Confidence Interval

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Outline

- The sampling variability of a percentage (Proportion).

- The sampling variability of a mean (Quantitative data).

Objectives

At the end of this lecture, students should be able to :

- Calculate confidence interval of a percentage.
- Calculate confidence interval of a mean

Introduction

- **Confidence intervals**, are used to express the statistical uncertainty of an estimate obtained from the data.

- Today's lectures introduce this concept for analysis of categorical data and quantitative measurements.

- Confidence intervals are linked by the concept of sampling variability.

- In the lecture we will illustrate this concept with practical examples.

The sampling variability of a percentage (proportion)

“Qualitative Data”

95% Confidence interval (CI) for a percentage

- When we observe a percentage in a random sample (e.g. 37%), it would be useful to be able to give an interval of possible values within which the true population percentage might lie.

- 95% CI is usually used to show the interval where the true population percentage lies with 95% degree of certainty (or confidence)

95% Confidence interval for a percentage

- A 95% confidence interval for a percentage=

  p ± 1.96 x standard error of p

Usually written

  p ± 1.96 x SE(p).

SE(P) = $\sqrt{\dfrac{p \times (100 - p)}{n}}$

So the 95% CI (p)= p - 1.96 x$\sqrt{\dfrac{p \times (100 - p)}{n}}$ to p + 1.96 x$\sqrt{\dfrac{p \times (100 - p)}{n}}$

p: percentage

n: number of observations (sample)

- If the 335 men were a random sample of all men in Erbil the true population percentage of smokers in Erbil has 95% confidence interval *(37.9% were smokers)*

$$37.9 \pm 1.96 \times \sqrt{\frac{37.9 \times (100-37.9)}{335}}$$

$$37.9 - 1.96 \times \sqrt{\frac{37.9 \times 62.1}{335}} = 32.7\%$$

$$37.9 + 1.96 \times \sqrt{\frac{37.9 \times 62.1}{335}} = 43.1\%$$

i.e. from 32.7% to 43.1%.

- These two values are the lower and upper confidence limits, respectively.

- The population percentage of smokers is very likely to lie between 32.7% to 43.1%.

Interpretation of 95% CI

- 95% confidence interval is the most common statistical technique for displaying the degree of uncertainty that should be attached to any percentage.

- We are 95% confident that the true population percentage lies within this interval

  (i.e. 32.7% to 43.1%).

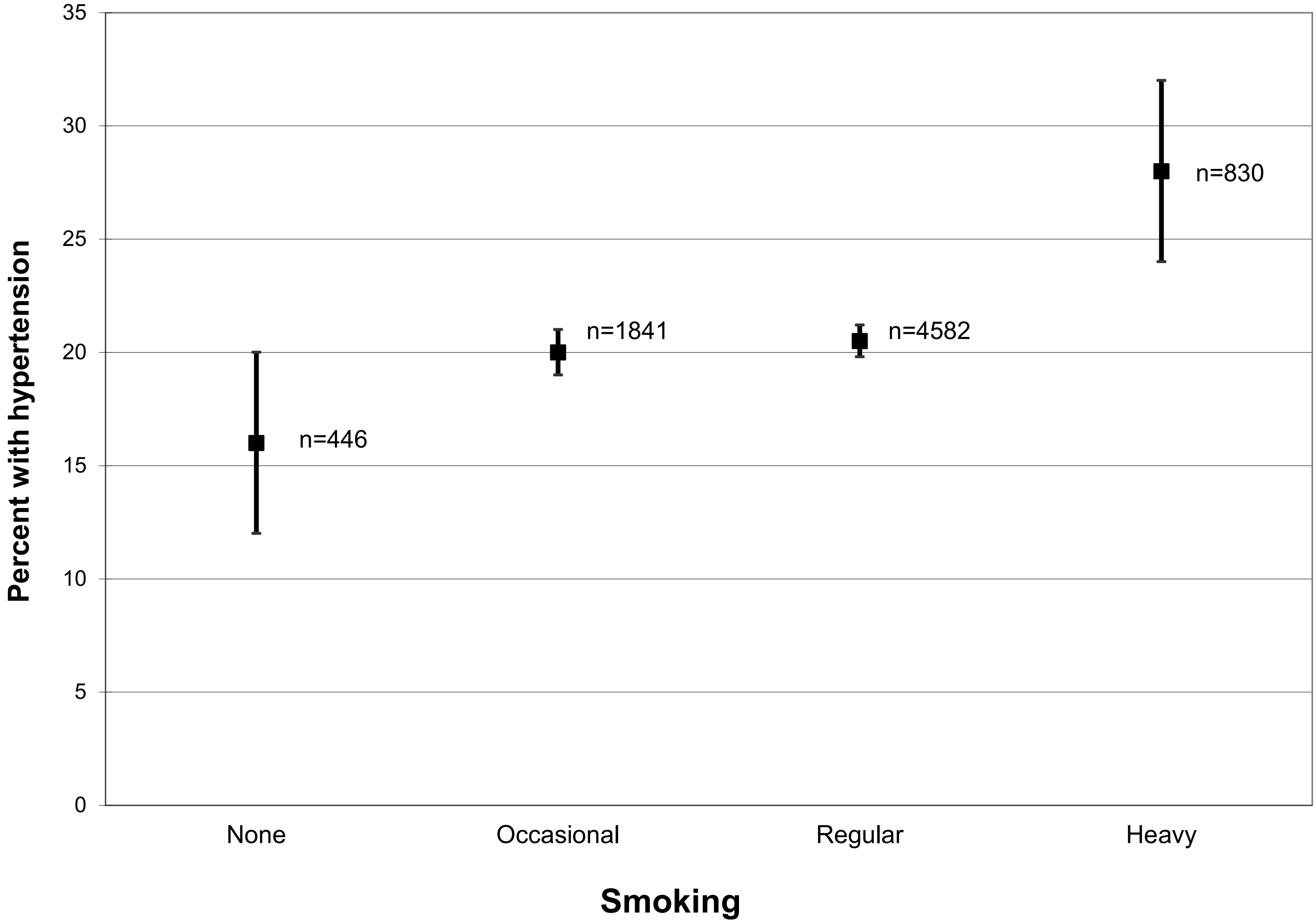
- Remember there exists a 5% risk that the true population percentage lies **outside** the interval.

Display of Confidence Intervals

- If we have percentage for two or more groups, we can display the percentages and their 95% confidence interval in a graph

- In a survey of over 7,000 men their smoking habits were classified into four categories: none, occasional, regular and heavy.

- The percentages of men in each group with systolic hypertension, together with the 95% confidence limits, are shown in the following figure



Percent with hypertension
35
30
25
20
15
10
5
0
n=446
n=1841
n=4582
n=830
None
Occasional
Regular
Heavy
Smoking

- The squares show the observed percentages, while the vertical lines show the 95% confidence intervals.
- Note how the confidence intervals become very narrow for a large sample (e.g. the 4582 regular smokers).
- Also, note that if two confidence intervals do not overlap, (e.g. heavy versus regular smokers) this is evidence of a real difference between two groups.

The sampling variability of a mean

“Quantitative data”

Notation

- A random sample of size (n) is taken from the population of interest.

- The mean ($\bar{x}$) and standard deviation (SD) of the quantitative variable in the sample are calculated

- E.g. of quantitative variables; height, weight, Blood pressure, serum cholesterol, haemoglobin level.

Question

- On the basis of $\overline{X}$ and SD, what can we say about true population mean ?

- **Example**

  The mean haemoglobin level of 25 persons sampled randomly from a population living in a town, with the following results:

  Mean = 13.6 gm/dl SD = 4.3

- What can be said about the true mean of haemoglobin level in this population?

Confidence Interval for a Mean

- To make inferences about the true mean of population, we construct a **confidence interval** using the same approach as that used for proportions.

- **Standard error** of the mean, or SE($\overline{x}$) = $\frac{SD}{\sqrt{n}}$

- 95% CI of mean = $\overline{x}$ ± 1.96SE($\overline{x}$)

- $\overline{x}$ ± 1.96 x $\frac{SD}{\sqrt{n}}$

- **Example**
- In the haemoglobin level example,

$n = 25, \qquad \bar{x} = 13.6, \qquad SD = 4.3$

- The 95% confidence interval:

$$13.6 \pm (1.96 \times 4.3/\sqrt{25})$$

which is 11.9 to 15.3 gm/dl

Example

- The weight of nine children in school is:

  32, 32, 31, 30, 28, 28, 27, 25, 18 kg

  Mean = 27·9 kg Standard deviation = 4·4
- *What is the confidence interval of the mean?*

  The number of children is 9. We can use the formula to calculate the standard error:

  $$4.4/\sqrt{9}$$

- This becomes:
- *SE = 4·4/3 = 1·467*
- The 95% confidence interval for the mean is:

  sample mean - 1·96 standard errors' to 'sample mean + 1·96 standard errors'

  or:

  27·9 - 1·96x1·467 to 27·9 + 1·96x1·467
- So, the 95% confidence interval of the mean is: *from 25·0 to 30·8 kg*

- We are 95% confident that the true mean weight of school children is between 25 and 30.8Kg

Summary

- Sample proportion or mean is only an estimate of population proportion or mean

- Confidence interval is one way to see how accurate our sample result (proportion or mean) is.

95% Confidence interval:

- Percentage p - 1.96 x $\sqrt{\frac{p \times (100 - p)}{n}}$ to p + 1.96 x $\sqrt{\frac{p \times (100 - p)}{n}}$

- Mean $\bar{x}$ ± 1.96 x $\frac{SD}{\sqrt{n}}$

Q1. In a survey of contraceptive use, a sample of 1200 women in a town found 25% were current users of contraception.

What is the 95% confidence interval ?

The standard error of this estimate is 1.25%.

The 95% confidence interval is from 22.55% to 27.45%

Interpret this 95%CI

We are 95% confident that 22.55 to 27.45% of women in this town use contraceptive

Q. The mean birth weight of a representative sample of 153 newborns is 3.250 Kg and the SD is 0.428 Kg. A 95% confidence interval for the population mean birth weight is:

from 3.182 to 3.318 Kg

Which one is true

a) about 95% of the individual newborn birth weights are between 3.182 and 3.318 Kg

b) the mean birth weight for these 153 newborns is probably between 3.182 and 3.318 Kg

c) the mean of the population from which the 153 newborns came is between 3.182 and 3.318 Kg

d) there is a 95% probability that the mean birthweight of the population from which the 153 newborns came is from 3.182 and 3.318 Kg

- Answer **d**

References

- Essential Medical Statistics, by Betty Kirkwood & Jonathan Sterne (Published by Blackwell)

  Statistics Without Tears, a Primer for Non-mathematicians, by Derek Rowntree (Published by Penguin)

Questions?