# LECTURE 06 : AM MODULATION AND DSB CHANNELS, POWER SPECTRUM

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**Communication Systems ME 229/A**

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# Todays Topics

- Modulators
- Typical Radio Receivers
- Commercial AM
  - Envelope detection
  - AM power
- Single Sideband AM (SSB)
  - SSB idea
  - SSB generation
  - SSD detection

# Modulators

- There are lots of ways to make modulators
- Often the problem is how not to make a modulator, say when you are designing an amplifier.
- We will look at some very common types
  - Just about any non-linearity
  - Multipliers such as choppers

## Modulators Using Nonlinearities

Suppose we have the non-linear input-output characteristic:

$$y(t) = ax(t) + bx^2(t)$$

Let

$$\begin{aligned} x_1(t) &= \cos(2\pi f_c t) + m(t) \\ x_2(t) &= \cos(2\pi f_c t) - m(t) \end{aligned}$$

Then, it we apply $x_1(t)$ and $x_2(t)$ to the non-linear modulator, and look at the difference

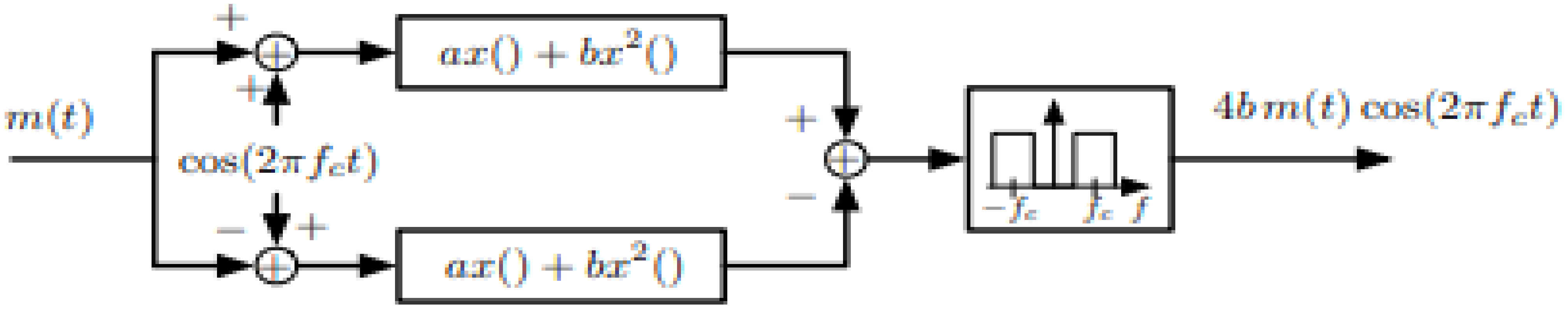
$$\begin{aligned} y_1(t) - y_2(t) = a(\cos(2\pi f_c t) + m(t)) + b(\cos(2\pi f_c t) + m(t))^2 \\ - a(\cos(2\pi f_c t) - m(t)) - b(\cos(2\pi f_c t) - m(t))^2 \\ = 2a\, m(t) + 4b\, m(t) \cos(2\pi f_c t) \end{aligned}$$

Convince yourself this is true!

From the previous page

$$y_1(t) - y_2(t) = 2a\,m(t) + 4b\,m(t)\cos(2\pi f_c t)$$

This has the term we want at $\omega_c = 2\pi f_c$, plus another copy of the message at baseband.
The unwanted baseband component is blocked by a bandpass filter. This could be the antenna or the amplifier.




Or we can just forget about the baseband signal, it won't propagate!

## Switching Modulators

Multiply message by a simple periodic function.

Suppose $w(t)$ is periodic with a fundament frequency $f_c$:

$$w(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi f_c n t}$$

This weighted sum of complex exponentials that are impulses at all multiples of $f_c$. Then

$$m(t)w(t) = \sum_{n=-\infty}^{\infty} D_n m(t) e^{j2\pi f_c n t}$$

By the convolution theorem, the spectrum of $m(t)w(t)$ consists of $M(f)$ shifted to $\pm f_c, \pm 2f_c, \pm 3f_c, \ldots$

For example if $w(t)$ is a 50% duty cycle square wave centered at $t = 0$,

$$w(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{j2\pi f_c n t}; \; n \text{ odd}$$

In general, the spectrum of $m(t)w(t)$ is then

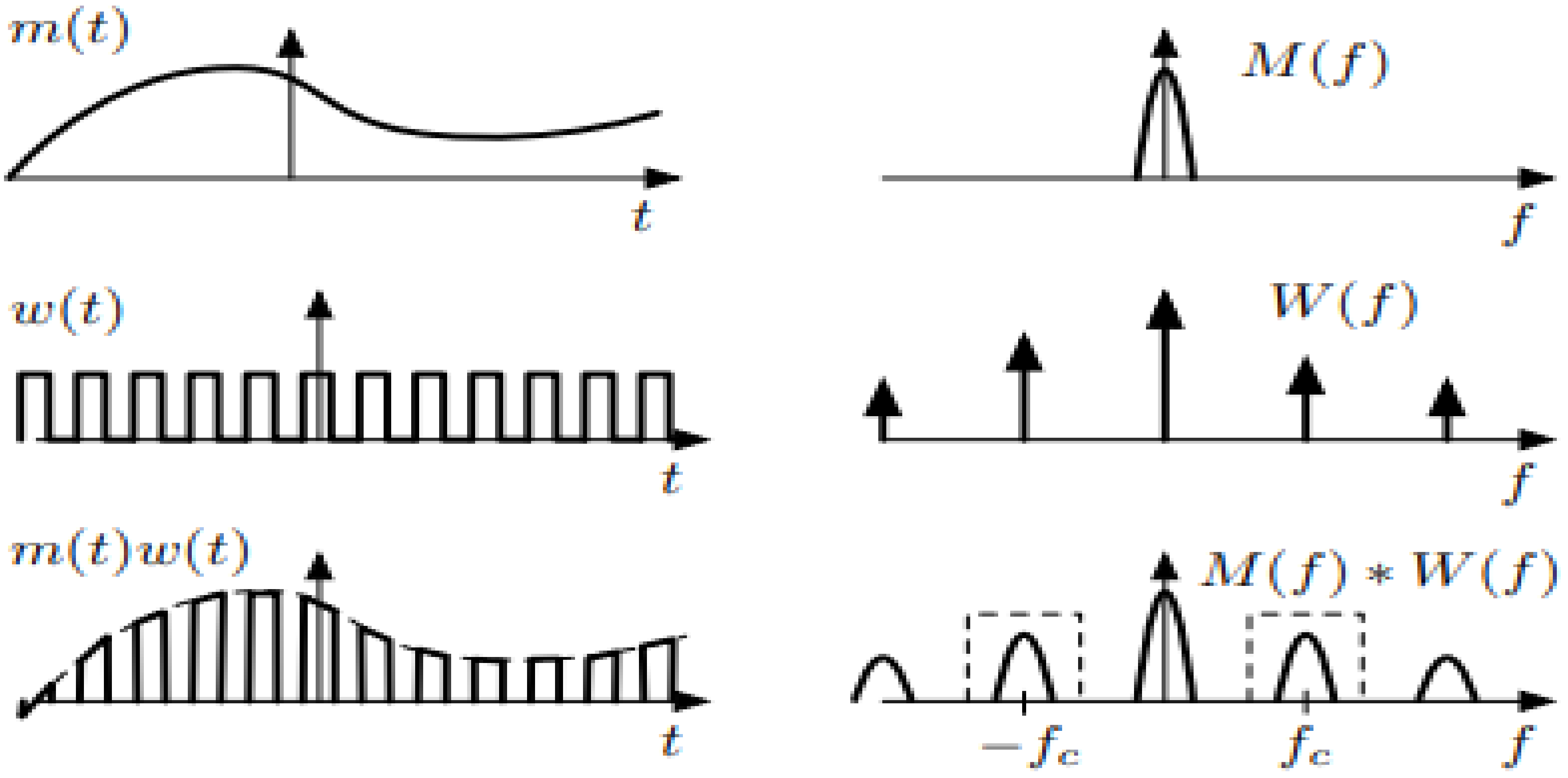
$$\begin{aligned} \mathcal{F}\{m(t)w(t)\} &= M(f) * W(f) \\ &= \sum_{n=-\infty}^{\infty} D_n M(f - nf_c) \end{aligned}$$

There are replicas at multiples of $f_c$.

I can choose any of these provided $D_n$ doesn't happen to be zero.

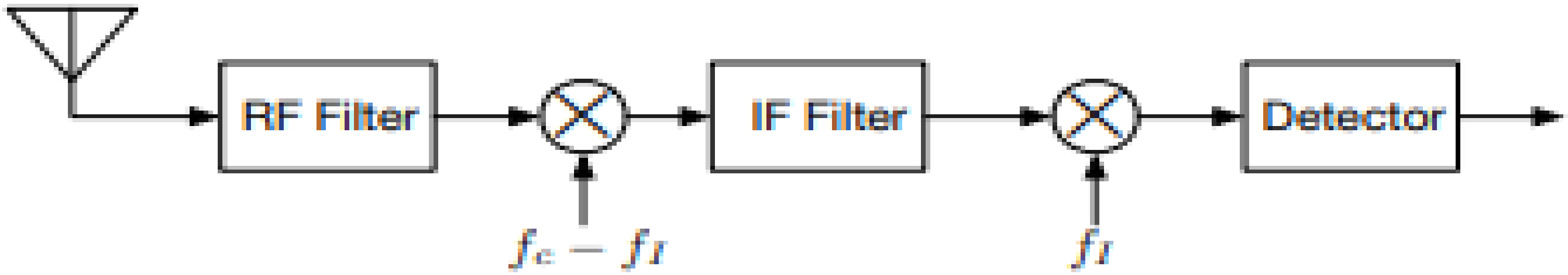
The next page illustrates this modulation method

# Switching Modulator



# Typical Radio Receiver

Assume we want to listen to a radio signal at $f_c$. This is a typical receiver.

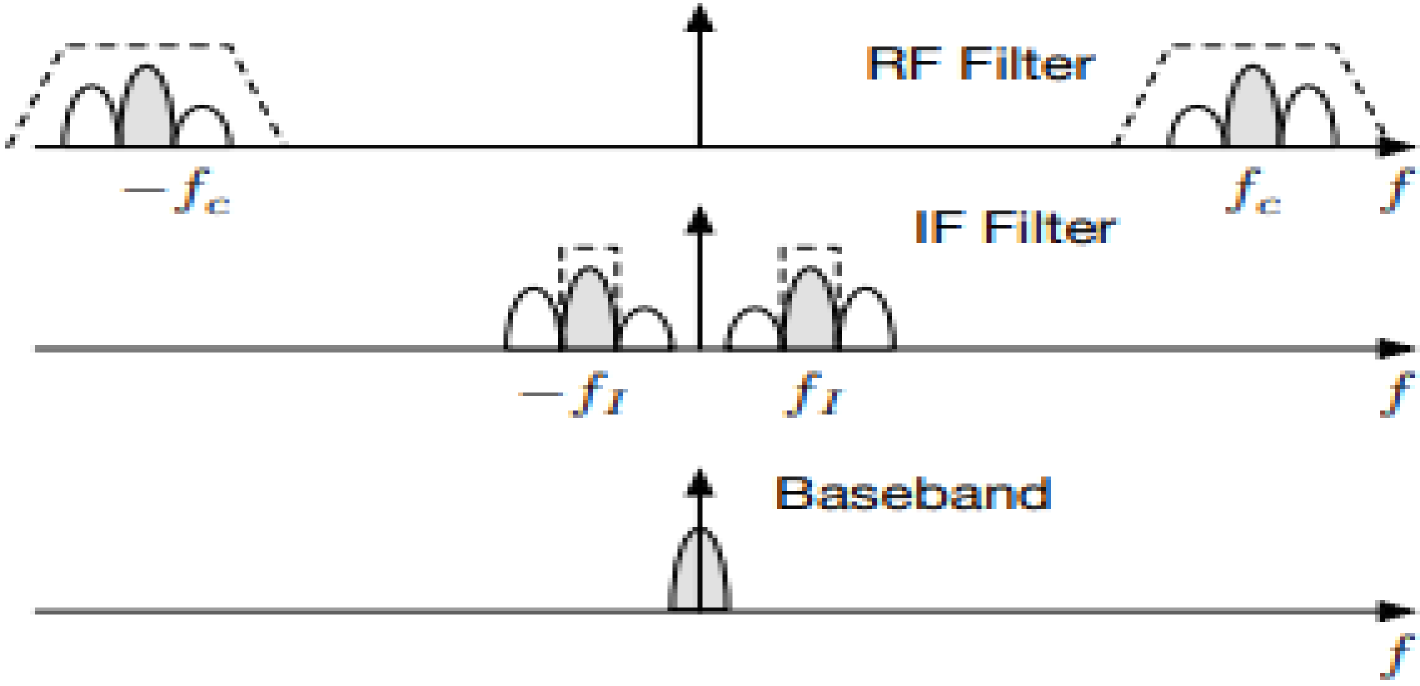



The input RF at $f_c$ is mixed down to a fixed intermediate frequency $f_I$

- RF filter is not very selective
- First modulation frequency is adjustable
- The IF filter is selective
- Everything from the IF filter onward doesn't change with tuning

# Typical Radio Receiver Spectrum

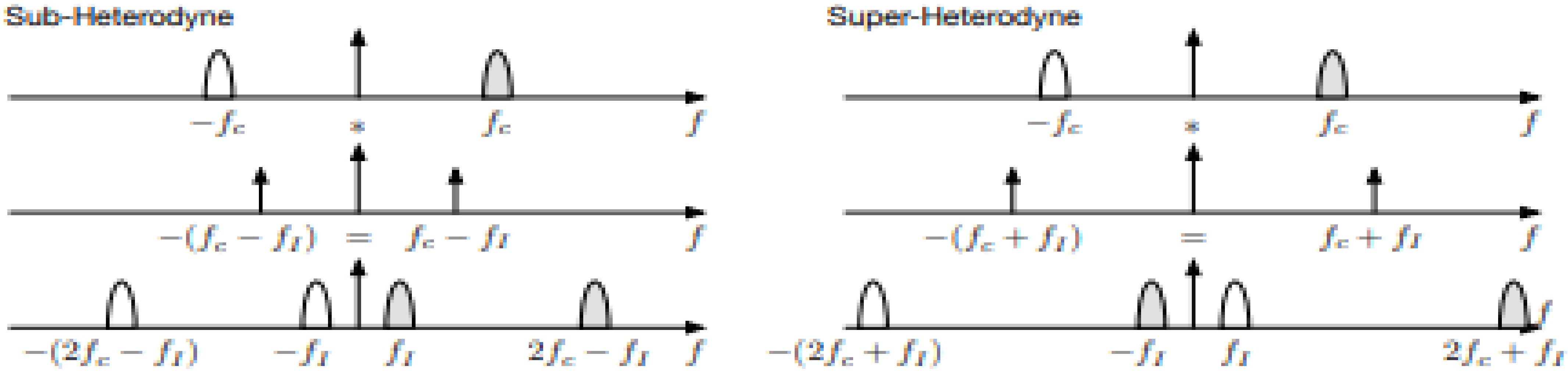
The spectrum of the signals in the receiver look like this:




The RF filter selects some part of the band of interest, while the IF filter selects the signal you are interested in.
For your SDR, you can sample directly instead of the IF filter, and do the rest in software.

# Frequency Translation

The key to this receiver is being able to translate signals in frequency.




- To help keep track of what is happening, one of the bands has been shaded gray. In fact, both are the same.
- Both produce the same IF signals.

# AM Modulation

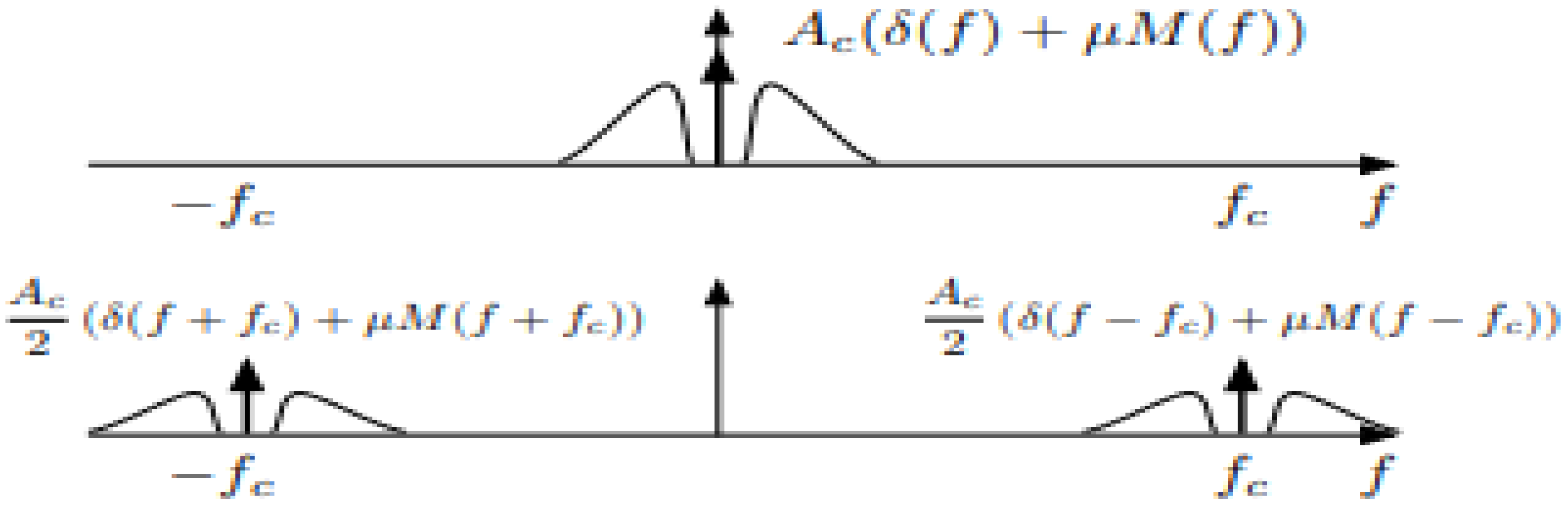
AM modulation is a form of amplitude modulation. For $\mu > 0$,

$$s(t) = (A_c + m(t))\cos(2\pi f_c t) = A_c(1 + \mu m(t))\cos(2\pi f_c t)$$

We need bandwidth of $m(t) \ll f_c$ and modulation index $\mu < 1$.

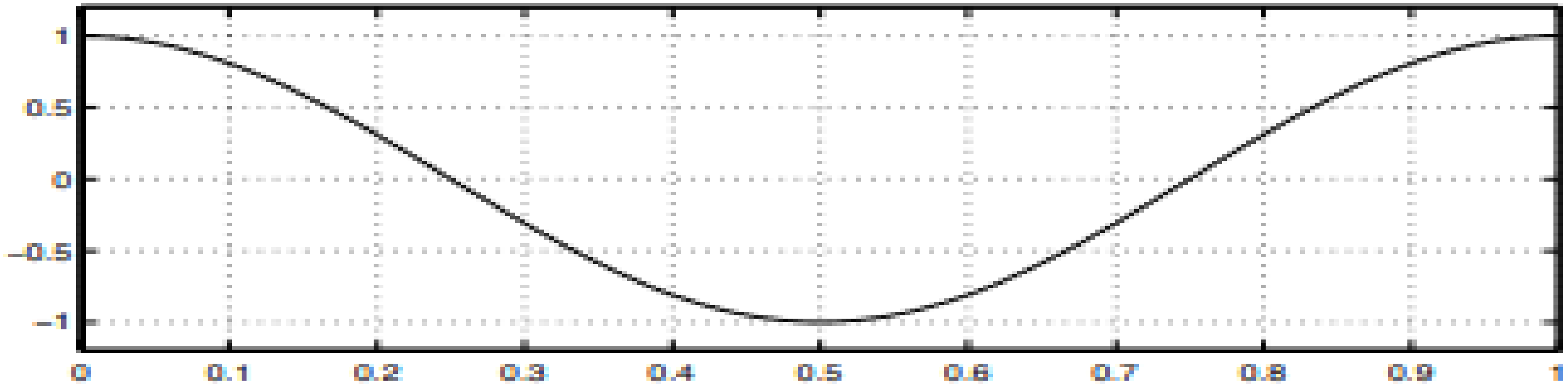
Spectrum of modulated signal:

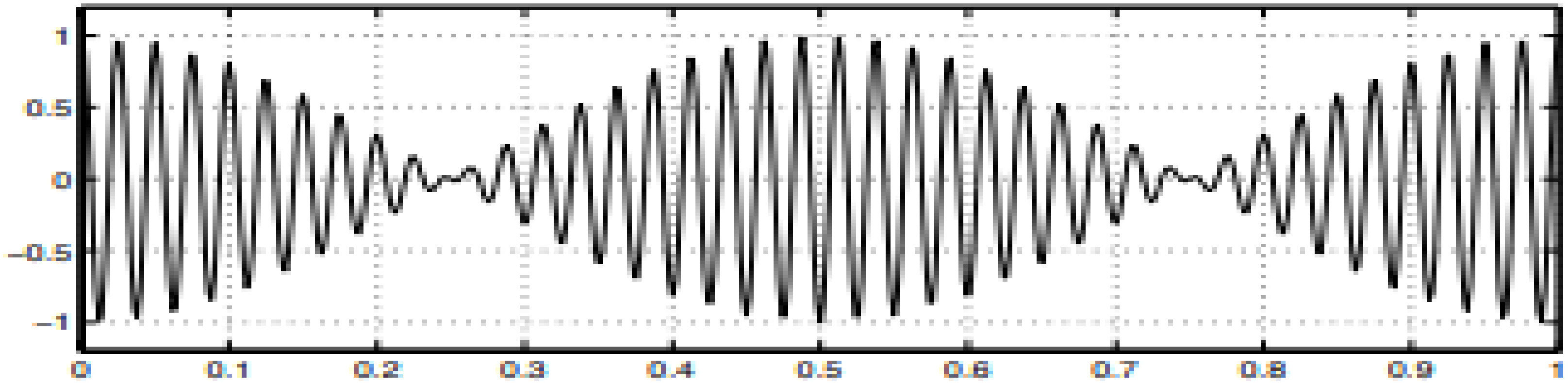
$$S(f) = \frac{A_c}{2}(\delta(f+f_c) + \delta(f-f_c)) + \frac{A_c\mu}{2}(M(f+f_c) + M(f-f_c))$$



# DSB-SC vs. AM

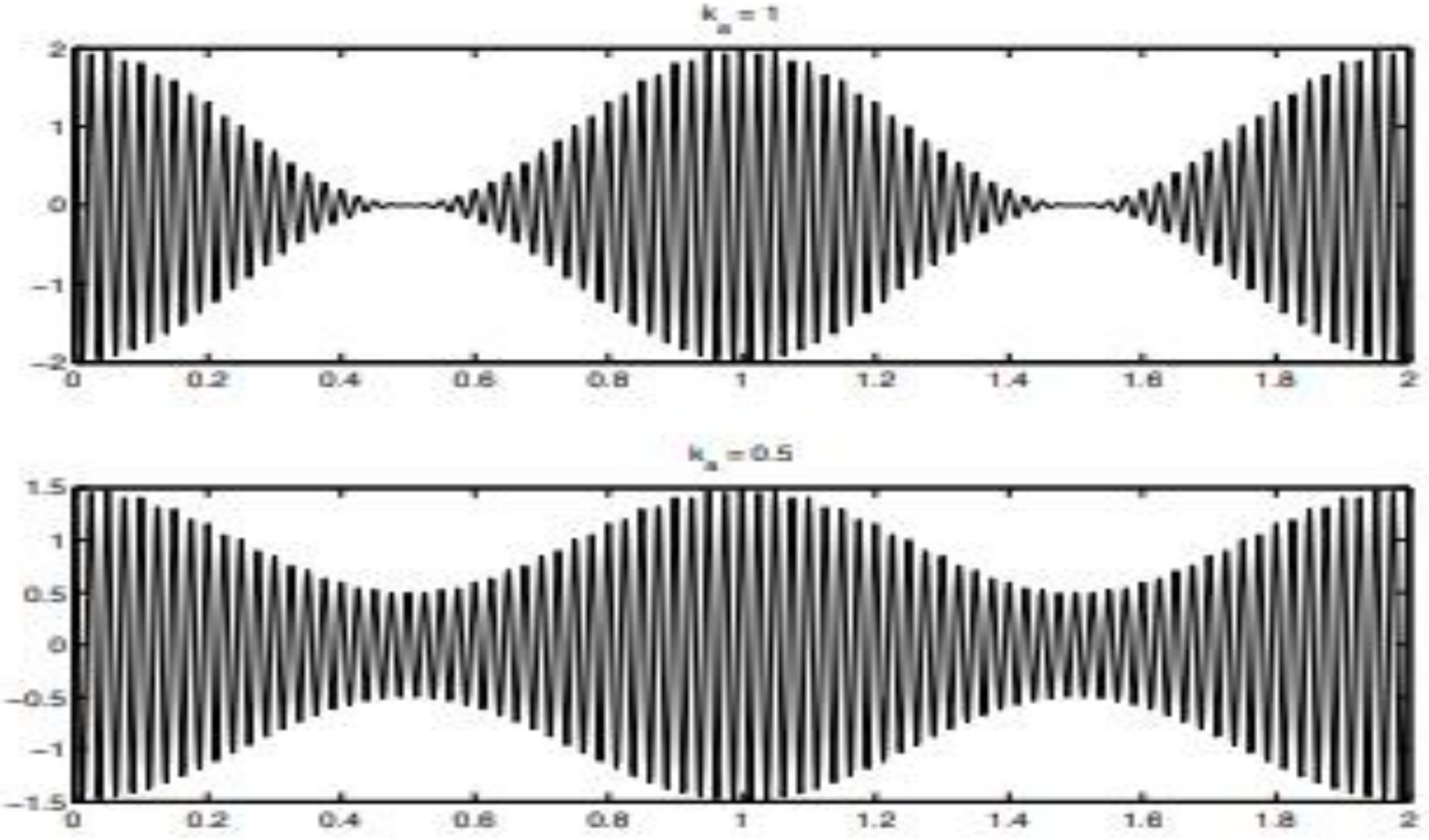
DSB-SC modulated signals undergo phase reversal when $m(t)$ changes sign. It is difficult to extract carrier from received signal.



# DSB-SC vs. AM (cont.)

In AM, the carrier signal is modulated by $A_c + m(t) = A_c(1 + \mu m(t))$.
Examples: $\mu = 1$ and $\mu = 0.5$.



# Envelope Detection of AM Signals

The term *detection* means extracting signal from received data. In some cases it means demodulation.

Suppose that a signal $x(t)$ can be written as

$$x(t) = E(t)\cos(2\pi f_c t)$$

where $E(t)$ varies slowly compared to the carrier $\cos(2\pi f_c t)$.

Then $|E(t)|$ is called the envelope of $x(t)$.

For envelope detection to work, we need

- $f_c \gg$ bandwidth of $m(t)$
  Otherwise positive and negative spectral components overlap.
- $A + m(t) \geq 0$
  Otherwise phase reversals occur when $A + m(t) < 0$.

# Modulation index

The maximum deviation of $m(t)$ from zero is

$$m_p = \max(|m(t)|)$$

The *modulation index* of the modulated signal is defined by

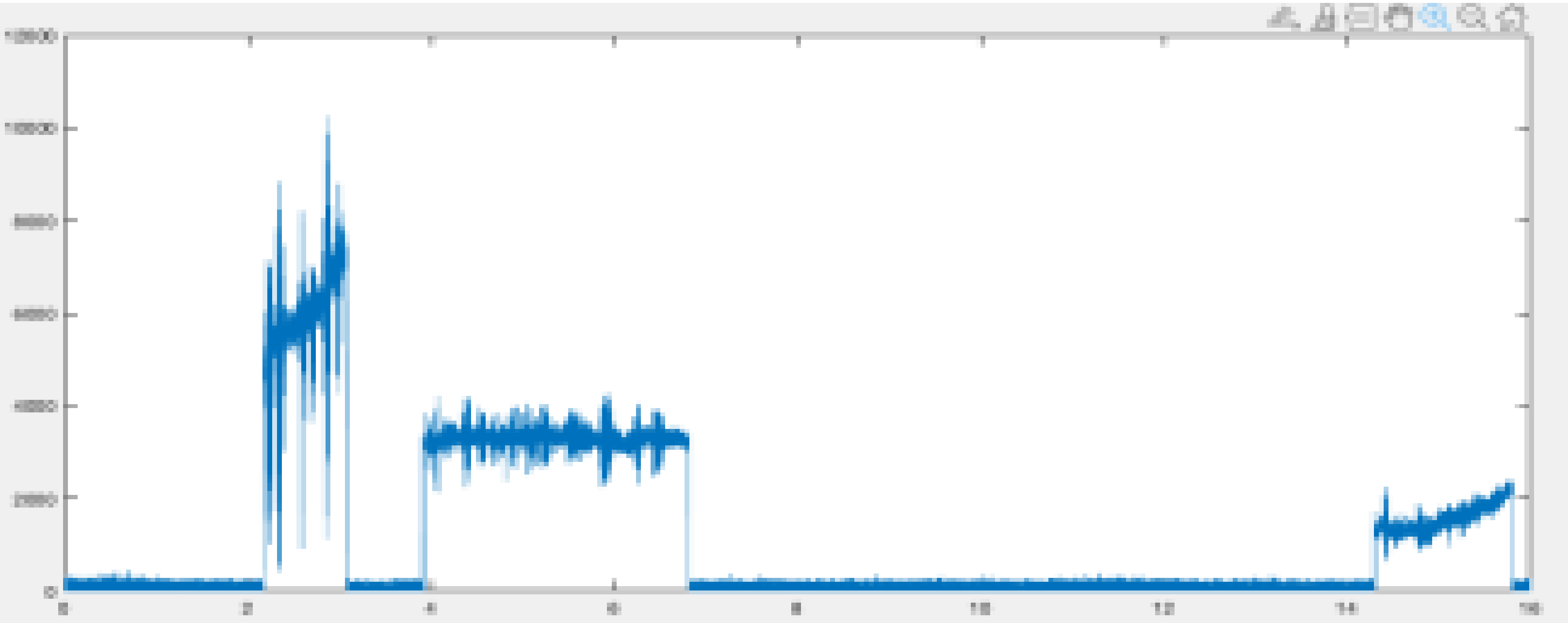
$$\mu = \frac{m_p}{A}$$

Larger modulation index reduces power but makes demodulation harder.

Broadcast AM stations use modulation index close to 1. Input signals are controlled using automatic gain control (AGC).

# Modulation Index Example

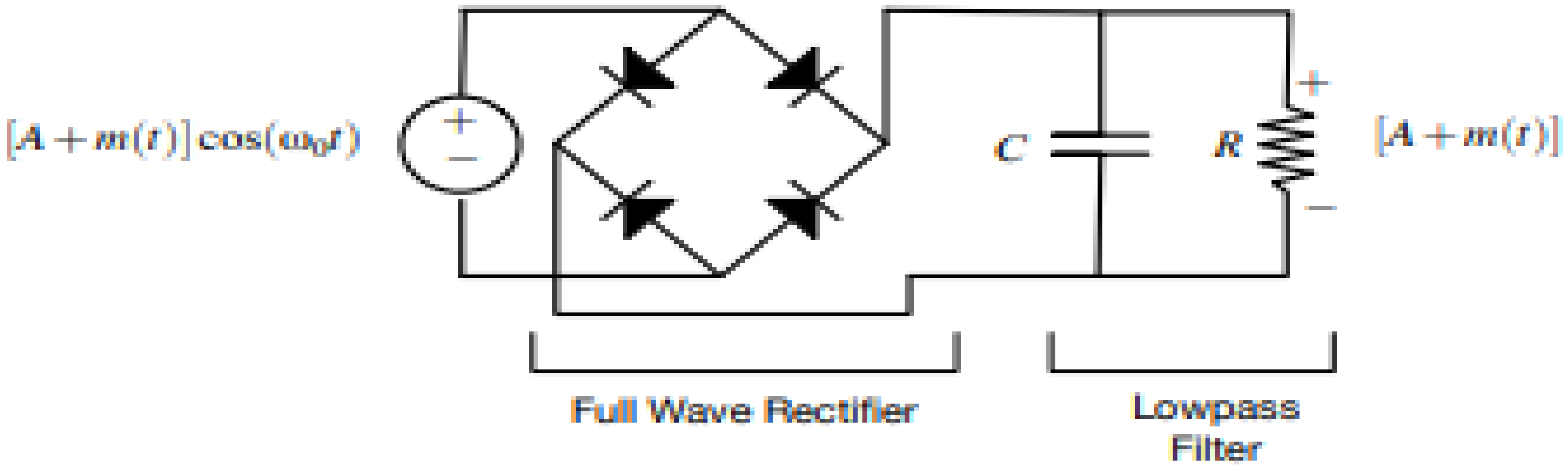
This is an captured airband signal

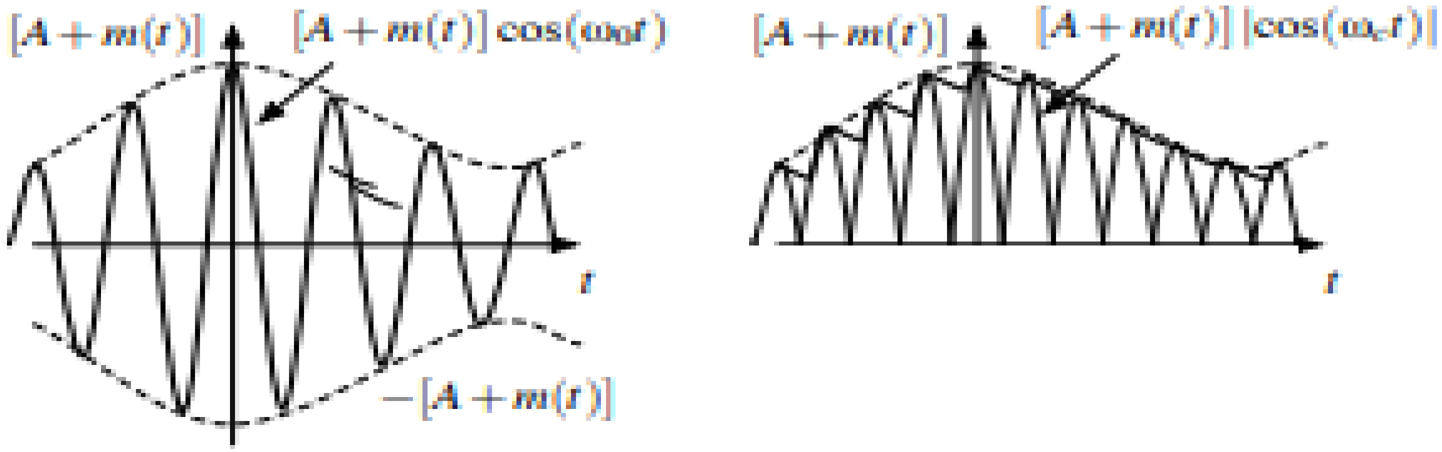


You can clearly see the carrier, and when it is keyed on and off.
The first transmission has a much higher modulation index than the second two. Why might this be?

# Envelope Detector for AM

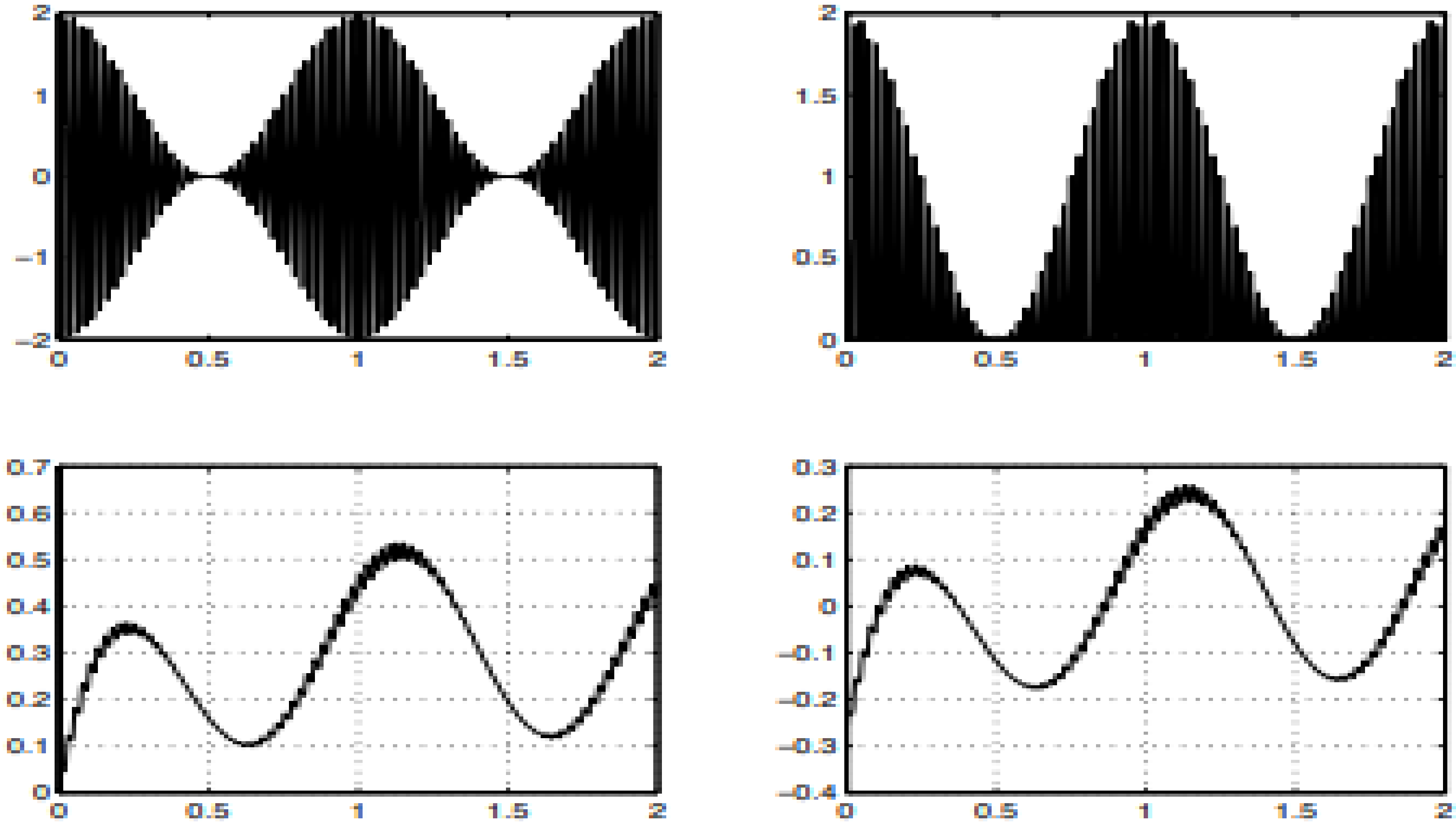
Rectify the RF signal, then lowpass filter:



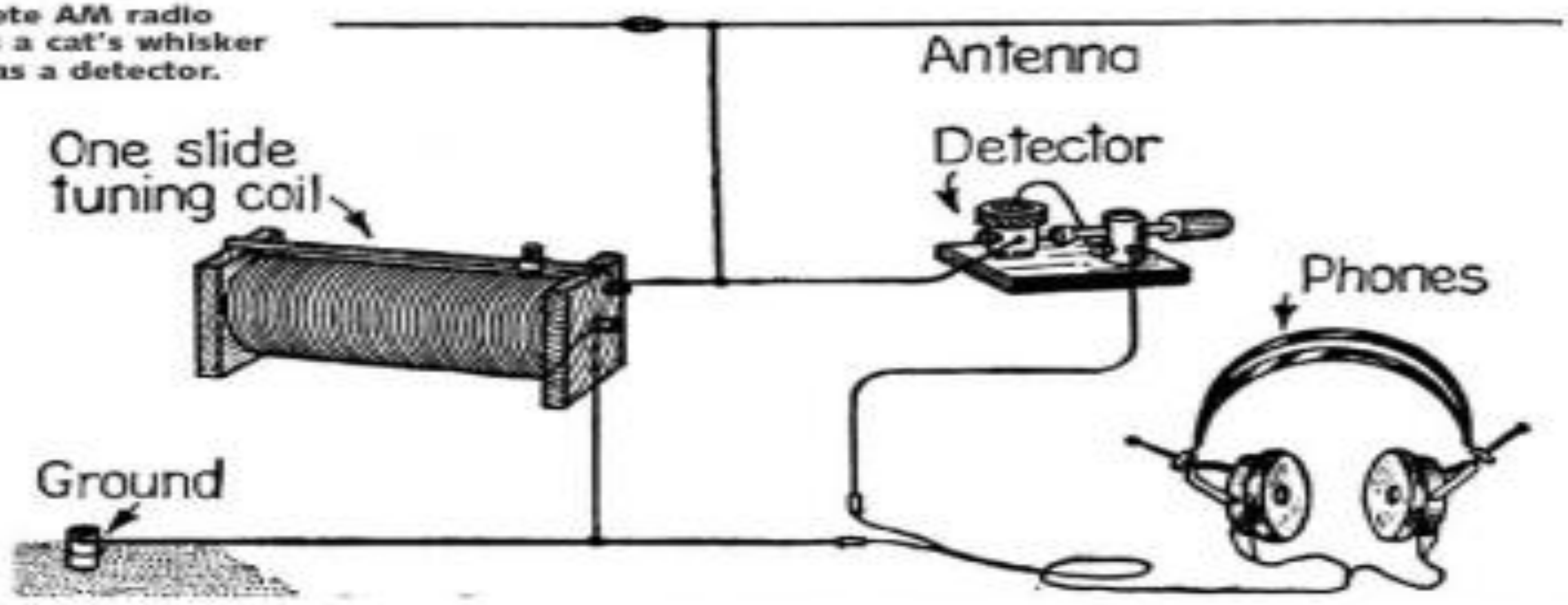
# AM Demodulation Experiment

$$m(t) = \cos 2\pi t\ ,\ f_c = 10\ ,\ h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$



# Cat's Whiskers (Crystal) Radio

This radio was powered only by received radio energy.




Wikipedia: Crystal radio wiring pictorial based on Figure 33 in Gernsback's 1922 book *Radio For All* (copyright expired) with "Aerial" changed to Antenna by J.A. Davidson.

The point-contact semiconductor detector was subsequently resurrected around World War II because of the military requirement for microwave radar detectors.

## Power of AM Signals

The power of an AM signal is the sum of the power of two components.

$$\varphi_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

The carrier and sideband signals are orthogonal, so powers add. Carrier power is

$$P_c = A^2 \int_0^T \cos^2(2\pi f_c t)\,dt = \frac{1}{2}A^2$$

Signal power after modulation is 1/2 the original message power

$$P_s = \tfrac{1}{2} P_m\,,$$

where message power is average power as $T$ gets large,

$$P_m = \overline{m^2(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} m^2(t)\,dt$$

E.g., the power of a tone $\cos(2\pi f_m t)$ is $\frac{1}{2}$.

# Power of AM Signals (cont.)

The carrier tone simplifies demodulation but carries no information.
The power efficiency is defined by

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

Examples: tone modulation $m(t) = \mu A \cos(2\pi f_c t)$ where $0 < \mu \leq 1$.

$$\eta = \frac{\frac{1}{2}(\mu A)^2/2}{A^2/2 + \frac{1}{2}(\mu A)^2/2} = \frac{\mu^2}{2 + \mu^2}$$

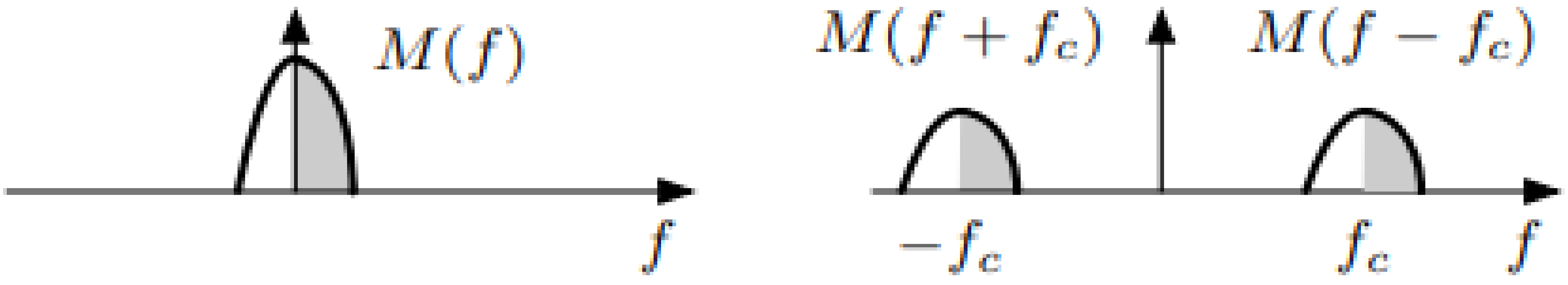
The efficiency increases with $\mu$; the maximum value is 1/3.
Efficiency falls off rapidly as $\mu$ decreases. For $\mu = 0.5$,

$$\eta = \frac{(0.5)^2}{2 + (0.5)^2} = \frac{1}{9}$$

AM is inefficient in both power and bandwidth.

# Single Sideband (SSB)

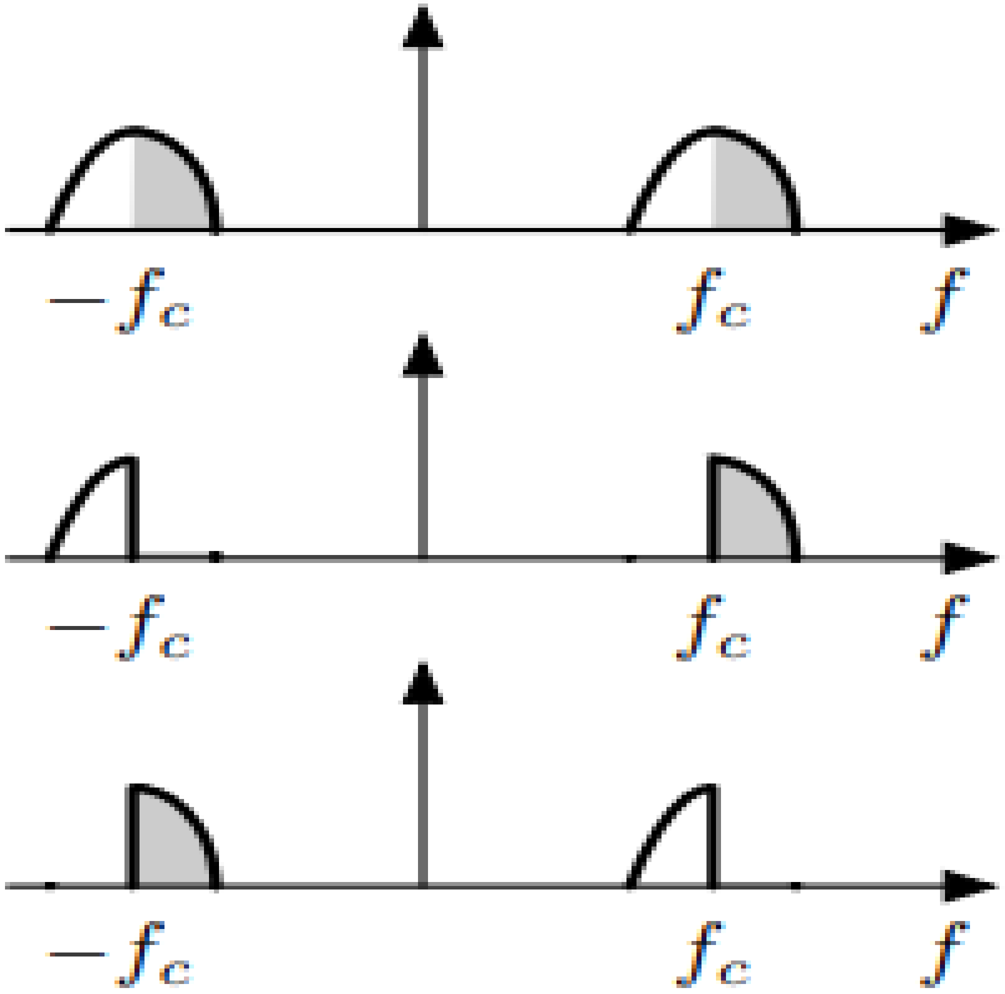
DSB-SC is spectrally inefficient. It uses twice the bandwidth of the message.




The signal can be reconstructed from either the upper sideband (USB) or lower sideband.

SSB transmits a bandpass filtered version of the modulated signal.

# Single Sideband (cont.)




Double Sideband

Upper Sideband

Lower Sideband

# Single Sideband Modulation and Demodulation

- SSB can be transmitted using a DSB-SC modulator with a narrower bandpass filter. For USB, center frequency is

  $$\tilde{f}_c = f_c + \tfrac{1}{2}B$$

  and cutoff frequency is $B/2$.

  The bandfilter must roll off quickly to eliminate unwanted contributions from the other sideband.

  Message frequencies near 0 will be affected by the nonideal filter.

- SSB demodulation can use a DSB-SC demodulator with no change.

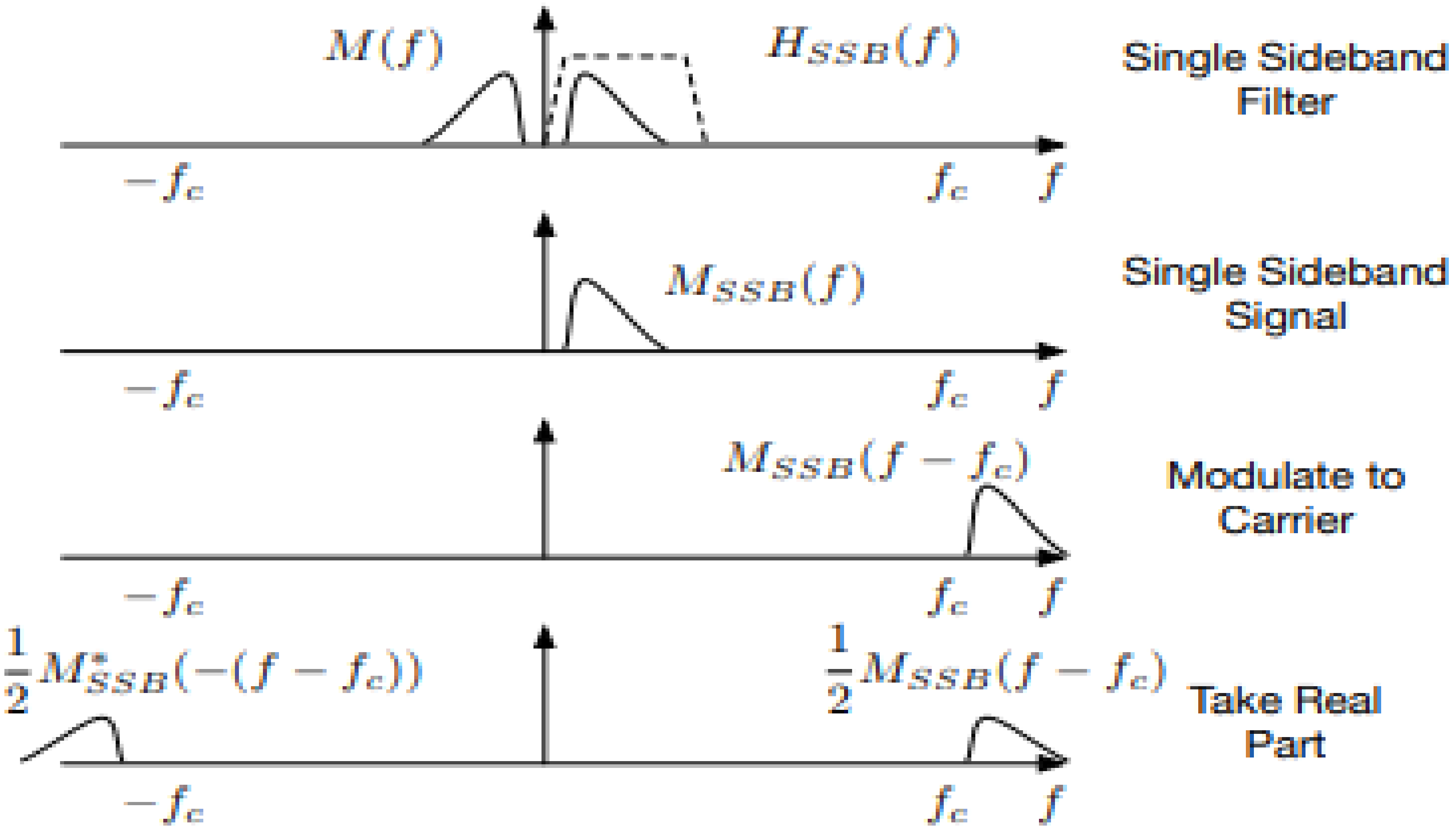
  The input to the lowpass filter is different from that of DSB-SC.

## Which SSB Sideband?

- Transmitter and receiver must agree on use of LSB vs. USB.
- SSB is common for amateur radio
  - Below 10 MHz : LSB
  - Above 10 MHz : USB
  - Exception for 5 MHz :USB
  - Exception for digital modes : USB
- SSB also common for shortwave
  - 120m (2300-2495 kHz): LSB
  - 90m (3200-3400 kHz): LSB
  - 75m (3900-4000 kHz): USB
  - 60m (4750-5060 kHz): LSB
  - 49m (5900-6200 kHz): USB
  - 41m (7200-7450 kHz): USB

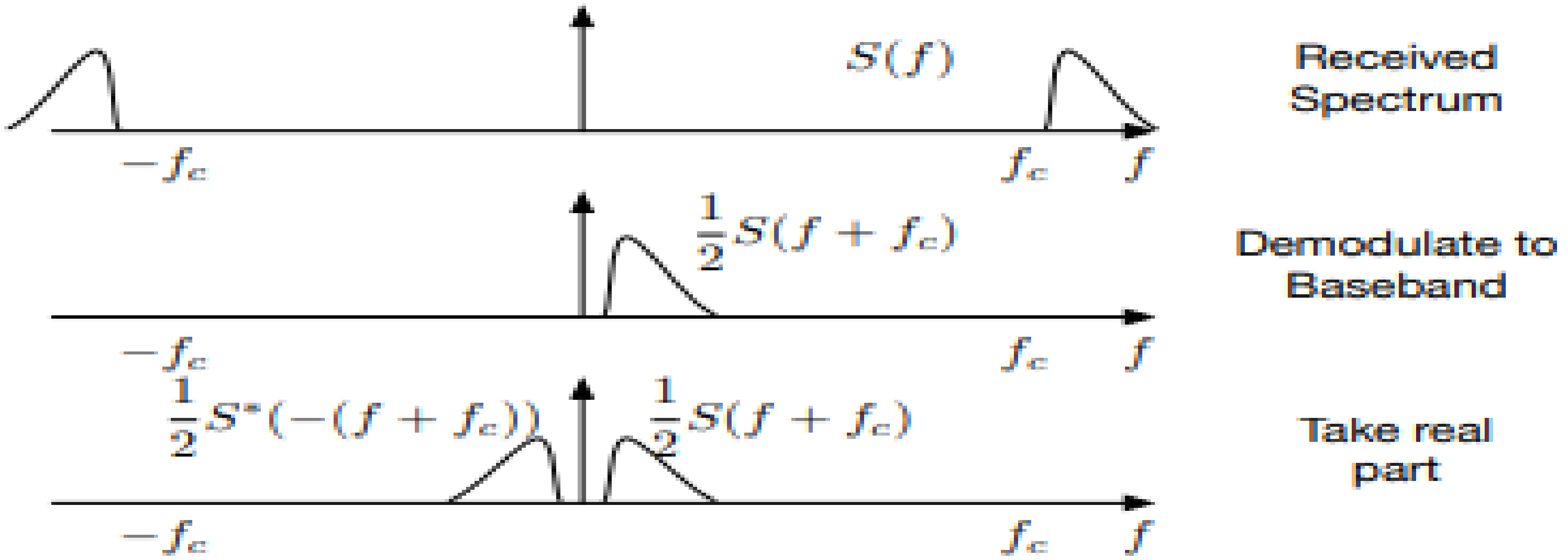
  Your radio knows!

# SSB Modulation



# SSB Demodulation

To decode the SSB signal, we just reverse the operations




- Ideally we want a synchronous demodulator
- In practice, $f_c$ is estimated by the sound of the signal
- An error of 50 Hz is quite noticeable

## SSB in Time Domain

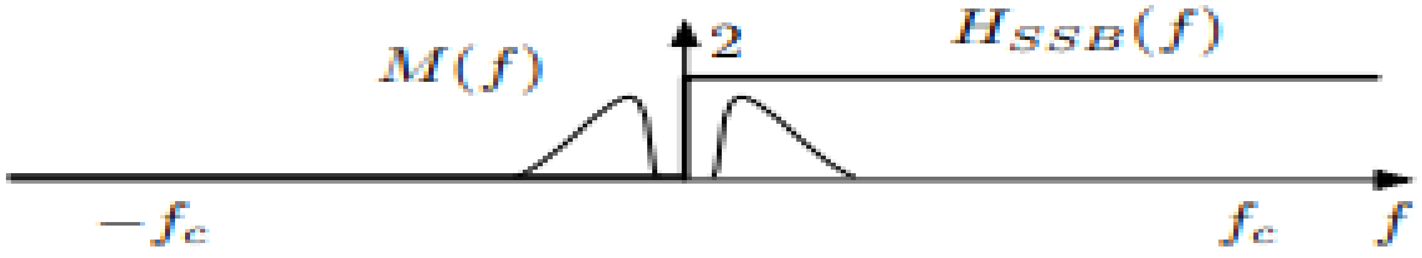
The upper sideband is the output of filtering a modulated signal $m(t)\cos\omega_c t$ with an ideal bandpass filter:

$$H_{\mathrm{SSB}}(f) = \begin{cases} 2 & f > 0 \\ 0 & f < 0 \end{cases}$$

This is

$$H_{\mathrm{SSB}}(f) = 2u(f)$$

This looks like




The impulse response of this filter is

$$h_{SSB}(t) = \mathcal{F}^{-1}\left\{2u(f)\right\}$$

# Hilbert Transform

We know

$$u(t) \rightleftharpoons \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

so by duality, and multiplying by 2

$$\delta(t) + \frac{j}{\pi t} \rightleftharpoons 2u(f)$$

The impulse response of the filter is

$$h_{SSB}(t) = \delta(t) + \frac{j}{\pi t}$$

If $m(t)$ is the input signal, the single sideband signal is

$$m(t) * h_{SSB}(t) = m(t) * \left(\delta(t) + \frac{j}{\pi t}\right) = m(t) + j\left(m(t) * \frac{1}{\pi t}\right)$$

The last term is the Hilbert transform of $m(t)$

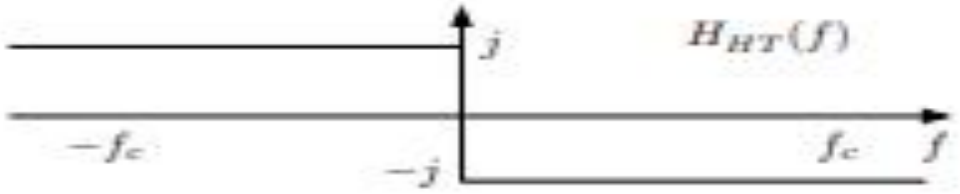
$$m_h(t) = m(t) * \frac{1}{\pi t}$$

# Hilbert Transform

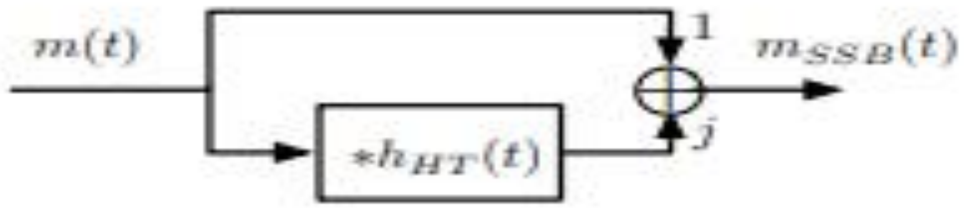
The transfer function of the Hilbert transform is

$$H_{HT}(f) = -j\,\mathrm{sgn}(f) = \begin{cases} -j = & f > 0 \\ j = & f < 0 \end{cases}$$

which looks like




What happens if $m(t) = \cos(2\pi f t)$, or $m(t) = \sin(2\pi f t)$?
The block diagram is



# Next time

AM receivers

Vestigial Sideband (VSB)

and Quadrature Amplitude

Modulation (QAM),

beginning of FM