TF and Block Diagrams

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ME 311 CONTROL SYSTEM (MAT LAB)

Semester No: 5

Week No:

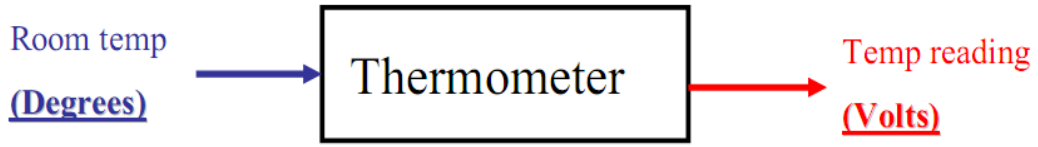
Transfer function:

Control system can be implemented as a combination of equipment and circuits connected together to perform a certain job or task, and every part of it can transfer energy from a certain form to another.

To understand and calculate the influence of any part of the control system to another, the transfer function concept will appear as a mathematical approach to find the relationship between the output and the input.

So a transfer function is the value of the output signal divided by the input signal

TF= output/ input



Control Systems – Feedback:

If either the output or some part of the output is returned to the input side and utilized as part of the system input, then it is known as feedback. Feedback plays an important role in order to improve the performance of the control systems.

Types of Feedback:

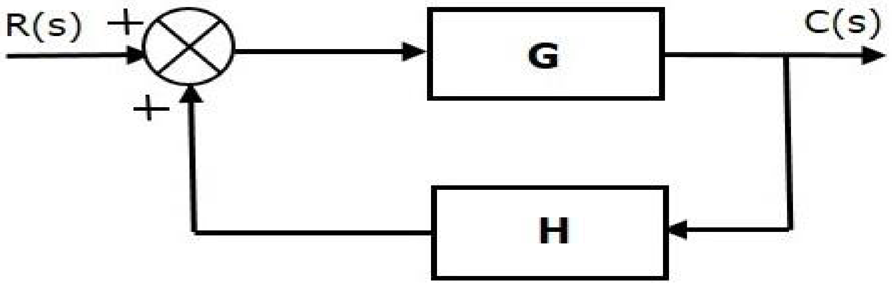
There are two types of feedback:

1- Positive feedback

2- Negative feedback

Positive Feedback:

The positive feedback adds the reference input, $R(s)$ and feedback output. The following figure shows the block diagram of positive feedback control system.



Calculations of total transfer function will be discussed in later lectures. For the time being, consider the transfer function of positive feedback control system is,

$T=G/(1-GH)$ ……………………………. (Equation 1)

Where,

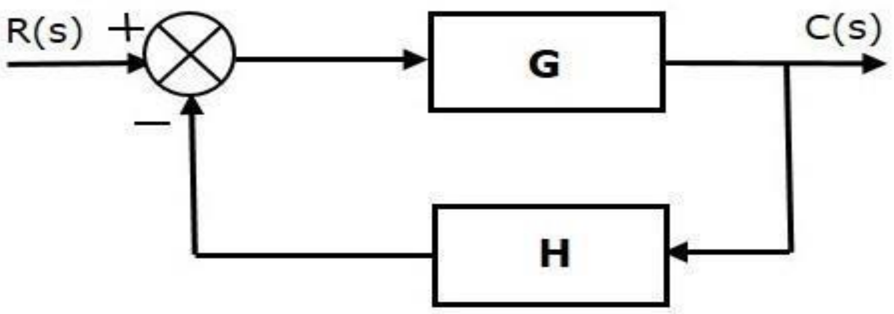
T is the transfer function or overall gain of positive feedback control system.

G is the open loop gain, which is function of frequency.

H is the gain of feedback path, which is function of frequency.

Negative Feedback

Negative feedback reduces the error between the reference input, $R(s)$ and system output. The following figure shows the block diagram of the negative feedback control system.



Transfer function of negative feedback control system is,

T=G / (1+GH) ……………………………..(Equation 2)

Where:

T: is the transfer function or overall gain of negative feedback control system.

G: is the open loop gain, which is function of frequency.

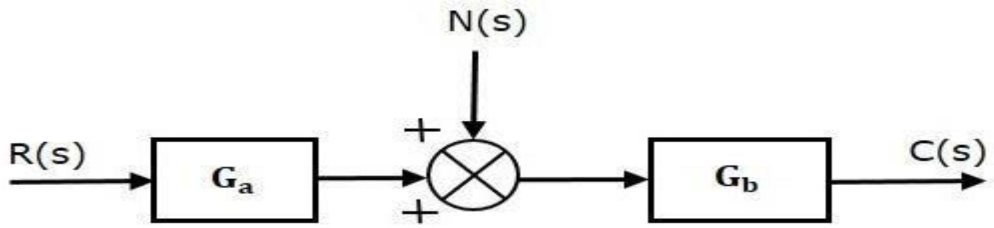
H: is the gain of feedback path, which is function of frequency.

Effect of Feedback on Stability

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. In Equation 2, if the denominator value is zero (i.e., GH = -1), then the output of the control system will be infinite. So, the control system becomes unstable.
Therefore, we have to properly choose the feedback in order to make the control system stable.

Effect of Feedback on Noise:

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.
Consider an open loop control system with noise signal as shown below.

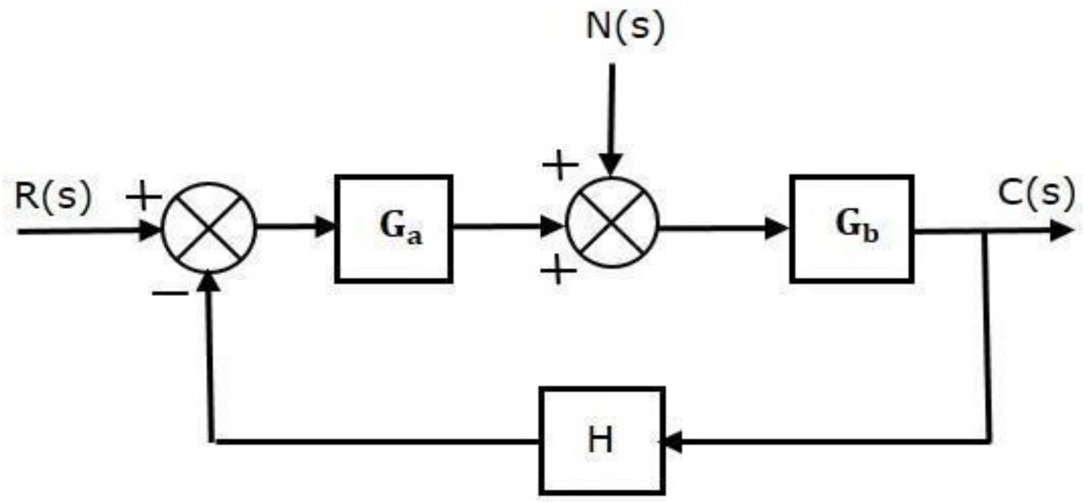


The open loop transfer function due to noise signal alone is

$C(s)/N(s)=Gb$ ………………………..(Equation 3)

It is obtained by making the other input $R(s)$ equal to zero.

Consider a closed loop control system with noise signal as shown below.



The closed loop transfer function due to noise signal alone is

$C(s)/N(s)=Gb/(1+GaGbH)$ ……………………………..(Equation 4)

It is obtained by making the other input $R(s)$ equal to zero.
Compare Equation 3 and Equation 4,
In the closed loop control system, the gain due to noise signal is **decreased by** a factor of **(1+*GaGbH*)** provided that the term (1+*GaGbH*) is greater than one.

Control Systems - Mathematical Models:

The control systems can be represented with a set of mathematical equations known as mathematical model. These models are useful for analysis and design of control systems.

Analysis of control system means finding the output when we know the input and mathematical model. Design of control system means finding the mathematical model when we know the input and the output

The following mathematical models are mostly used.

Differential equation model

Transfer function model

State space model

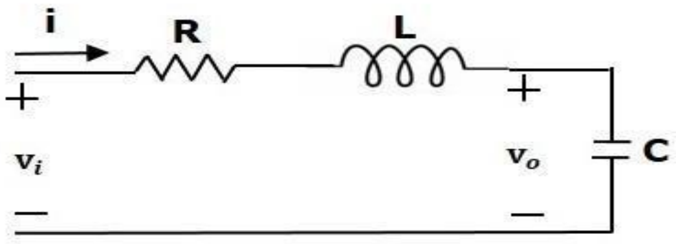
Let us discuss the first two models :

Differential Equation Model

Differential equation model is a **time domain** mathematical model of control systems. Follow these steps for differential equation model. Apply basic laws to the given control system. Get the differential equation in terms of input and output by eliminating the intermediate variable(s).

Example

Consider the following electrical system as shown in the following figure. This circuit consists of resistor, inductor and capacitor. All these electrical elements are connected in series. The input voltage applied to this circuit is V_i and the voltage across the capacitor is the output voltage V_o.



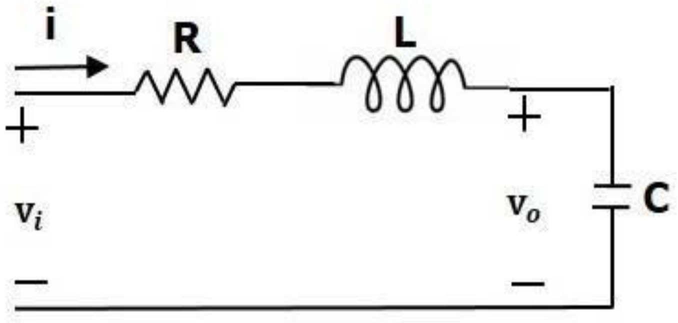
Mesh equation for this circuit is

$V_i = Ri + L\, di/dt + V_o$

Substitute, the current passing through capacitor $\boldsymbol{i = C\,(dV_o/dt)}$ in the above equation.

$\Rightarrow V_i = RC\,(dV_o/dt) + LC\,(d_2V_o/dt)^2 + V_o$

$\Rightarrow \mathbf{d_2V_o/dt^2 + (R/L)(dV_o/dt) + (1/LC)V_o = (1/LC)V_i}$



The above equation is a **second order** differential equation.

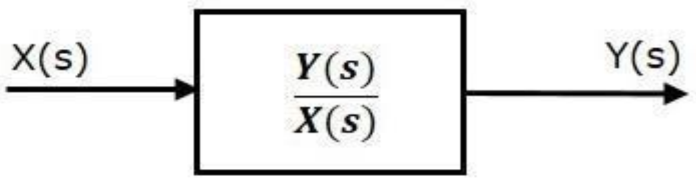
Transfer Function Model

Transfer function model is an **s-domain** mathematical model of control systems. The Transfer function of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.

If $x(t)$ and $y(t)$ are the input and output of an LTI system, then the corresponding Laplace transforms are $X(s)$ and $Y(s)$.

Therefore, the transfer function of LTI system is equal to the ratio of $Y(s)$ and $X(s)$. *i.e., Transfer Function* $=Y(s)/X(s)$

The transfer function model of an LTI system is shown in the following figure



Here, we represented an LTI system with a block having transfer function inside it. And this block has an input $X(s)$ & output $Y(s)$.

Example

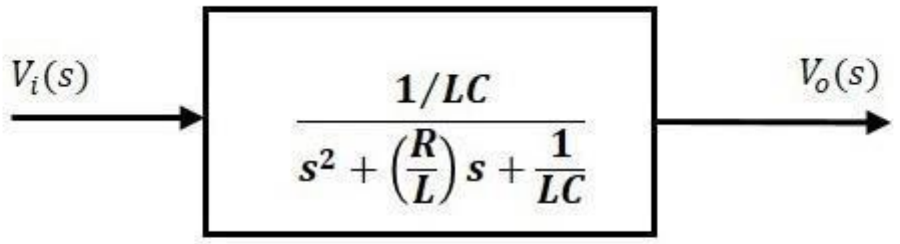
Previously, we got the differential equation of an electrical system as
$d_2V_o/dt^2+(R/L)dV_o\,/dt+(1/LC)V_o = (1/LC)V_i$
Apply Laplace transform on both sides.
$S^2\ Vo(s)+(SR/L)Vo(s)+(1/LC)Vo(s)=(1/LC)Vi(s)$

$\Rightarrow\{S^2+(R/L)S+1/LC\}Vo(s)=(1/LC)Vi(s)$

$\Rightarrow Vo(s)/Vi(s)=1/LC\,/[S^2+(R/L)S+1/LC]$

Where,
***Vi*(*s*)** is the Laplace transform of the input voltage V_i
***Vo*(*s*)** is the Laplace transform of the output voltage V_o

The above equation is a transfer function of the second order electrical system. The transfer function model of this system is shown in the figure.



Here, we show a second order electrical system with a block having the transfer function inside it. And this block has an input *Vi*(*s*) & an output *Vo*(*s*).