



# Lecture 1:

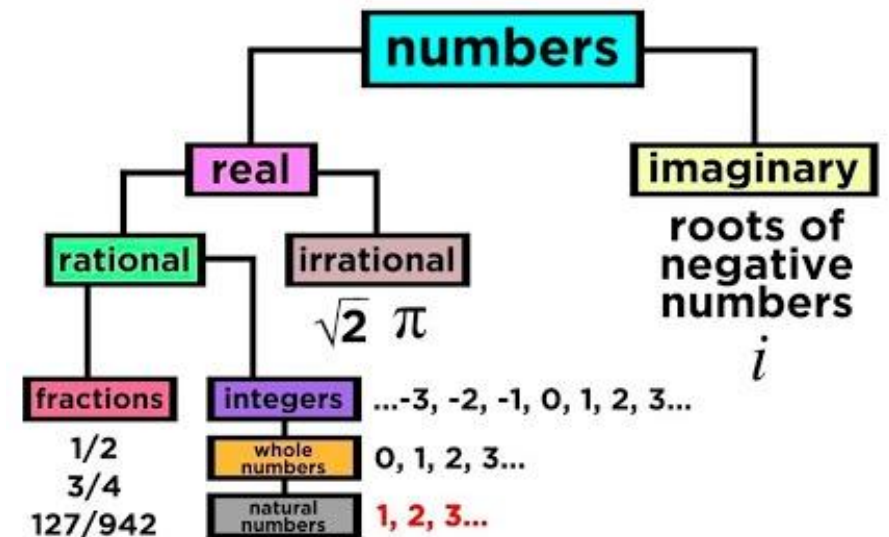
- Numbers
- Logical Operators
- Significant Figures
- Scientific Notation
- Factorial



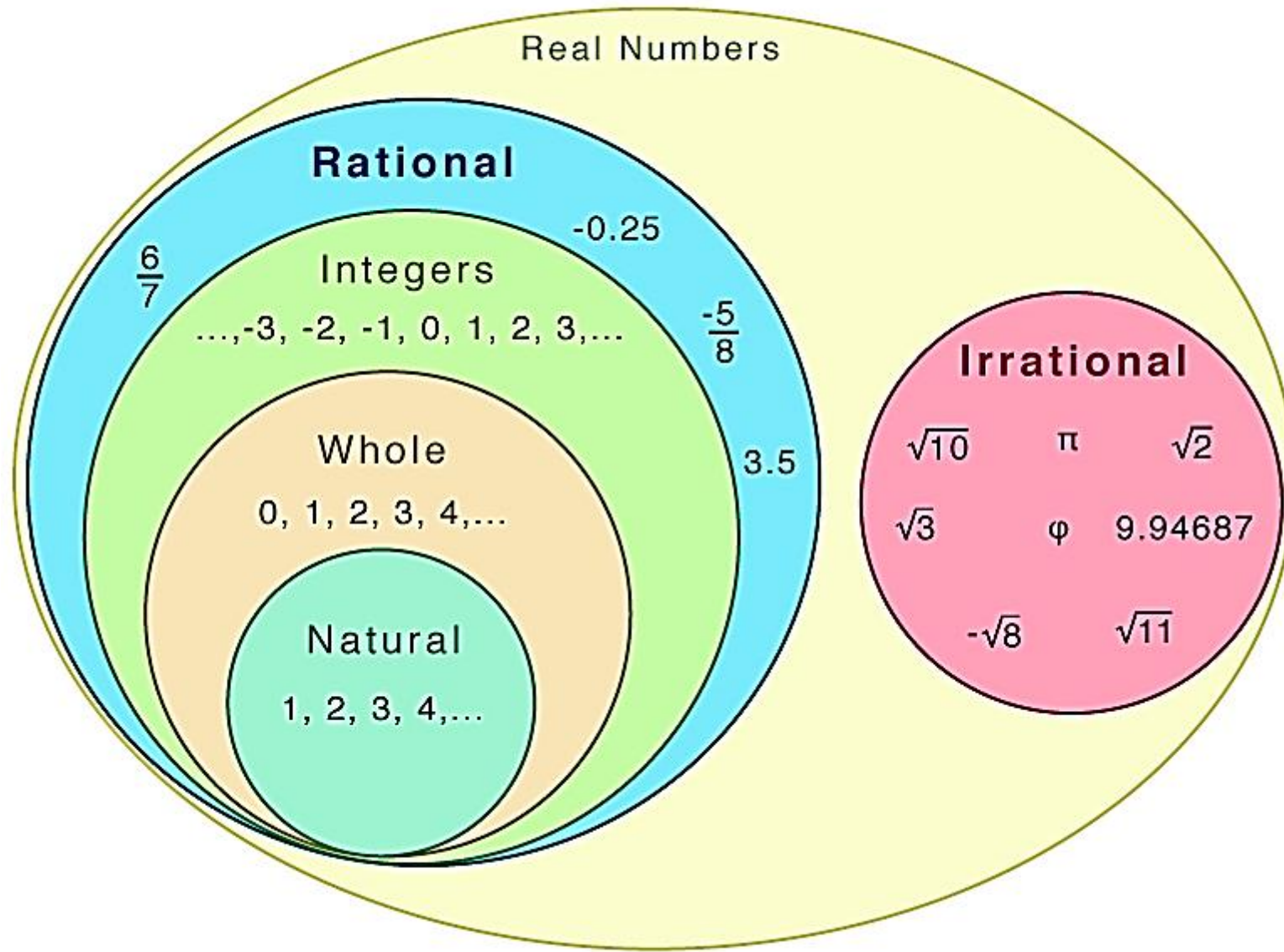
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Course Code: IT 161/A  
Semester 1  
Week 1-2  
Date: 10.12.2023

# Number Classifications

- ✓  $Q = \left\{ \frac{1}{2}, \frac{3}{4}, 2, 5, 0, -\frac{7}{11}, -12, \dots \right\}$  – rational numbers  $\left\{ \frac{a}{b} \mid a, b \in Z, b \neq 0 \right\}$
- ✓  $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$  – integers
  - $Z^+ = \{0, 1, 2, 3, \dots\}$  – non-negative integers
  - $Z^- = \{0, -1, -2, -3, \dots\}$  – non-positive integers
- ✓  $W = \{0, 1, 2, 3, \dots\}$  – whole numbers
- ✓  $N = \{1, 2, 3, \dots\}$  – natural numbers/counting numbers
- ✓  $\sqrt{2}, \pi, \sqrt{3}, \dots$  - irrational numbers



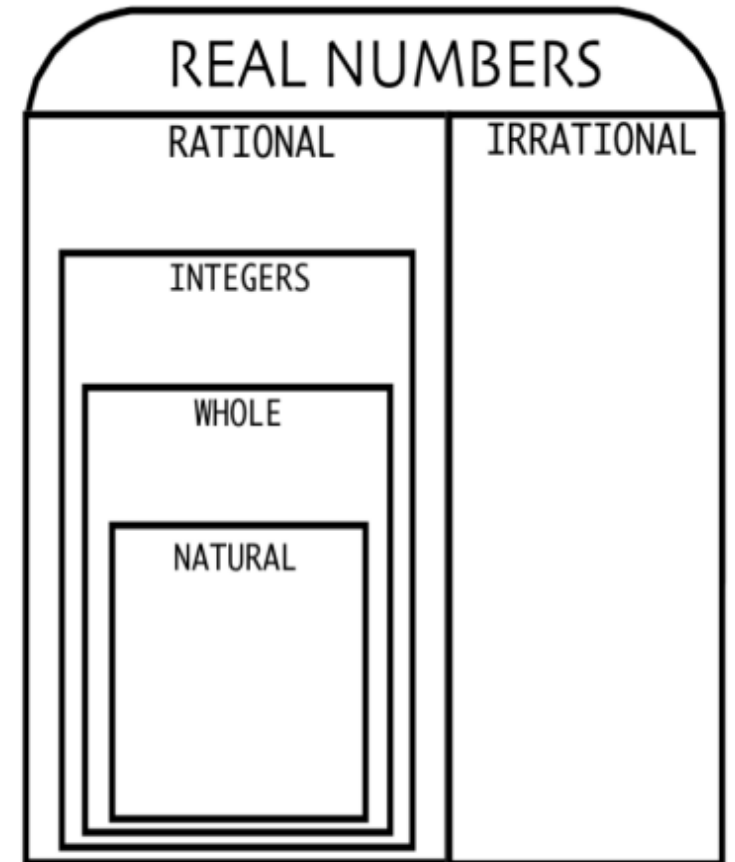
➤ **Real numbers** - set of irrational numbers & rational numbers





1) Re-write each number in the Venn Diagram where it belongs.

-19	$1.\bar{2}$	0	3
$\sqrt{10}$	$\sqrt{81}$	3.456	$-\frac{6}{11}$
-1.48298.....		$\pi + 3$	-44



2) List all classifications of the number.

a)  $\sqrt{10}$  \_\_\_\_\_

b)  $-44$  \_\_\_\_\_

c)  $3$  \_\_\_\_\_

d)  $-\frac{6}{11}$  \_\_\_\_\_

3) Check all boxes that apply to the number.

		Natural	Whole	Integer	Rational	Irrational	Real
a)	$\sqrt{81}$						
b)	$1.\bar{2}$						
c)	$0$						
d)	$13$						

*practice  
makes  
progress*

**Types of Numbers**

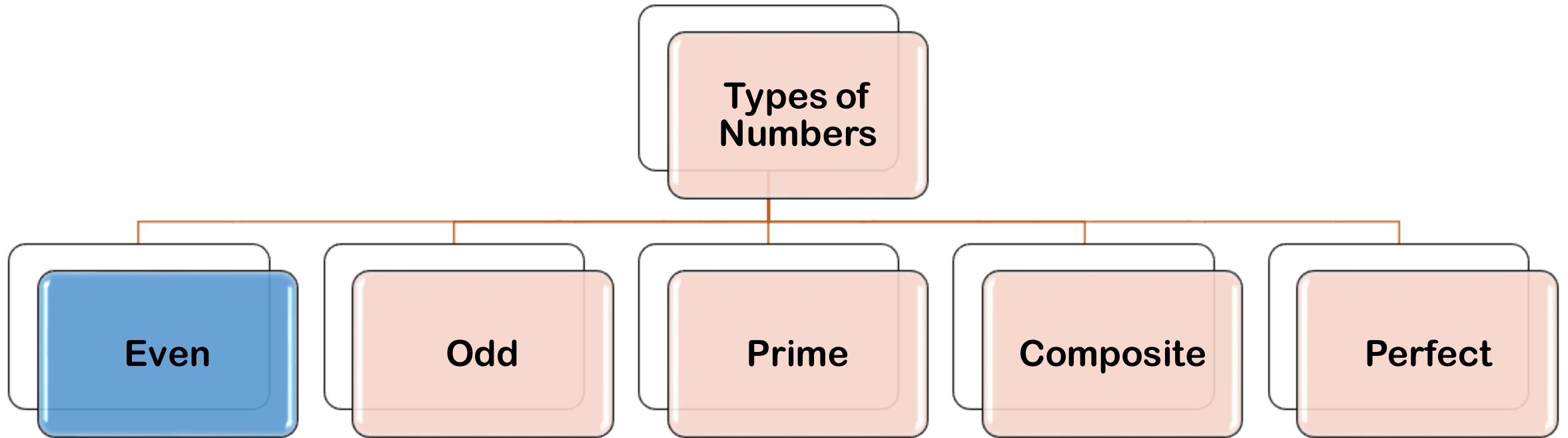
**Even**

**Odd**

**Prime**

**Composite**

**Perfect**



- A number that can be exactly divided by 2.
- Even numbers always end up with the last digit as 0, 2, 4, 6 or 8.
- The general form of even numbers is given by  $2k$ , where  $k \in \mathbb{Z}$

→ Ahmad has 30 pencils. He distributed 14 of those among his friends. Will he have an even number of pencils left? How do you know?

→ State true or false: 0 is an even number.

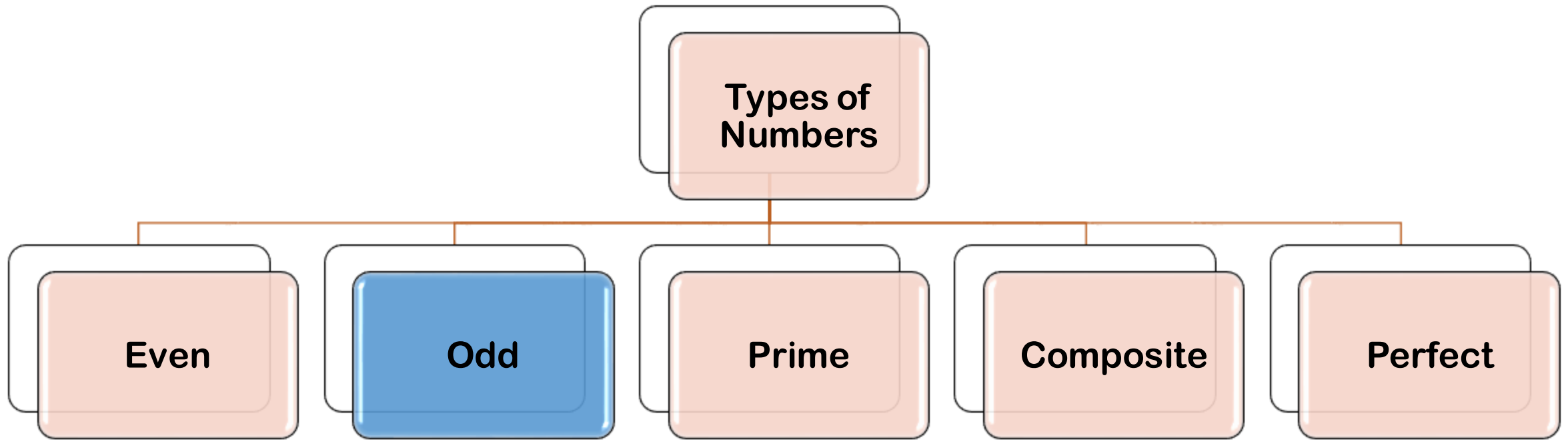
→ Select the pair of consecutive even numbers from the following:

- a) 24 and 28
- b) 91 and 93
- c) 84 and 86
- d) 39 and 42

→ When you buy a dozen bananas, are you getting an even number or an odd number of bananas?

→ Select the even numbers from the following:  
a.) 778  
b.) 912  
c.) 223





- A number which is not divisible by 2.
- An odd number always ends in 1, 3, 5, 7, or 9.
- The general form of odd numbers is given by  $2k + 1$ , where  $k \in \mathbb{Z}$

→ Determine whether 135 is an odd number or not.

→ Is 350 an odd number or an even number?

→ Will the sum of  $23 + 35$  result in an odd number?

→ Answer the following questions with reference to odd numbers:

a.) 1 is odd or even?

b.) Which is the smallest 4 digit odd number?

c.) What is the sum of any two odd numbers?

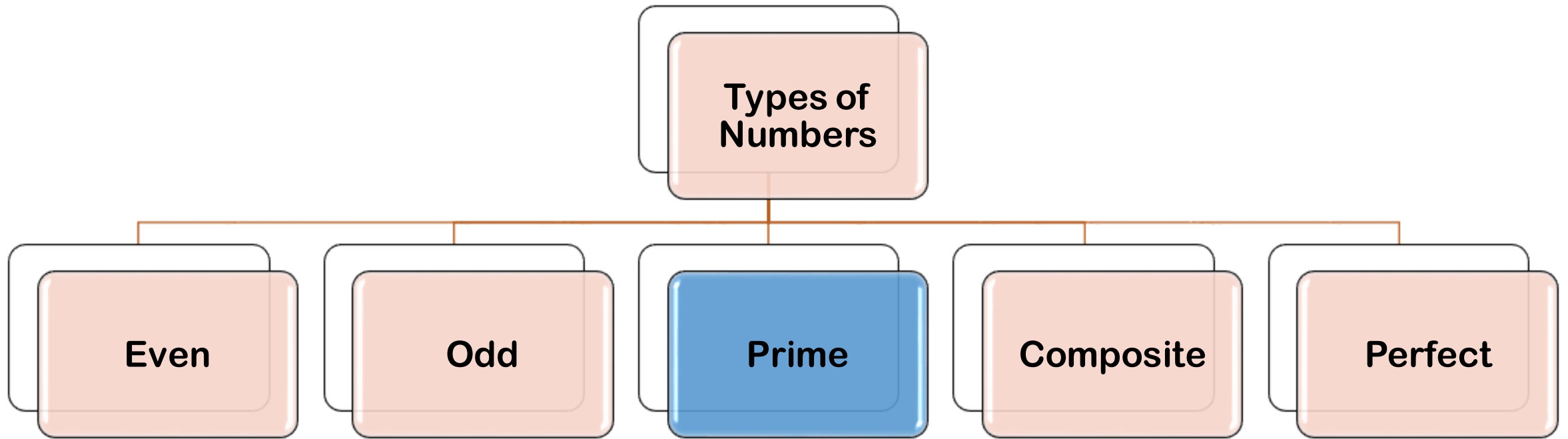
d.) Is 2 an odd number?

→ State true or false with respect to odd numbers.

a.) The sum of two odd numbers is always an even number.

b.) The smallest odd number is 5.

c.) 9 is an odd number.



- A natural number that are divisible by only 1 and the number itself.
- Ex: 2, 3, 5, 7, 11, 13, ...

→ Which of the two numbers is a prime number, 13 or 15?

→ Why is 20 not a prime number?

→ State true or false with respect to prime numbers.

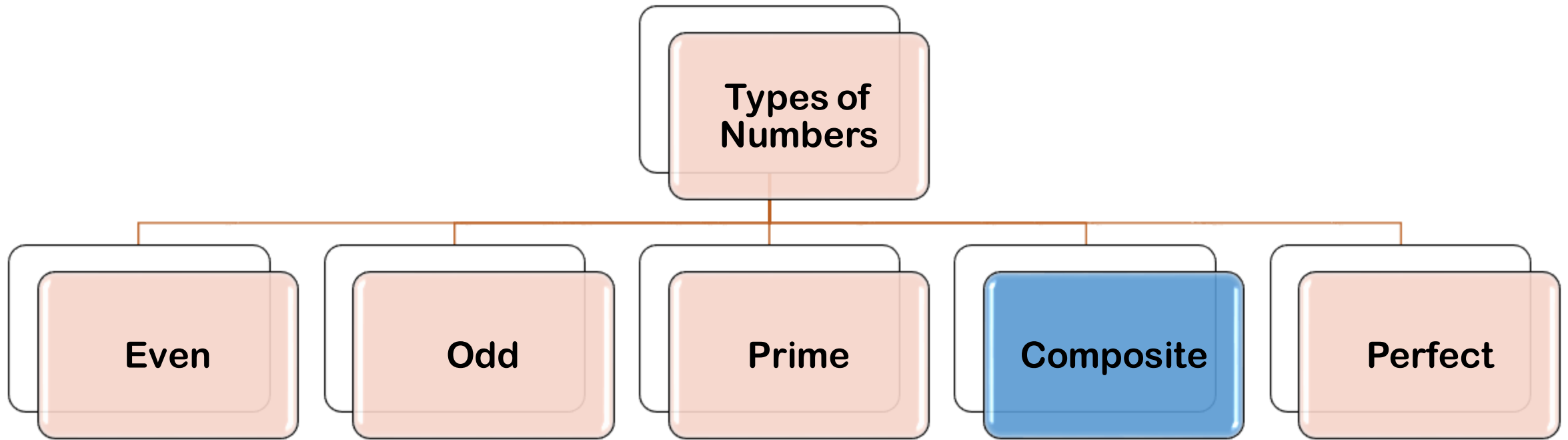
- a.) 1 is a prime number.
- b.) The only even prime number is 2.
- c.) The first five prime numbers are 2, 3, 5, 7, and 9.
- d.) All prime numbers are odd.

→ Which of the following numbers is a prime number?

- a) 4
- b) 10
- c) 33
- d) 43

→ Choose true/false against each statement.

	True	False
2 is the only even prime number.	<input type="radio"/>	<input type="radio"/>
3 is the smallest prime number.	<input type="radio"/>	<input type="radio"/>
97 is the largest prime number.	<input type="radio"/>	<input type="radio"/>
All prime numbers are odd.	<input type="radio"/>	<input type="radio"/>



- A natural number or a positive integer which has more than two factors.
- Ex: 15 has factors 1, 3, 5 and 15.

Always remember that **1** is neither prime nor composite

→ Which of the following is a composite number?

- a) 34
- b) 31
- c) 39

→ Fill in the blanks:

- a.) The smallest composite number is \_\_\_.
- b.) The smallest odd composite number is \_\_\_.

→ State true or false with respect to composite numbers.

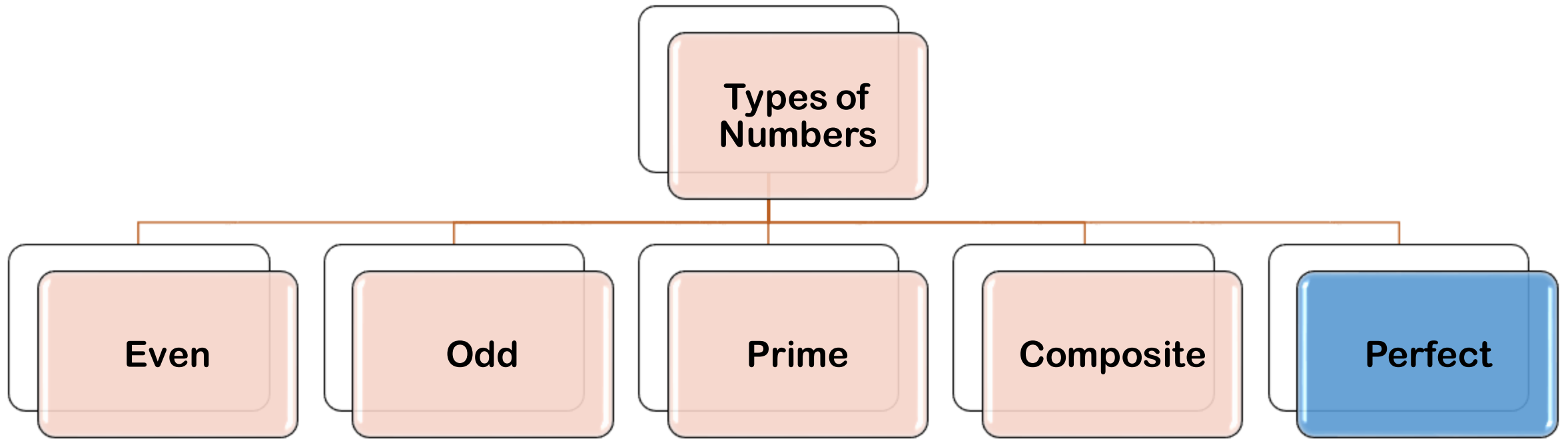
- a.) All even numbers are composite numbers.
- b.) 1 is a composite number.

→ Aya is listing all the composite numbers between 3 and 10. Can you help her choose the correct option?

- a) 4, 6, 8, 9
- b) 4, 9
- c) 4, 5, 6, 7, 8, 9
- d) 4, 8, 9

→ The smallest composite number is 2.

- a) True
- b) False



- A positive integer that is equal to the sum of its positive factors, excluding the number itself.
- Ex: 6, 28, 496, 8128, 33550336, ...
- All the perfect numbers are also complete numbers.

→ Is 28 a perfect number?

→ Select the perfect numbers from the following.

- a) 5
- b) 6
- c) 32
- d) 28
- e) 9

→ State true or false:

- a.) Perfect numbers are the positive integers that are equal to the sum of its factors except for the number itself.
- b.) All the perfect numbers are odd numbers.

→ Check whether the given numbers are perfect numbers or not by finding the sum of their factors:

- a.) 8
- b.) 25

→ Write true or false against each statement.

	True	False
All the perfect numbers known till now are even.	<input type="radio"/>	<input type="radio"/>
All perfect numbers can be written as the sum of its proper divisors.	<input type="radio"/>	<input type="radio"/>
The smallest perfect number is 9.	<input type="radio"/>	<input type="radio"/>



$$\text{Even} + \text{Even} = \text{Even}$$

$$\text{Odd} + \text{Odd} = \text{Even}$$

$$\text{Even} + \text{Odd} = \text{Odd}$$

$$\text{Even} \times \text{Even} = \text{Even}$$

$$\text{Even} \times \text{Odd} = \text{Even}$$

$$\text{Odd} \times \text{Odd} = \text{Odd.}$$



# Perfect Square Numbers

- Perfect squares are the squares of a whole number (when a number is multiplied by itself two times).

## Perfect Square Formula

$$N = X^2$$

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

→ Is 100 a perfect square number?

→ In an auditorium, the number of rows is the same as the number of columns. If there are 60 chairs in a row, how many chairs are there in the auditorium?

→ What smallest whole number is to be added to 75 to make it a perfect square?

→ Which of the following is not a perfect square?

a) 900

b) 800

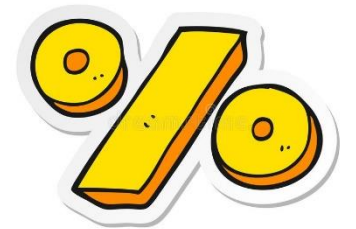
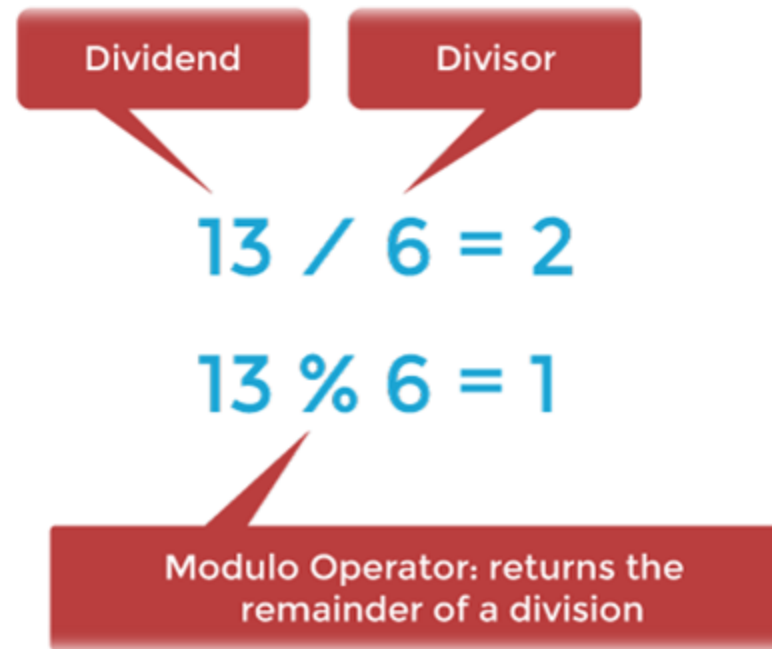
c) 400

d) 100

→ What will be the area of a square having a side of 16 meters?

# Modulo Operator

- Mod is also known as modulus or modulo.
- It gives the remainder after dividing one number by another number.
- Modulus of any real number  $x$  will always give **positive** value as it's output.





$$a : b \ (a \div b)$$



$$a, b, q, r \in \mathbb{Z}, \quad b \neq 0, \quad 0 \leq r \leq |b|$$

$$a = b \cdot q + r$$

$$27 \text{ mod } 4 = 27 : 4 \Rightarrow 4 \cdot 6 + \textcircled{3}$$

$$113 : (-3) \Rightarrow -37 \cdot (-3) + \textcircled{2}$$

$$-15 : 4 \Rightarrow -3 \cdot 4 + \textcircled{-3} = -15 \quad \text{✗}$$

$$-15 : (-7) \Rightarrow 3 \cdot (-7) + \textcircled{6}$$

$$0 \leq r \leq |b|$$

$$-5 \text{ mod } 9 = -1 \cdot 9 + \textcircled{4}$$

$$-15 : 4 \Rightarrow -4 \cdot 4 + \textcircled{1} = -15 \quad \text{✓}$$

$$-19 \text{ mod } 9 = -3 \cdot 9 + \textcircled{8}$$

$$3 \bmod 10 = 3$$
$$13 \bmod 10 = 3$$
$$23 \bmod 10 = 3$$
$$33 \bmod 10 = 3$$

$$-9 \bmod 9 = 0$$

$$-8 \bmod 9 = 1$$

$$-7 \bmod 9 = 2$$

$$-4 \bmod 9 = 5$$

$$-2 \bmod 9 = 7$$

$$-1 \bmod 9 = 8$$

$$\text{What is } -6 \bmod 18 ? = 12$$

$$\text{What is } -4 \bmod 9 ? = 5$$

$$\text{What is } -9 \bmod 6 ? = 3$$

$$\text{What is } -13 \bmod 1 ? = 0$$

$$\text{What is } 17 \bmod 7 ? = 3$$

$$\text{What is } -49 \bmod 5 ? = 1$$

$$\text{What is } -14 \bmod 2 ? = 0$$

$$\text{What is } -29 \bmod 4 ? = 3$$

$$\text{What is } -29 \bmod 3 ? = 1$$

$$\text{What is } 6 \bmod 18 ? = 6$$

$$\text{What is } 9 \bmod -6 ? = 3$$

$$\text{What is } 4 \bmod 9 ? = 4$$

$$\text{What is } -6 \bmod 18 ? = 12$$

$$\text{What is } 7 \bmod 6 ? = 1$$



Find the largest negative integer that when divided by nine leaves a remainder of one.

$$10:9 = 1 \cdot 9 + 1$$

$$1:9 = 0 \cdot 9 + 1$$

$$-8:9 = -1 \cdot 9 + 1$$

The largest negative integer is  $-8$ .



When a certain integer is divided by 12, the remainder is 5. What remainder is obtained when this number is divided by 4?



1) What is the remainder value:

- 108 is divided by 3.
- 129 is divided by 7.
  
- Find the product  $23 \cdot 43$  modulo 8
- Find  $11 \bmod 8$ .
- Find  $-3 \bmod 8$ .
- Find  $49 \bmod 5$ .

2) Perform the modular arithmetic operation

- What is  $13 \bmod 1 =$
- What is  $-17 \bmod 7 =$
- What is  $-4 \bmod 9 =$
- What is  $4 \bmod 9 =$
- What is  $-7 \bmod 6 =$
- What is  $49 \bmod 5 =$
- What is  $-49 \bmod 5 =$
- What is  $25+37 \bmod 12 =$

3) Given that  $5x \equiv 6 \pmod{8}$ , find  $x$ .





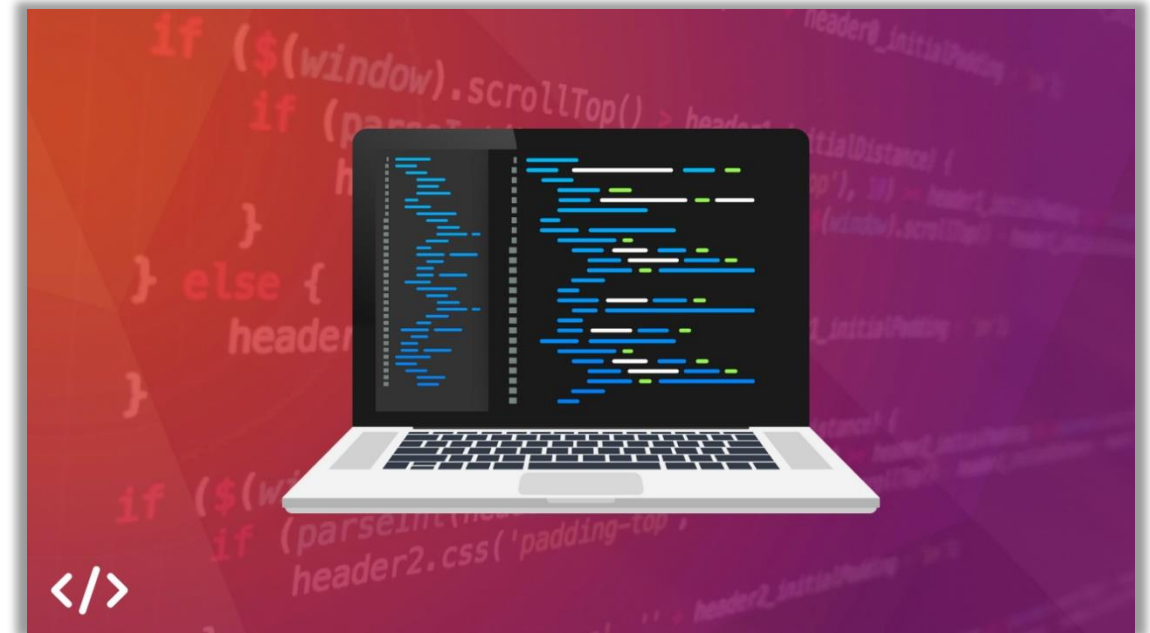
# MOD in programming languages and calculators

- Many programming languages, and calculators, have a mod operator, typically represented with the % symbol. If you calculate the result of a negative number, some languages will give you a negative result.

$$-5 \% 3 = -2$$

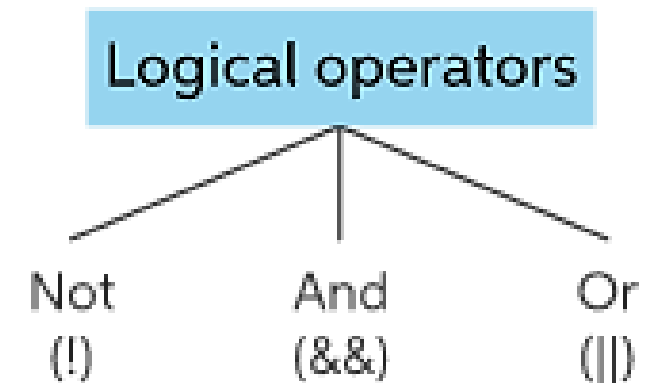


Modulus of any real number  $x$  will always give **positive** value as it's output.



# Logical Operators

- Logical operators are useful when we want to test multiple conditions.
- There are 3 types of logical operators and they work the same way as the boolean AND, OR and NOT operators.
- `&&` - Logical AND
  - ▣ All the conditions must be true for the whole expression to be true.
  - ▣ Example: `if (a == 10 && b == 9 && d == 1)` means the *if* statement is only true when `a == 10` **and** `b == 9` **and** `d == 1`.



# Logical Operators

- `||` - Logical OR
  - ▣ The truth of one condition is enough to make the whole expression true.
  - ▣ Example: `if (a == 10 || b == 9 || d == 1)`  
means the *if* statement is true when **either one** of *a*, *b* or *d* has the right value.
  
- `!` - Logical NOT (also called logical negation)
  - ▣ Reverse the meaning of a condition
  - ▣ Example: `if (!(points > 90))`  
means if points not bigger than 90.





# Logical Operator

Expression

Expression Equivalent

!(a == b)

a != b

!(a == b || a == c)

a != b && a != c

!(a == b && c > d)

a != b || c <= d

Answer for the following questions: **True** or **False**



If  $x = -2$ ,  $y = 5$ ,  $z = 0$ , and  $t = -4$ , what is the value of each of the following expressions:

1.  $x + y < z + 1$

2.  $x - 2 * y + y < z * 2/3$

3.  $3 * y/4 \% 5 < 8 \ \&\& \ y > = 4$

4.  $t > 5 \ || \ z < (y + 5) \ \&\& \ y < 3$

5.  $!(4 + 5 * y > = z - 4) \ \&\& \ (z - 2 < 7)$



If the numerator is **smaller than** the denominator, then the remainder is equal to the numerator.  $3 \% 10 = 3$

$$\begin{aligned} 3 \% 5 &= 3 \\ 5 \% 10 &= 5 \\ 78 \% 112 &= 78 \end{aligned}$$

If  $x = -2$ ,  $y = 5$ ,  $z = 0$ , and  $t = -4$ , what is the value of each of the following logical expressions?

1.  $x + y < z + 1$
2.  $x - 2 * y + y < z * 2 / 3$
3.  $3 * y / 4 < 8 \ \&\& \ y \geq 4$
4.  $t > 5 \ || \ z < 2$
5.  $x * y < 10 \ || \ y * z < 10$
6.  $(y + 2) / 3 > 3 \ \&\& \ t < 0$
7.  $x * 3 > 0 \ || \ y + 5 / t < 2$
8.  $!(x > 0)$
9.  $!(x * t < 10) \ || \ y / x * 4 < y * 2$
10.  $t > 5 \ || \ z < (y + 5) \ \&\& \ y < 3$
11.  $!(4 + 5 * y \geq z - 4) \ \&\& \ (z - 2 < 7)$



#### Order of Operations

- $()$
- $*$ ,  $/$ ,  $\%$  Multiplicative operators
- $+$ ,  $-$  Additive operators
- $<$ ,  $>$ ,  $\geq$ ,  $\leq$  Relational operators
- $==$ ,  $!=$  Then do any comparisons for equality and inequality
- $\&\&$  Logical and
- $\|\|$  Logical or
- $=$  Assignment operator

Write syntactically correct logical expressions for the following conditions:

1.  $m$  is less than 100
2.  $n$  is positive and greater than  $m$
3.  $m$  is between 5 and 10 (inclusive)
4.  $k$  is less than 1 or greater than 2
5.  $j$  and  $k$  are both negative
6.  $i$  is an even number



Given

`int a = 5, b = 7, c = 17 ;`

evaluate each expression as True or False.

1. `c / b == 2`

2. `c % b <= a % b`

3. `b + c / a != c - a`

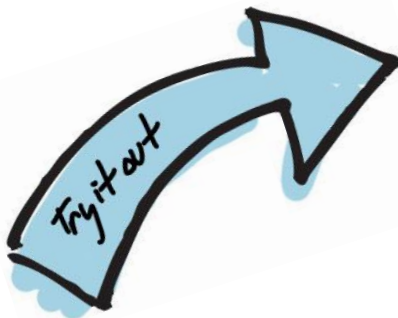
4. `(b < c) && (c == 7)`

5. `(c + 1 - b == 0) || (b = 5)`



- Assume  $a=5$ ,  $b=2$ ,  $c=4$ ,  $d=6$ , and  $e=3$ . Determine the value of each of the following expressions:

- $a > b$
- $a \neq b$
- $d \% b == c \% b$
- $a * c \neq d * b$
- $a \% b * c$



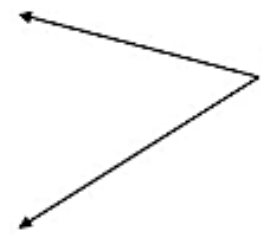
- $25 < 7 \parallel 15 > 36$
- $15 > 36 \parallel 3 < 7$
- $14 > 7 \ \&\& \ 5 \leq 5$
- $4 > 3 \ \&\& \ 17 \leq 7$
- $! \text{false}$
- $!(13 \neq 7)$
- $9 \neq 7 \ \&\& \ !0$
- $5 > 1 \ \&\& \ 7$





```
int x, y;  
X = 0 ;  
y = 1 ;  
if ( x < y || y < 5 && x == 3 )  
{  
    printf ("True \n");  
}  
else  
{  
    printf("False \n ");  
}
```

Which is printed?





$$7. \quad z + \frac{3}{5} = \frac{z}{5}$$

$$9. \quad \frac{7}{4}h = \frac{1}{4}h - 12w$$

$$11. \quad w + \frac{1}{7} = \frac{6w}{7} - 1$$

$$13. \quad \frac{y}{8} + 6 = 6 - \frac{5y}{8}$$

$$15. \quad 2 - \frac{n}{8} = 4n + \frac{5}{8}$$

$$17. \quad \frac{5y}{2} - 9 = \frac{2y}{3} + 2$$

$$19. \quad 1 - \frac{5}{8}x = 2 - \frac{2}{3}x$$

$$8. \quad \frac{y}{6} = y + 5$$

$$10. \quad \frac{4}{9}w + 5 = \frac{5}{9}$$

$$12. \quad 6 + \frac{x}{5} = \frac{4x}{5} - 3$$

$$14. \quad \frac{m}{2} + 2 = \frac{4m}{5} + 2$$

$$16. \quad \frac{w}{3} + 5 = \frac{8w}{3} - 2$$

$$18. \quad p - \frac{p}{8} = \frac{p}{4} - 10$$

$$20. \quad y + \frac{3}{4} = \frac{y}{4} + \frac{7}{8}$$



# Practice



Practice  
Makes  
Perfect

$$1) \frac{3}{5}(1 + p) = \frac{21}{20}$$

$$3) 0 = -\frac{5}{4}\left(x - \frac{6}{5}\right)$$

$$5) \frac{3}{4} - \frac{5}{4}m = \frac{113}{24}$$

$$7) \frac{635}{72} = -\frac{5}{2}\left(-\frac{11}{4} + x\right)$$

$$9) 2b + \frac{9}{5} = -\frac{11}{5}$$

$$11) \frac{3}{2}\left(\frac{7}{3}n + 1\right) = \frac{3}{2}$$

$$13) -\left(-\frac{5}{2}x - \frac{3}{2}\right) = -\frac{3}{2} + x$$

$$14) \frac{45}{16} + \frac{3}{2}n = \frac{7}{4}n - \frac{19}{16}$$

$$15) \frac{3}{2}\left(v + \frac{3}{2}\right) = -\frac{7}{4}v - \frac{19}{6}$$

$$16) \frac{47}{9} + \frac{3}{2}x = \frac{5}{3}\left(\frac{5}{2}x + 1\right)$$

$$2) -\frac{1}{2} = \frac{3}{2}k + \frac{3}{2}$$

$$4) \frac{3}{2}n - \frac{8}{3} = -\frac{29}{12}$$

$$6) \frac{11}{4} + \frac{3}{4}r = \frac{163}{32}$$

$$8) -\frac{16}{9} = -\frac{4}{3}\left(\frac{5}{3} + n\right)$$

$$10) \frac{3}{2} - \frac{7}{4}v = -\frac{9}{8}$$

$$12) \frac{41}{9} = \frac{5}{2}\left(x + \frac{2}{3}\right) - \frac{1}{3}x$$

$$17) -\frac{7}{2}\left(\frac{5}{3}a + \frac{1}{3}\right) = \frac{11}{4}a + \frac{25}{8}$$

$$18) -\frac{8}{3} - \frac{1}{2}x = -\frac{4}{3}x - \frac{2}{3}\left(-\frac{13}{4}x + 1\right)$$

$$19) \frac{1}{3}n + \frac{29}{6} = 2\left(\frac{4}{3}n + \frac{2}{3}\right)$$



# Answers to check

1)  $\frac{3}{4}$

2)  $-\frac{4}{3}$

3)  $\frac{6}{5}$

4)  $\frac{1}{6}$

5)  $-\frac{19}{6}$

6)  $\frac{25}{8}$

7)  $-\frac{7}{9}$

8)  $-\frac{1}{3}$

9)  $-2$

10)  $\frac{3}{2}$

11)  $0$

12)  $\frac{4}{3}$

13)  $-2$

14)  $16$

15)  $-\frac{5}{3}$

16)  $\frac{4}{3}$

17)  $-\frac{1}{2}$

18)  $-\frac{3}{2}$

19)  $\frac{3}{2}$





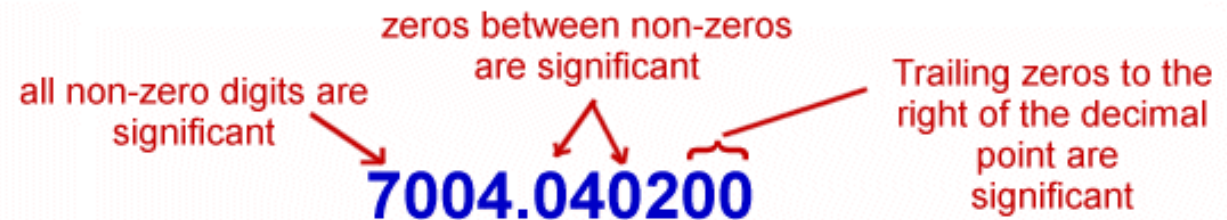
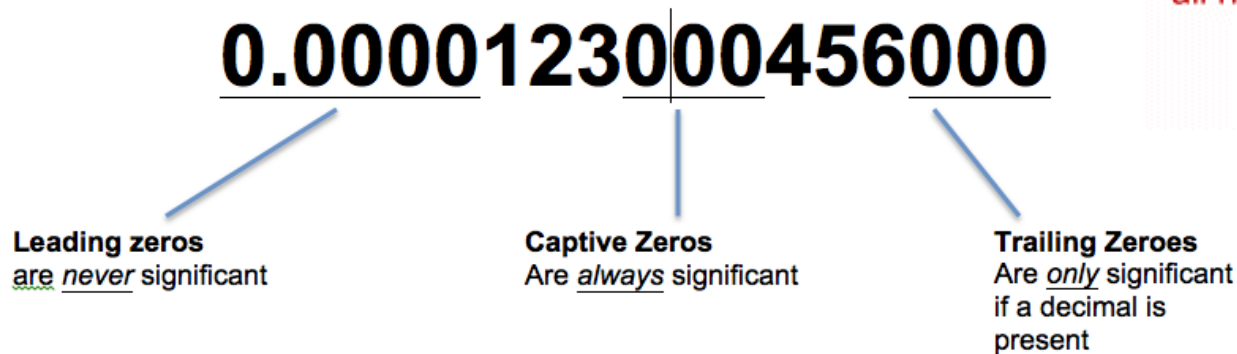
# SigFigs

Significant figures are important to show the precision of your answer. This is important in science and engineering because no measuring device can make a measurement with 100% precision. Using Significant figures allows the scientist to know how precise the answer is, or how much uncertainty there is.

2002 has two significant zeroes, but 0.0103 has only 1 significant zero.

# Significant Figures

- The number of digits counted to the right from the leftmost positive digit is called the *number of significant figures*. For example, 26.103, 0.00304, 202.000 and 0.003040 are quoted to 5, 3, 6, 4 significant figures respectively.



# Significant Figures Rules:

- All non-zero digits **DO** count.
  - $24 = 2$
  - $3.56 = 3$
- Leading zeros **DON'T** count.
  - (zeros in front of numbers)
  - $0.0025 = 2$
- Captive Zeros **DO** count.
  - (zeros between non-zero numbers)
  - $1502 = 4$        $1.008 = 4$
- Trailing Zeros **DO** count **IF** the number contains a **DECIMAL**.
  - (zeros at the end of numbers)
  - $100 = 1$        $2306.0 = 5$        $1.00 \times 10^3 = 3$



Sometimes, you'll be asked to round with significant figures. Significant figures have to do with the number of digits known with a degree of certainty. Keeping track of this number is important when gathering data from an experiment because it minimizes error. Here are the rules for significant figures:

1. All nonzero digits are significant.
2. All zeroes between nonzero digits are significant.
3. Trailing zeroes to the right of a decimal point are significant.
4. Leading zeroes to the left of the first non-zero number are not significant.

Here are some examples. Can you see which rule applies?

1.23 has 3 significant figures

1001 has 4 significant figures

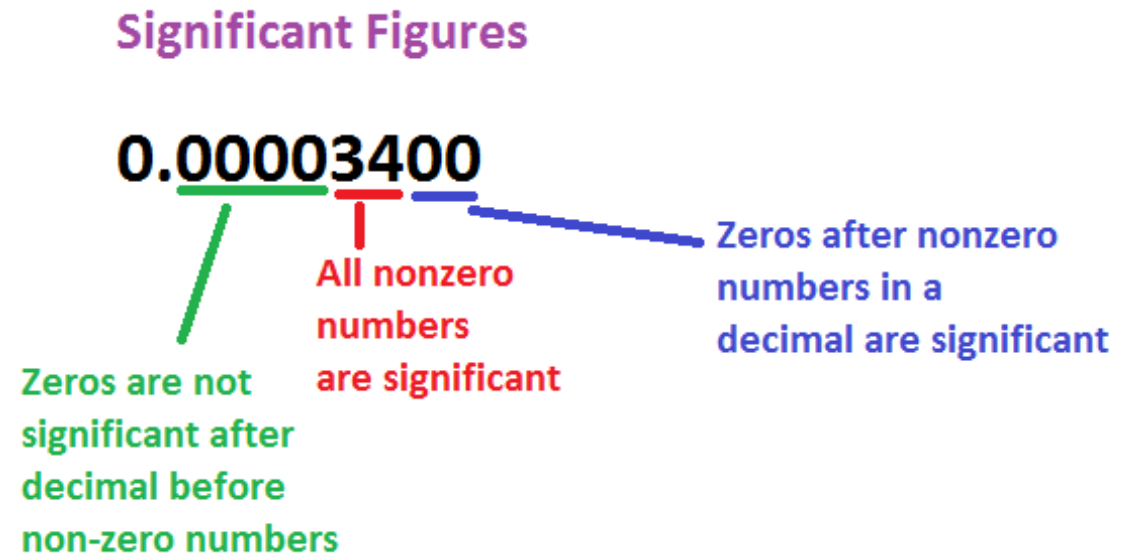
2.03 has 3 significant figures

0.033 has 2 significant figures

0.20 has 2 only significant figures

**Significant Figures**

**0.00003400**

A diagram illustrating the rules for significant figures using the number 0.00003400. The number is written in black. A green horizontal line underlines the leading zeros (0.0000), with a green line pointing to the text 'Zeros are not significant after decimal before non-zero numbers'. A red vertical line points to the digits '34', with red text 'All nonzero numbers are significant'. A blue horizontal line underlines the trailing zeros ('00'), with a blue line pointing to the text 'Zeros after nonzero numbers in a decimal are significant'.



**1. Find the number of significant figure in each of the following:**

(a) 7.3

(b) 162.5 m

(c) 306 g

(d) 3.57 m

(e) 7.005 kg

(f) 0.045 km

(g) 0.00234 l

(h) 82.030 mg

**2. Round off each of the following correct up to 3 significant figures:**

(a) 56.4517 g

(b) 5.20763 kg

(c) 33.311 km

(d) 50.001 cm

(e) 0.0012485 m

(f) 0.0013020 l

# Scientific Notation

- $193.034 = 1.93034 \times 10^2$
- $0.003040 = 3.040 \times 10^{-3}$

0.0050

The Number is a decimal less than 1, so the Exponent will be Negative.

= 0.0050  
3 places

Move the Decimal point to the RIGHT to create a number between 1 and 10.

= 0005.0

Remove Zeroes that are not needed. NEVER REMOVE ZEROES THAT CAME AFTER A DECIMAL POINT.

= 5.0 × 10<sup>-3</sup> ✓

2 Significant Figures

We moved 3 places so Power of 10 is three : 10<sup>-3</sup>

$$2 \times 10^9$$

$$2.000000000$$

1 2 3 4 5 6 7 8 9

$$2,000,000,000$$

$$284.6 = 2.846 \times 10^2$$

$$0.0245 = 2.45 \times 10^{-2}$$

$$3125000 = 3.125 \times 10^6$$

$$-0.0042 = -4.2 \times 10^{-3}$$

$$0.00056 = 5.6 \times 10^{-4}$$

$$245000 = 2.45 \times 10^5$$

$$240.06 = 2.4006 \times 10^2$$



*Convert the following numbers into scientific notation:*

- 1) 923              **9.23 x 10<sup>2</sup>**      \_\_\_\_\_
- 2) 0.00425            **4.25 x 10<sup>-3</sup>**      \_\_\_\_\_
- 3) 4523000            **4.523 x 10<sup>6</sup>**      \_\_\_\_\_
- 4) 0.94300            **9.4300 x 10<sup>-1</sup>**      \_\_\_\_\_
- 5) 6750.              **6.750 x 10<sup>3</sup>**      \_\_\_\_\_
- 6) 92.03              **9.203 x 10<sup>1</sup>**      \_\_\_\_\_
- 7) 7.80                **7.80 x 10<sup>0</sup>**      \_\_\_\_\_
- 8) 0.00000032         **3.2 x 10<sup>-7</sup>**      \_\_\_\_\_

*Convert the following numbers into standard notation:*

- 9) 3.92400 x 10<sup>5</sup>         **392400**      \_\_\_\_\_
- 10) 9.2 x 10<sup>6</sup>         **9200000**      \_\_\_\_\_
- 11) 4.391 x 10<sup>-3</sup>         **0.004391**      \_\_\_\_\_
- 12) 6.825 x 10<sup>-4</sup>         **0.0006825**      \_\_\_\_\_
- 13) 4.6978 x 10<sup>4</sup>         **46978**      \_\_\_\_\_
- 14) 8.36 x 10<sup>1</sup>         **83.6**      \_\_\_\_\_
- 15) 2.46 x 10<sup>-5</sup>         **0.0000246**      \_\_\_\_\_
- 16) 8.8 x 10<sup>2</sup>         **880**      \_\_\_\_\_

# Factorial



exclamation mark

$$0! = 1$$

$$1! = 1$$

$$2! = 2 \times 1 = 2$$

$$3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

**Example 1.**

**Simplify this factorial expression.**

$$3!$$

**Solution.**

- **Use this formula to calculate a factorial expression:**

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 1$$

- **Calculate the factorial expression.**

$$\begin{aligned} 3! &= 3 \cdot 2 \cdot 1 \\ &= 6 \end{aligned}$$

study  
now  
be proud  
later



Match each expression on the left with an equivalent expression on the right.

A	$\frac{14!}{13!}$
B	$\frac{52!}{51!}$
C	$\frac{101!}{99!}$
D	$20 \times 19!$
E	$90 \times 8!$
F	$30 \times 4!$

Letter		
	1	10100
	2	6!
	3	52
	4	10!
	5	14
	6	20!



Determine the value for each expression. Simplify fully before using a calculator.

a)  $\frac{10!}{5!}$

b)  $\frac{21!}{14!}$

c)  $\frac{9!}{3!6!}$

d)  $\frac{12!}{8!4!}$

e)  $\frac{7!}{2!5!} + \frac{7!}{4!3!}$

f)  $\frac{15!}{9!6!} + \frac{15!}{10!5!}$

g)  $2 \times \frac{5!}{2!3!}$

h)  $3 \times \frac{11!}{7!4!}$



$$\frac{(n-1)! \cdot n!}{(n!)^2}$$



$$\frac{88!}{90!}$$



$$\frac{(4-1)!}{4!}$$



$$\frac{38! \cdot 3!}{39!}$$



$$\frac{(n+5)!}{(n+1)!}$$



$$\frac{(2 \cdot 3)!}{3!}$$



$$\frac{77! \cdot 2!}{78!}$$

$$\Rightarrow \frac{10!}{12!}$$

$$\Rightarrow \frac{3!4!}{6!}$$

$$\Rightarrow \frac{16 \cdot 15 \cdot 14 \cdot 13}{20!}$$

$$\Rightarrow \frac{(8! + 7!)(6! + 5!)}{(8! - 7!)(6! - 5!)}$$

$$1) \frac{(6 - 2!)!}{4!}$$

$$2) 6! + (-3 \times 5!)$$

$$3) 9 - 2!$$

$$4) (3!)!$$

$$5) \frac{18!}{16!}$$

$$6) -35 + 0! + 7$$

$$7) 25 - 5! - 1!$$

$$8) 10 \times 3!$$

$$9) \frac{14!}{13!} \div \frac{7!}{6!}$$

$$10) 4! 2! + 40$$

$$11) 5! + 16$$

$$12) \frac{22!}{19! 8!}$$



1)  $4!$

2)  $8!$

3)  $7!$

4)  $\frac{4!}{3!}$

5)  $\frac{6!}{1!}$

6)  $\frac{6!}{4!}$

7)  $\frac{6!}{4!2!}$

8)  $\frac{5!}{2!2!}$

9)  $\frac{7!}{3!2!}$

10)  $\frac{6!}{(5-3)!3!}$

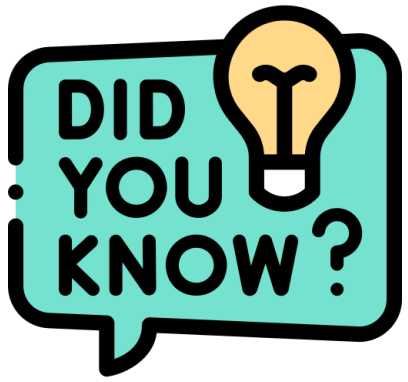
11)  $\frac{7!}{(7-4)!4!}$

12)  $\frac{4!}{(4-1)!1!}$



Answers: 1) 24 2) 40320 3) 5040 4) 4 5) 720 6) 30 7) 15 8) 30 9) 420 10) 60 11) 35 12) 4





**What makes a good life?  
Lessons from the longest study  
on happiness**



Robert Waldinger

What keeps us healthy and happy as we go through life?



<https://www.youtube.com/watch?v=8KkKuTCFvzI>