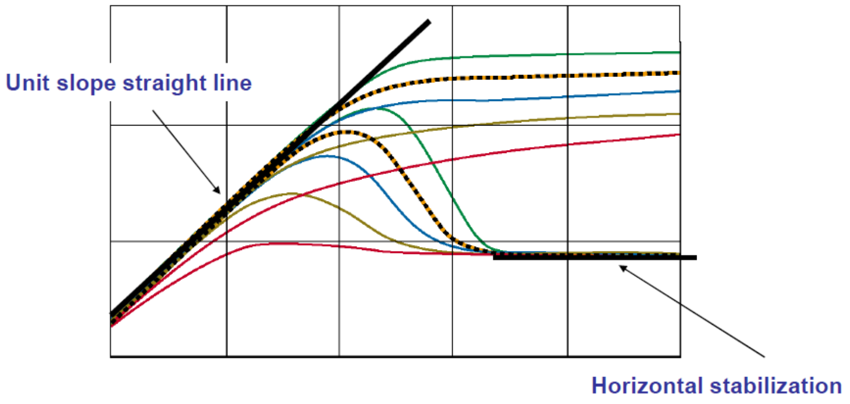
Bourdet derivative : $\Delta p' = \Delta t \frac{dC\Delta t}{d\Delta t} = C\Delta t = \Delta p$

Unit slope on the loglog plot

When IARF occurs : $\Delta P = m' \sup(\Delta t)$

Derivative with IARF : $\Delta P' = \frac{d\Delta P}{d(\sup \Delta t)} = m'$

In dimensionless terms : $P'_D = \frac{dP_D}{d(\ln t_D)} = 1/2$



PTA - Bourdet Derivate Flow Regimes and Models