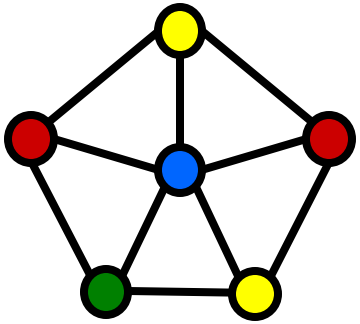
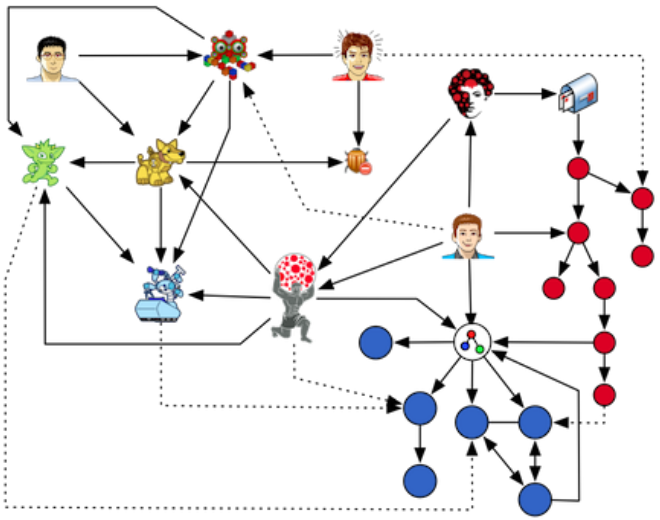
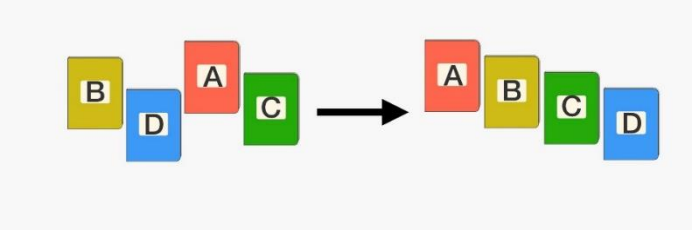
Introduction to Discrete Mathematics







Ms. Togzhan Nurtayeva
Course Code: IT 235/A
Semester 3
Week 1
Date: 04.10.2023

Basic Information

- **Instructor**: Ms. Togzhan
- **Lectures**: 3 hours a week
- **Homework:**
  - Will be posted on the SIS system
- **Lecture Notes**:
  - Will be posted on the Lecture Notes web page
- **Office Hours**: will be announced soon
- **Contact**: via email togzhan.nurtayeva@tiu.edu.iq

Course Materials

- Discrete Mathematics and its Applications, 8th edition by Kenneth H. Rosen
- Discrete Mathematics and its Applications, 8th edition by Richard Johnsonbaugh
- Discrete Mathematics with Applications, 5th edition by Susanna S. Epp

- Study links and references for specific topics will be given within Lecture Notes.

Class Regulations

Students have an obligation to arrive on time and remain in the classroom for the duration of scheduled classes and activities.

If students miss more than 10 minutes of any class period or leave before the instructor dismisses the class, they will be marked absent for the whole hour.

Students have an obligation to write, homework, quizzes and final examinations at the times scheduled by the teacher and university.

Students have an obligation to show respectful behavior and appropriate classroom deportment. Should a student be disruptive and/or disrespectful, the teacher has the right to exclude the disruptive student from learning activities (classes) and may refer the case to the Director of Student Services under the Student Code of Conduct.

Mobile Phones are not allowed in the classroom.

Should a student have an emergency case during lecture hours, he/she must raise his hand, and only after lecturer's permission can leave the class.

A Grading Scheme

- Participation/Discussion: **5%** (**not** attendance)
- Homework: **10%**
- Quizzes: **20%** (**25%** - for exempted students)
- Midterm: **25%**
- Final Exam **40%**

Course Content

01.10.2023-19.10.2023	Intro to Algorithms and Pseudo Code - Searching - Sorting - Optimization - Recursive - Big-O Notation
22.10.2023-26.10.2023	Time and Space Complexity of Algorithms
29.10.2023 – 02.11.2023	- Cartesian Products and Ordered Pairs - Relations
05.11.2023 – 09.11.2023	Revision Week (if topics above are covered fully)
11.11.2023 – 18.11.2023	MIDTERM
19.11.2023 – 23.11.2023	Proofs (direct, by case, induction, contradiction)
26.11.2023 – 30.11.2023	Proofs (direct, by case, induction, contradiction)
03.12.2023 – 07.12.2023	Intro to Graph Theory - Types - Applications
10.12.2023 – 21.12.2023	Trees / Dijkstra's, Kruskal's and Prim's Algorithms
24.12.2023 – 28.12.2023	Cryptography or Logic Laws (if topics above are covered fully)
31.01.2024 – 04.01.2024	Revision Week
06.01.2024 – 18.01.2024	FINALS

What is discrete math?

- Discrete math is the study of countable, distinct elements rather than continuous ones.

  So instead of smooth-running real numbers, you'll study:

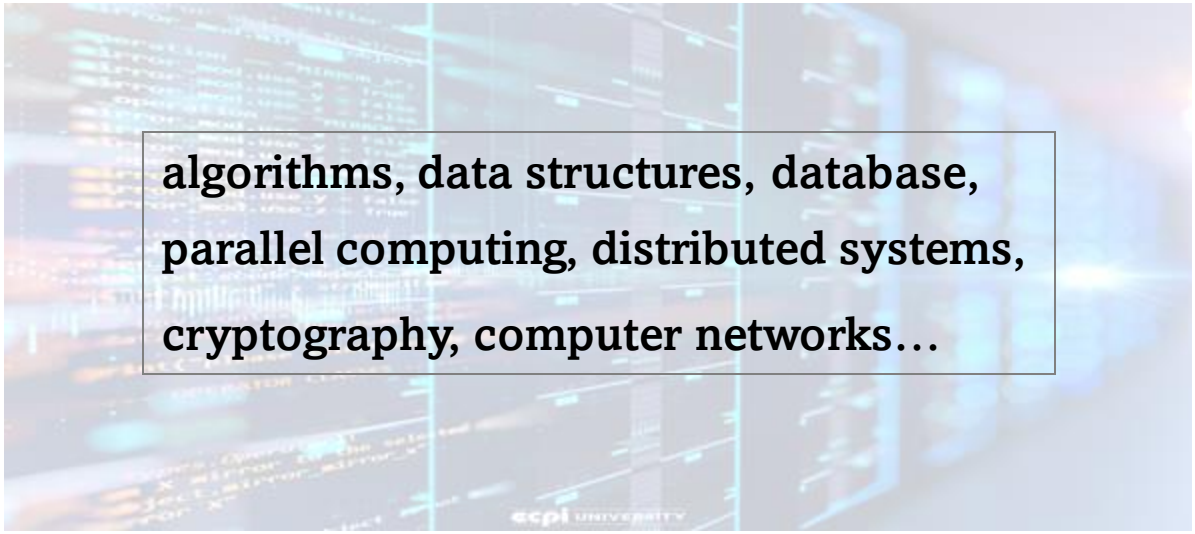
  - ☑ integers
  - ☑ graphs
  - ☑ statements

- Discrete math principles are commonly used in building algorithms for computer science and data science.

Why Discrete Mathematics?

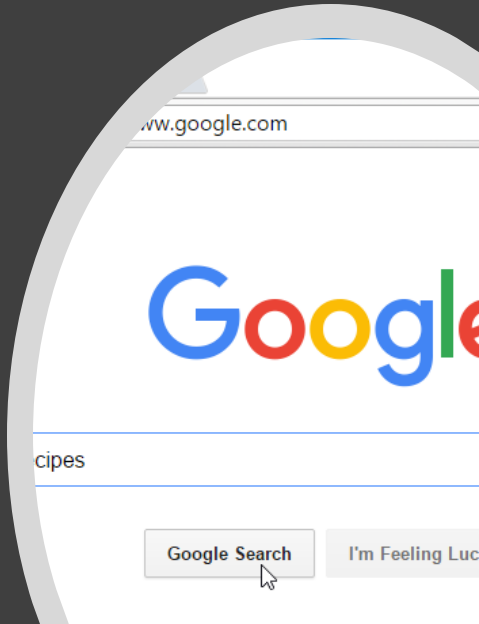
Design efficient computer systems.

- How did Google manage to build a fast search engine?
- What is the foundation of internet security?

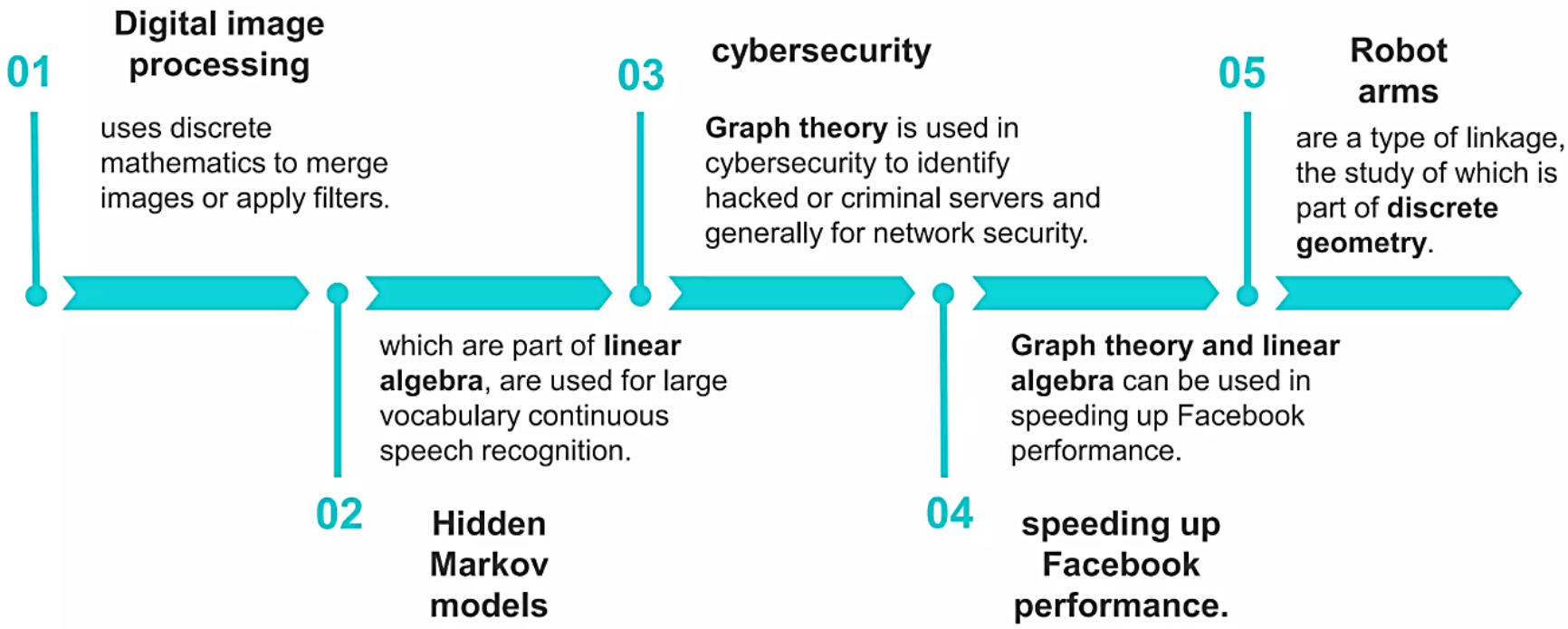


Logic, number theory, counting, graph theory…

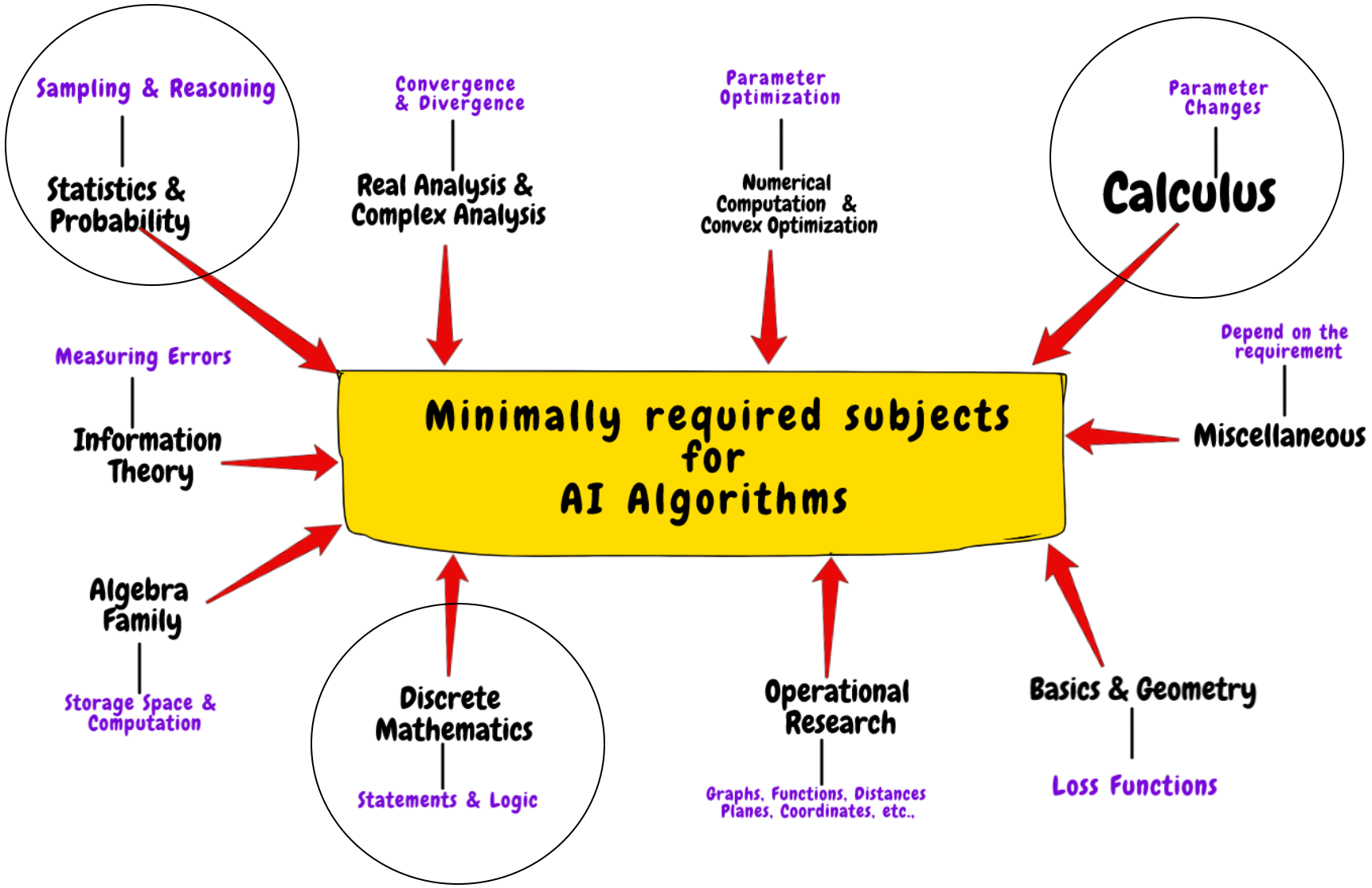
Applications



- Computer Graphics (e.g., video games)
- Cryptography (security)
- Networks (“following” on Twitter, “friending” on Facebook, etc.)
- Voting systems
- Logistics
- A programmer uses DM to design efficient algorithms.
- Web Search
- Database
- Scheduling
- Cell phone Communications
- Web Designing


01
Digital image processing
uses discrete mathematics to merge images or apply filters.
02
Hidden Markov models
which are part of **linear algebra**, are used for large vocabulary continuous speech recognition.
03
cybersecurity
Graph theory is used in cybersecurity to identify hacked or criminal servers and generally for network security.
04
speeding up Facebook performance.
Graph theory and linear algebra can be used in speeding up Facebook performance.
05
Robot arms
are a type of linkage, the study of which is part of **discrete geometry**.

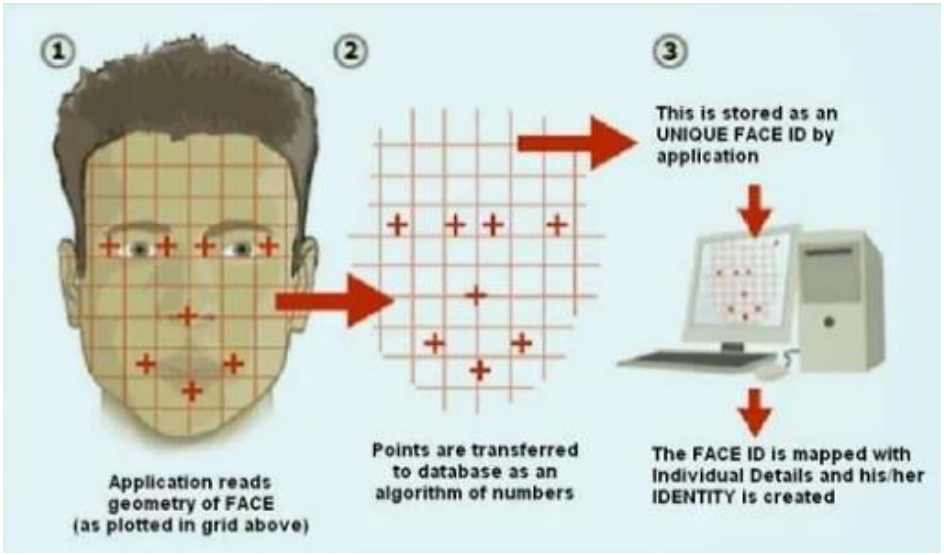

AI


Sampling & Reasoning
Statistics & Probability
Convergence & Divergence
Real Analysis & Complex Analysis
Parameter Optimization
Numerical Computation & Convex Optimization
Parameter Changes
Calculus
Measuring Errors
Information Theory
Minimally required subjects for AI Algorithms
Depend on the requirement
Miscellaneous
Algebra Family
Storage Space & Computation
Discrete Mathematics
Statements & Logic
Operational Research
Graphs, Functions, Distances Planes, Coordinates, etc.,
Basics & Geometry
Loss Functions

Digital Image Processing

In computer science, digital image processing is the use of computer algorithms to perform image processing on digital images to get an enhanced image or to extract some useful information from it.

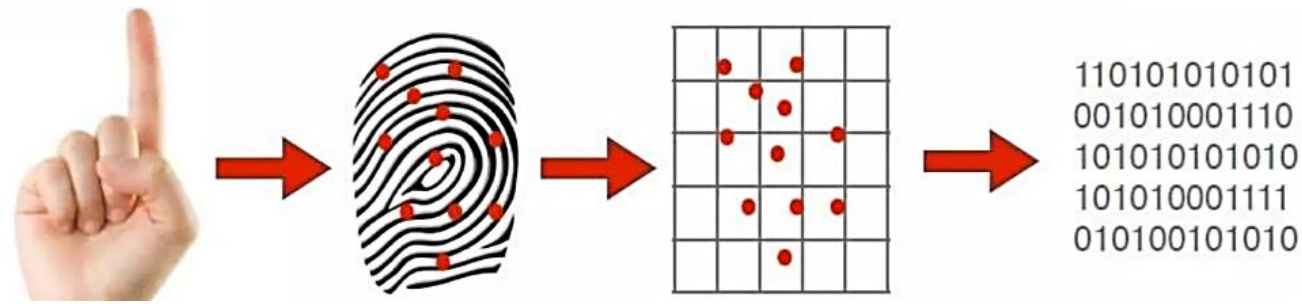
It allows a much wider range of algorithms to be applied to the input data and can avoid problems such as the build-up of noise and signal distortion during processing.

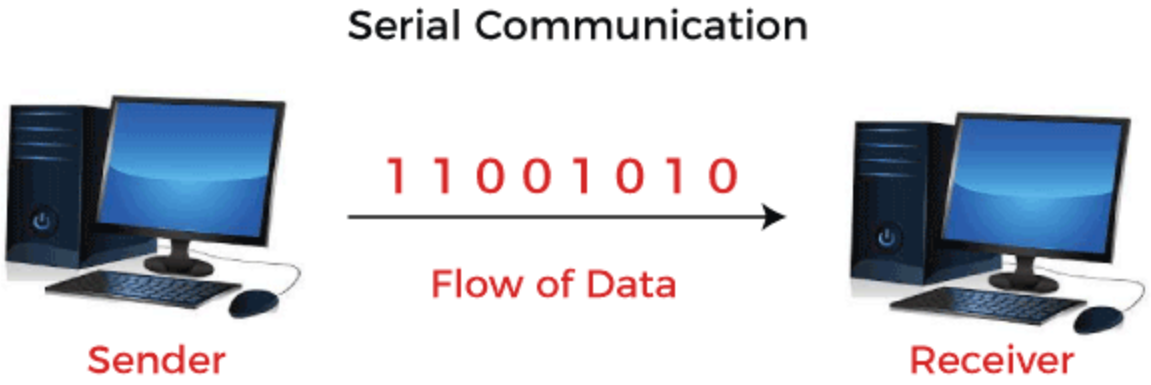


Fingerprint Optical Scanner

Fingerprint Optical Scanner measures your finger electrically.

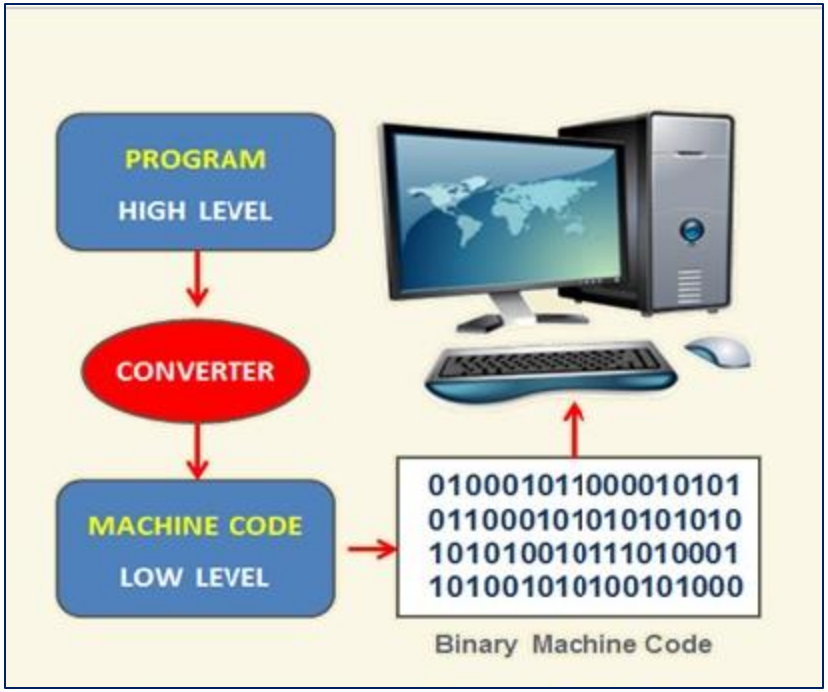
Only specific characteristics, which are unique to every fingerprint, are filtered and saved as an encrypted biometric key or mathematical representation. No image of a fingerprint is ever saved, only a series of numbers (a binary code), which is used for verification. The algorithm cannot be reconverted to an image, so no one can duplicate your fingerprints.





Computers

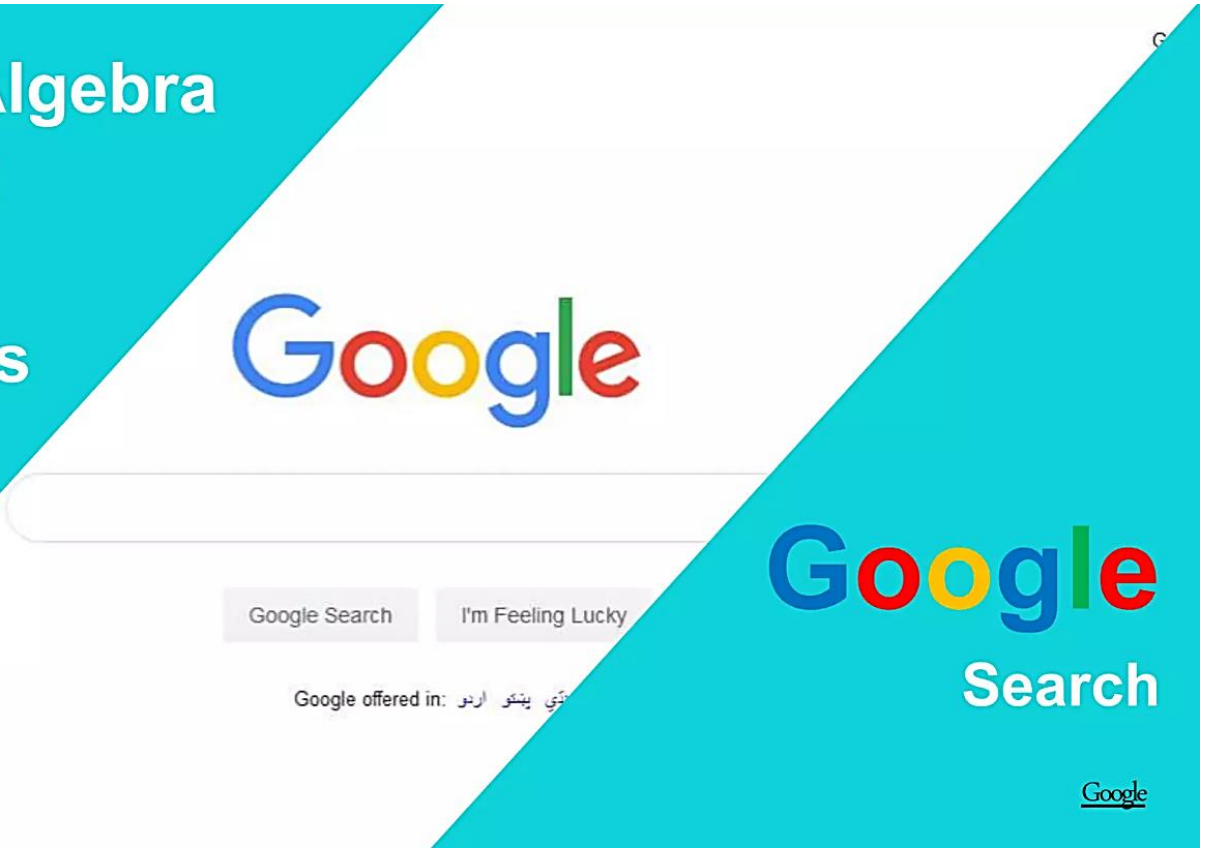
Run software and store files. The software and files are both stored as huge strings of 1s and 0s. Binary math is discrete mathematics.



Google Maps

- Uses discrete mathematics to determine fastest driving routes and times.
- Google Maps knows your position via the Global Positioning System.
- Edsger W. Dijkstra's Algorithm is used to calculate shortest route.
- Co-ordinate geometry is used.

Linear Algebra Behind Web Searches



Magic Behind Google Success

When Google went online in 1990's, one thing that set it apart from other search engines was its search result listings which always delivered "good stuff".

Search Engines like Google have to do three basic things :

01

Look the web and locate all web pages with public access.

02

Indexing of searched data for more efficient search.

03

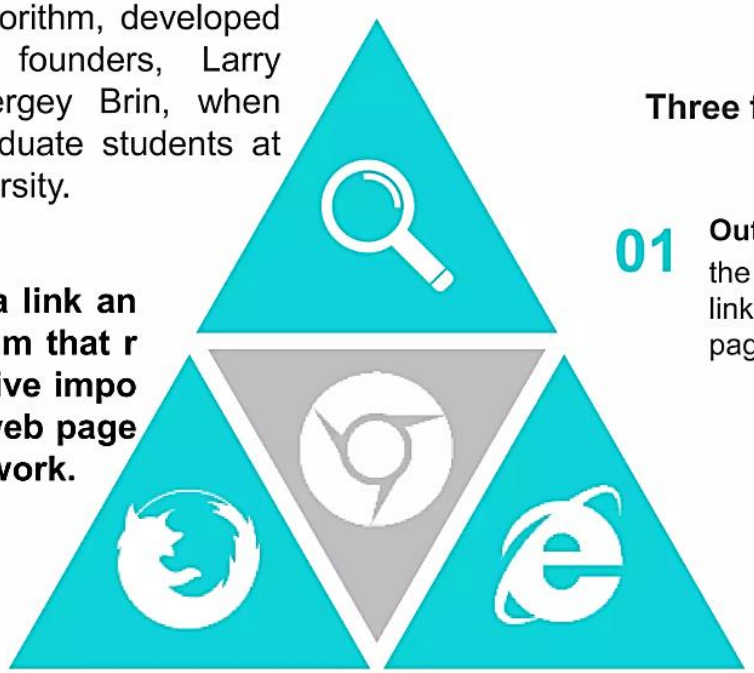
Rate the importance of each page in the database, so when the user does a search, the more important pages are presented first.

Big part of the MAGIC behind Google success is its PageRank Algorithm.

PageRank Algorithm

PageRank Algorithm, developed by Google's founders, Larry Page and Sergey Brin, when they were graduate students at Stanford University.

PageRank is a link an alysis algorithm that r anks the relative impo rtance of all web page s within a network.



Three features for determining PageRank :

01 **Outgoing Links**
the number of links found in a page

02 **Incoming Links -**
the number of times other pages have cited this page

03 **Rank**
A value representing the page's relative importance in the network.

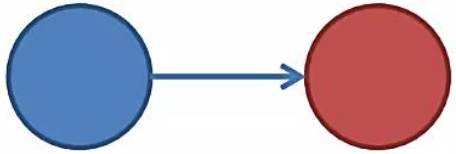
PageRank – How it Works ?

Mathematical Model of Internet:

- **Represent** Internet as Graph
- **Represent** Graph as Stochastic Matrix
- **Make** stochastic matrix more convenient ⇒ Google Matrix
- **Find Dominant** eigenvector of Google Matrix ⇒ PageRank

Internet as a Graph

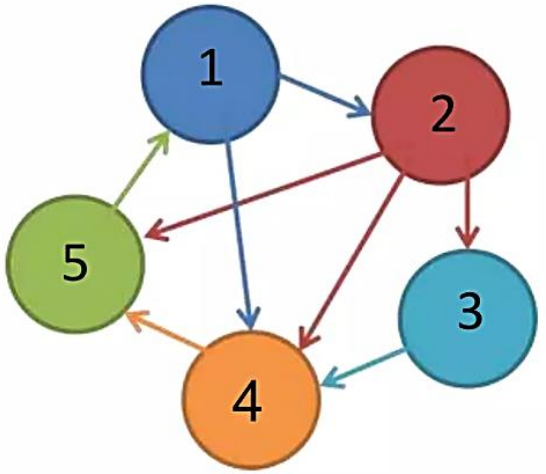
Link from one web page to another web page.



Web graph : Web pages = nodes, Links = edges

Continued…

Web graph as a Matrix



$$S = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

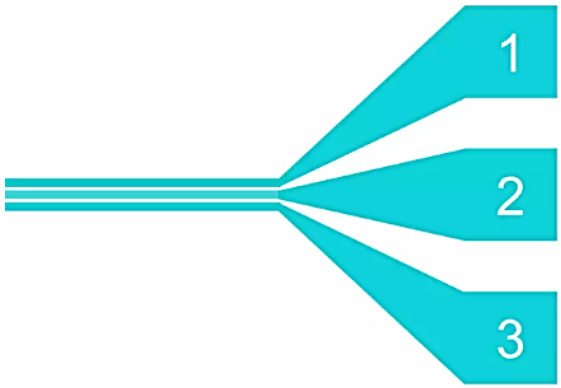
Links = nonzero elements in matrix

Every page 'i' has $l_i \geq 1$ outlinks. $S_{ij} = \begin{cases} 1/l_i & \text{if page I has link to page j} \\ 0 & \text{otherwise} \end{cases}$

S is a Sparse Matrix, as most of the entries are zero.

Probability that surfer moves from page i to page j.

Importance of Linear Algebra



1. Using techniques of Linear Algebra, one can compute a unique solution for PageRank Problem.
2. It gives importance of all webpages in terms of PageRank Eigenvector corresponding to each webpage.
3. No other successful technique other than Linear Algebra is available to solve this problem.

Topic 1: Algorithms

- Introduction to Algorithms and Pseudo Code
- Searching Algorithms
- Sorting Algorithms
- Optimization Algorithms
- Recursive Algorithms
- Big-O Notation

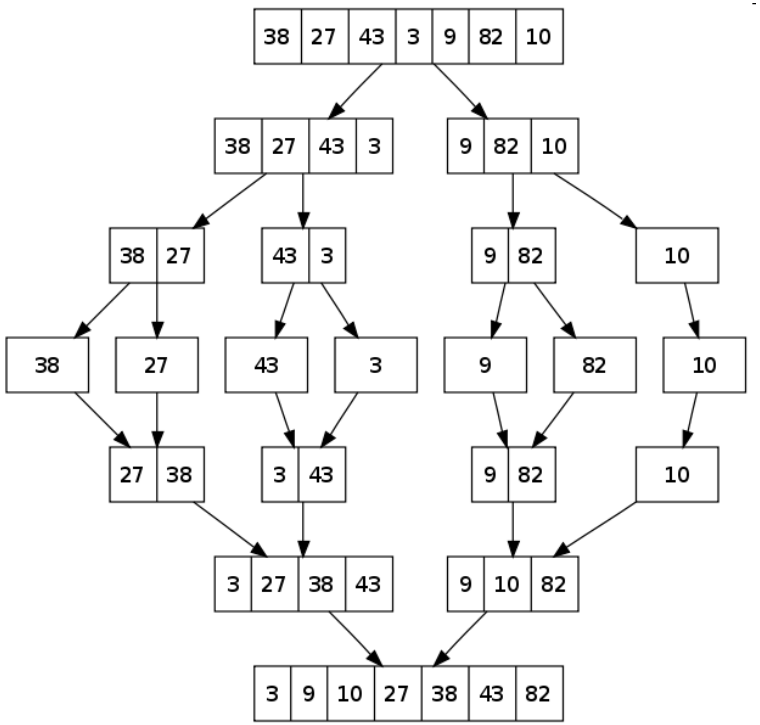
Algorithms, coding theory, data structures

How many steps are needed to sort n numbers?

Algorithm 1 (Bubble Sort):

Every iteration moves the i-th smallest number to the i-th position

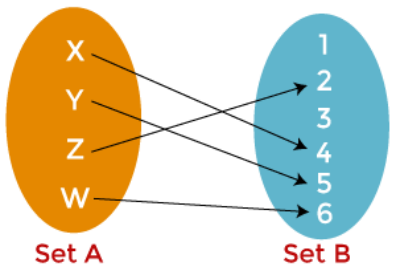
Algorithm 2 (Merge Sort):

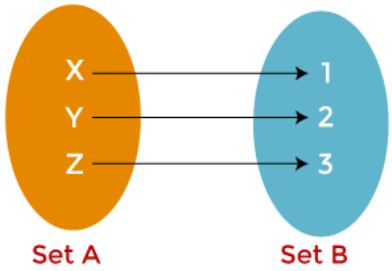


Which algorithm runs faster?

Topic 2: Number Theory

- Cartesian Products and Ordered Pairs
- Relations





Database, algorithms, data structures

Topic 3: Logic and Proofs

How do computers think?

Logic: propositional logic

Truth Tables

Conditionals and Negation

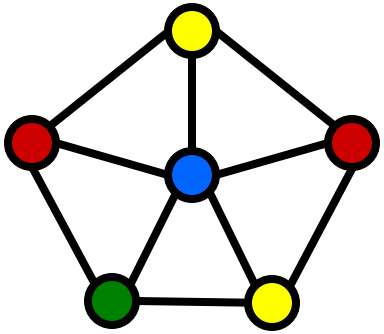
Proofs: direct, by case, induction, contradiction

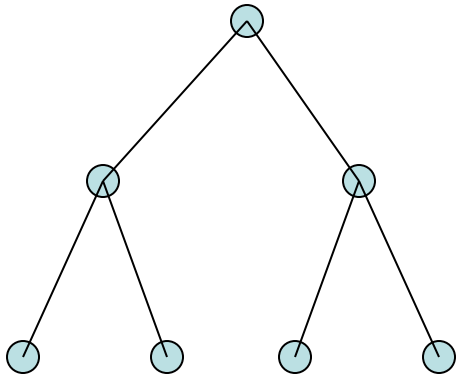
$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geq \sqrt[n]{x1 \cdot x_2 \cdots x_n}$$

Artificial intelligence, database, circuit, algorithms

Topic 4: Graph Theory

- Graphs
- Applications of Graphs
- Trees
- Dijkstra's Algorithm

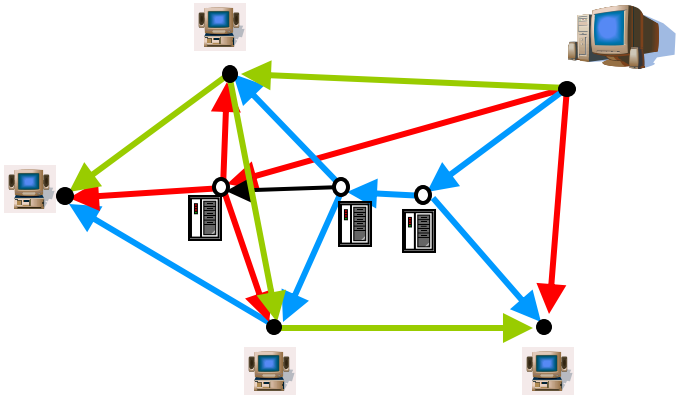




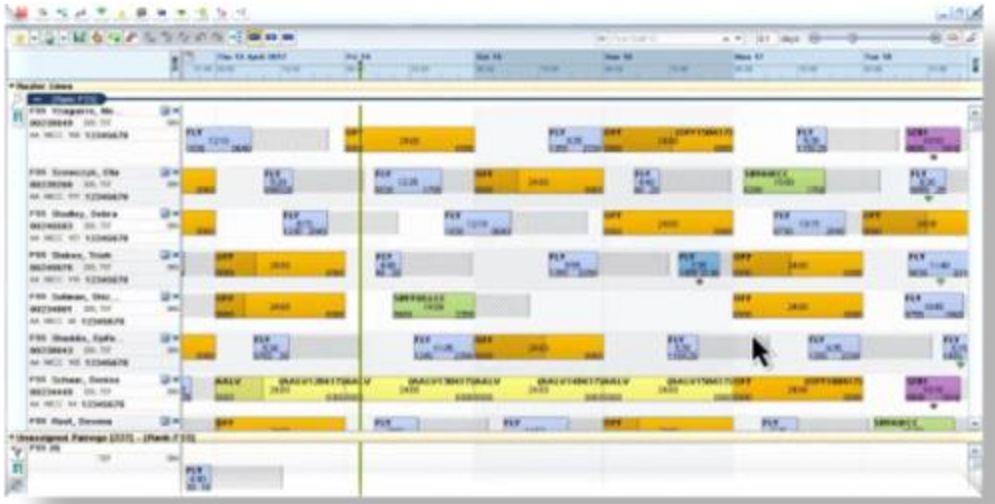
Computer networks, circuit design, data structures

Topic 4: Graph Theory

How to color a map?



How to send data efficiently?



Crew Scheduling

Objectives of This Course

- ✓ To learn mathematical concepts that are related to Computer Science
- ✓ To be familiar with formal mathematical reasoning, e.g. logic, proofs
- ✓ To improve problem solving skills
- ✓ To see the connections between discrete mathematics and computer science

Thank you for your attention.

Course Books' Links

- chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/http://www.toomey.org/tutor/text_books/Digital_Logic/Discrete%20Mathematics%20with%20Applications%20-%20Susanna%20S.%20Epp%20(2019).pdf
- chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://readyforai.com/download/discrete-mathematics-and-its-applications-8th-edition-pdf/?wpdmdl=1676&_wpdmkey=651a85506a31c
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