Tishk International University Department of Architecture 2023-2024

## Steel Structures

Dr. Yousif A.Hussein

## Course layout

## Chapter 01 Introduction to Structural Steel Design

## Chapter 02 Specifications, Loads, and Methods of Design

## Chapter 03 Analysis of Tension Members

## Chapter 04 Design of Tension Members

Chapter 05 Introduction to Axially Loaded Compression Members
Chapter 06 Design of Axially Loaded Compression Members
Chapter 07 Design of Axially Loaded Compression Members (Continued) Chapter 08 Introduction to Beams

## References:

Jack C . McCormac and Stephenf. Csernak (2012) "Structural Steel Design", Prentice Hall.

American Institute of Steel Construction (2005) "Steel onstruction Manual".

Alan Williams (2011) "Steel Structures Design: ASD/LRFD", McGraw Hill.

## CHAPTER1

## Introduction to Structural

## Steel Design

## INTRODUCTION

- There are endless number of steel bridges, buildings, towers, and other structures in the world.
- In the United States, the first steel framed building was the Rand McNally Building in Chicago, erected in 1890.
- The Royal Insurance Building in Liverpool designed by James Francis Doyle in 1895 (erected 1896-1903) was the first to use a steel frame in the United Kingdom.
- Eiffel tower ( 985 ft ) was constructed in 1889


## ADVANTAGES OF STEEL AS A STRUCTURAL MATERIAL

1. High Strength The high strength of steel per unit of weight means that the weight of structures will be small. This fact is of great importance for long-span bridges, tall buildings, and structures situated on poor foundations.
2. Uniformity The properties of steel do not change appreciably with time, as do those of a reinforced- concrete structure.

## 3. Elasticity

 Steel behaves closer to design assumptions than most materials because it follows Hooke's law up to fairly high stresses. The moments of inertia of a steel structure can be accurately calculated, while the values obtained for a reinforced-concrete structure are rather indefinite.

Figure Long span bridge



Figure definition of Hooke's law
4. Permanence Steel frames that are properly maintained will last indefinitely.
5. Ductility The property of a material by which it can withstand extensive deformation without failure under high tensile stresses is its ductility.

- When a mild or low-carbon structural steel member is being tested in tension, a considerable reduction in cross section and a large amount of elongation will occur at the point of failure before the actual fracture occurs.
- A material that does not have this property is generally unacceptable and is probably hard and brittle, and it might break if subjected to a sudden shock.
- In structural members under normal loads, high stress concentrations develop at various points. The ductile nature of the usual structural steels enables them to yield locally at those points, thus preventing premature failures.
- A further advantage of ductile structures is that when overloaded, their large deflections give visible evidence of impending failure (sometimes jokingly referred to as "running time").

6. Toughness

- Structural steels are tough-that is, they have both strength and ductility.
- A steel member loaded until it has large deformations will still be able to withstand large forces.
- The ability of a material to absorb energy in large amounts is called toughness.

7. Additions to Steel structures are quite well suited to having additions made to them. New Existing bays or even entire new wings can be added to existing steel frame Structures buildings, and steel bridges may often be widened.
8. Miscellaneous
a) Ability to be fastened together by several simple connection devices, including welds and bolts.
b) Adaptation to prefabrication.
c) Speed of erection;
d) Ability to be rolled into a wide variety of sizes and shapes.
e) Possible reuse after a structure is disassembled; and
f) Scrap value, even though not reusable in its existing form. Steel is the ultimate recyclable material.


DISADVANTAGES OF STEEL AS A STRUCTURAL MATERIAL

1. Corrosion Most steels are susceptible to corrosion when freely exposed to air and water, and therefore must be painted periodically.

2. Fireproofing The strength of structural members is tremendously reduced at temperatures Costs commonly reached in fires when the other materials in a building burn. As a result, the steel frame of a building may have to be protected by materials with certain insulating characteristics.


Fig. 2.1. Yield Strength Retention Factors for
Structural Steel at Elevated Temperatures
Structural Steel at Elevated Temperatures

## 3. Buckling

- As the length and slenderness of a compression member is increased, its danger of buckling increases.

- The use of steel columns is very economical because of their high strength-to-weight ratios. Occasionally, however, some additional steel is needed to stiffen them so they will not buckle. This tends to reduce their economy


## 4. Fatigue

Another undesirable property of steel is that its strength may be reduced if it is subjected to a large number of stress reversals or even to a large number of variations of tensile stress. (Fatigue problems occur only when tension is involved.)

5. Brittle Fracture Under certain conditions steel may lose its ductility, and brittle fracture may occur at places of stress concentration. Fatigue-type loadings and very low temperatures aggravate the situation. Triaxial stress conditions can also lead to brittle fracture.

$\qquad$

## STEEL SECTIONS

- Structural steel can be economically rolled into a wide variety of shapes and sizes without appreciably changing its physical properties.
- The most desirable members are those with large moments of inertia in proportion to their areas, such as the I, T, and C shapes.
- Steel sections are usually designated by the shapes of their cross sections.
- It is necessary to make a definite distinction between American standard beams (called S beams) and wide-flange beams (called W beams), as they are both I-shaped.

| S beams | W beams |
| :--- | :--- |
| The $S$ beams, which were the first beam | The inner surface of the flange of a W section |
| sections rolled in America, have a slope on | is either parallel to the outer surface or nearly |
| their inside flange surfaces of 1 to 6. | so, with a maximum slope of 1 to 20 on the <br> inner surface, depending on the manufacturer. |

The W and S sections are shown in the figure (below), together with several other familiar steel sections. The uses of these various shapes will be discussed in detail in the chapters to follow.
Fillet Slope 0 to $5 \%$

S beam
(American standard beam)

Unequal leg angle

Equal leg angle
$16 \frac{2}{3} \%$ slope
C section
(American standard channel)

WT section

Structural shapes are identified by a certain system described in the Manual for use in drawings, specifications, and designs. Examples of this identification system are as follows:

| 1 | W27 $\times 114$ | is a W section approximately 27 inch deep, weighing $114 \mathrm{lb} / \mathrm{ft}$. |
| :---: | :---: | :---: |
| 2 | $\mathrm{S} 12 \times 35$ | is an S section 12 inch deep, weig |
| 3 | HP12 $\times 74$ | is a bearing pile section approximately 12 inch deep, weighing $74 \mathrm{lb} / \mathrm{ft}$. Bearing piles are made with the regular W rolls, but with thicker webs to provide better resistance to the impact of pile driving. The width and depth of these sections are approximately equal, and the flanges and webs have equal or almost equal thickness. |
| 4 | M8 $\times 6.5$ | is a miscellaneous section 8 inch deep, weighing $6.5 \mathrm{lb} / \mathrm{ft}$. It is one of a group of doubly symmetrical H-shaped members that cannot by dimensions be classified as a W, S, or HP section, as the slope of their inner flanges is other than $162 / 3$ percent. |
| 5 | C10 $\times 30$ | is a channel 10 in |
| 6 | MC18 $\times 58$ | is a miscellaneous channel 18 inch deep, weighing $58 \mathrm{lb} / \mathrm{ft}$, which cannot be classified as a C shape because of its dimensions. |
| 7 | $\begin{aligned} & \mathrm{HSS} 14 \times 10 \times \\ & 5 / 8 \end{aligned}$ | is a rectangular hollow structural section 14 inch deep, 10 inch wide, with a $5 / 8$-inch wall thickness. It weighs $93.10 \mathrm{lb} / \mathrm{ft}$. Square and round HSS sections are also available. |
| 8 | L6 $\times 6 \times 1 / 2$ | is an equal leg angle, each leg being 6 inch long and $1 / 2$ inch thick. |
| 9 | WT18 $\times 151$ | is a tee obtained by splitting a W36 X 302 This type of section is known as a structural tee. |
| 10 | - | Rectangular steel sections are classified as wide plates or narrow bars. |

$\qquad$

## STRESS-STRAIN RELATIONSHIPS IN STRUCTURAL STEEL

- To understand the behavior of steel structures, an engineer must be familiar with the properties of steel.
- Stress-strain diagrams present valuable information necessary to understand how steel will behave in a given situation.
- If a piece of ductile structural steel is subjected to a tensile force, it will begin to elongate. If the tensile force is increased at a constant rate, the amount of elongation will increase linearly within certain limits. In other words, elongation will double when the stress goes from 6000 to 12,000 psi (pounds per square inch).

- When the tensile stress reaches a value roughly equal to three-fourths of the ultimate strength of the steel, the elongation will begin to increase at a greater rate without a corresponding increase in the stress.
- The largest stress for which Hooke's law applies, or the highest point on the linear portion of the stress-strain diagram, is called the proportional limit. The largest stress that a material can withstand without being permanently deformed is called the elastic limit.
- The stress at which there is a significant increase in the elongation, or strain, without a corresponding increase in stress is said to be the yield stress.
- The strain that occurs before the yield stress is referred to as the elastic strain; the strain that occurs after the yield stress, with no increase in stress, is referred to as the plastic strain.
- Plastic strains are usually from 10 to 15 times as large as the elastic strains.
- Following the plastic strain, there is a range in which additional stress is necessary to produce additional strain. This is called strain-hardening.
$\qquad$


## CHAPTER2

## Specifications, Loads,

## and Methods of Design

## SPECIFICATIONS AND BUILDING CODES

- The design of most structures is controlled by building codes and design specifications.
- Engineering specifications that are developed by various organizations present the best opinion of those organizations as to what represents good practice.
- Engineering specifications and codes are actually laws or ordinances specify minimum design loads, design stresses, construction types, material quality, and other factors.
- Several organizations publish recommended practices for regional or national use. Among these organizations are the AISC and AASHTO (American Association of State Highway and Transportation Officials). Nearly all municipal and state building codes have adopted the AISC Specification, and nearly all state highway and transportation departments have adopted the AASHTO Specifications.
- Another very important code, the International Building Code (IBC).


## LOADS

- Perhaps the most important and most difficult task faced by the structural engineer is the accurate estimation of the loads that may be applied to a structure during its life.
- After loads are estimated, the next problem is to determine the worst possible combinations of these loads that might occur at one time. For instance, would a highway bridge completely covered with ice and snow be simultaneously subjected to fastmoving lines of heavly loaded trailer trucks in every lane and to a 90 -mile lateral wind, or is some lesser combination of these loads more likely?
- AISC Specification states the nominal loads to be used for structural design. Also, the American Society of Civil Engineers (ASCE) provides a publication entitled Minimum Design Loads for Buildings and Other Structures.
$\qquad$

In general, loads are classified as dead loads, live loads, and environmental loads according to their character and duration of application.

Each of these types of loads are discussed in the next few sections.

## DEAD LOADS

Dead loads are loads of constant magnitude that remain in one position. They consist of the structural frame's own weight and other loads that are permanently attached to the frame. For a steel-frame building, the frame, walls, floors, roof, plumbing, and fixtures are dead loads.

The approximate weights of some common building materials for roofs, walls, floors, and so on are presented in Table 2.1.

TABLE 2.1 Typical Dead Loads for Some Common Building Materials

| Reinforced concrete | $150 \mathrm{lb} / \mathrm{cu} \mathrm{ft}$ |
| :--- | :--- |
| Structural steel | $490 \mathrm{lb} / \mathrm{cu} \mathrm{ft}$ |
| Plain concrete | $145 \mathrm{lb} / \mathrm{cu} \mathrm{ft}$ |
| Movable steel partitions | 4 psf |
| Plaster on concrete | 5 psf |
| Suspended ceilings | 2 psf |
| 5 -Ply felt and gravel | 6 psf |
| Hardwood flooring (7/8 in) | 4 psf |
| $2 \times 12 \times 16$ in double wood floors | 7 psf |
| Wood studs with $1 / 2$ in gypsum each side | 8 psf |
| Clay brick wythes (4 in) | 39 psf |

## LIVE LOADS

Live loads are loads that may change in position and magnitude. They are caused when a structure is occupied, used, and maintained.

Live loads include:

## 1. Floor loads:

- The minimum gravity live loads to be used for building floors are clearly specified by the applicable building code.
- A few of the typical values for floor loadings are listed in Table 2.2, and some typical concentrated loads are listed in Table 2.3.

TABLE 2.2 Typical Minimum Uniform Live Loads for Design of Buildings

| Type of building | LL (psf) |
| :--- | :---: |
| Apartment houses |  |
| $\quad$ Apartments | 40 |
| $\quad$ Public rooms | 100 |
| Dining rooms and restaurants | 100 |
| Garages (passenger cars only) | 40 |
| Gymnasiums, main floors, and balconies | 100 |
| Office buildings |  |
| Lobbies | 100 |
| Offices | 50 |
| Schools |  |
| $\quad$ Classrooms | 40 |
| Corridors, first floor | 100 |
| $\quad$ Corridors above first floor | 80 |
| Storage warehouses | 125 |
| Light | 250 |
| Heavy |  |
| Stores (retail) | 100 |
| First floor | 75 |
| Other floors |  |

TABLE 2.3 Typical Concentrated Live Loads for Buildings

| Hospitals-operating rooms, private rooms, and wards | 1000 lb |
| :--- | :--- |
| Manufacturing building (light) | 2000 lb |
| Manufacturing building (heavy) | 3000 lb |
| Office floors | 2000 lb |
| Retail stores (first floors) | 1000 lb |
| Retail stores (upper floors) | 1000 lb |
| School classrooms | 1000 lb |
| School corridors | 1000 lb |

2. Traffic loads for bridges: Bridges are subjected to series of concentrated loads of varying magnitude caused by groups of truck or train wheels.
3. Impact loads: Impact loads are caused by the vibration of moving or movable loads.
4. Longitudinal loads: Longitudinal loads are another type of load that needs to be considered in designing some structures. Stopping a train on a railroad bridge or a truck on a highway bridge causes longitudinal forces to be applied.
5. Other live loads: such as soil pressures, hydrostatic pressures, and blast loads.
$\qquad$

## ENVIRONMENTAL LOADS

Environmental loads are caused by the environment in which a particular structure is located. For buildings, environmental loads are caused by rain, snow, wind, temperature change, and earthquakes.

1. Snow: For roof designs, snow loads varying from 10 to 40 psf are commonly used. A load of approximately 10 psf might be used for $45^{\circ}$ (degree) slopes and a 40 -psf load for flat roofs.

2. Rain: If water on a flat roof accumulates faster than it runs off, the result is called ponding, because the increased load causes the roof to deflect into a dish shape that can hold more water, which causes greater deflections, and so on. This process continues until equilibrium is reached or until collapse occurs. The best method of preventing ponding is to have an appreciable slope of the roof ( $1 / 4 \mathrm{in} / \mathrm{ft}$ or more), together with good drainage facilities.
3. Wind loads: A survey of engineering literature for the past 150 years reveals many references to structural failures caused by wind. Wind forces act as pressures on vertical windward surfaces, pressures or suction on sloping windward surfaces (depending on the slope), and suction on flat surfaces.

For some common structures, uplift loads may be as large as 20 to 30 psf or even more.
4. Earthquake loads: Many areas of the world fall in "earthquake territory," and in those areas it is necessary to consider seismic forces in design for all types of structures.
$\qquad$


## LOAD AND RESISTANCE FACTOR DESIGN (LRFD)

AND ALLOWABLE STRENGTH DESIGN (ASD)
The AISC Specification provides two acceptable methods for designing structural steel members and their connections. These are Load and Resistance Factor Design (LRFD) and Allowable Strength Design (ASD).

## COMPUTATION OF LOADS FOR LRFD AND ASD

- With both the LRFD and the ASD procedures, expected values of the individual loads (dead, live, wind, snow, etc.) are first estimated in exactly the same manner as required by the applicable specification. These loads are referred to as service or working loads.
- Various combinations of these loads that feasibly may occur at the same time are grouped together. The largest load group (in ASD) or the largest linear combination of loads in a group (in LRFD) is then used for analysis and design.


## COMPUTING COMBINED LOADS WITH LRFD EXPRESSIONS

- Load factors are calculated to increase the magnitudes of service loads to use with the LRFD procedure.
- The purpose of these factors is to account for the uncertainties involved in estimating the magnitudes of dead and live loads.
- The AISC Manual provides the following load factors for buildings.
$1 \quad U=1.4 D$
$2 \quad U=1.2 D+1.6 L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
$3 \quad U=1.2 D+1.6\left(L_{r}\right.$ or $S$ or $\left.R\right)+\left(L^{*}\right.$ or $\left.0.5 w\right)$
$4 \quad U=1.2 D+1.0 W+L^{*}+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
$5 \quad U=1.2 D+1.0 E+L^{*}+0.2 S$
$6 \quad U=0.9 D+1.0 \mathrm{~W}$
$7 \quad U=0.9 D+1.0 E$
*The load factor on L in combinations (3.), (4.), and (5.) is to be taken as 1.0 for floors in places of public assembly, for live loads in excess of 100 psf and for parking garage live load. The load factor is permitted to equal 0.5 for other live loads.

In these load combinations, the following abbreviations are used:
$\mathrm{U}=$ the design or factored load
D = Dead load
L = Live load
$\mathrm{L}_{\mathrm{r}}=$ roof live load
S = Snow load
$R=$ nominal load due to initial rainwater or ice, exclusive of the ponding contribution.
W = Wind load
$\mathrm{E}=$ Earthquake load


Examples 2-1 to 2-3 show the calculation of the factored loads, using the applicable LRFD load combinations. The largest value obtained is referred to as the critical or governing load combination and is to be used in design.

## Example 2-1

The interior floor system shown in Figure (below) has W24 X 55 sections spaced 8 ft on center and is supporting a floor dead load of 50 psf and a live floor load of 80 psf . Determine the governing load in lb/ft that each beam must support.


## Solution:

Note that each foot of the beam must support itself (a dead load) plus $8 \times 1=8 \mathrm{ft}^{2}$ of the building floor

$$
\begin{gathered}
D=55 \frac{l b}{f t}+(8 f t)(50 p s f)=455 \frac{l b}{f t} \\
L=(8 f t)(80 p s f)=640 \mathrm{lb} / f t
\end{gathered}
$$

Computing factored loads, using the LRFD load combinations.
Note that with a floor live load of 80 psf a load factor of 0.5 has been added to load combinations (3.), (4.), and (5.) per the exception stated in ASCE 7-10 and this text for floor live loads.
$1 \quad W_{u}=(1.4)(455)=637 \mathrm{lb} / \mathrm{ft}$
$2 \quad W_{u}=(1.2)(455)+1.6(640)=1570 \mathrm{lb} / \mathrm{ft}$
$3 \quad W_{u}=(1.2)(455)+(0.5)(640)=866 \mathrm{lb} / \mathrm{ft}$
$4 \quad W_{u}=(1.2)(455)+(0.5)(640)=866 \mathrm{lb} / \mathrm{ft}$
$5 \quad W_{u}=(1.2)(455)+(0.5)(640)=866 \mathrm{lb} / \mathrm{ft}$
$6 \quad W_{u}=(0.9)(455)=409.5 \mathrm{lb} / \mathrm{ft}$
$7 \quad W_{u}=(0.9)(455)=409.5 \mathrm{lb} / f t$
Governing factored load $=1570 \mathrm{lb} / \mathrm{ft}$ to be used for design.

## Example 2-2

A roof system with W16 $\times 40$ section spaced 9 ft on center is to be used to support a dead load of 40 psf ; a roof live, snow, or rain load of 30 psf ; and a wind load of $\pm 32 \mathrm{psf}$. Compute the governing factored load per linear foot.

## Solution

$D=40 \mathrm{lb} / f t+(9 f t)(40 \mathrm{psf})=400 \mathrm{lb} / f t$
$L=0$
$L_{r}$ or $S$ or $R=(9 f t)(30 p s f)=270 \mathrm{lb} / f t$
$W=(9 f t)(32 p s f)=288 \mathrm{lb} / f t$
Substituting into the load combination expressions and noting that the wind can be downward, - or uplift, + in Equation 6, we derive the following loads:

1

$$
\begin{aligned}
& W_{u}=(1.4)(400)=560 \mathrm{lb} / \mathrm{ft} \\
& W_{u}=(1.2)(400)+0.5(270)=615 \mathrm{lb} / \mathrm{ft} \\
& W_{u}=(1.2)(400)+(1.6)(270)+(0.5)(288)=1056 \mathrm{lb} / f t \\
& W_{u}=(1.2)(400)+(1.0)(288)+(0.5)(270)=903 \mathrm{lb} / \mathrm{ft} \\
& W_{u}=(1.2)(400)+(0.2)(270)=534 \mathrm{lb} / f t \\
& W_{u}=(0.9)(400)+(1.0)(288)=648 \mathrm{lb} / \mathrm{ft} \\
& W_{u}=(0.9)(400)+(1.0)(-288)=72 \mathrm{lb} / \mathrm{ft}
\end{aligned}
$$

Governing factored load $=1056 \mathrm{lb} / \mathrm{ft}$ for design

## Example 2-3

The various axial loads for a building column have been computed according to the applicable building code, with the following results: dead load 200 k , load from roof= 50 k (roof live load); live load from floors (reduced as applicable for large area and multistory columns) $=250 \mathrm{k}$; compression wind $=128 \mathrm{k}$; tensile wind $=104 \mathrm{k}$; compression earthquake $=60 \mathrm{k}$; and tensile earthquake $=70 \mathrm{k}$.

Determine the critical design column load, Pu , using LRFD load combinations.

## Solution.

This problem solution assumes the column floor live load meets the exception for the use of the load factor of 0.5 in load combinations (3.), (4.), and (5.)
$1 \quad P_{u}=(1.4)(200)=280 k$
$2 P_{u}=(1.4)(200)+(1.6)(250)+(0.5)(50)=655 k$
3.(a) $\quad P_{u}=(1.2)(200)+(1.6)(50)+(0.5)(250)=445 k$
(b) $\quad P_{u}=(1.2)(200)+(1.6)(50)+(0.5)(128)=384 k$
4.(a) $\quad P_{u}=(1.2)(200)+(1.0)(128)+(0.5)(250)=518 k$
(b) $\quad P_{u}=(1.2)(200)-(1.0)(104)+(0.5)(250)=286$
5.(a) $\quad P_{u}=(1.2)(200)+(1.0)(60)+(0.5)(250)=425 k$
(b) $\quad P_{u}=(1.2)(200)-(1.0)(70)+(0.5)(250)=295 k$
6. (a) $\quad P_{u}=(0.9)(200)+(1.0)(128)=308 k$
(b) $\quad P_{u}=(0.9)(200)-(1.0)(104)=76 k$
7. (a) $\quad P_{u}=(0.9)(200)+(1.0)(60)=240 k$
(b) $\quad P_{u}=(0.9)(200)-(1.0)(70)=110 k$

## PROBLEMS FOR SOLUTION

For Probs. 2-1 through 2-4 determine the maximum combined loads using the recommended AISC expressions for LRFD.

2-1 $D=100 \mathrm{psf}, \mathrm{L}=70 \mathrm{psf}, \mathrm{R}=12 \mathrm{psf}, \mathrm{L}_{\mathrm{r}}=20 \mathrm{psf}$ and $\mathrm{S}=30 \mathrm{psf}($ Ans. 247 psf$)$
$2-2 \quad D=10,000 \mathrm{lb}, \mathrm{W}= \pm 32,000 \mathrm{lb}$
2-3 D $=9000 \mathrm{lb}, \mathrm{L}=5000 \mathrm{lb}, \mathrm{L}_{\mathrm{r}}=2500 \mathrm{lb}, \mathrm{E}= \pm 6500 \mathrm{lb}$ (Ans. 20,050 lb)
2-4
$\mathrm{D}=25 \mathrm{psf}, \mathrm{L}_{\mathrm{r}}=20 \mathrm{psf}$ and $\mathrm{W}= \pm 26 \mathrm{psf}$

## CHAPTER3

## Analysis of Tension Members

## INTRODUCTION

- Tension members are found in bridge and roof trusses, towers, and bracing systems, and in situations where they are used as tie rods.

- The selection of a section to be used as a tension member is one of the simplest problems encountered in design. As there is no danger of the member buckling, the designer needs to determine only the load to be supported. Then the area required to support that load is calculated, and finally a steel section is selected that provides the required area.
- One of the simplest forms of tension members is the circular rod, but there is some difficulty in connecting it to many structures.
- When rods are used in wind bracing, it is a good practice to produce initial tension in them, as this will tighten up the structure and reduce rattling and swaying.
A common rule of thumb is to detail the rods about $1 / 16$ in short for each 20 ft of length. Approximate stress is:

$$
f=\epsilon E=\frac{\left(\frac{1}{16} \mathrm{in}\right)}{\left(12 \frac{i n}{f t}\right)(20 \mathrm{ft})}\left(29 \times 10^{6} \mathrm{psi}\right)=7550 \mathrm{psi}
$$

Another very satisfactory method involves tightening the rods with some sort of sleeve nut or turnbuckle.


- Today, although the use of cables is increasing for suspended-roof structures, tension members usually consist of single angles, double angles, tees, channels, W sections, or sections built up from plates or rolled shapes.

A few of the various types of tension members in general use are illustrated in Fig. 3.1. In this figure, the dotted lines represent the intermittent tie plates or bars used to connect the shapes.


FIGURE 3.1 Types of tension members.

- The tension members of steel roof trusses may consist of single angles as small as $2 \times 1 / 2$ or $2 \times 1 / 4$ for minor members.
- For bridges and large roof trusses, tension members may consist of channels, W or S shapes, or even sections built up from some combination of angles, channels, and plates.
- Cross bracing is often done with tensile members as these members need only to act in tension.

- Steel cables are made with special steel alloy wire ropes that are colddrawn to the desired diameter. The resulting wires with strengths of about 200,000 to 250,000 psi can be economically used for suspension bridges, cable supported roofs, ski lifts, and other similar applications.

$\qquad$


## NOMINAL STRENGTHS OF TENSION MEMBERS

A ductile steel member without holes and subject to a tensile load can resist without fracture a load larger than its gross cross-sectional area times its yield stress $\left[\right.$ A. $\left.\sigma_{\text {yield }}\right]$ because of strain hardening.However, a tension member loaded until strain hardening is reached will lengthen a great deal before fracture-a fact that will, in all probability, end its usefulness and may even cause failure of the structural system of which the member is a part.

If, on the other hand, we have a tension member with bolt holes, it can possibly fail by fracture at the net section through the holes.


This failure load may very well be smaller than the load required to yield the gross section, apart from the holes.

## NET AREAS

The presence of a hole increases the unit stress in a tension member, even if the hole is occupied by a bolt.

There is still less area of steel to which the load can be distributed, and there will be some concentration of stress along the edges of the hole.

Tension is assumed to be uniformly distributed over the net section of a tension member, although photoelastic studies show there is a decided increase in stress intensity around the edges of holes, sometimes equaling several times what the stresses would be if the holes were not present.
$\qquad$


Figure. Stresses around hole (red color represents concentration of stresses)

- The term "net cross-sectional area," or simply, "net area," refers to the gross cross-sectional area of a member, minus any holes, notches, or other indentations.
- It is usually necessary to subtract an area a little larger than the actual hole. For instance, holes are punched to a diameter $1 / 16$ in larger than that of the bolts. When this practice was followed, the punching of a hole was assumed to damage or even destroy $1 / 16$ in more of the surrounding metal. As a result, the diameter of the hole subtracted was $1 / 8$ in larger than the diameter of the bolt. The area of the hole was rectangular and equalled the diameter of the bolt plus $1 / 8$ in times the thickness of the metal.
- Today, drills enable fabricators to drill very large numbers of holes. For such holes, only $1 / 16 \mathrm{in}$. is added to the bolt diameters for such holes.


## Example 3-1

Determine the net area of $3 / 8 \times 8$-in the plate shown in Fig. 3.2. The plate is connected at its end with two lines of $3 / 4$-in bolts.
$\qquad$


Fig. 3-2 Illustration of Example 3-1

$$
A_{n}=\left(\frac{3}{8} \mathrm{in}\right)(8 \mathrm{in})-2\left(\frac{3}{4} \mathrm{in}+\frac{1}{8} \mathrm{in}\right)\left(\frac{3}{8} \mathrm{in}\right)=2.34 \mathrm{in}^{2}\left(1510 \mathrm{~mm}^{2}\right)
$$

- The connections of tension members should be arranged so that no eccentricity is present.
- Should the connections have eccentricities, moments will be produced that will cause additional stresses in the vicinity of the connection.
- The centroidal axes of truss members meeting at a joint are assumed to coincide. Should they not coincide, eccentricity is present and secondary stresses are the result.
$\qquad$


FIGURE 3.3 Lining up centroidal axes of members.

## EFFECT OF STAGGERED HOLES

- Tensile members could fail transversely along line AB in either Fig. 3.4(a) or 3.4(b).


FIGURE 3.4 Possible failure sections in plates.

- Figure 3.4 (c) shows a member in which a failure other than a transverse one is possible. The holes are staggered, and failure along section ABCD is possible unless the holes are a large distance apart.

The strength of the member along section $A B C D$ can be determined accoridng to the AISC Specification, which offeres offer a very simple method for computing the net width of a tension member along a zigzag section. The method is to take
the gross width of the member, regardless of the line along which failure might occur, subtract the diameter of the holes along the zigzag section being considered, and add for each inclined line the quantity given by the expression $s^{2} / 4 g$. where: $s$ is the longitudinal spacing (or pitch) of any two holes and $g$ is the transverse spacing (or gage) of the same holes. The values of $s$ and $g$ are shown in Fig. 3.4(c).

There may be several paths, any one of which may be critical at a particular joint. Each possibility should be considered, and the one giving the least value should be used.The smallest net width obtained is multiplied by the plate thickness to give the net area.

Holes for bolts and rivets are normally drilled or punched in steel angles at certain standard locations.These locations or gages are dependent on the angleleg widths and on the number of lines of holes. Table 3.1 shows these gages.

TABLE 3.1 Workable Gages for Angles, in Inches

|  | Leg | 8 | 7 | 6 | 5 | 4 | $3 \frac{1}{2}$ | 3 | $2 \frac{1}{2}$ | 2 | $1 \frac{3}{4}$ | $1 \frac{1}{2}$ | $1 \frac{3}{8}$ | $1 \frac{1}{4}$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $g$ | $4 \frac{1}{2}$ | 4 | $3 \frac{1}{2}$ | 3 | $2 \frac{1}{2}$ | 2 | $1 \frac{3}{4}$ | $1 \frac{3}{8}$ | $1 \frac{1}{8}$ | 1 | $\frac{7}{8}$ | $\frac{7}{8}$ | $\frac{3}{4}$ | $\frac{5}{8}$ |
| $\stackrel{+}{\hat{V}_{2}}$ | $g_{1}$ | 3 | $2 \frac{1}{2}$ | $2 \frac{1}{4}$ | 2 |  |  |  |  |  |  |  |  |  |  |
| - | $g_{2}$ | 3 | 3 | $2 \frac{1}{2}$ | $1 \frac{3}{4}$ |  |  |  |  |  |  |  |  |  |  |

$\qquad$

## Example 3-2

Determine the critical net area of the $1 / 2$-in-thick plate shown in Fig. 3.5, using the AISC Specification. The holes are punched for 3/4-in bolts.


FIGURE 3.5

## Solution:

- The critical section could possibly be ABCD, ABCEF, or ABEF.
- Hole diameters to be subtracted are $3 / 4+1 / 8=7 / 8$ in.
- The net areas for each case are as follows:

$$
\begin{gathered}
A B C D=(11 \text { in })\left(\frac{1}{2} \text { in }\right)-2\left(\frac{7}{8} \text { in }\right)\left(\frac{1}{2} \text { in }\right)=4.63 \text { in }^{2} \\
A B C E F=(11 \text { in })\left(\frac{1}{2} \text { in }\right)-3\left(\frac{7}{8} \text { in }\right)\left(\frac{1}{2} \text { in }\right)+\frac{(3 \text { in })^{2}}{4(3 \text { in })}\left(\frac{1}{2} \text { in }\right)=4.56 \text { in }^{2} \leftarrow \\
A B E F=(11 \text { in })\left(\frac{1}{2} \text { in }\right)-2\left(\frac{7}{8} \text { in }\right)\left(\frac{1}{2} \text { in }\right)+\frac{(3 \text { in })^{2}}{4(6 \text { in })}\left(\frac{1}{2} \text { in }\right)=4.81 \text { in }^{2}
\end{gathered}
$$

## Example 3-3

For the two lines of bolt holes shown in Fig. 3.6, determine the pitch that will give a net area DEFG equal to the one along ABC. The problem may also be stated as follows: Determine the pitch that will give a net area equal to the gross area less one bolt hole. The holes are punched for 3/4-in bolts.


FIGURE 3.6

## Solution

The hole diameters to be subtracted are $3 / 4$ in $+1 / 8 \mathrm{in}=7 / 8 \mathrm{in}$.

$$
\begin{gathered}
A B C=6 \text { in }-(1)\left(\frac{7}{8} \mathrm{in}\right)=5.13 \mathrm{in} \\
D E F G=6 \mathrm{in}-2\left(\frac{7}{8} \mathrm{in}\right)+\frac{\mathrm{s}^{2}}{4(2 \mathrm{in})}=4.25 \mathrm{in}+\frac{\mathrm{s}^{2}}{8 \mathrm{in}} \\
A B C=D E F G \\
5.13=4.25 \mathrm{in}+\frac{s^{2}}{8 \mathrm{in}} \\
s=2.65 \mathrm{in}
\end{gathered}
$$

The AISC Specification does not include a method for determining the net widths of sections other than plates and angles. For channels, W sections, S sections, and others, the web and flange thicknesses are not the same. As a result, it is necessary to work with net areas rather than net widths. If the holes are placed in straight lines across such a member, the net area can be obtained by simply subtracting the crosssectional areas of the holes from the gross area of the member. If the holes are staggered, the $\frac{s^{2}}{4 g}$ values must be multiplied by the applicable thickness to change it to an area.

## Example 3-4

Determine the net area of the $W 12 \times 16\left(A_{g}=4.71 \mathrm{in}^{2}\right)$ shown in Fig. 3.7, assuming that the holes are for 1 -in bolts.


## FIGURE 3.7

Solution. Net areas: hole $\phi$ is 1 in $+\frac{1}{8}$ in $=1 \frac{1}{8}$ in

$$
\begin{aligned}
A B D E & =4.71 \mathrm{in}^{2}-2\left(1 \frac{1}{8} \mathrm{in}\right)(0.220 \mathrm{in})=4.21 \mathrm{in}^{2} \\
A B C D E & =4.72 \mathrm{in}^{2}-3\left(1 \frac{1}{8} \mathrm{in}\right)(0.220 \mathrm{in})+(2) \frac{(2 \mathrm{in})^{2}}{4(3 \mathrm{in})}(0.220 \mathrm{in})=4.11 \mathrm{in}^{2} \leftarrow
\end{aligned}
$$

$\qquad$

If the zigzag line goes from a web hole to a flange hole, the thickness changes at the junction of the flange and web.

In Example 3-5, the net area of a channel that has bolt holes staggered in its flanges and web has been computed. The channel is assumed to be flattened out into a single plate, as shown in parts (b) and (c) of Fig. 3.8. The net area along route $A B C D E F$ is determined by taking the area of the channel minus the area of the holes along the route in the flanges and web plus the $\frac{s^{2}}{4 g}$ values for each zigzag line times the appropriate thickness. For line $C D, \frac{s^{2}}{4 g}$ has been multiplied by the thickness of the web. For lines BC and DE (which run from holes in the web to holes in the flange), an approximate procedure has been used in which the $\frac{s^{2}}{4 g}$ values have been multiplied by the average of the web and flange thicknesses.

## Example 3-5

Determine the net area along route ABCDEF for the $C 15 \times 33.9\left(A_{g}=10.00 \mathrm{in}^{2}\right)$ shown in Fig. 3.8.Holes are for $3 / 4$ in bolts.


## Solution

Approximate net A along

$$
\begin{aligned}
A B C D E F=10.00 \mathrm{in}^{2} & -2\left(\frac{7}{8} \mathrm{in}\right)(0.650 \mathrm{in}) \\
& -2\left(\frac{7}{8} \mathrm{in}\right)(0.400 \mathrm{in}) \\
& +\frac{(3 \mathrm{in})^{2}}{4(9 \mathrm{in})}(0.400 \mathrm{in}) \\
& +(2) \frac{(3 \mathrm{in})^{2}}{(4)(4.60 \mathrm{in})}\left(\frac{0.650 \mathrm{in}+0.400 \mathrm{in}}{2}\right) \\
= & 8.78 \mathrm{in}^{2}
\end{aligned}
$$

$\qquad$

## EFFECTIVE NET AREAS

If the forces are not transferred uniformly across a member cross section, there will be a transition region of uneven stress running from the connection out long the member for some distance. This is the situation shown in Fig. 3.9(a), where a single angle tension member is connected by one leg only. At the connection more of the load is carried by the connected leg, and it takes the transition distance shown in part (b) of the figure for the stress to spread uniformly across the whole angle.

- In the transition region the stress in the connected part of the member may very well exceed and go into the strain-hardening range.
- In the transition region, the shear transfer has "lagged" and the phenomenon is referred to as shear lag.


FIGURE 3.9 Shear lag.

In such a situation, the flow of tensile stress between the full member cross section and the smaller connected cross section is not 100 percent effective.As a result, the AISC Specification states that the effective net area, $A_{e}$ of such a member is to be determined by multiplying an area $A$ (which is the net area or the gross area or the directly connected area) by a reduction factor $U$. The use of a factor such as $U$ accounts for the nonuniform stress distribution, in a simple manner.

$$
A_{e}=A_{n} U
$$

The value of the reduction coefficient, $U$, is affected by the cross section of the member and by the length of its connection.

Investigators have found that one measure of the effectiveness of a member such as an angle connected by one leg is the:

1. The distance $\overline{\boldsymbol{x}}$ measured from the plane of the connection to the centroid of the area of the whole section.
2. The length of its connection, $L$.

The effect of these two parameters, $\bar{x}$ and $L$, is expressed empirically with the reduction factor

$$
U=1-\frac{\bar{x}}{L}
$$



FIGURE 3-10

## Bolted Members

Should a tension load be transmitted by bolts, the gross area is reduced to the net area $A_{n}$ of the member, and $U$ is computed as follows:

$$
U=1-\frac{\bar{x}}{L}
$$

L is computed as follows

| One line of bolts | L is the distance between the first and last bolts in <br> the line. |
| :--- | :--- |
| Two or more lines of bolts | L is the length of the line with the maximum <br> number of bolts. |
| Blots be staggered | L is the out-to-out dimension between the extreme <br> bolts in a line. |

Table 3.2 provides a detailed list of shear lag or $U$ factors for different situations. [This table is a copy of Table D3.1 of the AISC Specification]

TABLE 3.2 Shear Lag Factors for Connections to Tension Members

| Case | Description of Element | Shear Lag Factor, $\boldsymbol{U}$ |  |
| :--- | :--- | :---: | :---: |
| 1 | All tension members where the tension <br> load is transmitted directly to each of the <br> cross-sectional elements by fasteners or <br> welds (except as in Cases 4,5 and 6). | $U=1.0$ |  |

$l=$ length of connection, in. (mm); $w=$ plate width, in. $(\mathrm{mm}) ; \bar{x}=$ eccentricity of connection, in. (mm); $B=$ overall width of rectangular HSS member, measured $90^{\circ}$ to the plane of the connection, in. (mm); $H=$ overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)
$\qquad$

In order to calculate U for a W section connected by its flanges only, we will assume that the section is split into two structural tees. Then the value of used will be the distance from the outside edge of the flange to the c.g. of the structural tee, as shown in parts (a) and (b) of Fig. 3.11.


FIGURE 3.11 Values of $\bar{x}$ for different shapes.

## Example 3-6

Determine the effective area of a $W 10 \times 45$ with two lines of $3 / 4$-in diameter bolts in each flange. There are assumed to be at least three bolts in each line 4-in on center, and the bolts are not staggered with respect to each other.

## Solution:

$$
A_{n}=13.3-4\left(\frac{3}{4}+\frac{1}{8}\right)(0.62)=11.13
$$

Referring to tables in Manual for ond-half of a $W 10 \times 45$ (or, that is a $W T 5 \times$ 22.5), we find that

$$
\bar{x}=0.907 \mathrm{in}
$$

Length of connection $L=8$ in
From Table 3.2 (case 2)

$$
U=1-\frac{\bar{x}}{L}=1-\frac{0.907}{8}=0.89
$$

But

$$
b_{f}=8.02>\frac{2}{3} d=\left(\frac{2}{3}\right) 10.1=6.73 \mathrm{in}
$$

$\therefore U$ from Table 3.2 (case 7) is 0.9

$$
A_{e}=U A_{n}=(0.9)(11.13)=10.02 \mathrm{in}^{2}
$$

## Example 3-7

Determine the effective area for $L 6 \times 6 \times 3 / 8$ that is connected at its ends with one line of four $7 / 8$-in-diameter bolts in standard holes 3 in on center in one leg of the angle. Knowing that $\left(A_{g}=4.38 \mathrm{in}^{2}, \bar{y}=\bar{x}=1.62 \mathrm{in}\right.$. $)$

## Solution:

$$
A_{n}=4.38-(1)\left(\frac{7}{8}+\frac{1}{8}\right)\left(\frac{3}{8}\right)=4.00 \operatorname{in}^{2}
$$

Length of connection, $L=9 \mathrm{in}$.

$$
U=1-\frac{\bar{x}}{L}=1-\frac{1.62}{9}=0.82
$$

From Table 3.2, case 8 , for 4 or more fastenrs in the direction of loading, $U=$ 0.80 .

Use calculated $U=0.82$

$$
A_{e}=U A_{n}=(0.82)(4.00)=3.28 \mathrm{in}^{2}
$$

## Welded Members

When tension loads are transferred by welds, the rules from AISC (Table 3.2 in this text) that are to be used to determine values for A and $\mathrm{U}\left(A_{e}\right.$ as for bolted connections $=A U$ ) are as follows:

1. Should the load be transmitted only by longitudinal welds to other than a plate member, or by longitudinal welds in combination with transverse welds, $A$ is to equal the gross area of the member $A_{g}$ (Table 3.2, Case 2).
2. Should a tension load be transmitted only by transverse welds, A is to equal the area of the directly connected elements and $U$ is to equal 1.0 (Table 3.2, Case 3).
3. For flat plates or bars connected by longitudinal fillet welds, use the values of $U$ listed in Table 3.2, Case 4.


Length, $\ell$
$\underline{U}$

$$
\begin{aligned}
& A_{g}=w t \\
& A_{e}=U A_{g}
\end{aligned}
$$

$\ell>2 w$
$2 w>\ell>1.5 w$
1.0
0.87
$1.5 w>\ell>w$
0.75

FIGURE 3.12 a

$\overline{\mathrm{x}}=$
distance from the centroid of the shape to the
plane of the connection, in.
$\ell=$ weld length, in.

## FIGURE 3.12 b

## Example 3-8

The $1 \times 6$ in plate shown in Fig. 3.13 is connected to a $1 \times 10$ in plate with longitudinal fillet welds to transfer a tensile load. Determine the effecitve area.


FIGURE 3.13

## Solution:

$$
\begin{gathered}
(1.5 w=1.5 \times 6=9 \text { in } .)>(L=8 \text { in. })>(w=6) \\
\therefore U=0.75 \text { from Table } 3.2 \text { case } 4 \\
A_{e}=U A_{n}=(0.75)(6.0)=4.5 \mathrm{in}^{2}
\end{gathered}
$$

Sometimes an angle has one of its legs connected with both longitudinal and transverse welds, but no connections are made to the other leg.To determine U from Table 3.2 for such a case is rather puzzling. sometimes Case 2 of Table 3.2 (that is, $U=1-\frac{\bar{x}}{L}$ ) is be used for this situation.This is done in Example 3-9.

## Example 3-9

Compute the net area for the angle shown in the Figure below. It is welded on the end (transverse) and sides (longitudinal) of the 8-in leg only.


## Solution:

Nominal or available tensile strength of the angle

$$
\begin{gathered}
U=1-\frac{\bar{x}}{L}=1-\frac{1.56}{6}=0.74 \\
A_{e}=U A_{g}=(0.74)(9.99)=7.39 \mathrm{in}^{2}
\end{gathered}
$$

Example Determine $A_{e}$ for the $1 \times 5 \mathrm{in}$. plate shown below if (a) $l=7$ in., (b) $l=8.5 \mathrm{in}$., (c) $l=11 \mathrm{in}$.


Solution. $A_{e}=U A_{g}$

$$
\begin{aligned}
A_{g} & =1 \mathrm{in} . \times 5 \mathrm{in} .=5 \mathrm{in} .^{2} \\
w & =5 \mathrm{in} ., 1.5 w=7.5 \mathrm{in} ., 2 w=10 \mathrm{in} .
\end{aligned}
$$

a) Since $1.5 w>l>w$ in this case, then $U=0.75$ and $A_{e}=0.75 \times 5=3.75 \mathrm{in}^{2}{ }^{2}$
b) Since $2 w>l>1.5 w$ in this case, then $U=0.87$ and $A_{e}=0.87 \times 5=4.35 \mathrm{in}^{2}$
c) Since $l>2 w$, then $U=1.0$ and $A_{e}=5 \mathrm{in} .{ }^{2}$

## Example Determine $A_{e}$ for the WT $5 \times 15$ shown below.



Solution.

$$
\begin{aligned}
U & =1-\frac{\bar{x}}{l} \\
\bar{x} & =1.1 \mathrm{in} . \text { (see AISCM Properties Section) } \\
l & =6 \mathrm{in} . \\
U & =1-\frac{1.1}{6}=0.82 \\
A_{e} & =U A_{g}=0.82 \times 4.42=3.61 \mathrm{in.}^{2}
\end{aligned}
$$

## CHAPTER4

## Design of Tension Members

## SELECTION OF SECTIONS

The determination of the net and effective areas of various tension members was presented in Chapter 3. In this chapter, the selection of members to support given tension loads is described.

The choice of member type is often affected by the type of connections used for the structure.

- Tension members consisting of angles, channels, and W or S sections will probably be used when the connections are made with bolts.
- Plates, channels, and structural tees might be used for welded structures.
- Specifications usually recommend that slenderness ratios be kept below 300. used.
- The recommended maximum slenderness ratio of 300 is not applicable to tension rods. Maximum L/r values for rods are left to the designer's judgment.
- The AASHTO Specifications provide mandatory maximum slenderness ratios of 200 for main tension members and 240 for secondary members. (A main member is defined by the AASHTO as one in which stresses result from dead and/or live loads, while secondary members are those used to brace structures or to reduce the unbraced length of other membersmain or secondary.). The AASHTO also requires that the maximum slenderness ratio permitted for members subjected to stress reversal be 140.
$\qquad$

Accoridng to the LRFD equations, the design strength of a tension member is the least of , $\phi_{t} F_{y} A_{g}, \phi_{t} F_{u} A_{e}$. In addition, the slenderness ratio should, preferably, not exceed 300 .
a. To satisfy the first of these expressions, the minimum gross area must be at least equal to

$$
\min A_{g}=\frac{P_{u}}{\phi_{t} F_{y}}
$$

b. To satisfy the second expression, the minimum value of $A_{e}$ must be at least

$$
\min A_{e}=\frac{P_{u}}{\phi_{t} F_{u}}
$$

And since $A_{e}=U A_{n}$ for a bolted member, the minimum value of $A_{n}$ is

$$
\min A_{n}=\frac{\min A_{e}}{U}=\frac{P_{u}}{\phi_{t} F_{u} U}
$$

Then the minimum $A_{g}$ is

$$
\begin{aligned}
& =\min A_{n}+\text { estimated area of holes } \\
& =\frac{P_{u}}{\phi_{t} F_{u} U}+\text { estimated ared of holes }
\end{aligned}
$$

The designer can substitute into above Equations, taking the larger value of so obtained for an initial size estimate. It is, however, well to notice that the maximum preferable slenderness ratio $\mathrm{L} / \mathrm{r}$ is 300

$$
\min r=\frac{L}{300}
$$

Usually, for the examples, $D$ and $L$ loads are specified so that we will not have to go through all of the load combination expressions. For such problems, then, we will need only to use the following load combinations:

$$
\begin{gathered}
P_{u}=1.4 D \\
P_{u}=1.2 D+1.6 L
\end{gathered}
$$

$\qquad$

## Example 4-1

Select a 30 -ft-long W12 steel to support a tensile service dead load $P_{D}=130 k$ and a tensile service live load $P_{L}=110 k$. As shown in Fig. 4.1, the member is to have two lines of bolts in each flange for $7 / 8$-in bolts (at least three in a line 4 in on center). $\left[F_{y}=50 \mathrm{kis}, F_{u}=65 \mathrm{ksi}\right]$.


FIGURE 4.1 Cross section of member for Example 4-1.

## Solution

(a) considering load the necessary load combinaitons

$$
\begin{gathered}
P_{u}=1.4 D=1.4(130 k)=182 k \\
P_{u}=1.2 D+1.6 L=(1.2)(130 k)+(1.6)(110 k)=332 k
\end{gathered}
$$

(b) computing the mininum $A_{g}$ required, using LRFD equations

1. $\min A_{g}=\frac{P_{u}}{\phi_{t} F_{y}}=\frac{332 \mathrm{k}}{(0.9)(50 \mathrm{ksi})}=7.38 \mathrm{in}^{2}$
2. $\min A_{g}=\frac{P_{u}}{\phi_{t} F_{u} U}+$ estimated area of holes

Assume that $\mathrm{U}=0.85$ from Table 3.2, Case 7, and assume that flange thickness is about 0.380 in after looking at W12 sections in the LRFD Manual which have
areas of $7.38 \mathrm{in}^{2}$ or more. $\mathrm{U}=0.85$ was assumed since $b_{f}$ appears to be less than $2 / 3 \mathrm{~d}$.
$\min A_{g}=\frac{332 k}{(0.75)(65 k s i)(0.85)}+(4)\left(\frac{7}{8}\right.$ in $\left.+\frac{1}{8} \mathrm{in}\right)(0.381 \mathrm{in})=9.53 \mathrm{in}^{2} \leftarrow$
(c) Preferable miniumum $r$
$\min r=\frac{L}{300}=\frac{(12 \mathrm{in} / \mathrm{ft})(30 \mathrm{ft})}{300}=1.2 \mathrm{in}$
Try $W 12 \times 35\left(A_{g}=10.3 \mathrm{in}^{2}, d=12.5 \mathrm{in}, b_{f}=6.56 \mathrm{in}\right.$.

$$
t_{f}=0.520 \mathrm{in}, r_{\min }=r_{y}=1.54 \mathrm{in}
$$

## Checking

(a) Gross section yielding

$$
\begin{gathered}
P_{u}<\phi_{t} P_{n} \\
P_{n}=F_{y} A_{g}=(50 \mathrm{ksi})\left(10.3 \mathrm{in}^{2}\right)=515 k \\
\phi_{t} \stackrel{P_{n}}{ }=(0.9)(515 \mathrm{k})=463.5 \mathrm{k}>332 k[\therefore \text { OK }]
\end{gathered}
$$

(b) Tensile rupture strength

From Table 3.2, case 2
$\bar{x}$ for half of $W 12 \times 35$ or, that is, a $W T 6 \times 17.5=1.30$ in

$$
\begin{gathered}
L=(2)(4 \mathrm{in})=8 \mathrm{in} \\
U=\left(1-\frac{\bar{x}}{L}\right)=\left(1-\frac{1.3}{8}\right)=0.84
\end{gathered}
$$

From table 3.2, case 7
$U=0.85$, since $b_{f}=6.56<\frac{2}{3} d=\left(\frac{2}{3}\right)(12.50 \mathrm{in})=8.33 \mathrm{in}$

$$
\begin{gathered}
A_{n}=10.3 \mathrm{in}^{2}-(4)\left(\frac{7}{8} \mathrm{in}+\frac{1}{8} \mathrm{in}\right)(0.520 \mathrm{in})=8.22 \mathrm{in}^{2} \\
A_{e}=(0.85)\left(8.22 \mathrm{in}^{2}\right)=6.99 \mathrm{in}^{2}
\end{gathered}
$$

$$
\begin{gathered}
P_{u}<\phi_{t} P_{n} \\
P_{n}=F_{u} A_{e}=(65 \mathrm{ksi})\left(6.99 \mathrm{in}^{2}\right)=454.2 \mathrm{k}
\end{gathered}
$$

LRFD with $\phi_{t}=0.75$

$$
\phi_{t} P_{n}=(0.75)(454.2 k)=340.7 k>332 k[\because O K]
$$

(c) Slenderness ratio

$$
\frac{L}{r_{y}}=\frac{(12 \mathrm{in} / f t)(30 \mathrm{ft})}{1.54}=234<300[\because \mathrm{OK}]
$$

## $\therefore$ OK to use W12 $\times 35$

## Example 4-2

Design a 9 -ft single-angle tension member to support a dead tensile working load of 30 k and a live tensile working load of 40 k . The member is to be connected to one leg only with $7 / 8$-in bolts (at least four in a line 3 in on center). Assume that only one bolt is to be located at any one cross section. Use $\left.F_{y}=36 \mathrm{kis}, F_{u} 58 \mathrm{ksi}\right]$.
(a) considering load the necessary load combinaitons

$$
\begin{gathered}
P_{u}=1.4 D=1.4(30 k)=42 k \\
P_{u}=1.2 D+1.6 L=(1.2)(30 k)+(1.6)(40 k)=100 k
\end{gathered}
$$

(b) computing the mininum $A_{g}$ required, using LRFD equations

1. $\min A_{g}=\frac{P_{u}}{\phi_{t} F_{y}}=\frac{100 \mathrm{k}}{(0.9)(36 \mathrm{ksi})}=3.09 \mathrm{in}^{2}$
2. $\min A_{g}=\frac{P_{u}}{\phi_{t} F_{u} U}+$ estimated aread of holes

Assume that $U=0.8$ [case 8, Table 3.2]

$$
\min A_{g}=\frac{100}{0.75(58)(0.8)}+\left(\frac{7}{8}+\frac{1}{8}\right) \mathrm{t}
$$

$\Rightarrow \min A_{g}=2.87+\mathrm{t}$
(c) miniumum $r$
$\min r=\frac{L}{300}=\frac{(12 \mathrm{in} / \mathrm{ft})(9 \mathrm{ft})}{300}=0.36 \mathrm{in}$

| Angle <br> $(\mathrm{t})$ | thickness | $\min A_{g}=2.87+\mathrm{t}$ | Lightest angle avaibale |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  | 3.18 | section | Area (in ${ }^{2}$ ) | least $r(\mathrm{in})$ |  |
| $3 / 8$ | 3.25 | $6 \times 6 \times \frac{5}{16}$ | 3.67 | 1.20 |  |
| $7 / 16$ | 3.30 | $6 \times 3 \frac{1}{2} \times \frac{3}{8}$ | 3.42 | 0.763 |  |
|  |  | $4 \times 4 \times \frac{7}{16}$ | 3.31 | 0.777 |  |
| $1 / 2$ | 3.37 | $5 \times 3 \times \frac{7}{16}$ | 3.31 | 0.65 |  |
| $5 / 8$ | 3.50 | $4 \times 3 \frac{1}{2} \times \frac{1}{2}$ | 3.50 | 0.716 |  |

Try $4 \times 4 \times \frac{7}{16}$

## Checking

(a) Gross section yielding

$$
P_{u}<\phi_{t} P_{n}
$$

$$
\begin{aligned}
P_{n} & =F_{y} A_{g}=(36 \mathrm{ksi})\left(3.31 \mathrm{in}^{2}\right)=119.1 k \\
\phi_{t} P_{n} & =(0.9)(119.1 k)=107.2 k>100 k[\therefore O K]
\end{aligned}
$$

(b) Tensile rupture strength
$\bar{x}$ for $4 \times 4 \times \frac{7}{16}=1.15$ in

$$
\begin{gathered}
L=(3)(3 \mathrm{in})=9 \mathrm{in} \\
U=\left(1-\frac{\bar{x}}{L}\right)=\left(1-\frac{1.15}{8}\right)=0.86
\end{gathered}
$$

From table 3.2, case $8, U=0.80$
Use $U=0.86$

$$
A_{e}=U A_{n}=(0.86)\left(2.86 \mathrm{in}^{2}\right)=2.46 \mathrm{in}^{2}
$$

$$
\begin{gathered}
P_{u}<\phi_{t} P_{n} \\
P_{n}=F_{u} A_{e}=(58 \mathrm{ksi})\left(2.46 \mathrm{in}^{2}\right)=142.7 \mathrm{k}
\end{gathered}
$$

LRFD with $\phi_{t}=0.75$

$$
\phi_{t} P_{n}=(0.75)(142.7 k)=107 k>100 k[\therefore O K]
$$

(c) Slenderness ratio

$$
\frac{L}{r_{z}}=\frac{(12 \mathrm{in} / \mathrm{ft})(9 \mathrm{ft})}{0.777-1 O}=139<300[\therefore O K]
$$

$\therefore$ OK to use $4 \times 4 \times \frac{7}{16}$

## BUILT-UP TENSION MEMBERS

Sections D4 and J3.5 of the AISC Specification provide a set of definite rules describing how the different parts of built-up tension members are to be connected together.

## RODS AND BARS

When rods and bars are used as tension members, they may be simply welded at their ends, or they may be threaded and held in place with nuts. The AISC nominal tensile design stress for threaded rods, $F_{n t}=0.75 F_{u}$.

The gross area of the rod area, $A_{D}$, required for a particular tensile load can be calculated as:

$$
A_{D} \geq \frac{P_{u}}{\phi 0.75 F_{u}}
$$

## Example 4-3

Using the AISC Specification, select a standard threaded steel rod to support a tensile working dead load of 10 k and a tensile working live load of $20 \mathrm{k} .\left[F_{u}=\right.$ 58 ksi].

Solution

$$
\begin{gathered}
P_{u}=1.4 D=1.4(10 k)=14 k \\
P_{u}=1.2 D+1.6 L=(1.2)(10 k)+(1.6)(20 k)=44 k \\
A_{D} \geq \frac{P_{u}}{\phi 0.75 F_{u}}=\frac{44 k}{(0.75)(0.75)(58 \mathrm{ksi})}=1.35 \mathrm{in}^{2}
\end{gathered}
$$

From AISC Tables, try $1 \frac{3}{8}$ in steel rod [has $A_{D}=1.49 \mathrm{in}^{2}$ ]

## Checking

Gross section yielding

$$
P_{u}<\phi_{t} R_{n}
$$

$$
\begin{gathered}
R_{n}=0.75 F_{u} A_{D}=(0.75)(58 k s i)\left(1.49 i n^{2}\right)=64.8 k \\
\phi_{t} R_{n}=(0.9)(64.8 k)=48.6 k>44 k[\therefore \text { OK }]
\end{gathered}
$$

## Summary

## Design of Tension Members

$$
\begin{array}{ll}
\text { 1. } \min A_{g}=\frac{P_{u}}{\phi_{t} F_{y}} & \phi_{t}=0.9 \\
\text { 2. } \min A_{g}=\frac{P_{u}}{\phi_{t} F_{u} U}+\text { estimated area of holes } & \phi_{t}=0.75 \\
\text { 3. } \min r=\frac{L}{300} &
\end{array}
$$

## Checking

(a) Gross section yielding

$$
\begin{gathered}
P_{u}<\phi_{t} P_{n} \\
P_{n}=F_{y} A_{g} \text { and } \phi_{t}=0.9
\end{gathered}
$$

(b) Tensile rupture strength

$$
\begin{gathered}
P_{u}<\phi_{t} P_{n} \\
P_{n}=F_{u} A_{e} \text { and } \phi_{t}=0.75
\end{gathered}
$$

(c) Slenderness ratio

$$
\frac{L}{r}<300
$$

## CHAPTER5

## Introduction to Axially Loaded Compression Members

## GENERAL

There are three general modes by which axially loaded columns can fail. These are flexural buckling, local buckling, and torsional buckling. These modes of buckling are briefly defined as follows:

1. Flexural buckling (also called Euler buckling): Members are subject to flexure, or bending, when they become unstable.
2. Local buckling occurs when some part or parts of the cross section of a column are so thin that they buckle locally in compression before the other modes of buckling can occur.
3. Flexural torsional buckling: Columns fail by twisting (torsion) or by a combination of torsional and flexural buckling.

## SECTIONS USED FOR COLUMNS



FIGURE 5.1 Types of compression members.

## THE EULER FORMULA

For a column to buckle elastically, it will have to be long and slender. Its buckling load P can be computed with the Euler formula that follows:

$$
\frac{P}{A}=\frac{\pi^{2} E}{(L / r)^{2}}=F_{e}
$$

Example 5-1 illustrates the application of the Euler formula to a steel column. If the value obtained for a particular column exceeds the steel's proportional limit, the elastic Euler formula is not applicable.

## Example 5-1

(a) A W10 $\times 22$ is used as a 15 - ft long pin-connected column. Using the Euler expression, determine the column's critical or buckling load. Assume that the steel has a proportional limit of 36 ksi .
(b) Repeat part (a) if the length is changed to 8 ft .

## Solution

(a) Using a $15-\mathrm{ft}$ long $\mathrm{W} 10 \times 22\left(A=6.49 \mathrm{in}^{2}, r_{x}=4.27 \mathrm{in}, r_{y}=1.33 \mathrm{in}\right)$

Minimum $r=r_{y}=1.33$ in

$$
\frac{L}{r}=\frac{(12 \mathrm{in} / \mathrm{ft})(15 \mathrm{ft})}{1.33 \mathrm{in}}=135.34
$$

Elastic or buckling stress $F_{e}=\frac{\left(\pi^{2}\right)\left(29 \times 10^{3} \mathrm{ksi}\right)}{(135.34)^{2}}$

$$
=15.63 \mathrm{ksi}<\text { the proportional limit of } 36 \mathrm{ksi}
$$

OK column is in elastic range
Elastic or buckling load $=(15.63 \mathrm{ksi})\left(6.49 \mathrm{in}^{2}\right)=101.4 \mathrm{k}$
(b) Using an 8 -ft long W10 $\times 22$,

$$
\frac{L}{r}=\frac{(12 \mathrm{in} / \mathrm{ft})(8 \mathrm{ft})}{1.33 \mathrm{in}}=72.18
$$

Elastic or buckling stress $F_{e}=\frac{\left(\pi^{2}\right)\left(29 \times 10^{3} \mathrm{ksi}\right)}{(72.18)^{2}}=54.94 \mathrm{ksi}>36 \mathrm{ksi}$
$\therefore$ column is in inelastic range and Euler equation is not applicable.

## END RESTRAINT AND EFFECTIVE LENGTHS OF COLUMNS

- End restraint and its effect on the load-carrying capacity of columns is a very important subject indeed.
- In steel specifications, the effective length of a column is referred to as $K L$, where $K$ is the effective length factor. $K$ is the number that must be multiplied by the length of the column to find its effective length.

Columns with different end conditions have entirely different effective lengths. For this initial discussion, it is assumed that no sidesway or joint translation is possible between the member ends. Sidesway or joint translation means that one or both ends of a column can move laterally with respect to each other. Should a column be connected with frictionless hinges, as shown in Fig. 5.2(a), its effective length would be equal to the actual length of the column and K would equal 1.0. If there were such a thing as a perfectly fixed-ended column, its points of inflection (or points of zero moment) would occur at its one-fourth points and its effective length would equal L / 2, as shown in Fig. 5.2(b). As a result, its $K$ value would equal 0.50 .


FIGURE 5.2 Effective length ( $K L$ ) for columns in braced frames (sidesway prevented).


TABLE 5.1
Approximate Values of Effective Length Factor, $K$


## LONG, SHORT, AND INTERMEDIATE COLUMNS

A column subject to an axial compression load will shorten in the direction of the load. If the load is increased until the column buckles, the shortening will stop and the column will suddenly bend or deform laterally and may at the same time twist in a direction perpendicular to its longitudinal axis.

Columns are sometimes classed as being long, short, or intermediate.A brief discussion of each of these classifications is presented in the paragraphs to follow.

| 1. Long Columns | The Euler formula predicts very well the strength of long <br> columns where the axial buckling stress remains below <br> the proportional limit. Such columns will buckle <br> elastically. |
| :--- | :--- |
| 2. Short Columns | For very short columns, the failure stress will equal the <br> yield stress and no buckling will occur. (For a column to <br> fall into this class, it would have to be so short as to have <br> no practical application.Thus, no further reference is <br> made to them here.) |
| 3. Intermediate | For intermediate columns, some of the fibers will reach <br> the yield stress and some will not.The members will fail <br> by both yielding and buckling, and their behavior is said to <br> be inelastic. Most columns fall into this range. |

## COLUMN FORMULAS

The AISC Specification provides one equation (the Euler equation) for long columns with elastic buckling and an empirical parabolic equation for short and intermediate columns.With these equations, a flexural buckling stress, $F_{c r}$, is determined for a compression member. Once this stress is computed for a particular member, it is multiplied by the cross-sectional area of the member to obtain its nominal strength $P_{n}$. The LRFD design strength of a column may be determined as follows:

$$
P_{n}=F_{c r} A_{g}
$$

LRFD compression strength $\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}$
[ $\phi_{c}=0.90$ ]
The following expressions shsow how $F_{c r}$, the flexureal bucking stressof a column, may be determined
a) If $\frac{K L}{R} \leq 4.71 \sqrt{\frac{E}{F_{y}}} \quad \Rightarrow F_{c r}=\left[0.658^{\frac{F_{y}}{F_{e}}}\right] F_{y}$
b) If $\frac{K L}{R}>4.71 \sqrt{\frac{E}{F_{y}}} \quad \Rightarrow F_{c r}=0.877 F_{e}$
where $F_{e}=\frac{\pi^{2} E}{\left(\frac{K L}{r}\right)^{2}}$


## Example 5-2

a. Using the column critical stress values in Table 4-22 of the Manual, determine the LRFD design strength $\phi_{c} P_{n}$ for the column shown below, if a 50-ksi steel is used.
b. Repeat the problem, using Table 4-1 of the Manual.
c. Calculate $\phi_{c} P_{n}$ using AISC equations.


## Solution

From thable $5.1 K=0.8$

$$
\begin{gathered}
\left(\frac{K L}{r}\right)_{x}=\frac{(0.8)(12 \times 15)}{5.31}=27.11 \\
\left(\frac{K L}{r}\right)_{y}=\frac{(0.8)(12 \times 15)}{3.04}=47.37 \leftarrow \text { controls }
\end{gathered}
$$

From Table 4-22 (MANUAL), by stright line interpolation, $\phi_{c} F_{c r}=38.19 \mathrm{ksi}$ For LRFD, $\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(38.19)(21.1)=805.8 k$
(b) from Table 4-1 (MANUAL), for $K L=(0.8)(15)=12 f t, \Rightarrow \phi_{c} P_{n}=807 k$

(c) Elastic critical bucking stress

$$
\begin{aligned}
& \left(\frac{K L}{r}\right)_{y}=47.37[\text { from part (a)] } \\
& \qquad F_{e}=\frac{\pi^{2} E}{\left(\frac{K L}{r}\right)^{2}}=\frac{\pi^{2}(29000)}{(47.37)^{2}}=127.55 \mathrm{ksi}
\end{aligned}
$$

$$
\begin{aligned}
& 4.71 \sqrt{\frac{E}{F_{y}}}=\sqrt{\frac{29000 k s i}{50 k s i}}=113.3>\left(\frac{K L}{r}\right)_{y}=47.37 \\
\Rightarrow & F_{c r}=\left[0.658^{\frac{F_{y}}{F_{e}}}\right] F_{y}=\left[0.658^{\left(\frac{50}{127.55}\right)}\right] 50=42.43 \mathrm{ksi}
\end{aligned}
$$

LRFD method, $\phi_{c} F_{c r}=(0.90)(42.43)=38.19 k s i$ $\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(38.19)(21.1)=805.8 k$

## Example 5-3

An HSS $16 \times 16 \times \frac{1}{2}$ with $F_{y}=46 \mathrm{ksi}$ is used for an 18 -ft-long column with simple end supports.
(a) Determine $\phi_{c} P_{n}$ with the appropriate AISC equations.
(b) Repeat part (a), using Table 4-4 in the AISC Manual.

## Solutuion

(a) Using an HSS

$$
\begin{aligned}
& 16 \times 16 \times \frac{1}{2}\left(A=28.3 \mathrm{in}^{2}, t_{\mathrm{wall}}=0.465 \mathrm{in}, r_{x}=r_{y}=6.31 \mathrm{in}\right) \\
& K=1.0 \\
& \left(\frac{K L}{r}\right)_{x}=\left(\frac{K L}{r}\right)_{y}=\frac{(1.0)(12 \times 18) \mathrm{in}}{6.31 \mathrm{in}}=34.23 \\
& \quad<4.71 \sqrt{\frac{E}{F_{y}}}=4.71 \sqrt{\frac{29,000}{46}}=118.26
\end{aligned}
$$

## $\therefore$ Use AISC Equation for $F_{c r}$

$$
\begin{aligned}
F_{e} & =\frac{\pi^{2} E}{\left(\frac{K L}{r}\right)^{2}}=\frac{\left(\pi^{2}\right)(29,000)}{(34.23)^{2}}=244.28 \mathrm{ksi} \\
F_{c r} & =\left[0.658^{\frac{F_{r}}{r_{c}}}\right] F_{y}=\left[0.658^{\frac{46}{2428}}\right] 46 \\
& =42.51 \mathrm{ksi}
\end{aligned}
$$

$$
\begin{gathered}
\text { LRFD } \phi_{c}=0.90 \\
\phi_{c} F_{c r}=(0.90)(42.51)=38.26 \mathrm{ksi} \\
\phi_{c} P_{n}=\phi_{c} F_{c r} A=(38.26)(28.3)=1082 \mathrm{k}
\end{gathered}
$$

(b) from Table 4-4 (MANUAL), for $K L=(1.0)(18)=18 \mathrm{ft}, \Rightarrow \phi_{c} P_{n}=1080 k$

## Example 5-4

Determine the LRFD design strength $\phi_{c} P_{n}$ for the 50 ksi axially loaded $W 14 \times 90$ shown below.

Because of its considerable length, this column is braced perpendicular to its weak, or $y$, axis at the points shown in the figure. These connections are assumed to permit rotation of the member in a plane parallel to the plane of the flanges. At the same time, however, they are assumed to prevent translation or sidesway and twisting of the cross section about a longitudinal axis passing through the shear center of the cross section.


## Solution

Using W14 $\times 90\left(A=26.5 \mathrm{in}^{2}, r_{x}=6.14 \mathrm{in}, r_{y}=3.70 \mathrm{in}\right)$
Determining effective lengths

$$
\begin{aligned}
& K_{x} L_{x}=(0.80)(32)=25.6 \mathrm{ft} \\
& K_{y} L_{y}=(1.0)(10)=10 \mathrm{ft} \leftarrow \text { governs for } K_{y} L_{y} \\
& K_{y} L_{y}=(0.80)(12)=9.6 \mathrm{ft}
\end{aligned}
$$

Computing slenderness ratios

$$
\begin{aligned}
& \left(\frac{K L}{r}\right)_{x}=\frac{(12)(25.6)}{6.14}=50.03 \leftarrow \\
& \left(\frac{K L}{r}\right)_{y}=\frac{(12)(10)}{3.70}=32.43
\end{aligned}
$$

$\phi_{c} F_{c r}=37.49 \mathrm{ksi} \quad$ from Manual, Table 4-22, $F_{y}=50 \mathrm{ksi}$

$$
\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(37.49)(26.5)=993 \mathrm{k}
$$

## CHAPTER6

## Design of Axially Loaded Compression Members

## INTRODUCTION

The design of columns by formulas involves a trial-and-error process.The LRFD design stress $\phi_{c} F_{c r}$ is not known until a column size is selected, and vice versa.

There are two method for design of compression memebers
1- Trial and error method (Ilustrated in Example 6-1)
2- AISC Tables method (Illustrated in Example 6-2)
Example 6-1 (Tiral and error method)
Using $F_{y}=50 k s i$, select the lightest W14 available for the service column loads $P_{D}=130 \mathrm{k}$ and $P_{L}=210 \mathrm{k}, \mathrm{KL}=10 \mathrm{ft}$.

## Solution

$P_{u}=1.2(130)+1.6(210)=492 k$
Assume $\frac{K L}{r}=50$
From table 4-22, for $F_{y}=50 \mathrm{ksi}$ and $\frac{K L}{r}=50$, we get $\phi_{c} F_{c r}=37.5 \mathrm{ksi}$
Now caclulate required area of section,

$$
\begin{array}{r}
\because P_{u}=\phi_{c} F_{c r} A \Rightarrow A=\frac{P_{u}}{\phi_{c} F_{c r}} \\
A=\frac{492 \mathrm{k}}{37.5 \mathrm{ksi}}=13.12 \mathrm{in}^{2}
\end{array}
$$

Try $W 14 \times 48\left(A=14.1 \mathrm{in}^{2}, r_{x}=5.85 \mathrm{in}, r_{7}=1.91 \mathrm{in}\right)$

Check :

$$
\frac{K L}{r}=\frac{(12)(10)}{1.91}=62.63
$$

From table 4-22, for $F_{y}=50 \mathrm{ksi}$ and $\frac{K L}{r}=62.63$, we get $\phi_{c} F_{c r}=33.75 \mathrm{ksi}$

$$
\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(33.75)(14.1)=476 k<492 \therefore \text { NOT GOOD }
$$

Try next larger section, $W 14 \times 53\left(A=15.6\right.$ in $^{2}, r_{x}=5.89$ in,$\left.r_{y}=1.92 \mathrm{in}\right)$ Check:

$$
\frac{K L}{r}=\frac{(12)(10)}{1.92}=62.5
$$

From table 4-22, for $F_{y}=50 k s i$ and $\frac{k L}{r}=62.5$, we get $\phi_{c} F_{c r}=33.85 k s i$

$$
\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(33.85)(15.6)=528 k>492 \therefore O K
$$

## (1) Table 4-22 (:ontinued) Available Critical Stress for Compression Members

(2)

| $F_{y}=35 \mathrm{ksi}$ |  |  | $F_{y}=36 \mathrm{ksi}$ |  |  | $F_{y}=42 \mathrm{ksi}$ |  |  | $F_{y}=46 \mathrm{ksi}$ |  |  | $F_{v}=50 \mathrm{ksi}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $F_{c r} / \Omega_{c}$ | $\phi_{c} F_{c r}$ |  | $F_{c r} / \Omega_{c}$ | $\phi_{c} F_{c r}$ |  | $F_{c r} / \Omega_{c}$ | $\phi_{c} F_{\text {cr }}$ |  | $F_{c r} / \Omega_{c}$ | $\phi_{c} F_{c r}$ |  | $F_{c r} / \Omega_{c}$ | $\phi_{c} F_{c r}$ |
|  | ksi | ksi | K | ksi | ksi | KI | ksi | ksi | KI | ksi | ksi | $\underline{K I}$ | ksi | ksi |
|  | ASD | LRFD |  | ASD | LRFD |  | ASD | LRFD |  | ASD | LRFD |  | ASD | LRFD |
| 41 | 19.2 | 3.9 | 41 | 19.7 | 9.7 | 41 | 22.7 | 34.1 | 41 | 24.6 | 37.0 | 41 | 26.5 | 39.8 |
| 42 | 19.2 | 28.8 | 42 | 19.6 | 29.5 | 42 | 22.6 | 33.9 | 42 | 24.5 | 36.8 | 42 | 26.3 | 39.5 |
| 43 | 19.1 | 28.7 | 43 | 19.6 | 9.4 | 43 | 2.5 | 33. | 43 | 24.3 | 36 | 43 | 26.2 | 39.3 |
| 44 | 19.0 | 28.5 | 44 | 19.5 | 29.3 | 44 | 22.3 | 33.6 | 44 | 24.2 | 36.3 | 44 | 26.0 | 39.1 |
| 45 | 18.9 | 28.4 | 45 | 19.4 | 29.1 | 45 | 22.2 | 33.4 | 45 | 24.0 | 36.1 | 45 | 25.8 | 38.8 |
| 46 | 18.8 | 28.3 | 46 | 19.3 | 29.0 | 46 | 22. | 33.2 | 46 | 23.9 | 35.9 | 46 | 25.6 | 38.5 |
| 47 | 18.7 | 28.1 | 47 | 19.2 | 28.9 | 47 | 22.0 | 33.0 | 47 | 23.8 | 35.7 | 47 | 25.5 | 38.3 |
| 48 | 18.6 | 28.0 | 48 | 19.1 | 28.7 | 48 | 21.8 | 32.8 | 48 | 23.6 | 35.4 | 48 | 25.3 | 38.0 |
| 49 | 18.5 | 27.9 | 49 | 19.0 | 28.5 | 49 | 21.7 | 32.6 | 49 | 23.4 | 35.1 | 49 | 25.1 | 37.7 |
| 50 | 18.4 | 27.7 | 50 | 18.9 | 28.4 | 50 | 21.6 | 32.4 | 50 | 23.3 | 35.0 | 50 | 24.9 | 37.5 |
| 51 | 18.3 | 27.6 | 51 | 18.8 | 28.3 | 51 | 21.4 | 32.2 | 51 | 23.1 | 34.8 | 51 | 24.8 | 37.2 |
| 52 | 18.3 | 27.4 | 52 | 18.7 | 28.1 | 52 | 21.3 | 32.0 | 52 | 23.0 | 34.5 | 52 | 24.6 | 36.9 |



## Example 6-2 (AISC Tables method)

If you know that $P_{D}=130 k$ and $P_{L}=210 k, K L=10 \mathrm{ft}$, Use the AISC column tables (LRFD method) for the designs to follow.
a) Select the lightest W section available for the loads, steel, $F_{y}=50 \mathrm{ksi}$
b) Select the lightest satisfactory rectangular HSS sections, $F_{y}=46 \mathrm{ksi}$
c) Select the lightest satisfactory square HSS sections, $F_{y}=46 \mathrm{ksi}$
d) Select the lightest satisfactory round HSS section, $F_{y}=42 \mathrm{ksi}$.

## Solution

$P_{u}=1.2(130)+1.6(210)=492 k$
(a)

From Table 4-1, for $K L=10 \mathrm{ft}$ and $F_{y}=50 \mathrm{ksi}$
For $\boldsymbol{W 8} \times 48$ we get $\phi_{c} P_{n}=497 k s i>492[0 \mathrm{~K}]$

(b) From Table 4-3, for $K L=10 \mathrm{ft}$ and $F_{y}=46 \mathrm{ksi}$

For $\boldsymbol{H S S} \mathbf{1 2} \times \mathbf{8 \times 3 / 8}$ we get $\phi_{c} P_{n}=498 \mathrm{ksi}>492[\mathrm{OK}]$

| HSS12 | Table 4-3 (continued) <br> Available Strength in Axial Compression, kips |  |  |  |  |  |  |  |  | $\begin{aligned} & (2) \\ & F_{y}=46 \mathrm{ksi} \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Shape | HSS12×8× |  |  |  |  |  |  |  | HSS12×6 $\times$ |  |  |  |
|  | /8 |  | $5 / 16^{\text {c }}$ |  | $1 / 4^{\text {c }}$ |  | 3/16 ${ }^{\text {c }}$ |  | 5/8 |  | 1/2 |  |
| $t_{\text {design, }}$ in. | 0.349 |  | 0.291 |  | 0.233 |  | 0.174 |  | 0.581 |  | 0.465 |  |
| Wt/ft | 47.8 |  | 40.4 |  | 32.6 |  | 24.8 |  | 67.6 |  | 55.5 |  |
| Desig ${ }^{7}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ | $P_{n} / \Omega_{c}$ | $\phi_{c} P_{n}$ |
|  | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| 0 | 362 | 545 | 296 | 444 | 218 | 327 | 136 | 204 | 515 | 774 | 422 | 634 |
| 6 | 351 | 527 | 288 | 433 | 213 | 320 | 134 | 201 | 484 | 728 | 398 | 598 |
| 7 | 347 | 521 | 286 | 429 | 211 | 317 | 133 | 200 | 474 | 712 | 390 | 586 |
| (4) 8 | 342 | 514 | 283 | 425 | 209 | 314 | 132 | 199 | 462 | 694 | 380 | 571 |
| 19 | 337 | 506 | 279 | 420 | 207 | 311 | 131 | 198 | 449 | 674 | 370 | 556 |
| 2 10 | 331 | 498 | 275 | 414 | 204 | 307 | 130 | 196 | 435 | 653 | 359 | 539 |
| 長 11 | 325 | $488$ | 271 | 408 | 202 | 303 | 129 | 194 | 419 | 630 | $346$ | $521$ |
| 5 11 | 318 | $478$ | 267 | $401$ | 199 | 299 | $128$ | $192$ | $403$ | $606$ | $334$ | 502 |

(c) From Table 4-4, for $K L=10 \mathrm{ft}$ and $F_{y}=46 \mathrm{ksi}$

For HSS $\mathbf{1 0 \times 1 0 \times 3 / 8}$ we get $\phi_{c} P_{n}=511 k s i>492[0 K]$

(d) From Table 4-5, for $K L=10 \mathrm{ft}$ and $F_{y}=46 \mathrm{ksi}$

For HSS 16.000 $\times \mathbf{0 . 3 1 2}$ we get $\phi_{c} P_{n}=528 \mathrm{ksi}>492[\mathrm{OK}]$


## CHAPTER7

## Design of Axially Loaded Compression Members (Continued)

## INTRODUCTION

- In this chapter, the available axial strengths of columns used in unbraced steel frames are considered.
- In this chapter, the available strength of compression members, $\phi P_{n}$, will be determined in building frames calculating $K L$ using the Effective Length Method.


## Effective Length of a column in steel frames:

- The true effective length of a column is a property of the whole structure, of which the column is a part.
- Theoretical mathematical analyses may be used to determine effective lengths, but such procedures are too lengthy and too difficult.
- The most common method for obtaining effective lengths is to employ the charts shown in Fig. 7.1.

(a) Sidesway inhibited (Braced Frame)
(b) Sidesway uninhibited (Moment Frame)

Figure 7.1 alignment charts for effective lengths of columns in continous frames.

When we say sidesway is inhibited, we mean there is something present other than just columns and girders to prevent sidesway or the horizontal translation of the joints.That means we have a definite system of lateral bracing, or we have shear walls, see Figure 7.2. If we say that sidesway is uninhibited, we are saying that resistance to horizontal translation is supplied only by the bending strength and stiffness of the girders and beams of the frame in question, with its continuous joints.


Figure 7.2 Sidesway inhibited.

The rotational restraint at the end of a particular column is proportional to the ratio of the sum of the column stiffnesses to the girder stiffnesses meeting at that joint, or

$$
G=\frac{\sum \frac{E I}{L} \text { for columns }}{\sum \frac{E I}{L} \text { for } \text { girders }}==\frac{\sum \frac{E_{c} I_{c}}{L_{c}}}{\sum \frac{E_{g} I_{g}}{L_{g}}}
$$

where

| $E_{c}$ | Modulus of Elasticity of column |
| :---: | :--- |
| $E_{g}$ | Modulus of Elasticity of girder |
| $I_{c}$ | Moment of Inertia of column |
| $I_{g}$ | Moment of Inertia of girder |
| $L_{c}$ | Length column |
| $L_{g}$ | Legnth of girder |



The recommended values of $G$ factors at the column bases are:

1. For pinned columns, $G$ is theoretically infinite, such as when a column is connected to a footing with a frictionless hinge. Since such a connection is not frictionless, it is recommended that G be made equal to 10.
2. For rigid connections of columns to footings, $G$ theoretically approaches zero, but from a practical standpoint, a value of 1.0 is recommended, because no connections are perfectly rigid.

## Example 7-1

Determine the effective length factor for each of the columns of the frame shown in Fig. 7.3 if the frame is not braced against sidesway.


Solution. Stiffness factors: $E$ is assumed to be $29,000 \mathrm{ksi}$ for all members and is therefore neglected in the equation to calculate $G$.

| Member | Shape | $I$ | $L$ | I/L |
| :---: | :---: | :---: | :---: | :---: |
| Columns | W8 $\times 24$ | 82.7 | 144 | 0.574 |
|  | W8 $\times 24$ | 82.7 | 120 | 0.689 |
|  | W8 $\times 40$ | 146 | 144 | 1.014 |
|  | W8 $\times 40$ | 146 | 120 | 1.217 |
|  | W8 $\times 24$ | 82.7 | 144 | 0.574 |
|  | W8 $\times 24$ | 82.7 | 120 | 0.689 |
| Girders $\left\{\begin{array}{l}E F \\ E H \\ F I\end{array}\right.$ | W18 $\times 50$ | 800 | 240 | 3.333 |
|  | W16 $\times 36$ | 448 | 240 | 1.867 |
|  | W18 $\times 97$ | 1750 | 360 | 4.861 |
|  | W16 $\times 57$ | 758 | 360 | 2.106 |

$G$ factors for each joint:

| Joint | $\Sigma\left(I_{c} / L_{c}\right) / \Sigma\left(I_{g} / L_{g}\right)$ | G |
| :---: | :---: | :---: |
| A | Pinned Column, $G=10$ | 10.0 |
| B | $\frac{0.574+0.689}{3.333}$ | 0.379 |
| C | $\frac{0.689}{1.867}$ | 0.369 |
| D | Pinned Column, $G=10$ | 10.0 |
| $E$ | $\frac{1.014+1.217}{(3.333+4.861)}$ | 0.272 |
| F | $\frac{1.217}{(1.867+2.106)}$ | 0.306 |
| G | Pinned Column, $G=10$ | 10.0 |
| H | $\frac{0.574+0.689}{4.861}$ | 0.260 |
| I | $\frac{0.689}{2.106}$ | 0.327 |

Column $K$ factors from chart [Fig. 7.1 b ]

| Column | $G_{A}$ | $G_{B}$ | $K^{\star}$ |
| :---: | :---: | :---: | :---: |
| $A B$ | 10.0 | 0.379 | 1.76 |
| $B C$ | 0.379 | 0.369 | 1.12 |
| $D E$ | 10.0 | 0.272 | 1.74 |
| $E F$ | 0.272 | 0.306 | 1.10 |
| $G H$ | 10.0 | 0.260 | 1.73 |
| $H I$ | 0.260 | 0.327 | 1.10 |

*It is a little difficult to read the charts to the three decimal places shown by the author. He has used a larger copy of Fig. 7.2 for his work. For all practical design purposes, the $K$ values can be read to two places, which can easily be accomplished with this figure.

## C H A P T ER 8

## Introduction to Beams

## INTRODUCTION

Beams are usually said to be members that support transverse loads.They are probably thought of as being used in horizontal positions and subjected to gravity or vertical loads, but there are frequent exceptions-roof rafters, for example.


## SECTIONS USED AS BEAMS

The W shapes will normally prove to be the most economical beam section, and they have largely replaced channels and $S$ sections for beam usage.


## DETERMINING THE ALLOWABLE BENDING STRESS

Bending stress, $f_{b}$, in a beam is determined by the flexure formula

$$
f_{b}=\frac{M}{S}
$$

Where $M$ is bending moment.

## $S$ is section modulus

Example 8.1 calculate the maxium bending stress, $f_{b}$ due to a 170 ft .k mombet about the strong axis on:
a) $W 12 \times 65$ section
b) $W 14 \times 61$ section

## Solution

a) for a $W 12 \times 65$ section, $S_{x}=87.9 \mathrm{in}^{3}$

$$
f_{b, x}=\frac{170 \mathrm{ft} . \mathrm{k} \times 12 \mathrm{in} . / \mathrm{ft}}{87.9 \mathrm{in}^{3}}=23.2 \mathrm{ksi}
$$

b) for a $W 14 \times 61$ section, $S_{x}=92.1 \mathrm{in}^{3}$

$$
f_{b, x}=\frac{170 \mathrm{ft} . \mathrm{k} \times 12 \mathrm{in} . / \mathrm{ft}}{92.1 \mathrm{in}^{3}}=22.1 \mathrm{ksi}
$$

Example 8.2 Determine the bending stress on a $W 12 \times 79$ subjected to a moment of $80 \mathrm{ft} . \mathrm{k}$ about (a) the strong axis (b) the weak axis.

## Soltuion

for a $W 12 \times 79$ section, $S_{x}=107 \mathrm{in}^{3}, S_{y}=35.8 \mathrm{in}^{3}$
a)

$$
f_{b, x}=\frac{80 \mathrm{ft} . \mathrm{k} \times 12 \mathrm{in} . / f t}{107 \mathrm{in}^{3}}=8.97 \mathrm{ksi}
$$

b)

$$
f_{b, y}=\frac{80 \mathrm{ft} . \mathrm{k} \times 12 \mathrm{in} . / f t}{35.8 \mathrm{in}^{3}}=26.8 \mathrm{ksi}
$$

