Chapter 2

Static stability - 6 DOF

2.1 Definitions

Many of the Forces on an aircraft are produced by Lifting Surfaces.

• Major exception is propulsion.

Lifting surfaces are characterized by:

Planform Shape: The shape of the wing when viewed from above.

• Surface area, tapering, etc.

• determines magnitude of forces.

Airfoil: The cross-section of the wing.

- Determines type of forces and moment (Positivity, location, etc)
- 2.1.1 Planform Shapes: The planform shape of the wing will affect
 - Lift
- Drag
- Moment



Figure 2.1: Planform Shapes

Rectangular wing planform. Correction factors can be used for rounded or swept-wing configurations.



Figure 2.2:

Chord, c: The width of the surface at some point. This determines the size of the airfoil. **Root Chord, Cr**: The width of the surface where joined to the airplane.

Tip Chord, Ct: The width of the surface at free-stream.

Span, b: The total length of the surface.

Quarter-Chord Line: The line connecting the points of 1/4 chord along the span of the surface. The 1/4 chord point is approximately the aerodynamic center of an airfoil - to be discussed **Sweep:** The angle the 1/4-chord line makes with the horizontal.

2.2 Coordinate Systems



Figure : 2.3

2.2.1 The Body-Fixed Frame:

- The origin is the center of mass.
- The *x*-axis points toward the front of the aircraft.
- The *z*-axis points down.
- The *y*-axis is perpendicular to the x z plane.
- Use the "right-hand rule" to define y



Orientation between reference Frames

Euler angles: Euler angles represent three composed and finite rotations given in a preestablish order that move a reference frame to a given referred frame. This is equivalent to saying that any orientation can be achieved by composing three elemental and finite rotations (rotations around a single axis of a basis), and also equivalent to saying that any rotation matrix can be decomposed as a product of three elemental rotation matrices.

<u>**Transformation or rotation matrix:**</u> If the three components of a vector \mathbf{A} in \mathbf{F}_{I} are known, the transformation or rotation matrix \mathbf{L}_{FI} expresses a vector \mathbf{A} in the reference \mathbf{F}_{F} system as follows:



The matrix is called a rotation matrix.

Rotation matrices can be used to calculate the effect of ANY rotation.

Roll Angle
$$\phi$$
 :Pitch Angle θ :Yaw Angle ψ : $\vec{v}' = R_1(\phi)\vec{v}$ $\vec{v}' = R_2(\theta)\vec{v}$ $\vec{v}' = R_3(\psi)\vec{v}$

Remember to use the right-hand rule to determine what is a positive rotation.

The rotation matrices are (for reference):

Roll Rota	ation	(ϕ) :	Pitch Rotatio	on (θ):	Yaw Rotati	on (ψ):	
$R_1(\phi)$			$R_2(heta)$		$R_3(\psi)$		
$=\begin{bmatrix}1\\0\\0\end{bmatrix}$	$0 \\ \cos \phi \\ \sin \phi$	$\begin{bmatrix} 0 \\ -\sin\phi \\ \cos\phi \end{bmatrix}$	$= \begin{bmatrix} \cos \theta \\ 0 \\ -\sin \theta \end{bmatrix}$	$\begin{array}{cc} 0 & \sin\theta \\ 1 & 0 \\ 0 & \cos\theta \end{array}$	$= \begin{bmatrix} \cos\psi\\ \sin\psi\\ 0 \end{bmatrix}$	$-\sin\psi\\\cos\psi\\0$	$\begin{bmatrix} 0\\0\\1 \end{bmatrix}$

Rotation matrices, can be used to calculate a sequence of rotations: Roll-Pitch-Yaw:

Roll-Pitch-Yaw:

$$\vec{v}_{RPY} = R_3(\psi)R_2(\theta)R_1(\phi)\vec{v}$$

Note the order of multiplication is critical.

$$\vec{v}_{RPY} = \left(R_3(\psi) \left(R_2(\theta) \left(R_1(\phi) \vec{v} \right)_1 \right)_2 \right)_3$$

Example Consider a pure lift force of 10MN after a pitch up of 10 deg and a yaw of 20 deg.

$$\vec{L} = \begin{bmatrix} 0\\0\\-10 \end{bmatrix}$$
$$\vec{L}_{PY} = \begin{bmatrix} \cos 20 & -\sin 20 & 0\\\sin 20 & \cos 20 & 0\\0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 10 & 0 & \sin 10\\0 & 1 & 0\\-\sin 10 & 0 & \cos 10 \end{bmatrix} \begin{bmatrix} 0\\0\\-10 \end{bmatrix}$$
$$= \begin{bmatrix} 0.9397 & -0.3420 & 0\\0.3420 & 0.9397 & 0\\0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 0.9848 & 0 & 0.1736\\0 & 1.0000 & 0\\-0.1736 & 0 & 0.9848 \end{bmatrix} \begin{bmatrix} 0\\0\\-10 \end{bmatrix}$$
$$= \begin{bmatrix} 0.9254 & -0.3420 & 0.1632\\0.3368 & 0.9397 & 0.0594\\-0.1736 & 0 & 0.9848 \end{bmatrix} \begin{bmatrix} 0\\0\\-10 \end{bmatrix} = \begin{bmatrix} -1.6318\\-0.5939\\-9.8481 \end{bmatrix}$$

Compare this to the same rotations in the reverse order (Yaw, then Pitch)

$$\vec{L}_{YP} = \begin{bmatrix} \cos 10 & 0 & \sin 10 \\ 0 & 1 & 0 \\ -\sin 10 & 0 & \cos 10 \end{bmatrix} \begin{bmatrix} \cos 20 & -\sin 20 & 0 \\ \sin 20 & \cos 20 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$
$$= \begin{bmatrix} -1.7365 \\ 0 \\ -9.8481 \end{bmatrix}$$

Which is still in the x - z plane!!! Why?

Compare to Pitch, Yaw value:

$$\vec{L}_{PY} = \begin{bmatrix} -1.6318\\ -0.5939\\ -9.8481 \end{bmatrix}$$

Coordinate Rotations (Roll-Pitch-Yaw)

There are 3 basic rotations an aircraft can make:

- p = Rotation rate about x-axis (rad/s) roll rate <math>rate Rolling = Rotation about x-axis
- q = Rotation rate about y-axis (rad/s) pitch rate Pitch = Rotation about y-axis
- $r = Rotation rate about z-axis (rad/s) yaw rate \Rightarrow Yaw = Rotation about z-axis$

(Each rotation is a one-dimensional transformation.)

(Any two coordinate systems can be related by a sequence of 3 rotations of the)

- A Roll to the right is a positive rotation.
- An upward pitch is positive.
- A yaw to the right is positive

2.2.2 Forces and Moments

A-Forces: These forces and moments have standard labels. The Forces are:

- X Axial Force Net Force in the positive x-direction
- Y Side Force Net Force in the positive y-direction
- Z Normal Force Net Force in the positive z-direction
- **B Moments:** The Moments are called, intuitively:
 - L Rolling Moment Net Moment in the positive p-direction

M - Pitching Moment Net Moment in the positive q-direction

N - Yawing Moment Net Moment in the positive r-direction.

The perturbation variables are shown in Fig. 2.3 and summarised in Table 2.1.

Most of these forces scale in a linear way with something called Dynamic Pressure.



 Table 2.1
 Summary of motion variables

Table 2.2 The perturbation variables

	Trimmed equilibrium			Perturbed		
Aircraft axis	ox	oy	OZ	ox	oy	oz
Force	0	0	0	X	Ŷ	Ζ
Moment	0	0	0	L	M	N
Linear velocity	U_e	V_e	W_{e}	U	V	W
Angular velocity	0	0	0	р	q	r
Attitude	0	θ_e	0	ϕ	$\hat{\theta}$	ψ

A simple description of the perturbation variables is given in Table 2.2. The intention is to provide some insight into the physical meaning of the many variables used in the model. Note that the components of the total linear velocity perturbations (U, V, W) are given by the sum of the steady equilibrium components and the transient perturbation components (u, v, w) thus,

U = Ue + u V = Ve + v W = We + w

X	Axial "drag" force	Sum of the components of
Y	Side force	aerodynamic, thrust and
Ζ	Normal "lift" force	weight forces
L	Rolling moment	Sum of the components of
M	Pitching moment	aerodynamic, thrust and
N	Yawing moment	weight moments
р	Roll rate	Components of angular
q	Pitch rate	velocity
r	Yaw rate	
U	Axial velocity	Total linear velocity
V	Lateral velocity	components of the cg
W	Normal velocity	-

2.3 Dynamic Pressure:

Dynamic Pressure, Q, refers the pressure of the air moving over the aircraft and is given by $Q = (1/2) \rho V^2$

where $\bullet \rho$ is the density of the air (kg/m3 or slug/f t3)

 \bullet v is the magnitude of the velocity of the aircraft with respect to the air (m/s or f t/s)

Among other things, Lift is usually proportional to dynamic pressure.

Something like $Lif t = C_LQS$

where \bullet C_L is a non-dimensional lift coefficient which depends primarily on the airplane configuration and angle-of-attack

• S is surface area of the plane (or another reference area).

In any case, this provides a convenient way to quantify the forces and moments without having to account for the effect of altitude and airspeed.

 $X = C_x QS$ Axial Force Net Force in the positive x-direction $Y = C_y QS$ Side Force Net Force in the positive y-direction $Z = C_z QS$ Normal Force Net Force in the positive z-direction

Thus the forces on the aircraft are defined by the quantities Cx, Cy, and Cz.

Moments are similarly defined

$L = C_l QSl_w$	Rolling Moment	Net Moment in the positive <i>p</i> -direction
$M = C_m QSl_w$	Pitching Moment	Net Moment in the positive <i>q</i> -direction
$N = C_n Q S l_c$	Yawing Moment	Net Moment in the positive <i>r</i> -direction

where • S again is surface area of the plane (or another reference area).

- lw is the wingspan
- lc is the mean chord

2.4 Sideslip and Angle of Attack

Two quantities which heavily influence Cx, Cy, Cz, Cl, Cm, and Cn are angle of attack, α , and sideslip angle, β .

• Let $\sim V$ be the velocity vector of the aircraft with respect to the free-stream and expressed in the body-fixed frame.

• If \sim Vis projected onto the x-z plane, then α is the angle between the x-axis and this projection.

• If $\sim V$ is projected onto the x-y plane, then β is approximately the angle between the x-axis and this projection.



Figure 2.4: Side slip and angle of attak

If we introduce the notation:

$u = \vec{\mathbf{v}} \cdot \vec{x}$	velocity in the \vec{x} direction
$v = \vec{\mathbf{v}} \cdot \vec{y}$	velocity in the $ec{y}$ direction
$w = \vec{\mathbf{v}} \cdot \vec{z}$	velocity in the \vec{z} direction

Then α and β can be quantified as

$$\alpha = \tan^{-1} \frac{w}{u}$$
 and $\beta = \sin^{-1} \frac{v}{V}$

Where V is the magnitude of the velocity: $V=\sqrt{u^2+v^2+w^2}.$ In radians, this approximates as:

$$lpha \cong rac{w}{u}$$
 and $eta \cong rac{v}{u}$

Example: Suppose an airplane is flying at 20 km at a speed of 200 m/s. The surface area is 30 m^2 . The aircraft has a wingspan of 10 m. Suppose we have the following data

Cx = 1.1 and Cy = 0.1 and Cz = 2.3 the density of air is .08891 kg/m3.

The wind attach the airplane from the direction:

$$\vec{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 180 \\ 10 \\ 86.6 \end{bmatrix} m/s$$

Calculate Q, Z, X, lift to drag ratio (L/D), α , and β

Solution

Then the dynamic pressure is

$$Q = \frac{1}{2}\rho V^2 = \frac{1}{2}.00891 * 200^2 = 178 \frac{kg}{ms^2} = 178 \frac{N}{m^2}$$

The Lift force is about Z = Cz * Q * S = 2.3 * 178 * 30 = 12, 282N

Likewise, the drag is about

$$X = Cx * Q * S = 1.1 * 178 * 30 = 5,874N$$

This gives a Lift-to-Drag ration of about L/D = Cz/Cx = 2.1

Then we can find angle of attack and sideslip as approximately

 $\alpha \cong \frac{w}{u} = .48rad = 27.5 \deg \qquad \text{and} \qquad \beta \cong \frac{v}{u} = .055rad = 3.18 \deg$ Which contrast with the exact values of

$$\alpha = \tan^{-1} \frac{w}{u} = 25.7 \deg$$
 and $\beta = \sin^{-1} \frac{v}{V} = 2.866 \deg$

2.5 Airfoils:

Chord Line: A line connecting the leading edge to the trailing edge.

Camber Line: A line connecting the points halfway between the top and bottom surfaces. **Camber:** Camber refers to the difference between the chord line and the camber line. Camber determined the moment produced by a wing. Most wings have positive camber.



2.2.1 Aerodynamic Center: The point at which the pitching moment does not vary with angle of attack.

• Convenient since C_M is now static. • Typically located at the 1/4-chord line.

Forces and Moments: The motion of air creates forces and moments.

• Lift and Drag are measured at the aerodynamic center.

• Moment is measured as the moment about the aerodynamic center. • Usually take standard form

$$L = C_L QS$$
, $D = C_D QS$, and $M = C_M QSI$

 \bullet CL (Lift coefficient) and CD (Drag coefficient) will depend on angle of attack and airfoil geometry.

• C_M will (hopefully) depend only on airfoil geometry, especially camber.



2.2.2 Lift Coefficient:

Lift is given by $\mathbf{L} = \mathbf{C}\mathbf{L}\mathbf{Q}\mathbf{S}$: $\mathbf{C}\mathbf{L} = \mathbf{C}_{\mathbf{L}\mathbf{0}} + \mathbf{C}_{\mathbf{L}\alpha}\alpha$

where • C_{L0} is the lift produced at steady-level flight. We define $C_{L0} = 0$ for an airfoil. However, for the aircraft overall, we want $C_{L0} > 0$ (*Don't want to fly nose-up all the time*).

• $C_{L\alpha} > 0$ is determined by the airfoil type and other factors (*Sweep, planform shape, winglets, Mach number, etc.*).



Figure 2.5: Lift and Drag

2.2.3 Drag Coefficient: $D = C_DQS$ The drag coefficient, C_D , of an airfoil is related to the lift coefficient, C_L . It can be approximated as $C_D = C_{D0} + KC_L^2$ where • C_{D0} and K are determined by airfoil type and other factors (Mach number, thrust coefficient, etc.)

2.2.4 Moment Coefficient: Positive pitching moment is given by

$$\mathbf{M} = \mathbf{C}_{\mathbf{M}} \mathbf{Q} \mathbf{S} \mathbf{l}$$

General form: $C_M = C_{M0} + C_{M\alpha} \alpha$

where $\bullet C_{M0}$ is the moment produced at steady-level flight.

• For the aircraft overall, we typically want $C_{M0} > 0$ (negative camber), but most airfoils have positive camber.

• By definition $C_{M\alpha} = 0$ for an airfoil if we are considering moment about the aerodynamic center.

2.3 Stability:

Two conditions are necessary for an airplane to fly its mission successfully:

1. The airplane must be able to achieve equilibrium flight and

2. It must have the capability to maneuver for a wide range of flight velocities and altitudes.

The stability and control characteristics of an airplane are referred to as the vehicle's handling or flying qualities.

In the study of airplane stability and control, we are interested in:

- what makes an airplane stable?
- how to design the control systems?
- what conditions are necessary for good handling (good flying qualities)?

2.3.1 Static and dynamic Stability :

Is a property of an equilibrium state. To discuss stability we must first define what is meant by equilibrium. If an *airplane is to remain in steady uniform flight, the resultant force as well as the resultant moment about the center of gravity must both be equal to 0.* An airplane satisfying this requirement is said to be in a *state of equilibrium or flying at a trim condition.* On the other hand, if the forces and moments do not sum to 0, the airplane will be subjected to translational and rotational accelerations.

The subject of airplane stability is generally divided into:

1- **Static stability**: is the initial tendency of the vehicle to return to its equilibrium state after a disturbance. An example of the various types of static stability is illustrated in Figure 2.6.

Static stability is classified according to the nature tendency of an aircraft's response to disturbance from its original steady flight path (equilibrium):

- a- **Positive** when, subsequent to the displacement, the forces and moments acting on the aircraft return it to its original steady flight path (statically stable).
- b- **Neutral** if the forces and moments cause the aircraft to take up a new flight path of constant relationship to the original.

c- **Negative** if the aircraft is caused to diverge from the original steady flight path (an Statically unstable condition).



Figure 2.6: Illustrating various conditions of static stability.

2- **dynamic stability:** concerned with the time history of the motion of the vehicle after it is disturbed from its equilibrium point. Figure 2.7 shows several airplane motions that could occur if the airplane were disturbed from its equilibrium conditions.







FIGURE 2.7 Examples of stable and unstable dynamic motions.

Of particular interest to the pilot and designer is the degree of dynamic stability. Dynamic stability usually is specified by the time it takes a disturbance to be damped to half of its initial amplitude or, in the case of an unstable motion, the time it takes for the initial amplitude of the disturbance to double. In the case of an oscillatory motion, the frequency and period of the motion are extremely important.

Control: deals with the issue of whether the aerodynamic and propulsive controls are adequate to trim the vehicle (i.e., produce an equilibrium state) for all required states in the flight envelope.

In order to study this evolution we need to solve the system of equation describing the 6DOF movement of the aircraft.

2.4 Static Stability and Control

2.4.1 Definition of Longitudinal Static Stability

In the first example we showed that to have static stability we need to develop a restoring moment on the ball when it is displaced from its equilibrium point. The same requirement exists for an airplane. Let us consider the two airplanes and their respective pitching moment curves shown in Figure 2.8. The pitching moment curves have been assumed to be linear until the wing is close to stalling.



FIGURE 2.8 Pitching moment coefficient versus angle of attack.

In Figure 2.8, both airplanes are flying at the trim point denoted by B; that is, CmCK = 0. Suppose the airplanes suddenly encounter an upward gust such that the angle of attack is increased to point C. At the angle of attack denoted by C, *airplane 1 would develop a negative* (*nose-down*) *pitching moment* that would tend to *rotate the airplane back toward its equilibrium* point. However, for the same disturbance, **airplane 2 would develop a positive** (**nose-up**) pitching moment that would tend to *rotate the aircraft away from the equilibrium* point. If we were to encounter a disturbance that reduced the angle of attack, say, to point A, we would find that airplane 1 would develop a nose-up moment that would rotate the aircraft back toward the equilibrium point. On the other hand, airplane 2 would develop a nose-down moment that would rotate the aircraft away from the equilibrium point. On the basis of this simple analysis, we can *conclude that to have static longitudinal stability the aircraft pitching moment curve must have a negative slope.* That is,

$$rac{dc_{M,cg}}{dlpha}=c_{Mlpha}<0.$$
(2.1)

through the equilibrium point.

Another point that we must make is illustrated in Figure 2.9. Here we see two pitching moment curves, both of which satisfy the condition for static stability. However, only curve 1 can be trimmed at a positive angle of attack. Therefore, in addition to having static stability, we also must have a positive intercept, that is, Cmo > 0 to trim at positive angles of attack. Although we developed the criterion for static stability from the C_m versus α curve, we just as easily could have accomplished the result by working with a C_m versus C, curve. In this case, the requirement for static stability would be as follows:



Figure 2.9: Flow field around an airplane created by the wing.

The two conditions are related by the following expression:

$$C_{m_{\alpha}} = \frac{\mathrm{d}C_{m}}{\mathrm{d}\alpha} = \frac{\mathrm{d}C_{m}}{\mathrm{d}C_{L}}\frac{\mathrm{d}C_{L}}{\mathrm{d}\alpha} \qquad \dots 2.3$$

which shows that the derivatives differ only by the slope of the lift curve. $C_{m\alpha}$ depends, among other, of the center of gravity of the aircraft.

In the condition of equilibrium, the sum of moments around the center of gravity is null, and therefore it must be fulfilled that

$$C_m = C_{m0} + C_{m\alpha} \alpha + C_{m\delta e} \delta_e = 0.$$

Two main problems can be derived in the longitudinal control:

1. Determine the deflection angle of the elevator, , to be able to fly in equilibrium at a given angle of attack, :

$$\delta_e = rac{-c_{M0}-c_{Mlpha}lpha_e}{c_{M\delta_e}}$$

2. Determine the angle of attack to fly in equilibrium, α_e , for a known deflection of the elevator, :

$$lpha_e = rac{-c_{M0}-c_{M\delta_e}\delta_e}{c_{Mlpha}}$$

Figure 2.10 shows the effects of the elevator's deflection in the angle of attack of equilibrium. Simplifying, for a , the angle of attack of equilibrium at which the aircraft flies increases and so does the coefficient of lift. Since the lift must be equal to weight, the aircraft must fly slower. In other words, the elevator is used to modify the velocity of a steady horizontal flight.



Figure 7.15: Effects of elevator on moments coefficient.

As we have pointed out before, the geometric condition of the aircraft vary during the flight. Therefore it is necessary to recalculate this conditions and modify the variables continuously. This is made using control

2.4.2 Longitudinal Balancing

The longitudinal balancing is the problem of determining the state of equilibrium of a longitudinal movement in which the lateral and directional variables are considered uncoupled. For the longitudinal analysis, one must consider forces on z-axis (Fz) and torques around y-axis (My). Generally, it is necessary to consider external actions coming from aerodynamics, propulsion, and gravity. However, it is common to consider only the gravity and the lift forces in wing and horizontal stabilizer. Additional hypotheses include: no wind; mass and velocity are constant.

The equations to be fulfilled are:

 $\sum \mathbf{F} \mathbf{z} = \mathbf{0},$ $\sum \mathbf{M} \mathbf{y} = \mathbf{0}.$

Which results in: mg-L-Lt = 0,

-Mca + Lxcg - Ltl = 0

Where \mathbf{Lt} is the lift generated by the horizontal stabilizer, \mathbf{Mca} is the pitch torque with respect to the aerodynamic center, $\mathbf{x_{cg}}$ is the distance between the center of gravity and 1 the aerodynamic center, and is the distance between the center of gravity and the aerodynamic center of the horizontal stabilizer.

2.4.3 Lateral - directional Stability and control

Consider again an aircraft in horizontal, steady, linear flight. Suppose in this case that the lateral-directional elements (vertical stabilizer, rudder, ailerons) do not produce forces nor moments, so that there not exists a primary problem of balancing (as there was in the longitudinal case) since we have a longitudinal plane of symmetry. In this case, the lateral-direction control surfaces (rudder and ailerons) fulfill a mission of secondary balancing since they are used when there exists an asymmetry (propulsive or aerodynamic). For instance, aircraft must be able to fly under engine failure, and thus the asymmetry must be compensated with the rudder. Another instance could be the landing operation under lateral wind, which must be also compensated with the rudder deflection. Notice that the center of gravity lays on the plane of symmetry, so that its position does not affect the lateral-directional control. Further mathematical analysis will be studied in posterior courses.

AXIS	MOTION	CONTROL	STABILITY
Longitudinal	Rolling	Ailerons	Lateral
			Stability
			around the
			Longitudinal
			Axis.
Lateral	Pitching	Elevators	Longitudinal
			Stability
			around the
			Lateral Axis.
Normal	Yawing	Rudder	Directional
			Stability
			around the
			Normal
			Axis.

Question #1 -	When the restoring forces are absent and the airplane will neither return from its disturbed position, nor move further away is known as?			
	A Static stability			
	Neutral stability			
	С	Positive stability		
	D	Dynamic stability		
Question #2 - Longitudinal stability is pitch stabilit around or about what axis?				
	Α	A Longitudinal		
1. State	В	Lateral		
	Normal			
D Vertical				

Question #	3 - Na	ame a type of lateral stability?		
	A	Keel effect		
10.00	В	Centre of gravity		
1.11	С	Fin		
1.000	D	Horizontal stabilizer		
Question #4 - What stability is around or about longitudinal axis?				
ADirectionalBLateral				
				С
	D	Longitudinal		