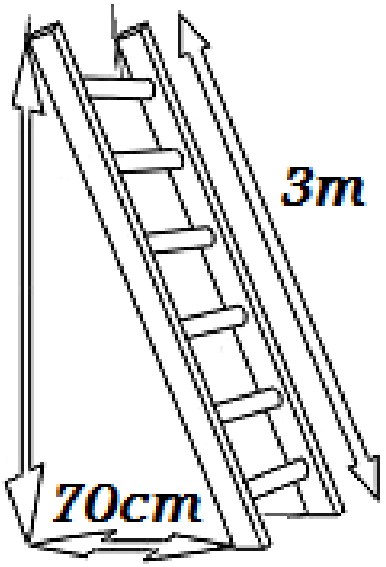
Problems for Revision

- Find the measure of the hypotenuse of a right-angled triangle whose sides measure square root of 2 and square root of 3.

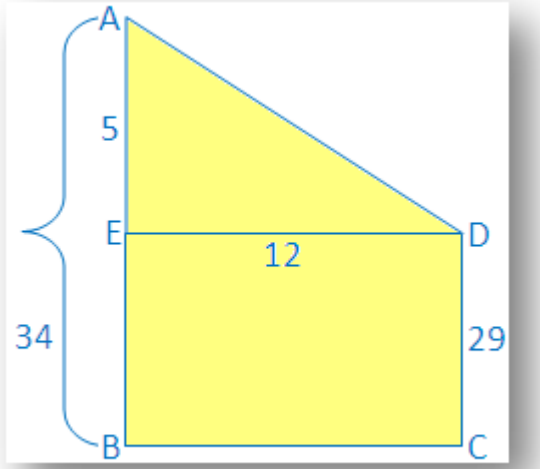
- Calculate the height that we can reach with a 3m ladder leaning against a wall if the bottom of the ladder is 70cm from the wall.



Factorization of zero: $(3\sqrt{4}y - 4)(\frac{4}{5}y + 12)$

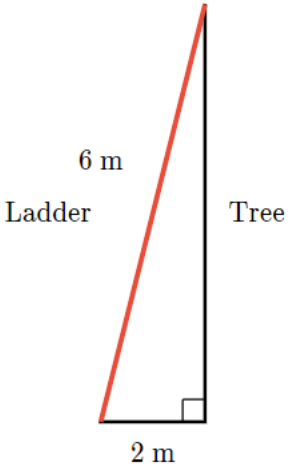
Rationalize the denominator: $\frac{2}{\sqrt{5}-\sqrt{x}}$

The height of two building is 34 m and 29 m respectively. If the distance between the two building is 12 m, find the distance between their tops.



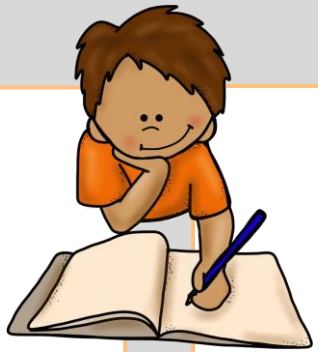
Find the perimeter of a rectangle whose length is 150 m and the diagonal is 170 m.

Ahmed has a 6 meter ladder that he wants to use to get his cat out of a tree. He puts the base of the ladder 2 meters away from the base of the tree. How high into the tree will the ladder reach?



A flying squirrel lives in a nest that is 16 feet high in a tree. To reach a fallen acorn that is 12 feet from the base of the tree, how far will the flying squirrel have to glide?

Negative six times a number decreased by 19 is no more than the seven times the number increased by 23. Find the number.



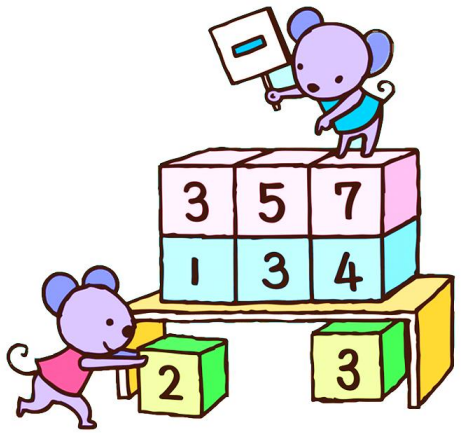
- $2(4x-1)<38$
- $-9-\frac{x}{2}\geq\frac{x}{2}+1$
- $1-\frac{3}{2}x\geq x-4$
- $4(x+1)<2x+3$

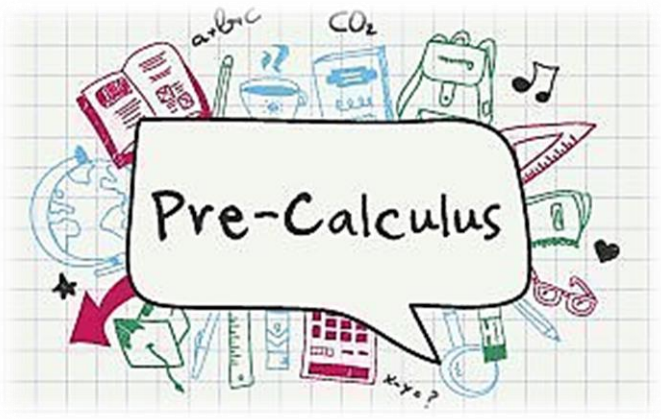
- $\frac{6}{2-\sqrt{3}}$
- $\frac{3}{2-\sqrt{2}}$
- $\frac{2}{-3+\sqrt{7}}$
- $\frac{5}{\sqrt{5}+4}$

- $(2\sqrt{3}x + 5)\,(x^2 - 9)(12x - 10)\left(\frac{3}{4}x + \frac{11}{16}\right) = 0$

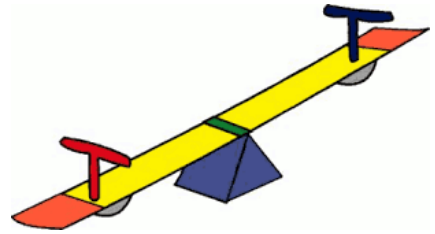
- $\left(\frac{7}{8}y - 5\right)(3x^3 - 81)\left(3x + 1\frac{5}{13}\right)\left(\sqrt{3}x - 1\right)(12x + 12) = 0$

- $\left(3x - \frac{2}{5}\right)\left(\sqrt{5}x - 10\right)(x^3 - 64) = 0$

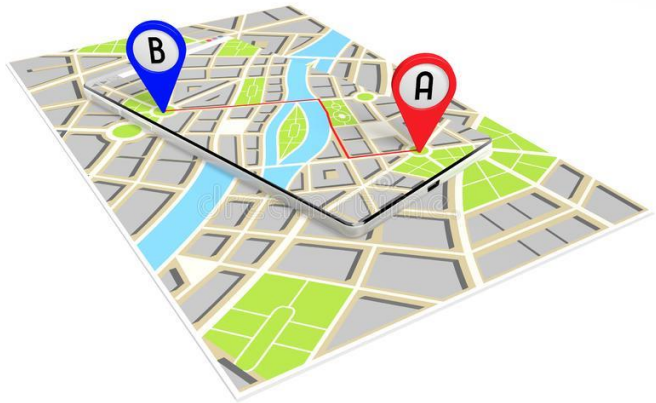




Lecture 3



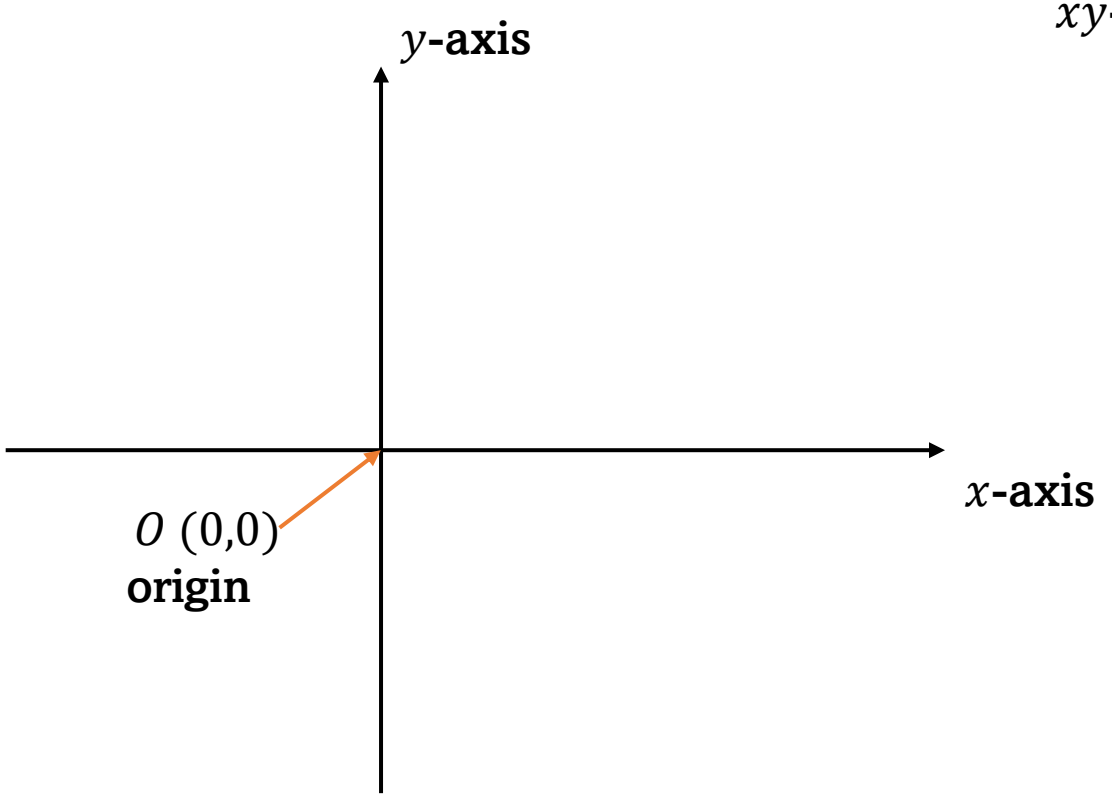
- Distance and Midpoint
- Absolute Value: equation & inequality
- Line Equation
- Systems of Equations



$$|x|$$

Ms. Togzhan Nurtayeva
Course Code: IT 161/A
Semester 1
Week 5-6
Date: 09.01.2024

The Cartesian Plane



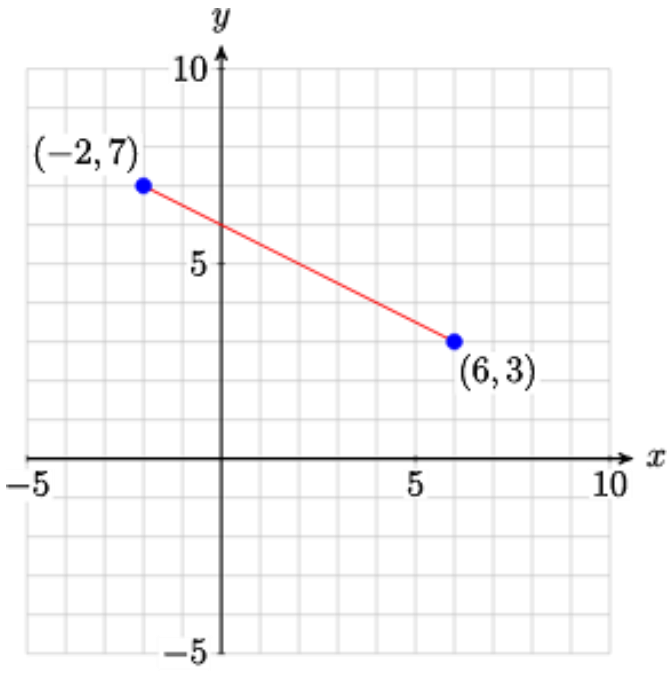
xy-plane – Cartesian plane

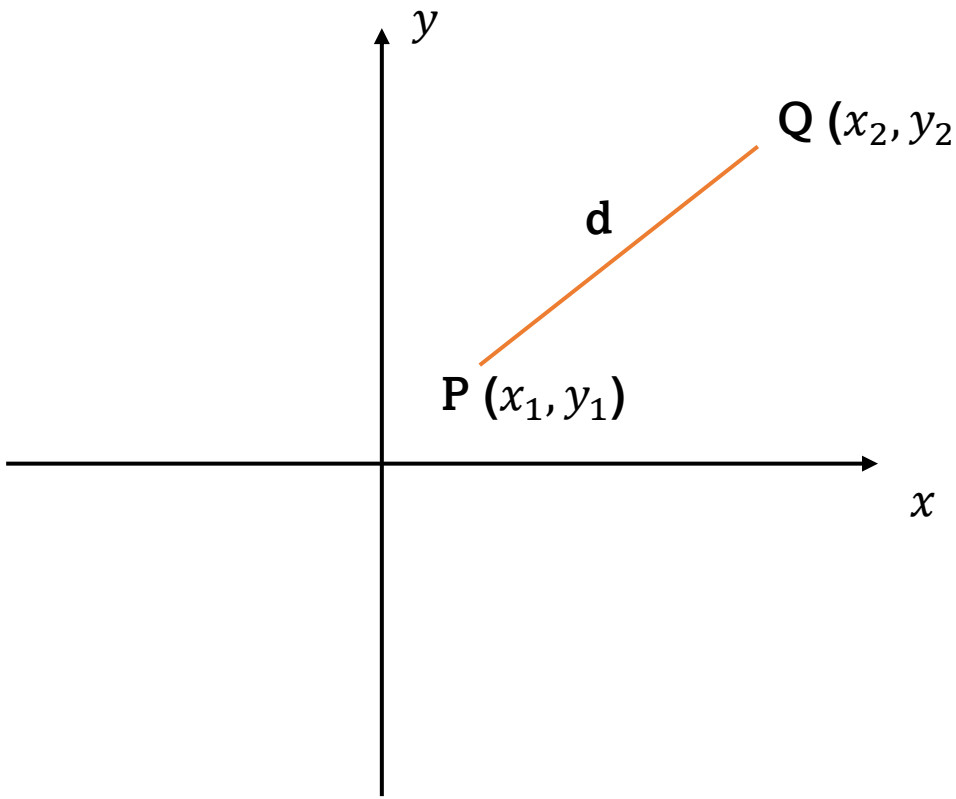
Plot points on a graph:

$P\ (2, 1)$
$Q\ (6, 4)$
$R\ (6, 1)$

Distance

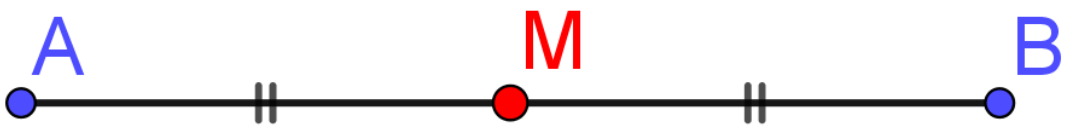
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$





Midpoint

$$\mathbf{m}\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



Find midpoint of a segment PQ, where $P\ (-2, 4)$ and $Q\ (4, -2)$.

- P (1, 2)
  Q (5, 6)
  R (a, 2) – at this point occurs right-angle
  Find a. Find the length of PQ, QR and PR.

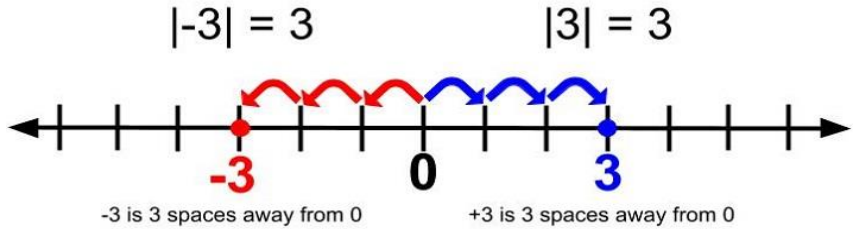
- A (0, 2)
  B (4, 5)
  C (4, 2)
  Find the length of AB, BC, and AC.

- Find distance between two points (1, 3) and (3, 12).

- Find distance between two points (2, 3) and (8, 27). Find midpoint of the line segment.

- Find midpoint of the line segment between two points (1, 4) and (3, 10).

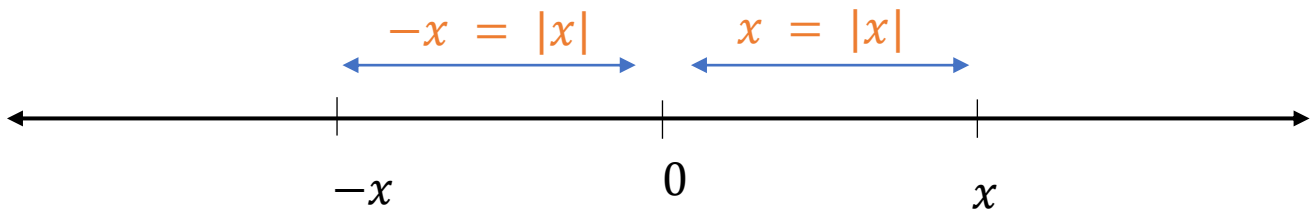
Distance and Absolute Value

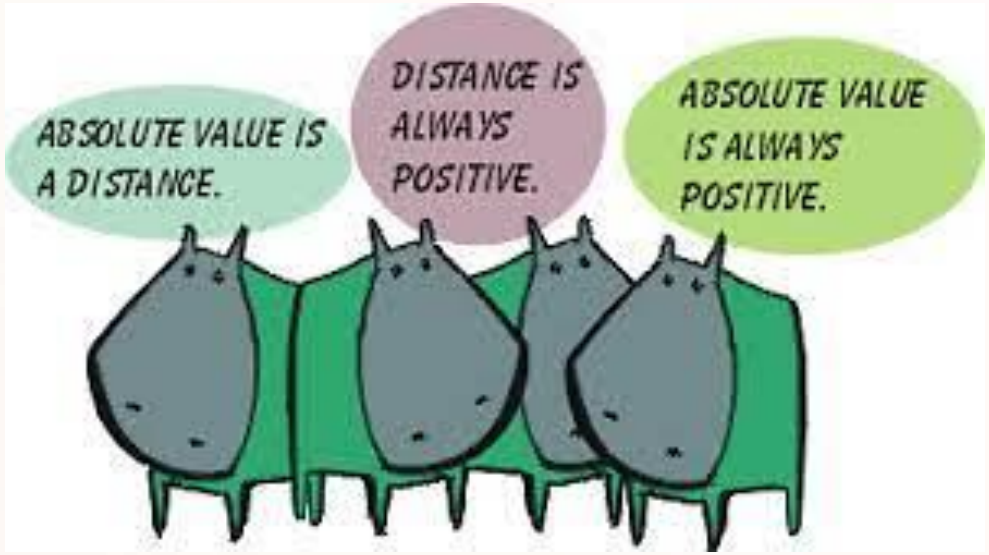


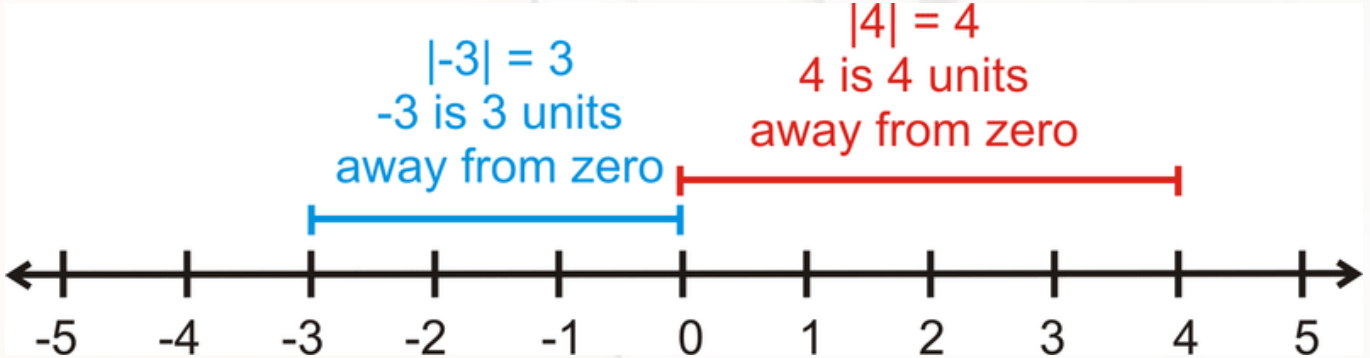
- **Let** $x \in \mathbb{R}$
- **Define the absolute value or magnitude of** x **to be**

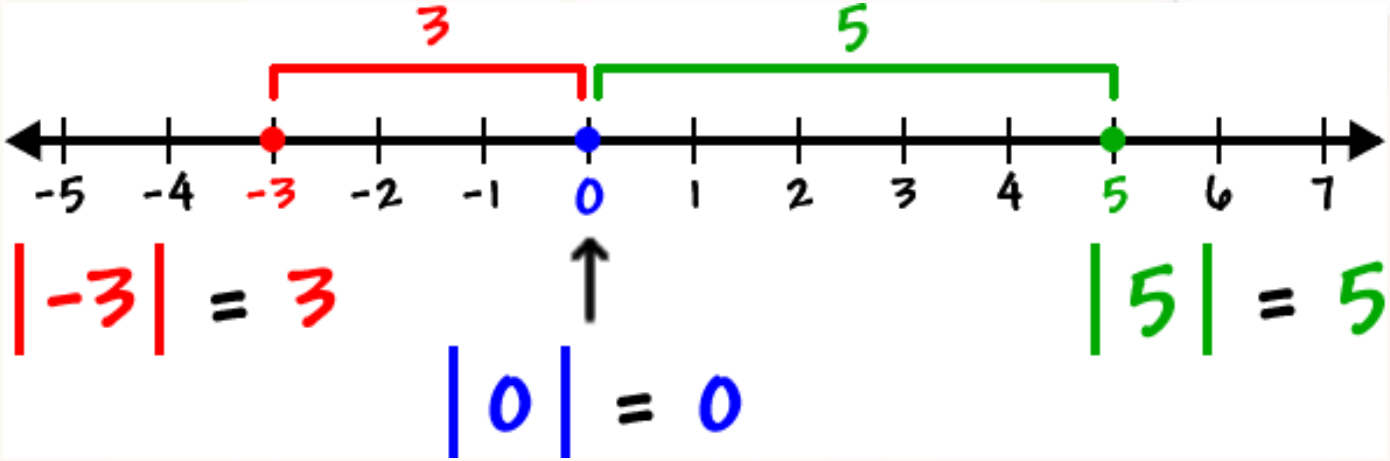
$$|x| = \begin{cases} x, if\ x \geq 0 \\ x, if\ x < 0 \end{cases}$$

= $= |x|$ **distance between** x **and** 0 **on the real line.**




ABSOLUTE VALUE IS A DISTANCE.
DISTANCE IS ALWAYS POSITIVE.
ABSOLUTE VALUE IS ALWAYS POSITIVE.


|-3| = 3
-3 is 3 units
away from zero
|4| = 4
4 is 4 units
away from zero
-5 -4 -3 -2 -1 0 1 2 3 4 5


3
5
-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
|-3| = 3
|5| = 5
|0| = 0

Absolute Value Equation

$$|5 - 2x| - 11 = 0$$

$$|5 - 2x| = 11$$

Isolate the absolute value

Split the equation up into two separate equations

$$5 - 2x = 11$$
$$-2x = 6$$
$$x = -3$$

$$5 - 2x = -11$$
$$-2x = -16$$
$$x = 8$$

Solve each of the equations

Absolute Value Inequality

$$|x - 4| < 7$$

$$x - 4 < 7$$

$$x - 4 + 4 < 7 + 4$$

$$x < 11$$

$$|9 - 4| = |5| = 5$$

$$5 < 7$$

$$x - 4 > -7$$

$$x - 4 + 4 > -7 + 4$$

$$x > -3$$

$$|-2 - 4| = |-6| = 6$$

$$6 < 7$$

$$-3 < x < 11$$

Absolute Value on both sides

$$|2x + 1| = |3x - 5|$$

$$|number| = |number|$$

Check each case that is true:

$\|number\| = \|number\|$	$\|number\| = \|-number\|$
$\|-number\| = \|-number\|$	$\|-number\| = \|number\|$

IMPORTANT

$|2x+1| \geq -5$ **All real numbers**. The absolute value will always be greater than zero.

$|8-x| \leq -3$ **No solution**. The absolute value will never be less than zero. Just like absolute value cannot be = to a negative number.

Example:

$$|x+3| < -2$$

$$|5+3| < -2$$

$$|8| < -2$$

$$8 < -2 \quad \text{False}$$

If an absolute value equation is equal to zero,

$|x+5| = 0$

there is one solution.

$|x| = -3$

NO SOLUTION
An absolute value can never equal a negative number

Two Solutions	One Solution	No Solutions
$\|x\| = 6$	$\|x\| = 0$	$\|x\| = -6$
$\|2x-5\| = 8$	$\|2x-5\| = 0$	$\|2x-5\| = -8$
$\left\|\frac{2}{3}x-7\right\| = 23$	$\left\|\frac{2}{3}x-7\right\| = 0$	$\left\|\frac{2}{3}x-7\right\| = -23$

Solve the equation.

$\frac{1}{4}|2x-6|+1=2$

$-3|x-1|-6=3$

$|x-7|+2=2$

$|3x+2|=|x-6|$

$|x-4|=|4-x|$

Solve the inequality.

$2|x-9|+6>6$

$-4|3x-1|\geq 8$

$-5|2x+2|-3\geq -3$

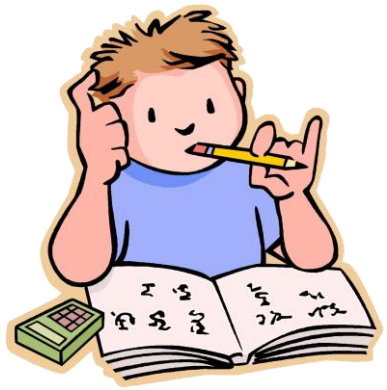
$-10+\frac{1}{2}|x-4|\geq -10$

$3\left|\frac{1}{2}x+2\right|+6<15$

$\frac{1}{4}\|2x-6\|+1=2$	$\{1,5\}$
$-3\|x-1\|-6=3$	No Solution
$\|x-7\|+2=2$	$\{7\}$
$\|3x+2\|=\|x-6\|$	$\{-4,1\}$
$\|x-4\|=\|4-x\|$	$\mathbb{R}$

$2\|x-9\|+6>6$	$(-\infty, 9)\cup(9,\infty)$
$-4\|3x-1\|\geq 8$	No Solution
$-5\|2x+2\|-3\geq -3$	$\{-1\}$
$-10+\frac{1}{2}\|x-4\|\geq -10$	$\mathbb{R}$
$3\left\|\frac{1}{2}x+2\right\|+6<15$	$(-10,2)$

- $|\,6x + 4\,| = 8x + 10$
- $|\,3x - 1\,| + 5 = 3$
- $|\,x - 2\,| = 2x - 3$
- $|\,12 - 6x\,| = 42$
- $|\,2b - 4\,| = 2b - 4$
- $|\,2a + 1\,| = a + 5$
- $-3|x + 5| + 1 = 7|x + 5| + 8$
- $5\,|\,c - 2\,| = 30$
- $|x - 5| > 7$
- $|2x + 3| \leq 12$
- $2|x - 6| \geq 24$
- $|4x - 1| - 11 < 20$
- $-3|x + 2| \leq -9$
- $\frac{|3x-3|}{-5} > -12$
- $8 + |4v - 7| \geq 17$



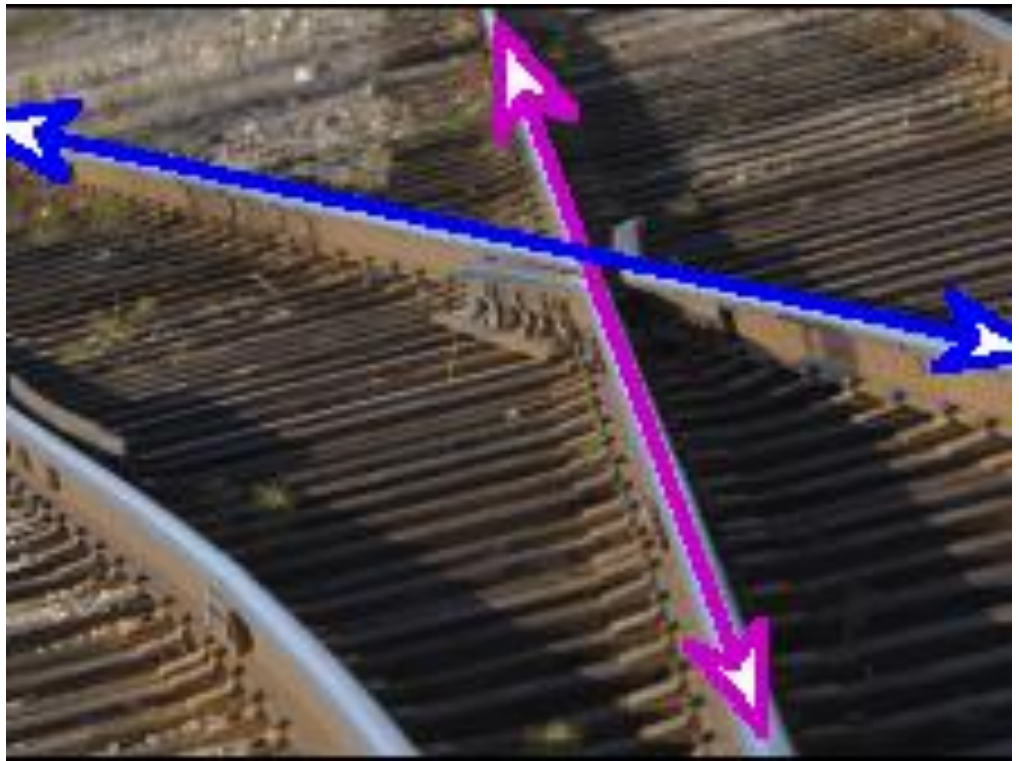
Lines

- A line is a one-dimensional figure, which has length but no width.
- A line is a set of collinear points with no curves and extends limitlessly in opposite directions is called a Straight line.

- General equation:

$$ax + by = c \qquad a, b, c \ - constants$$

variables



How do we find the equation of a line?

- $ax + by = c$
- $by = -ax + c$
- $y = \frac{-ax+c}{b}, \qquad b \neq 0$
- $y = \left(-\frac{a}{b}\right)x + \frac{c}{b}$ $\rightarrow$ $y = mx + k$

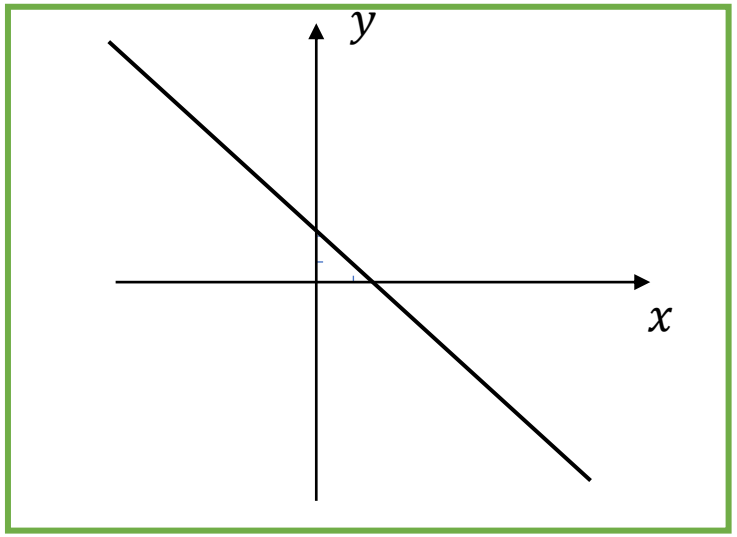
Lines

- Sketch the line.

$$2x + 3y = 6$$

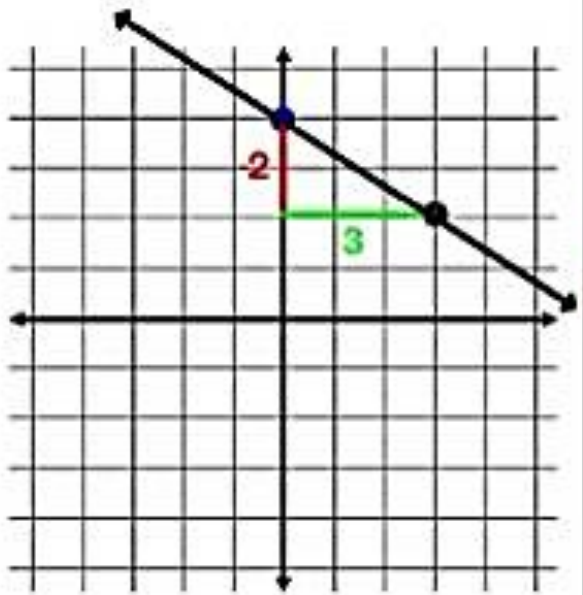
$$3y = 6 - 2x$$

$$\boxed{y = -\frac{2}{3}x + 2}$$



- A line is determined by two points.

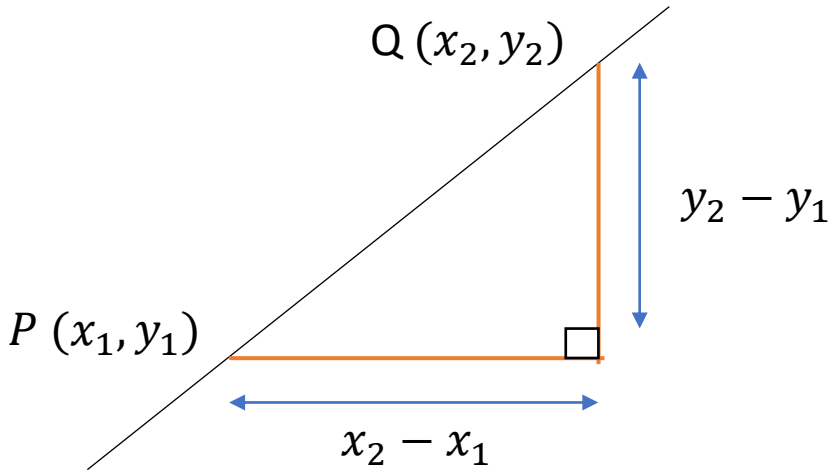
x	y
0	2
1	$\frac{4}{3}$

Graph	Equation / Verbal Explanation	Table
	vertical change $y = \frac{-2}{3}x + 4$ horizontal change y-intercept The line has a slope of $\frac{-2}{3}$ and passes through the (0 , 4)	horizontal change x / vertical change y 0, 4 3, 2 (3 → -2) 6, 0 9, -2

Slope

The slope or gradient of a line is a number that describes both the direction and the steepness of the line.

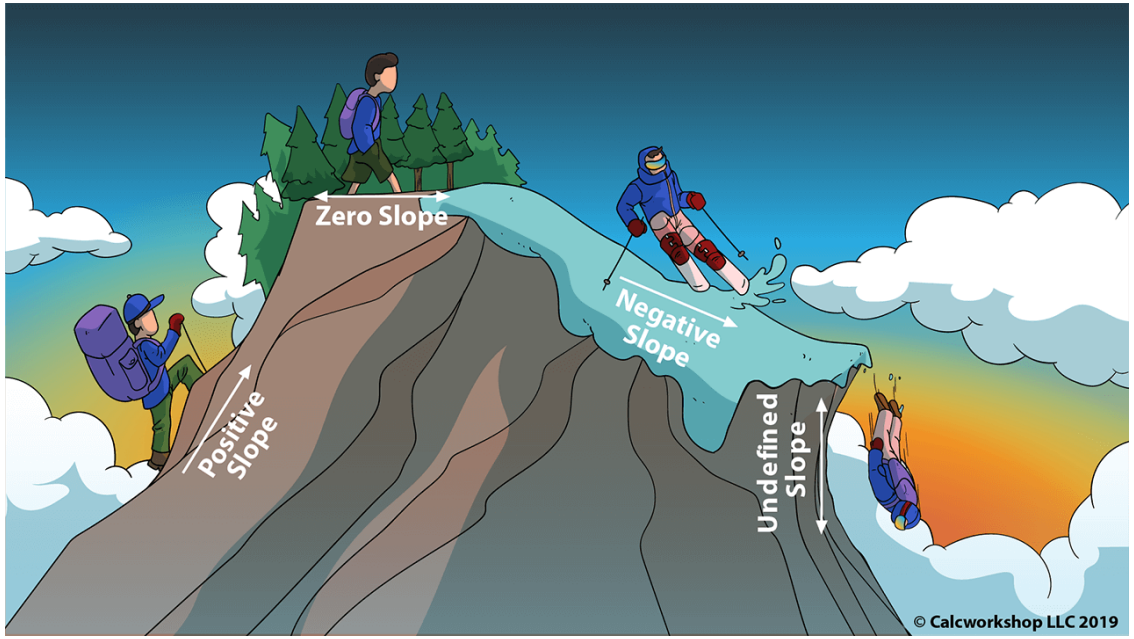
$$m = slope = \frac{vertical\ rise}{horizontal\ run}$$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In mathematics, the slope or gradient of a line is a **number that describes both the direction and the steepness of the line.**

Slope tells you how steep a line is, or how much y increases as x increases. The slope is constant (the same) anywhere on the line.



Some real life examples of slope include:

- ✓ in building roads one must figure out how steep the road will be.
- ✓ skiers/snowboarders need to consider the slopes of hills in order to judge the dangers, speeds, etc.
- ✓ when constructing wheelchair ramps, slope is a major consideration.

- **Find the slope of the line passing through P (2, 2) and Q (5, 6)**

P(x_1, y_1) Q (x_2, y_2)

$$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{5 - 2} = \frac{4}{3}$$

- **Find the slope where P (-1, 2) and Q (5, -4)**

Slope

- Positive
- Negative
- Zero
- Infinite

The y-intercept and the x-intercept

Intercepts are where lines on graphs cross axes.

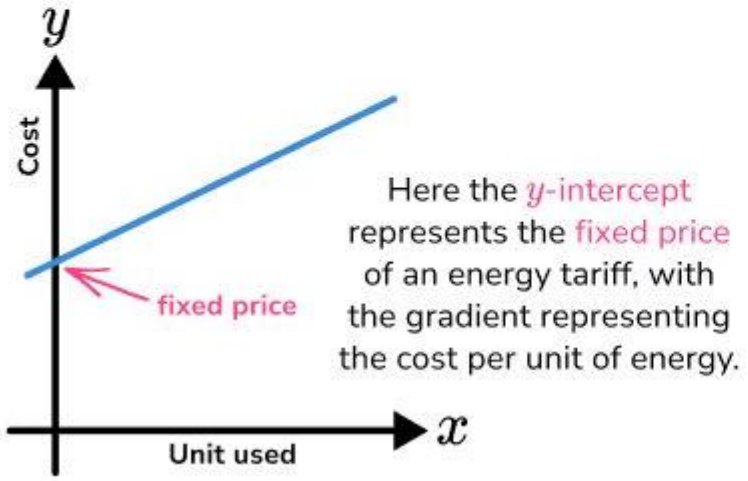
The y-intercept is the value of y when the x coordinate is 0

To find the y-intercept, substitute $x = 0$

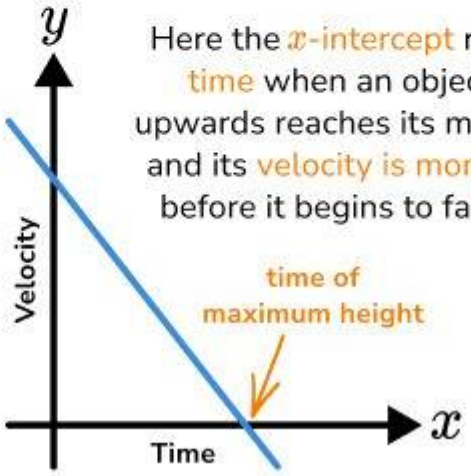
The x-intercept is the value of x when the y coordinate is 0

To find the x-intercept, substitute $y = 0$

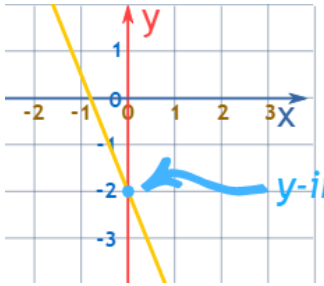
Examples



Here the y-intercept represents the fixed price of an energy tariff, with the gradient representing the cost per unit of energy.



Here the x-intercept represents the time when an object projected upwards reaches its maximum height and its velocity is momentarily zero, before it begins to fall back down.



Find the equation of line L passing through P (2, 3) and Q (7, 13)

1) find a slope.

2) put coordinates of any of given points and find constant k.

1. $m = \frac{13-3}{7-2} = \frac{10}{5} = 2 \quad (slope)$

2. $y = mx + k$ – *line equation*

- $y = 2x + k \qquad m = 2$
- *P(2,3):* $3 = 2 \times 2 + k$

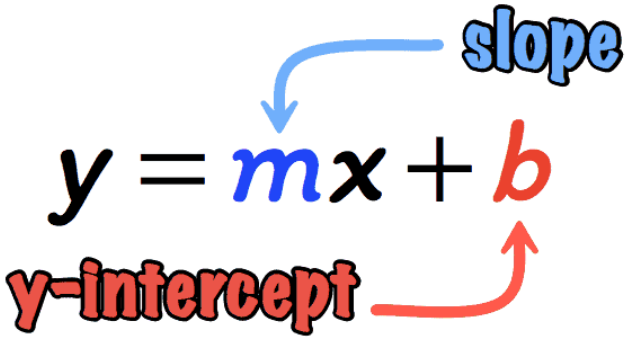
$$3 = 4 + k$$

$$k = 3 - 4$$

$$k = -1$$

- $m = 2, k = -1$

$y = 2x - 1$

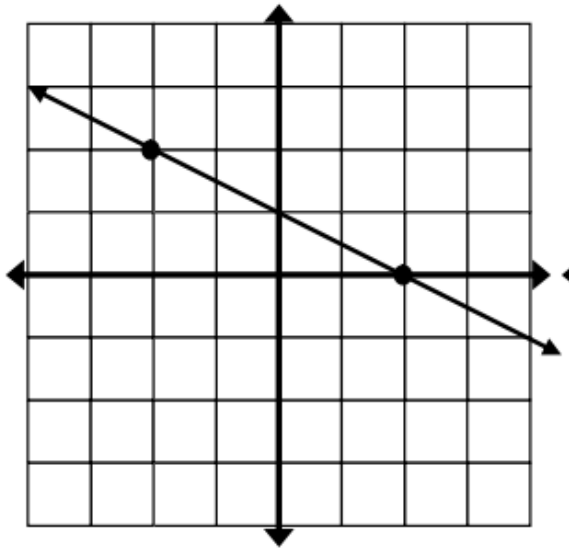


- P (2, 5) and Q (4, 9) – find a line equation.
- A (2, 0) and B (8, 3) – find a line equation.
- W (-2, 5) and Y (4, -9) – find slope.

- What is the slope of a line that runs through points: (-2, 5) and (1, 7)? Write the line equation.
- A line passes through the points (–3, 5) and (2, 3). What is the slope of this line and line equation?

1) For each graph: Write the equation of the line in SLOPE-INTERCEPT FORM.

a.

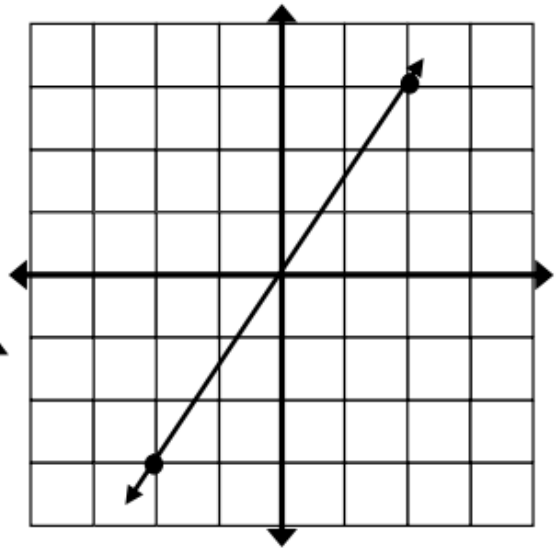


Slope = _____

y-intercept = _____

equation: _____

b.

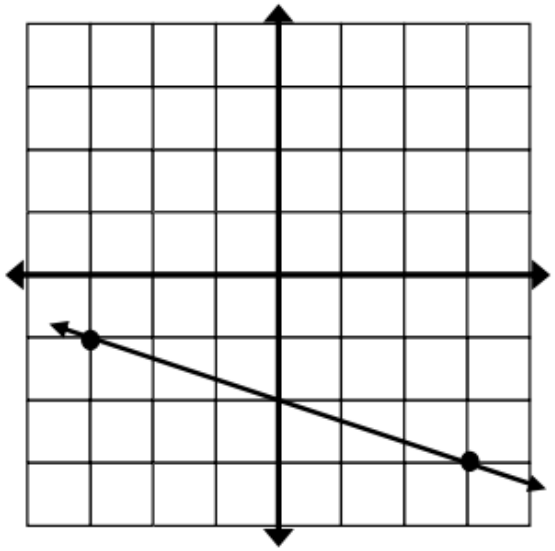


Slope = _____

y-intercept = _____

equation: _____

c.

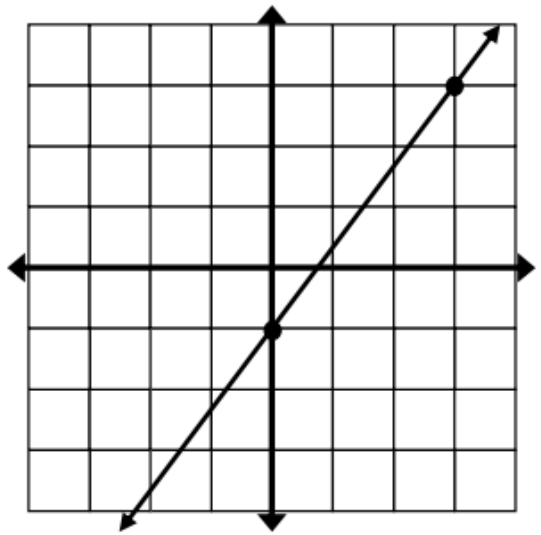


Slope = _____

y-intercept = _____

equation: _____

d.

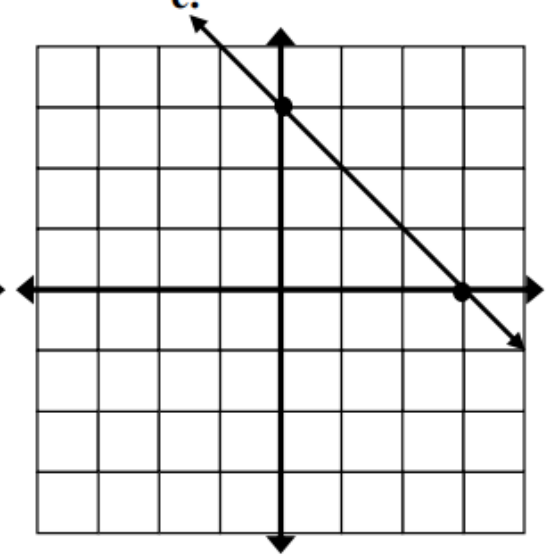


Slope = _____

y-intercept = _____

equation: _____

e.

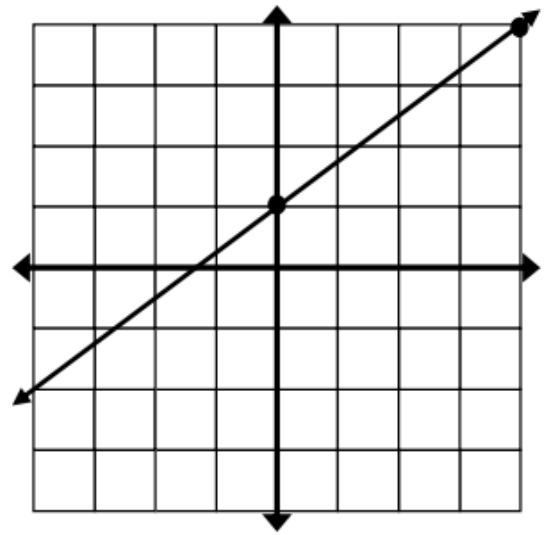


Slope = _____

y-intercept = _____

equation: _____

f.



Slope = _____

y-intercept = _____

equation: _____

2) Find the slope of the line through each pair of points. $Slope = \frac{y_2 - y_1}{x_2 - x_1}$

a. (8, -7) and (5, -3)

b. (-5, 9) and (5, 11)

c. (-8, -4) and (-4, -9)

d. (-4, 3) and (-6, -8)

e. (-7, -1) and (-7, 2)

f. (9, 4) and (-6, 4)

3) Tell whether each slope is positive, negative, zero, or undefined.

a.

_____

b.

_____

c.

_____

d.

_____

4) **For each linear equation, identify the slope (m) and the y-intercept (b)**

a. $y = 4x - 5$

b. $y = 11 + \frac{2}{3}x$

c. $y = \frac{2}{3} - x$

d. $6 - \frac{9}{2}x = y$

e. $y = \frac{5}{2}x - \frac{19}{8}$

f. $-\frac{5}{4} - \frac{2}{7}x = y$

Practice Problems

1. Find the equation of the line that passes through the point (1, 4) and has a slope of 12.
2. Find the equation of the line that passes through the point (1, 4) and has a slope of 2.
3. Find the equation of the line that passes through the point (27, 4) and has a slope of $\frac{-2}{9}$.
4. Find the equation of the line that passes through the point $(-11, 2)$ and has a slope of $\frac{-5}{11}$.
5. Find the equation of the line that passes through the point (10, 6) and has a slope of $\frac{1}{5}$. What is the y-intercept of the line?
6. Find the equation of the line that passes through the point (3, 29) and has a slope of 6. What is the y-intercept of the line?

Systems of Equations

$$\begin{cases} 2x+3y=-12 \\ -x-3y=18 \end{cases}$$

$$\begin{cases} x-y=11 \\ 2x+y=19 \end{cases}$$

$$\begin{cases} 7x-y=-10 \\ -7x+5y=-6 \end{cases}$$

$$\begin{cases} x+3y=18 \\ -x-4y=-25 \end{cases}$$

$$\begin{cases} -6x+5y=1 \\ 6x+4y=-10 \end{cases}$$

$$\begin{cases} -7x-y=13 \\ 8x+y=-14 \end{cases}$$

Systems of Equations

① $\begin{cases} -6x - 8y = -28 \\ 9x + 5y = -14 \end{cases}$

② $\begin{cases} -9x + 3y = 27 \\ -3x + 4y = 27 \end{cases}$

③ $\begin{cases} -30x + 4y = 2 \\ 15x - 12y = -81 \end{cases}$

④ $\begin{cases} -5x + 5y = -25 \\ 3x + 2y = 10 \end{cases}$

⑤ $\begin{cases} -6x = 18 - y \\ 9 = -3x - 8y \end{cases}$

⑥ $\begin{cases} 10x + 12y = -26 \\ -6x + 6y = -24 \end{cases}$

⑦ $\begin{cases} 18x - 6y = 30 \\ -9x - y = -19 \end{cases}$

⑧ $\begin{cases} 3x - 5y = -17 \\ 2x + 15y = 7 \end{cases}$

[1] $\begin{cases} 5x + y = 9 \\ 10x - 7y = -18 \end{cases}$

[2] $\begin{cases} 5x - 3y = 2 \\ -5x + 3y = 8 \end{cases}$

[3] $\begin{cases} 2x = -3y + 16 \\ 5x - 4y = -6 \end{cases}$

[4] $\begin{cases} 6x + 6y = -6 \\ 5x + y = -13 \end{cases}$

[5] $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

[6] $\begin{cases} -2x - 9y = -25 \\ -4x - 9y = -23 \end{cases}$

[7] $\begin{cases} 4x + 8y = 12 \\ 2x + 4y = -6 \end{cases}$

[8] $\begin{cases} -7x + y = -19 \\ -2x + 3y = -19 \end{cases}$

Solve and show a solution set for each problem.

- $|-6x+3|=27$
- $2|3x-1|-1=7$
- $2|3x-1|-1\leq 7$
- $|-2x+7|+5\geq 14$

- $A(5,-6)\ and\ B\ (-3,1)$

$find\ midpoint\ coordinates\ and\ distance\ between\ given\ two\ points.$