LECTURE 09 : FREQUENCY MODULATION NBFM & WBFM

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Communication Systems ME 229/A

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Lecture 09 : Frequency Modulation NBFM & WBFM

Angle Modulation, II

Lecture topics

- FM bandwidth and Carson's rule
- Spectral analysis of FM
- Narrowband FM Modulation
- Wideband FM Modulation

Based on lecture notes from John Gill

Bandwidth of Angle-Modulated Waves

Angle modulation is nonlinear and complex to analyze.

Early developers thought that bandwidth could be reduced to 0.
They were wrong. FM has infinite bandwidth.

Two approximations for FM:

- Narrowband approximation (NBFM)
- Wideband approximation (WBFM)

These depend on whether the FM modulation is larger than the signal bandwidth.

If we define

$$a(t) = \int_{-\infty}^{t} m(u)\, du$$

Then the frequency modulated signal is

$$\varphi_{\mathrm{FM}}(t) = \cos(2\pi f_c t + k_f a(t))$$

since phase modulation is the integral of frequency modulation.

For the narrow band case

$$|k_f a(t)| \ll 1$$

We will show that the NBFM bandwidth is the same as the signal bandwidth

$$2B_s \approx 2B_m$$

and

$$\varphi_{\mathrm{FM}}(t) \approx A\big(2\pi f_c t - k_f a(t)\sin\omega_c t\big)$$

This is what we saw last time. The NBFM case looks like a small modulation in quadrature with the carrier.
For the wideband case, if the peak frequency modulation is

$$\Delta f = \max|k_f m(t)|$$

then the bandwidth of the WBFM signal is

$$2B_s = 2\Delta f + 2B_m$$

This is known as known as Carson's rule.
J.R. Carson, Proc. IRE, 1922.

Narrowband FM

Recall that

$$a(t) = \int_{-\infty}^{t} m(u)\, du$$

The *complex FM signal* from last time is

$$\begin{aligned}\hat{\varphi}_{\mathrm{FM}}(t) &= Ae^{j(2\pi f_c t + k_f a(t))} \\ &= Ae^{jk_f a(t)} e^{j2\pi f_c t}\end{aligned}$$

The transmitted signal is just the real part, $\varphi_{\mathrm{FM}}(t) = \mathrm{Re}\left(\hat{\varphi}_{\mathrm{FM}}(t)\right)$.

Using the Maclaurin power series for the exponential $e^{jk_f a(t)}$ in $\hat{\varphi}_{\mathrm{FM}}(t)$:

$$\hat{\varphi}_{\mathrm{FM}}(t) = A\left(1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \cdots + j^n \frac{k_f^n}{n!}a^n(t) + \cdots\right)e^{j2\pi f_c t}$$

If $a(t)$ has a bandwidth $2B$ Hz, then the nth term has a bandwidth $n2B$!

This expansion for $\hat{\varphi}_{\mathrm{FM}}(t)$ shows that the bandwidth is infinite.

However, things aren't quite that bad ...

Since $k_f^n/n! \to 0$, all but a small amount of power is in a finite band.
Using $\varphi_{\text{FM}}(t) = \mathcal{R}\{\hat{\varphi}_{\text{FM}}(t)\}$, the FM signal is

$$\begin{aligned}\varphi_{\text{FM}}(t) &= \mathcal{R}\left\{A\left(1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \cdots +\right)(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right\} \\ &= A\Big(\cos 2\pi f_c t - k_f a(t)\sin 2\pi f_c t - \frac{k_f^2}{2!}a^2(t)\cos 2\pi f_c t + \cdots\Big)\end{aligned}$$

If $|k_f a(t)| \ll 1$ then all but first two terms are negligible.
The narrowband FM approximation is

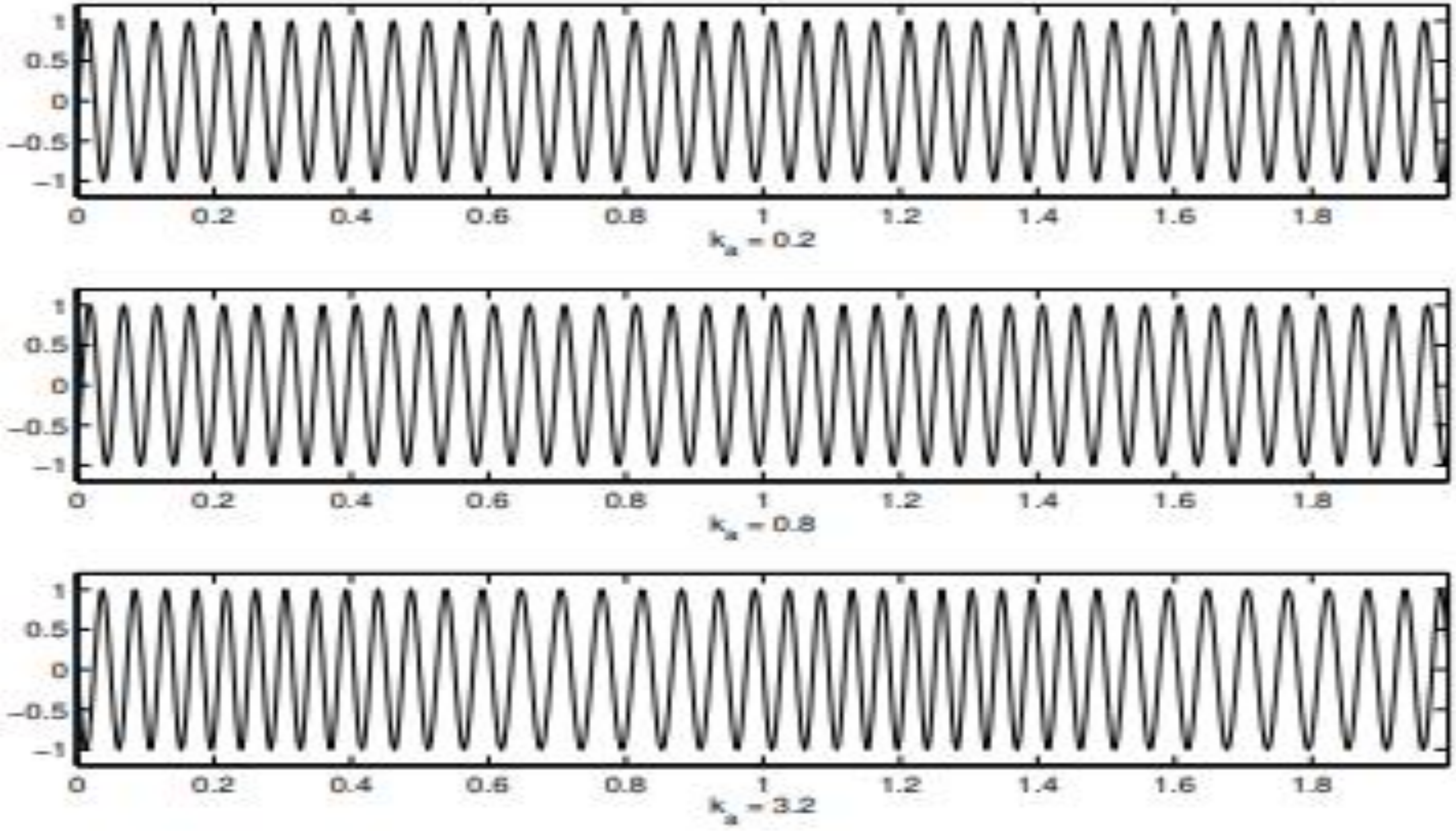
$$\varphi_{\text{FM}}(t) \approx A\big(\cos(2\pi f_c t) - k_f a(t)\sin(2\pi f_c t)\big)$$

NBFM signal has bandwidth $2B$, same as bandwidth of AM.
NBFM has power $\frac{1}{2}A^2$, which does not depend directly on $m(t)$.
A narrowband argument for phase modulation gives is similar result:

$$\varphi_{\text{PM}}(t) \approx A\big(\cos(2\pi f_c t - k_p m(t)\sin(2\pi f_c t)\big)$$

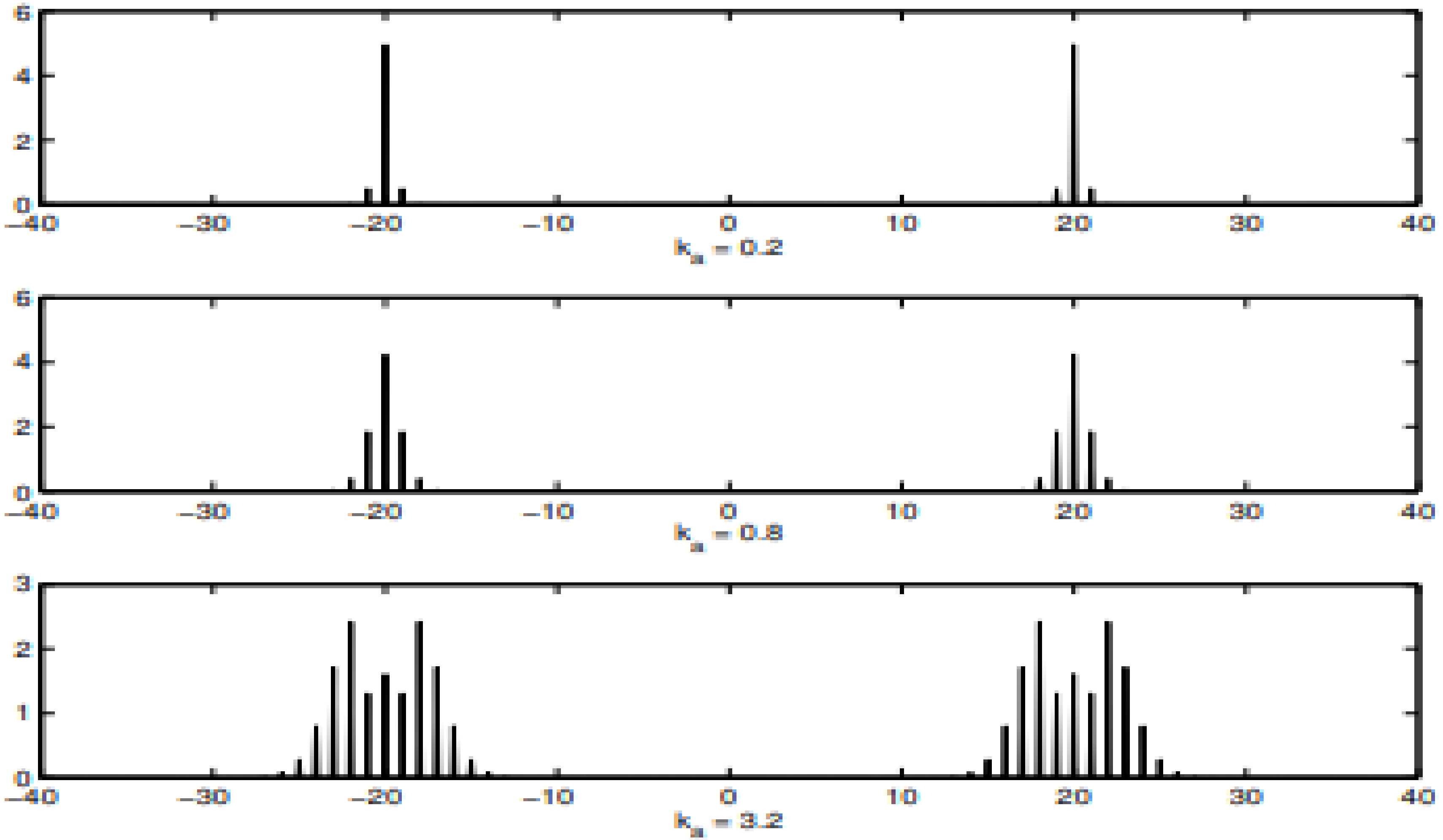
Tone Frequency Modulation, $f_c = 20$, $f_m = 1$

$$\varphi_{\text{FM}}(t) = \cos\left(2\pi f_c t + k_a a(t)\right), |k_f a(t)| = 0.2, 0.8, 3.2$$



Fourier Transforms of Tone FM

$$\varphi_{\rm FM}(t) = \cos\left(2\pi f_c t + k_a a(t)\right), |k_f a(t)| = 0.2, 0.8, 3.2$$



Wideband FM (WBFM) Bandwidth

For wideband FM, the frequency deviation contributes to the FM bandwidth. If the message signal is $m(t)$, and the FM signal is

$$\varphi_{\mathrm{FM}}(t) = \cos(2\pi f_c t + k_f a(t))$$

where again

$$a(t) = \int_{\infty}^{t} m(\tau) d\tau.$$

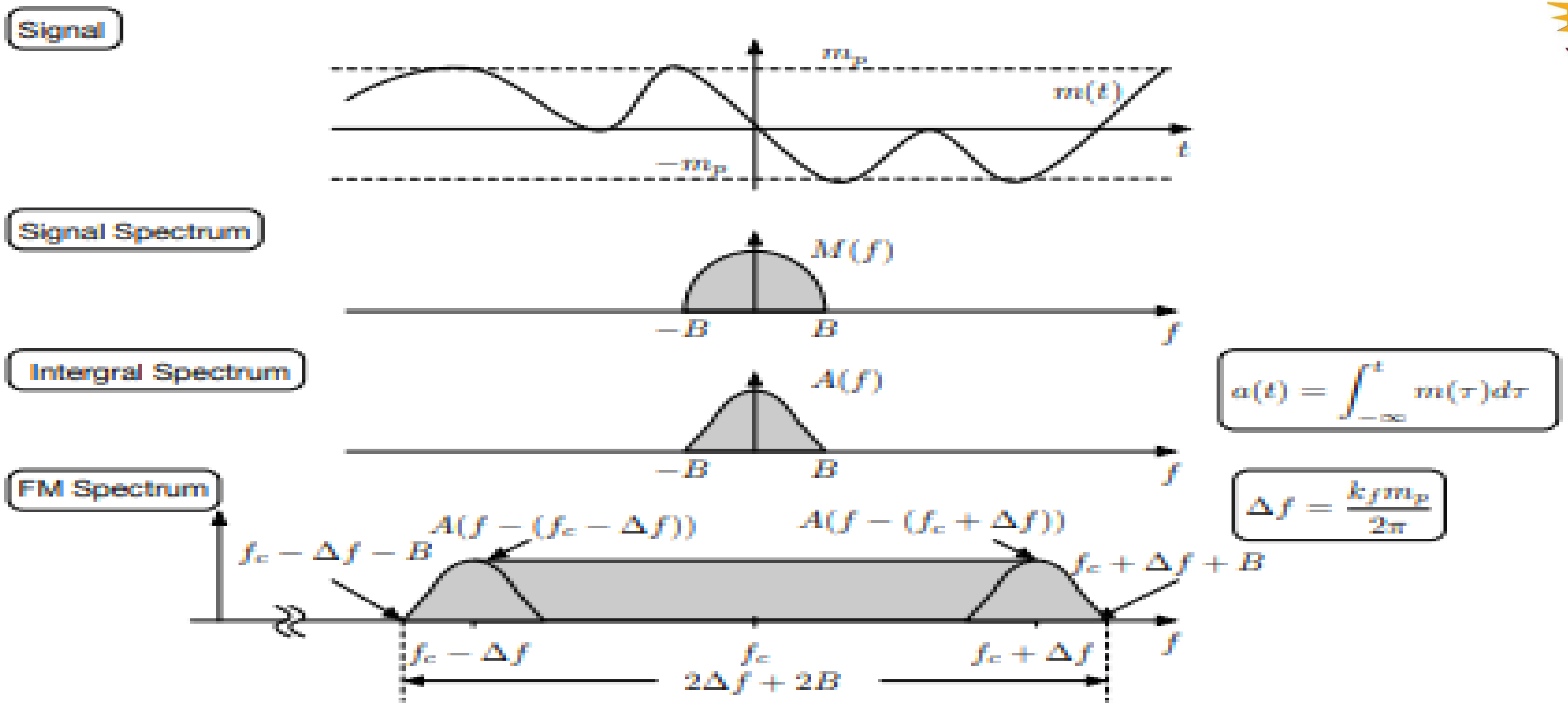
If $m(t)$ has a bandwidth $2B$ Hz, then $a(t)$ also has a bandwidth of $2B$ Hz.

What is the bandwidth of $\varphi_{\mathrm{FM}}(t)$? This is a difficult question in general. There are explicit solutions for only a few signals, such as sinusoids.

In practice, there are two contributors to the bandwidth

- Signal bandwidth $2B$
- FM deviation frequency $\Delta f = \frac{k_f m_p}{2\pi}$

This leads to Carson's rule.



Carson's Rule for the FM bandwidth is then

$$B_{FM} = 2\Delta f + 2B$$

where B_{FM} is the total signal bandwidth (not the half bandwidth).

Frequency Modulation of Tone

Spectral analysis of FM is difficult/impossible for general signals.

The special case of a sinusoidal input $m(t) = \cos(2\pi f_m t)$ is tractable. In this case $B_m = f_m$, and

$$a(t) = \int_{-\infty}^{t} m(u)\,du = \frac{1}{2\pi f_m}\sin(2\pi f_m t)$$

assuming $a(-\infty) = 0$. Then

$$\hat{\varphi}_{\mathrm{FM}} = Ae^{j2\pi f_c t}\, e^{j\beta \sin 2\pi f_m t}$$

where $\beta = k_f \,/\, 2\pi f_m$ is frequency deviation ratio (also called FM modulation index).

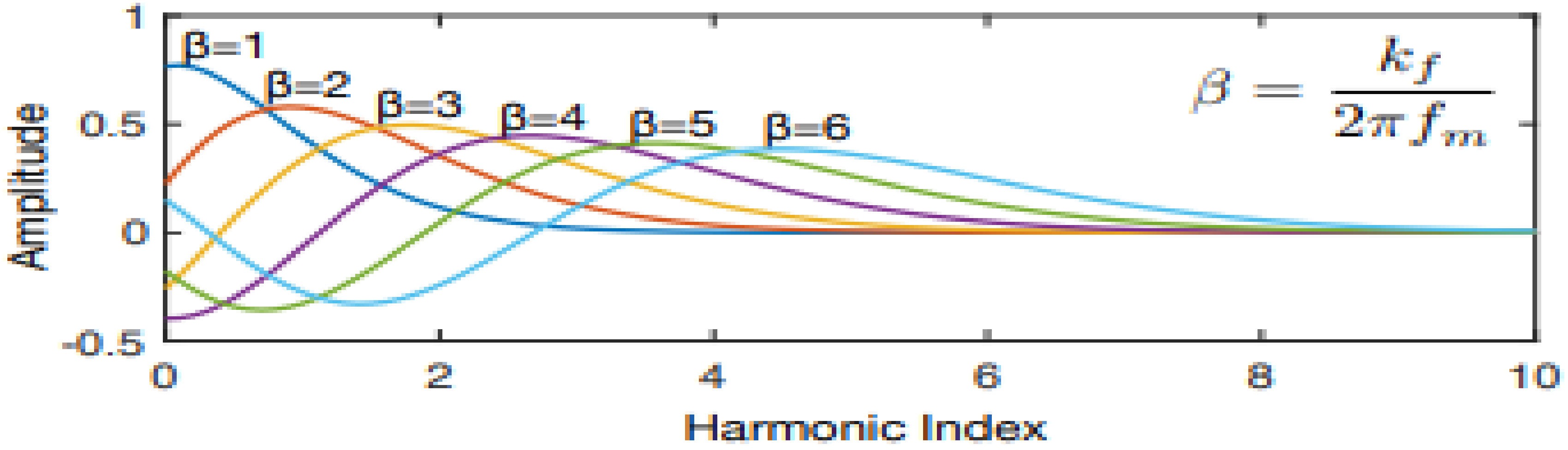
Since $e^{j\beta \sin(2\pi f_m t)}$ is periodic with a fundamental frequency f_m we can compute it's Fourier series as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_n J_n(\beta) e^{jn2\pi f_m t}$$

where the coefficients are

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin t - nt)}\, dt$$

Each frequency component n has a coefficient that is $J_n(\beta)$, where n is the order of the Bessel function, and β is the frequency deviation ratio. $J_n(\beta)$ is negligible if $n > \beta + 1$.



US Broadcast FM

- Frequency range: 88.0 – 108.0 MHz
- Channel width: 200 KHz (100 channels)
- Channel center frequencies: 88.1, 88.3, . . . , 107.9
- Frequency deviation: ±75 KHz
- Signal bandwidth: high-fidelity audio requires ±20 KHz, so bandwidth is available for other applications:
  - Muzak (elevator music) (1936)
  - Stock market quotations
  - Interactive games
- Stereo uses sum and difference of L/R audio channels

FM radio was assigned the 42–50 MHz band of the spectrum in 1940. In 1945, at the behest of RCA (David Sarnoff CEO), the FCC moved FM to 88–108 MHz, obsoleting all existing receivers.

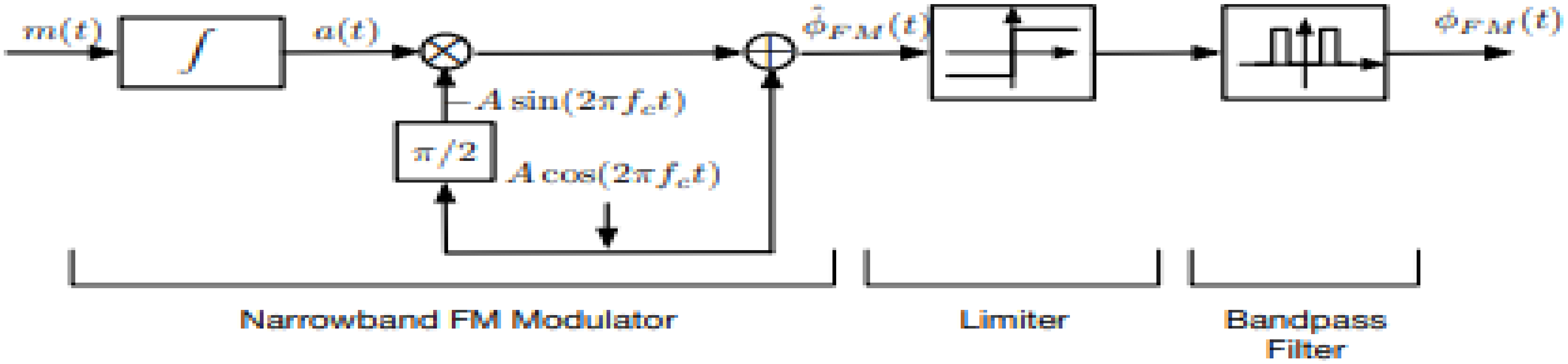
NBFM Modulation

For narrowband signals, $|k_f a(t)| \ll 1$ and $|k_p m(t)| \ll 1$,

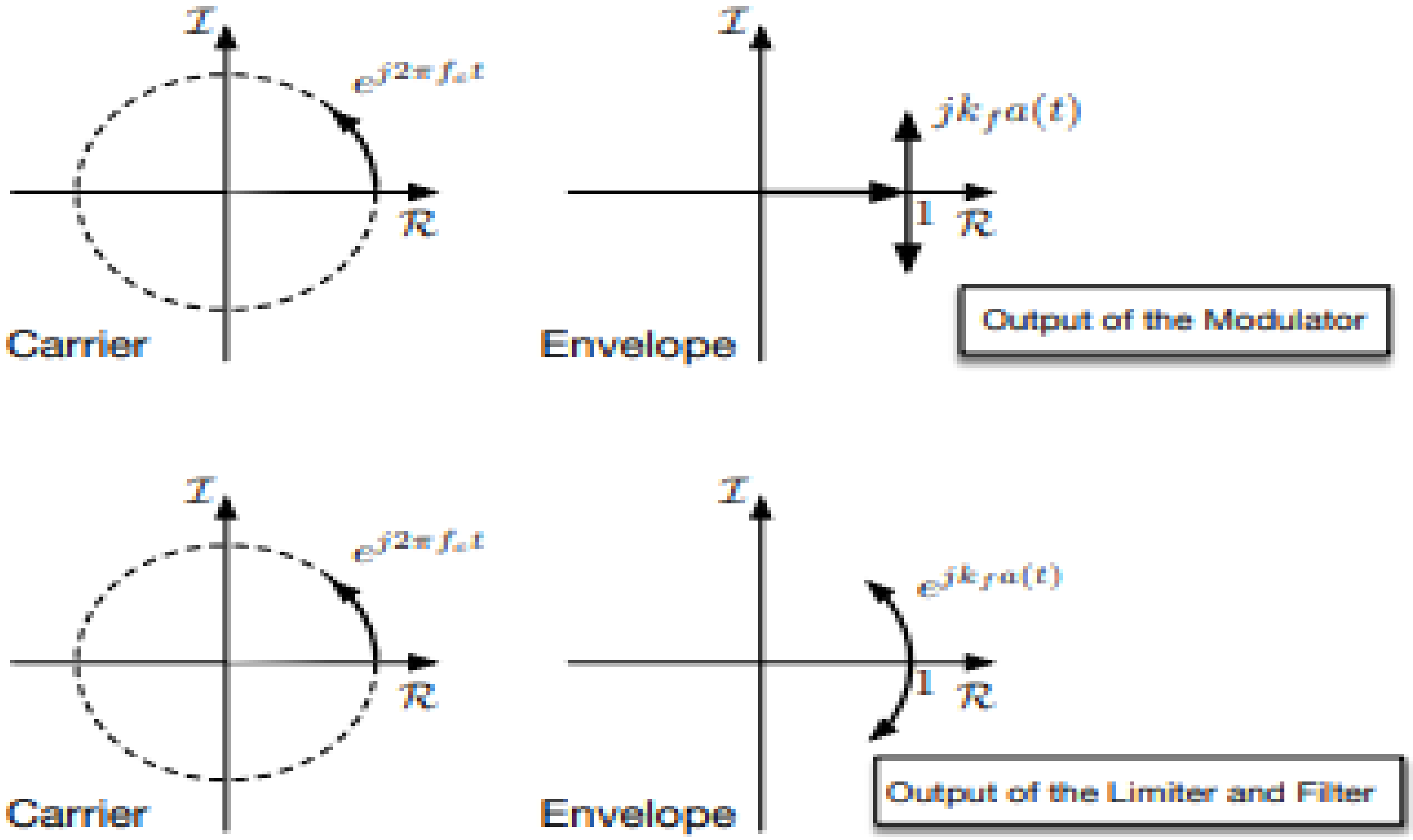
$$\hat{\varphi}_{\text{NBFM}} \approx A(\cos(2\pi f_c t) - k_f a(t)\sin(2\pi f_c t))$$

We can use a DSB-SC modulator with a phase shifter. In practice, this modulation will not be perfect, and there will be some amplitude modulation remaining.

To fix this up, follow with a limiter and a bandpass filter. For the case of FM,



NBFM Envelope and Carrier

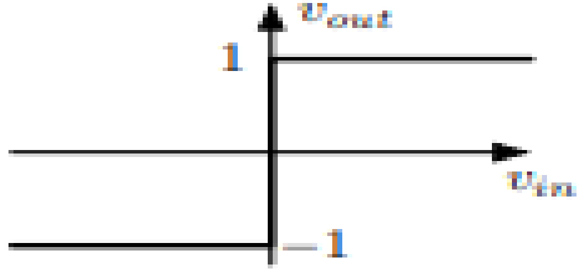


NBFM: Bandpass Limiter

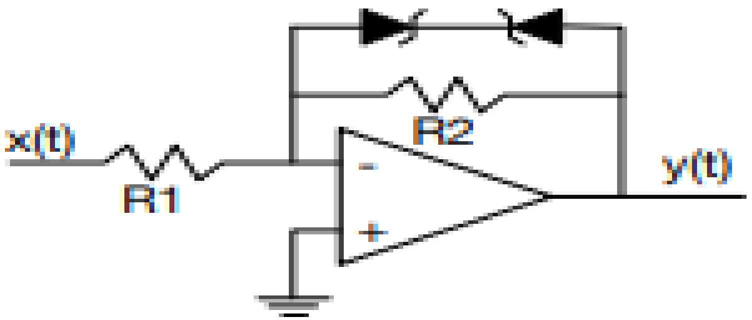
The input-output diagram for an ideal hard limiter is

$$v_o(t) = \begin{cases} +1 & v_i(t) > 0 \\ -1 & v_i(t) < 0 \end{cases}$$

This is a signum function, the output of a comparison against 0.



A hard limiter can be implemented by an op amp inverting amplifier, with back-to-back zener diodes to limit the output amplitude.



NBFM: Bandpass Limiter (cont.)

Input to bandpass limiter is

$$v_i(t) = A(t)\cos\theta(t)\,, \text{ where } \theta(t) = 2\pi f_c t + k_f a(t)$$

Ideally, $A(t)$ is constant, but it may vary slowly. We assume $A(t) > 0$. The input to the bandpass filter is

$$v_o(\theta) = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

which is periodic in θ with period 2π. Its Fourier series is

$$\begin{aligned} v_o(\theta) &= \frac{4}{\pi}\left(\cos\theta - \tfrac{1}{3}\cos 3\theta + \tfrac{1}{5}\cos 5\theta + \cdots\right) \\ &= \frac{4}{\pi}\Big(\cos\Big(2\pi f_c t + k_f a(t)\Big) - \tfrac{1}{3}\cos 3\Big(2\pi f_c t + k_f a(t)\Big) + \cdots\Big) \end{aligned}$$

The bandpass filter eliminates all but the first term.
Note that the angle modulation for the third term is three times greater.
We'll return to this next time.

WBFM Modulation: Direct Generation Using VCO

A voltage controlled oscillator generates a signal whose instantaneous frequency proportional to an input $m(t)$:

$$f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$$

The signal with frequency $f_i(t)$ is bandpass filtered, then used in a modulator.

VCO can be constructed by using input voltage to control one or more circuit parameters in an oscillator

One example is using a reverse-biased diode as a variable capacitor in the tank circuit of an oscillator.

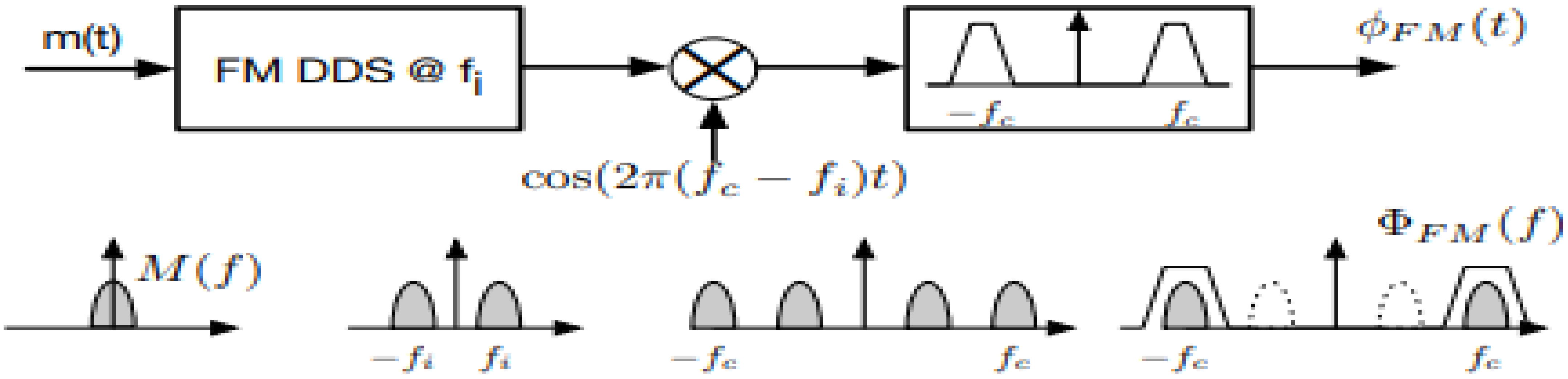
- Input voltage is the message signal, and it modulates the reverse bias potential
- This changes the capacitance, and hence the frequency of the oscillator

FM Direct Digital Synthesis

Currently, it is very common to synthesize the FM waveform digitally either at an intermediate frequency f_i and then mix it up to the desired carrier, or to directly synthesize the waveform at the carrier frequency.

This is Direct Digital Synthesis, or DDS.

Typical intermediate frequencies are a few MHz, so that the calculations are accurate but manageable, and the undesired sidebands can be suppressed easily.



Next time

- FM detection, commercial FM, pre-emphasis
- decoding commercial FM signals