Attempt all problems. Show all of your work.

Practice and Theory Questions.

Question 1:

a) Input: $arr[\,] = \{13,\ 10,\ 94,\ 20,\ 4, 14\}$; using algorithm finding *min* element (pseudocode) show every step of finding minimum value of given array list. Draw a flowchart.

b) Write a pseudocode for the sum of first 5 natural numbers. Draw a flowchart.

Question 2:

$18,\ 11,\ 5,\ 6,\ 7,\ 3,\ 9,\ 1$

Using linear search algorithm find the location of $x = 9$ and $x = 12$ (show every step using pseudocode). Create a flowchart of your program.

Question 3:

$20,\ 13,\ 5,\ 6,\ 7,\ 3,\ 9,\ 1$

Using binary search algorithm find the location of $x = 9$ and $x = 12$ (show every step using pseudocode). Create a flowchart of your program.

Question 4:

$5, 2, 6, 1, 7$

Using bubble sort algorithm sort given array list in decreasing order (show every step using pseudocode). Create a flowchart of your program.

Question 5:

Write an algorithm that finds the second-smallest element among a, b, and c. Assume that the values of a, b, and c are distinct. (Hint: use (both) sorting algorithms and then find second-smallest element; create 2 pseudocodes)

Question 6:

Describe an algorithm that takes as input a list of n integers and finds the location of the last even integer in the list or returns 0 if there are no even integers in the list.

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Question 7:

Describe an algorithm that locates the first occurrence of the largest element in a finite list of integers, where the integers in the list are not necessarily distinct.

Question 8:

5, 2, 6, 1, 7, 3

Using insertion sort algorithm sort given array list in increasing order (show every step using pseudocode). Create a flowchart of your program.

Question 9:

Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for:

a) 51 cents
b) 69 cents
c) 76 cents
d) 60 cents

Question 10:

Use Algorithm 7 to schedule the largest number of talks in a lecture hall from a proposed set of talks, if the starting and ending times of the talks are 9:00 A.M. and 9:45 A.M.; 9:30 A.M. and 10:00 A.M.; 9:50 A.M. and 10:15 A.M.; 10:00 A.M. and 10:30 A.M.; 10:10 A.M. and 10:25 A.M.; 10:30 A.M. and 10:55 A.M.; 10:15 A.M. and 10:45 A.M.; 10:30 A.M. and 11:00 A.M.; 10:45 A.M. and 11:30 A.M.; 10:55 A.M. and 11:25 A.M.; 11:00 A.M. and 11:15 A.M.

Question 11:

Write a pseudocode for the Factorial of a Number and Fibonacci series using Recursion.

Question 12:

Show that $2x^3 - 7x^2 + 5$ is O(x^3) by finding the witnesses, C and k.

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Question 13:

Suppose a computer can perform 10^{12} bit operations per second. Find the largest problem size that could be solved in 1 second if an algorithm requires:

a) n^5 bit operations
b) 3^n bit operations

Question 14:

What is time and space complexity of given algorithms below?

a) $procedure\ function()$: [Hint: dependent loop. Create a table and find a formula for the output.]
$int\ a[m][m]$
$sum\ =\ 0$
$for\ i\ =\ 1\ to\ m\ \{$
$for\ j\ =\ i\ to\ m$
$sum\ +=\ a[i][j]$
$print(sum)\ \}$

b) $procedure\ function(n)$
$answer\ =\ 0$
$while\ (n\ >\ 0)\ \{$
$answer\ +=\ n$
$n\ /=\ 7;$
$print(answer)\ \}$

c) $procedure\ function()$ [Hint: dependent loop. Create a table and find a formula for the output.]
$answer\ =\ 0$
$for\ i\ =\ 1\ to\ n\ \{$
$for\ j\ =\ 1\ to\ log(i)$
$answer\ +=\ 1$
$print(answer)\ \}$

d) {
$int\ i;$
$for\ (i = 1;\ i < \mathrm{n}^5;\ i = i*7)$
$print$ (“$Hello\ World$”); }

e) for (i = 1; i ≤ n; i++){
for (k = 1; k <= i; k = k + 1)
$print$ (“$Hello\ World$”); }

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f)
```
{
int i;
for (i = 1; i^3 < n^2; i ++)
print ("Hello World"); }
```

g)
```
for (int i = 0; i * i < n; i + +){
  for(int j = 0; j * j < n; j + +){
    //do something
  }
}
```

h)
```
for (i = 1; i ≤ 10 i + +){
for (k = 1; k <= 15; k = k + 1)
print ("Hello World"); }
```

i)
```
for (i = 1; i ≤ n; i + +){
//do something
}

for (j = 1; j <= k; j = j + 1){
//do something
}
```

Question 15:

Explain all possible algorithms for greedy optimization algorithm for scheduling and which one is the most optimal and why.

Question 16:

a) Give a formula for triangular numbers (show first 7 elements of triangular numbers with explanation).
b) Give a formula for Fibonacci numbers (show first 10 elements with explanation).
c) How does Hanoi Tower work? (**explain**).
d) Give a **pseudocode and diagram** for the Factorial (show how it works).

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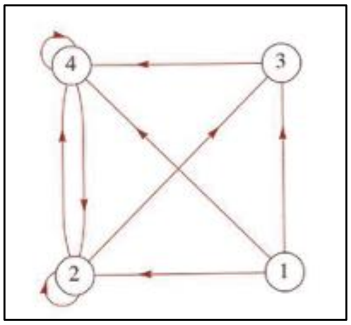
Question 17:

a) Write a relation from a given digraph. b) Determine properties. c) Write in a matrix form.



Question 18:

Using proof methods, prove statements below. (Solve at least 8 proofs (2 from each method))

a) Prove that if p, q are positive integers such that $p|q$ and $q|p$, then $p = q$. (direct)
b) Prove that if n is odd, then n^3 is odd. (direct)
c) Prove that $\sum_{i=1}^{n}(2i-1) = n^2$. (induction)
d) Prove that $\sum_{i=1}^{n}\frac{1}{\sqrt{i}} \le 2\sqrt{n}$. (induction)
e) Let $n \in Z$. Prove that if $1 - n^2 > 0$, then $3n - 2$ is an even integer. (direct)
f) $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ (induction)
g) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using a proof by contradiction.
h) Prove that $\sqrt{3}$ is irrational by giving a proof by contradiction.
i) Prove $(2n+9)^2 - (2n+5)^2$ is always a multiple of 4. (direct)
j) The expression $2k^2 - 1$ is odd for all integers k. (by cases)
k) Give a proof by cases that for integers m, n, we have $|mn| = |m||n|$.
l) Prove that $|x-1| + |x+5| \ge 6$ for all real numbers x. (by cases)
m) There exist no integers a and b for which $21a + 30b = 1$. (by contradiction)

Question 19:

a) Why do we need relations? Where do we use it?
b) Why do we need proofs? Where do we use it?

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Question 20:

Corncob College elects 10 students to serve as officers on 8 committees. The list of the members of each of the committees is:

- Corn Feed Committee: *Darcie, Barb, Kyler*
- Dorm Policy Committee: *Barb, Jack, Anya, Kaz*
- Extracurricular Committee: *Darcie, Jack, Miranda*
- Family Weekend Committee: *Kyler, Miranda, Jenna, Natalie*
- Homecoming Committee: *Barb, Jenna, Natalie, Skye*
- Off Campus Committee: *Kyler, Jenna, Skye*
- Parking Committee: *Jack, Anya, Miranda*
- Student Fees Committee: *Kaz, Natalie*

They need to schedule meetings for each of these committees, but two committees cannot meet at the same time if they have any members in common.

a) Draw a graph representing this situation. (Hint: let the vertices represent the committees.)

b) How many different meeting times will we need?

Question 21:

The mathematics department has six committees, each meeting once a month. How many different meeting times must be used to ensure that no member is scheduled to attend two meetings at the same time if the committees are

C1 = {Arlinghaus, Brand, Zaslavsky},

C2 = {Brand, Lee, Rosen},

C3 ={Arlinghaus, Rosen, Zaslavsky},

C4 = {Lee, Rosen, Zaslavsky},

C5 = {Arlinghaus, Brand},

C6 = {Brand, Rosen, Zaslavsky}?

Question 22:

Suppose Adam, Ben, Chris, David and Eric are training for tasks at work. Adam and Chris are training for task 1, Ben, Chris and Eric are training for Task 2, David is training for task 3, Chris and Eric are training for task 4 and Eric is training for task 5. Create a graph to model this, then determine if a **matching** is possible.

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Question 23:

A chain letter starts when a person sends a letter to five others. Each person who receives the letter either sends it to five other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. **How many people receive the letter, and how many do not send it out?**

Question 24:

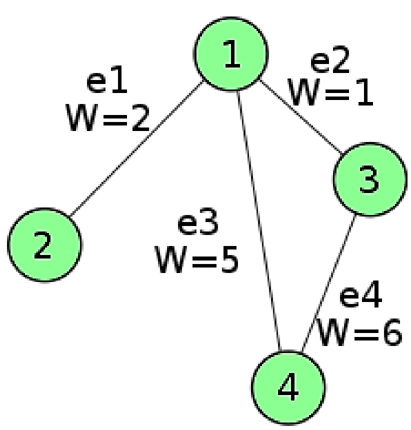
Suppose someone starts a chain letter. Each person is asked to send the letter to four other people. Everyone does it. How many people have seen the letter, including the original sender, if no one receives more than one letter, and the chain ends after 100 people read it but did not send it on. **How many people sent out the letter?**

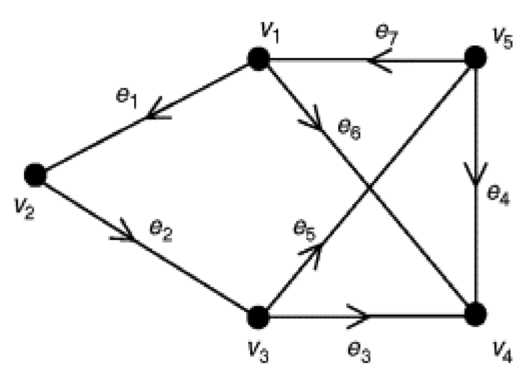
Question 25:

There are 256 players in a chess tournament (singles). Two players play a match. Matches are played on a knockout basis, the loser is eliminated after each match. How many matches need to be played to declare a winner? There is no draw.

Question 26:

Find Incidence matrix:





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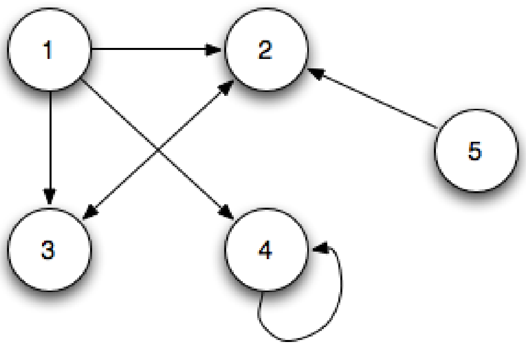
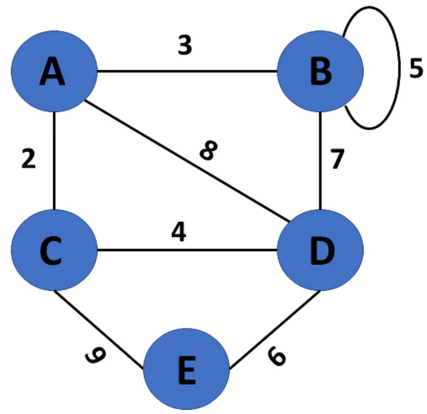
Question 27:

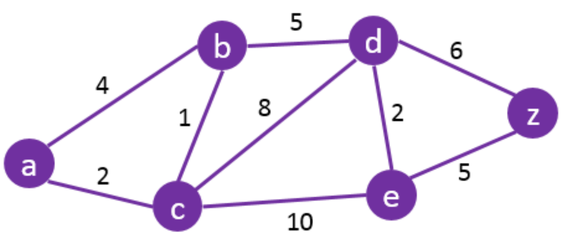
Find adjacency matrix:

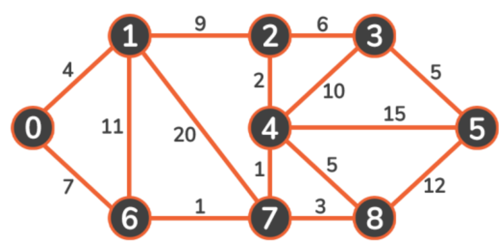




Question 28:

Find the shortest path from vertex A to all other vertices.



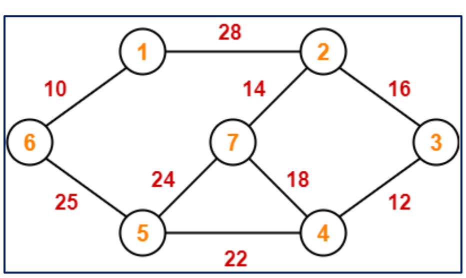


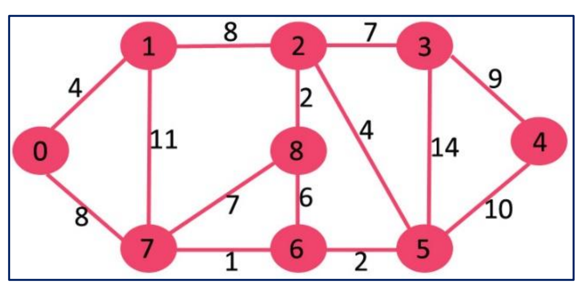
Question 29:

Find MST for both graphs using:

a) Prim's Algorithm

b) Kruskal's Algorithm





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Question 30:

Theory questions:

a) What is the height of a rooted tree?
b) What is balanced and unbalanced trees? Which one makes for the better search time and what is it?
c) What is a full m-ary tree? What is a m-ary tree? What is the difference between them?
d) Define Graph Applications.
e) Give example for the handshaking theorem.
f) What is a flowchart?
g) Define types of graphs.
h) What is a primary key in relations?
i) What is a selection operator?
j) Give example for projections in relations.
k) How join operation works?
l) What is the cardinality of a set? Give examples.
m) Define truth sets. Provide examples.
n) Define quantifiers. Give examples.
o) Find all possible properties of relations for (a, a).
p) Find all possible properties of relations for ∅.
q) Define reflexive, symmetric, antisymmetric, transitive relations. Give examples for each.
r) Define types of proofs.
s) What are the applications of graph coloring?
t) What are the applications of graphs?
u) What are the applications/uses of Dijkstra's, Prim's and Kruskal's algorithms?
v) What is a time and space complexity? Give examples.
w) What is a big O notation?
x) Define greedy algorithms.
y) What is the most optimal scheduling algorithm?
z) Define WOA.
aa) Find a factorial of a number using recursion.
bb) Find a Fibonacci series using recursion.

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Multiple Choice questions:

Select the correct answer.

1) Find a reflexive relation:
   a) (a, a), (a, b), (a, c)
   b) (a, a), (b, b), (c, c)
   c) (a, b), (a, c), (b, c)
   d) (b, a), (a, b), (c, c)

……………………………………………………………

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Matching:

Match each numbered item with the most closely related lettered item. Write your answers in the spaces provided:

Concept		Description
a. An ordered pair of a set $(1, 2)$ is	________	1. $a_{n-1} + a_{n-2}$
b. Towers of Hanoi	________	2. for every
c. $\exists x$ means	________	3. $2^n - 1$
d. Triangular Numbers	________	4. $\{\{a\}, \{a, b\}\}$
e. $\forall x$ means	________	5. $\frac{n(n+1)}{2}$
f. Fibonacci sequence	________	6. there exists

…………………………….…………………………….

True or False questions:

Answer the following with True or False:

a) There are four ways to represent relations: ordered pairs, mapping diagram, table, graph.

b) a divides $b = a|b = b * k = a$

c) In functions same domain may have several ranges.

…………………………………………………………….

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