AIRCRAFT STABILITY AND CONTROL

CONTENTS

References

1. **David A. Caughey "Introduction to Aircraft Stability and Control" Cornell University**
2. **Dr. Robert C. Nelson "Flight Stability and Automatic Control " WCB/McGraw-Hill, 1998**

Chapter 1

Introduction

1. Generalities

1.1 Engineering:

engineering can be defined as: ***The application of scientific, economic, social, and practical knowledge in order to design, build, and maintain structures, machines, devices, systems, materials, and processes.***

It may encompass using insights to conceive, model, and scale an appropriate solution to a problem or objective. The discipline of engineering is extremely broad, and encompasses a range of more specialized fields of engineering, each with a more specific emphasis on particular areas of technology and types of application**.**

The foundations of engineering lays on mathematics and physics, but more important, it is reinforced with additional study in the natural sciences and the humanities. Therefore, attending to the previously given definition, engineering might be briefly summarized with the following six statements:

- to adapt scientific discovery for useful purposes;
- to create useful devices for the service of society;
- to invent solutions to meet society's needs;
- to come up with solutions to technical problems;
- to utilize forces of nature for society's purposes;
- to convert energetic resources into useful work.

Aerospace engineering can be defined as:

A primary branch of engineering concerned with the research, design, development, construction, testing, and science and technology of aircraft and spacecraft.

It is divided into two major and overlapping branches:

a- **Aeronautical engineering** : deals with aircraft that operate in Earth's atmosphere,
b- **Astronautical engineering:** deals with spacecraft that operates outside it.

An aerospace engineering education should produce engineers capable of the following:

- **Conceive**: conceptualize technical problems and solutions.
- **Design**: study and comprehend processes that lead to solutions to a particular problem including verbal, written, and visual communications.
- **Development**: extend the outputs of research.
- **Testing**: determine performance of the output of research, development, or design.
- **Research**: solve new problems and gain new knowledge.
- **Manufacturing**: produce a safe, effective, economic final product.
- **Operation and maintenance**: keep the products working effectively.
- **Marketing and sales**: look for good ideas for new products or improving current products in order to sell.
- **Administration (management)**: coordinate all the above.

Thus, the student as a future aerospace engineer, will develop his or her professional career accomplishing some of the above listed capabilities in any of the activities that arise within the aerospace industry.

1.2 Parts of the aircraft:

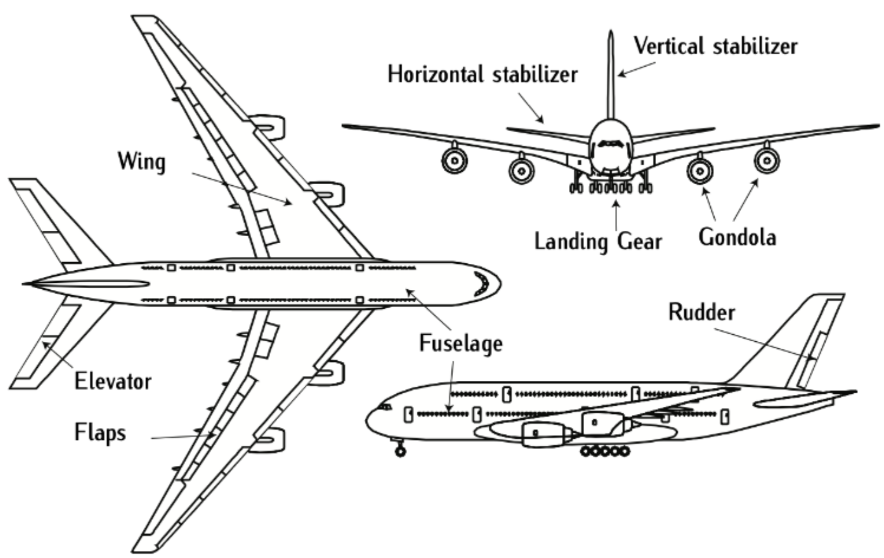


Figure 1: Parts of an aircraft.

Fuselage: is the central body of the airplane, which hosts the crew and the payload (passengers, luggage, and cargo),

The wing: is the main contributor to lift force. The surfaces situated at the tail or empennage of the aircraft is referred to as horizontal stabilizer and vertical stabilizer.

Fuselage: *The fuselage is the aircraft's central body that accommodates the crew and the payload (passengers and cargo) and protects them from the exterior conditions.* The fuselage also gives room for the pilot's cabin and its equipment's, and serves as main structure to which the rest of structures (wing, stabilizers, etc.) are attached. Its form is a tradeoff between an aerodynamic geometry (with minimum drag) and enough volume to fulfill its mission.

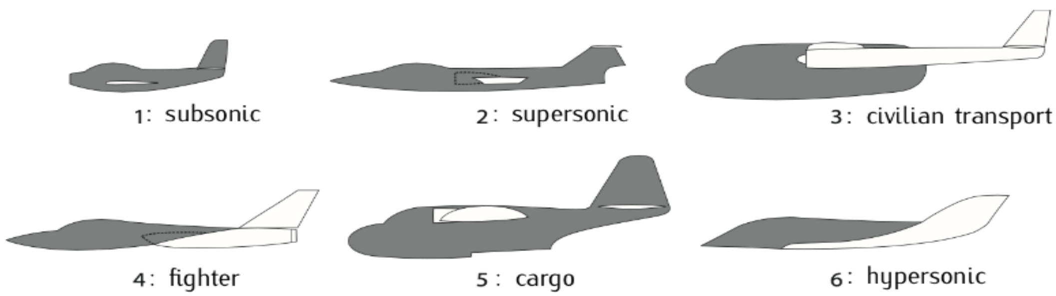


Figure 2: Types of fuselages.

Wing: wing is an airfoil that has an aerodynamic cross-sectional shape producing a useful lift to drag ratio. A wing's aerodynamic quality is expressed as its lift-to-drag ratio. The lift that a wing generates at a given speed and angle of attack can be one to two orders of magnitude greater than the total drag on the wing.

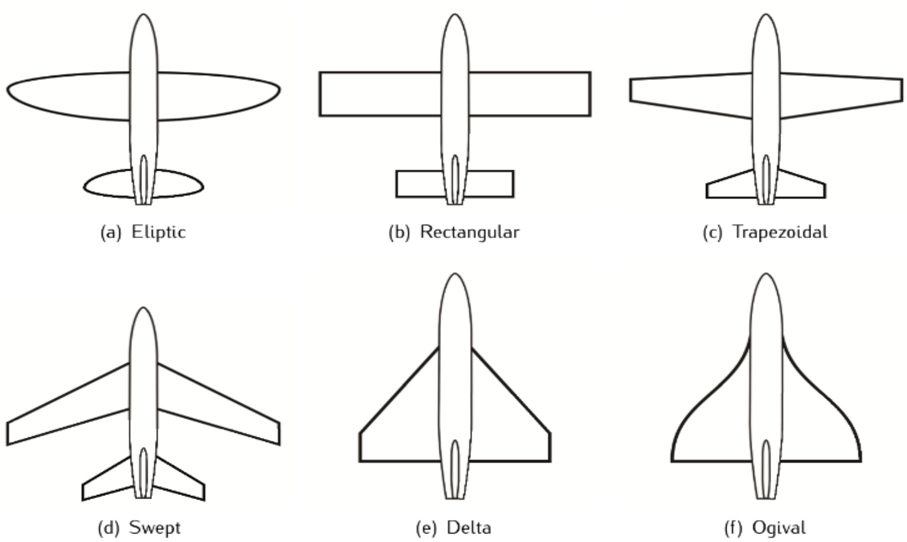


Figure 3: Aircraft's plant-form types.

Attending at the vertical position, the wing can also be classified as; **high, medium, and low**. High wings are typical of cargo aircraft. It allows the fuselage to be nearer the floor, and it is easier to execute load and download tasks. On the contrary, it is difficult to locate space for the retractile landing gear (also referred to as undercarriage). The low wing is the typical one in commercial aviation. It does not interfere in the passenger cabin, diving the deck into two spaces. It is also useful to locate the retractile landing gear. The medium wing is not typical in commercial aircraft, but it is very common to see it in combat aircraft with the weapons bellow the wing to be dropped.

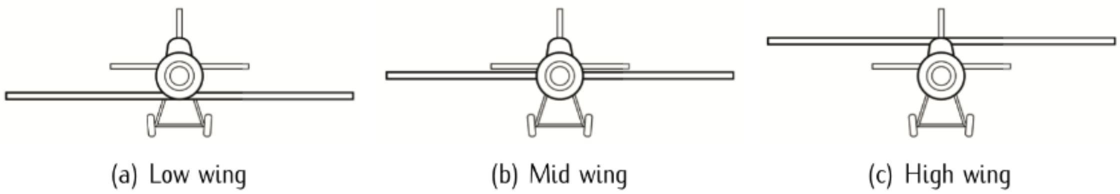


Figure 4: Wing vertical position.

Usually, aircraft's wings have various devices, such as **flaps or slats**, that the pilot uses to modify the shape and surface area of the wing to change its aerodynamic characteristics in flight, **or ailerons**, which are used as control surfaces to make the aircraft roll around its longitudinal axis. Another kind of devices are the **spoilers** which typically used to help braking the aircraft after touching down. Spoilers are deflected so that the lift gets reduced in the semi-wing they are acting, and thus they can be also useful to help the aircraft rolling. If both are deflected at the same time, the total lift of the aircraft drops and can be used to descent quickly or to brake after touching down.

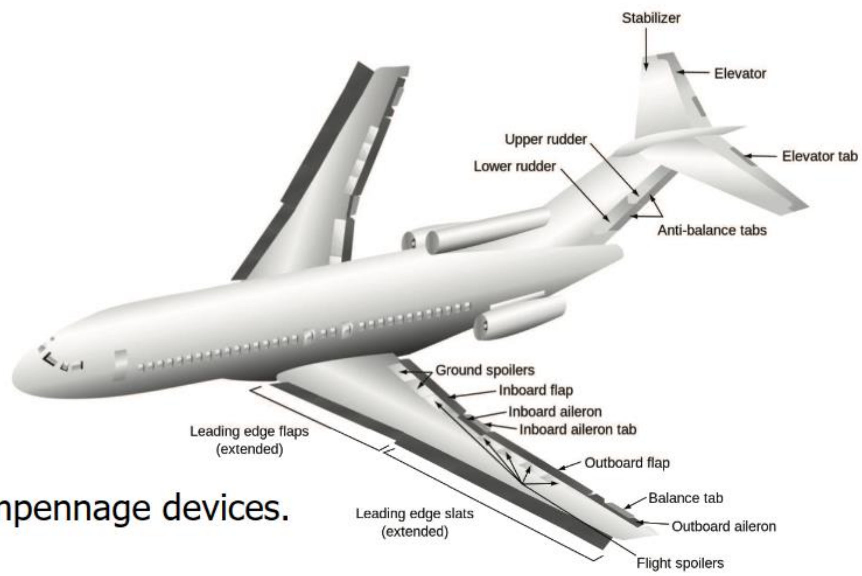


Figure 5 : Wing and empennage devices.

Main control surfaces; The main control surfaces of a fixed-wing aircraft are attached to the airframe on hinges or tracks so they may move and thus deflect the air stream passing over them. This redirection of the air stream generates an unbalanced force to rotate the plane about the associated axis.

The main control surfaces are: **ailerons, elevator, and rudder**.

Ailerons are mounted on the trailing edge of each wing near the wingtips and move in opposite directions. When the pilot moves the stick left, the left aileron goes up and the right aileron goes down. A raised aileron reduces lift on that wing and a lowered one increases lift, so moving the stick left causes the left wing to drop and the right wing to rise. This causes the aircraft to roll to the left and begin to turn to the left. Centering the stick returns the ailerons to neutral maintaining the bank angle. The aircraft will continue to turn until opposite aileron motion returns the bank angle to zero to fly straight.

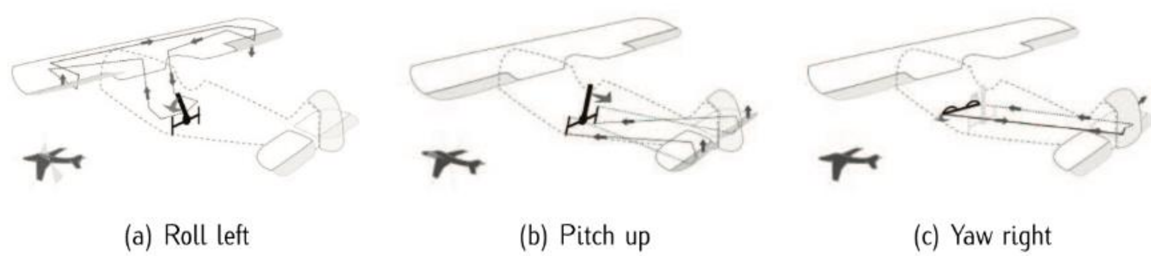


Figure 6: Actions on the control surfaces.

An elevator is mounted on the trailing edge of the horizontal stabilizer on each side of the fin in the tail. They move up and down together. When the pilot pulls the stick backward, the elevators go up. Pushing the stick forward causes the elevators to go down. Raised elevators push down on the tail and cause the nose to pitch up. This makes the wings fly at a higher angle of attack, which generates more lift and more drag. Centering the stick returns the elevators to neutral position and stops the change of pitch.

The rudder is typically mounted on the trailing edge of the fin, part of the empennage. When the pilot pushes the left pedal, the rudder deflects left. Pushing the right pedal causes the rudder to deflect right. Deflecting the rudder right pushes the tail left and causes the nose to yaw to the right. Centering the rudder pedals returns the rudder to neutral position and stops the yaw.

1.3 Standards (Hypotheses)

Standard atmosphere: The basic standards of ISA are:

- Complies with the perfect gas equation: $p=\rho RT$ ….. (1)

where R is the perfect gas constant for air R=287.053[J/kg K], p is the pressure, ρ is the density, and T is the temperature.

- In the troposphere the temperature gradient is constant.

Troposphere ($0 \leq h < 11000$[m]),

$$T = T_0 - \alpha h, \quad \ldots\ldots(2)$$

where T_0=288.15[K], α=6.5[K/km]

- In the Tropopause and the inferior stratosphere the temperature is constant.

Tropopause and inferior stratosphere ($11000[m] \leq h < 20000[m]$)

$$T = T_{11} = 216.65[K]. \quad \ldots\ldots\ldots\textbf{(2b)}$$

where T_{11}=216.65[K].

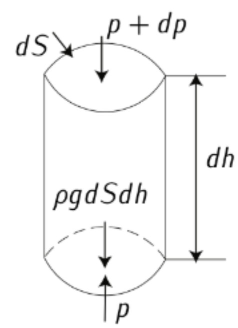
- The air pressure at sea level (h=0) is p_0=101325[Pa]

  The air density at sea level yields ρ_0=1.225[kg/m3]

- . The acceleration due to gravity is constant (g = 9.80665[m/s2]).
- The atmosphere is in calm with respect to Earth.

Fluid-static equation: Fluid statics (also called hydrostatics) is the science of fluids at rest, and is a sub-field within fluid mechanics. It embraces the study of the conditions under which fluids are at rest in stable equilibrium.

Figure 7 : Differential cylinder of air.



If we assume the air at rest as in Hypothesis above, we can formulate the equilibrium of a differential cylindrical element where only gravitational volume forces and pressure surface forces act (see Figure 7):

$$pdS - (p + dp)dS = \rho g\, dS\, dh \qquad (3)$$

Which gives rise to the equation of the fluid statics:

$$dp/dh = -\rho g. \qquad (4)$$

ISA equations: Considering the above mentioned equations, the variations of **ρ** and **p** within altitude can be obtained for the different layers of the atmosphere that affect atmospheric flight:

Troposphere (0≤ h < 11000[m]:

Introducing Equation (1) and Equation (2) in Equation (4), it yields:

$$\frac{dp}{dh} = -\frac{p}{R(T_0 - \alpha h)} g. \quad \text{.... (5)}$$

Integrating between a generic value of altitude and the altitude at sea level ( ), the variation of pressure with altitude yields:

$$\frac{p}{p_0} = (1 - \frac{\alpha}{T_0} h)^{\frac{g}{R\alpha}}. \quad \text{.....(6)}$$

Entering (6) in the equation of perfect gas (1), the variation of density with altitude yields:

$$\frac{\rho}{\rho_0} = (1 - \frac{\alpha}{T_0} h)^{\frac{g}{R\alpha} - 1}. \quad \text{.......(7)}$$

Introducing now the numerical values, it yields:

T[k] = 288.15 −0.0065h[m];

$\rho[kg/m^3] = 1.225(1 - 22.558 \times 10^{-6} \times h[m])^{4.2559h}$

$p[Pa] = 101325(1 - 22.558 \times 10^{-6} \times h[m])^{5.2559}$

Tropopause and Inferior part of the stratosphere (11000 < h < 20000): Insert equation (1)and Equation (2b) in Equation (4), and integrating between a generic altitude (h > 11000 m) and the altitude at the Tropopause (h11 =11000 m)

$$\frac{p}{p_{11}} = \frac{\rho}{\rho_{11}} = e^{-\frac{g}{RT_{11}}(h - h_{11})} \quad \text{....... (8)}$$

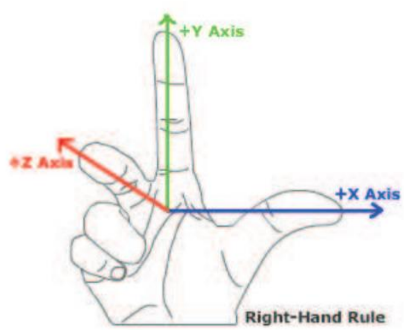
Introducing now the numerical values, it yields:

$$T[k] = 216.65; \quad \rho[kg/m^3] = 0.3639e^{-157.69 \cdot 10^{-6}(h[m]-11000)};$$
$$p[Pa] = 22632e^{-157.69 \cdot 10^{-6}(h[m]-11000)}.$$

1.4 System Reference; The atmospheric flight mechanics uses different coordinates references to express the positions, velocities, accelerations, forces, and torques. It is useful to define some of the most important ones:

a. Inertial Reference Frame: According to classical mechanics, an inertial reference from is either a non-accelerated frame with respect to a quasi fixed reference star, or either a system which for a punctual mass is possible to apply the second Newton's law:

$$\sum \vec{F}_I = \frac{d(m \cdot \vec{V}_I)}{dt} \quad \text{......(9)}$$



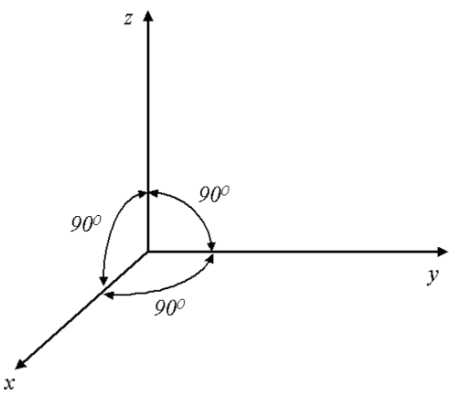


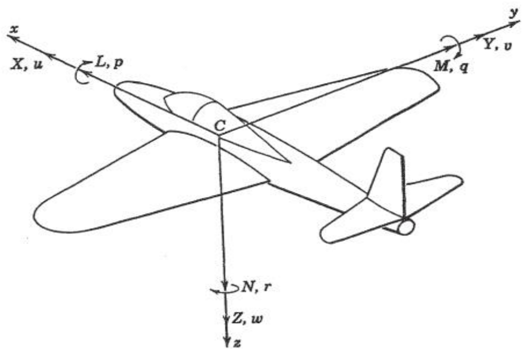
Figure 8

b. **Earth Reference Frame:** An earth reference frame Fe**(Oe, x, y, z)**is a rotating topocentric (measured from the surface of the earth) system. The origin **Oe** is any point on the surface of earth defined by its latitude **(θe)** a and longitude **(γe)**. Axis **(ze)** points to the center of earth; **(xe)** lays in the horizontal plane and points to a fixed direction (typically north); **(ye)** forms a right-handed trihedral (typically east).

Such system is sometimes referred to as navigational system since it is very useful to represent the trajectory of an aircraft from the departure airport.

c. **Body Axes Frame:** A body axes frame (Ob, xb, yb, zb) represents the aircraft as a rigid solid model. It is a system of axes centered in any point of the symmetry plane (assuming there is one) of the aircraft, typically the center of gravity.

Axis (x_b) lays in to the plane of symmetry and it is parallel to a reference line in the aircraft (for instance, the zero-lift line), pointing forwards according to the movement of the aircraft. Axis (z_b) also lays in to the plane of symmetry, perpendicular to and pointing down according to regular aircraft performance. Axis (y_b) is perpendicular to the plane of symmetry forming a right-handed thrihedral (points then to the right wing side of the aircraft).



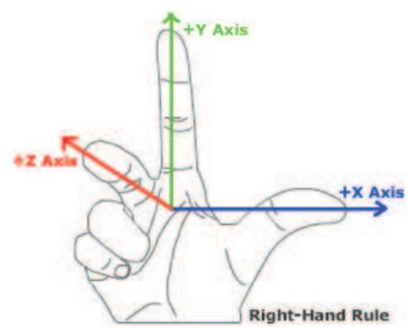


Figure 9

d. **Wind Axes Frame;** A wind axes frame **Fe** (O_w, x_w, y_w, z_w) is linked to the instantaneous aerodynamic velocity of the aircraft. It is a system of axes centered in any point of the symmetry plane (assuming there is one) of the aircraft, typically the center of gravity. Axis (x_w) points at each instant to the direction of the aerodynamic velocity of the aircraft **V** . Axis (z_w) lays in to the plane of symmetry, perpendicular to x_w and pointing down according to regular aircraft performance. Axis (yb) forms a right-handed thrihedral. Notice that if the aerodynamic velocity lays in to the plane of symmetry, **yw = yb**.

The standard notation for describing the motion of, and the aerodynamic forces and moments acting upon, a flight vehicle are indicated in Fig.10

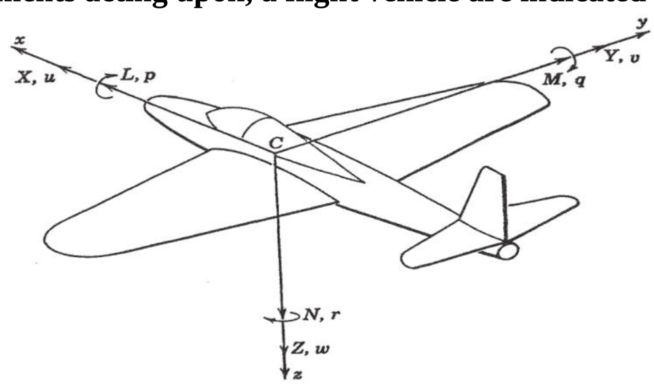


Figure 10: Body axes system, forces, moments and linear and angular velocities

The quantity V is the velocity vector. The quantities X, Y, Z are the components of the resultant aerodynamic force, along OXb, OYb and OZb axes respectively. L, M, N are the rolling moment, pitching moment and yawing moment respectively about OXb, OYb and OZb;

Figure 10 also shows the positive directions of L, M and N. The convention is that an aerodynamic moment is taken positive in clock-wise sense when looking along the axis about which the moment is taken. u,v,w are the components , along OXb, OYb and OZb of the velocity vector (V). The angular velocity components are indicated by p,q,r or ωx ωy ωz

1.5 Attitudes

The airplane is treated as a rigid body. **A rigid body** *has six degrees of freedom and hence, six coordinates are needed to describe the position of the airplane with respect to an earth fixed system.*

In flight dynamics, the six coordinates' employed to prescribe the position are

(a) the three coordinates describing the instantaneous position of the c.g. of the airplane with respect to the earth fixed system.

(b) the attitude of the airplane described by the angular orientations of OXbYbZb system with respect to the OXeYeZe system.

This is done with the help of Euler angles. To arrive at the OXbYbZb system, we need to rotate the **EXeYeZe** system through only three angles which are called **Euler angles**.

At this stage, simpler cases are considered. When an airplane climbs along a straight line its attitude is given by the angle 'γ ' between the axis OXb and the horizontal (Fig.11). When an airplane executes a turn, the projection of the OXb axis, in the horizontal plane makes an angle Ψ with reference to fixed horizontal axis (Fig.12). When an airplane is banked, the axis OYb makes an angle φ with respect to the horizontal and the axis OZb makes an angle φ with vertical (Fig.13).

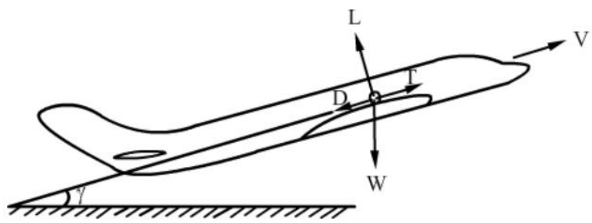
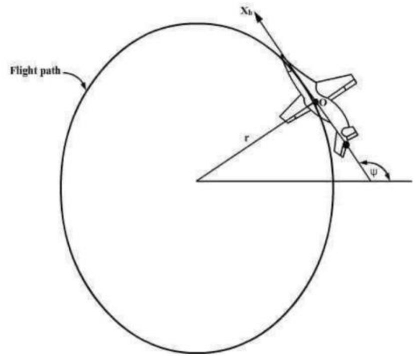


Fig.11 Airplane in a climb



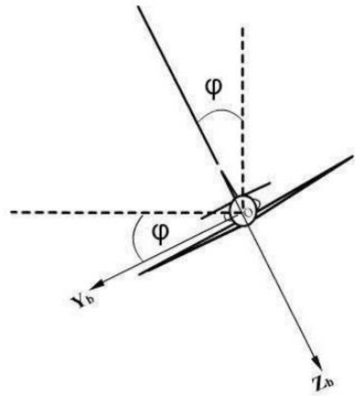


Fig.12 Airplane in a turn – view from top **Fig.13** Angle of bank (φ)

1.6 Flight path: The flight path, also called the trajectory, means the path or the line along which the c.g. of the airplane moves. The tangent to this curve at a point gives the direction of flight velocity at that point on the flight path. The relative wind is in a direction opposite to that of the flight velocity.

1.7 Angle of attack and angle side slip: The concept of the angle of attack of an airfoil is well known. While discussing the forces acting on an airfoil, we take the chord of the airfoil as the reference line and the angle between the chord line and the relative wind is the angle of attack (α). The aerodynamic forces namely lift (L) and drag (D), produced by the airfoil, depend on the angle of attack (α) and are respectively perpendicular and parallel to relative wind direction (Fig.14).

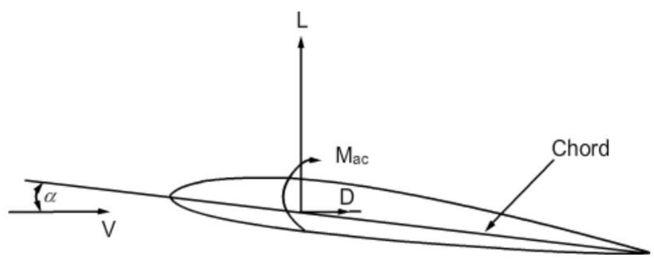


Fig.14 Angle of attack and forces on a airfoil

In the case of an airplane, the flight path, as mentioned earlier, is the line along which c.g. of the airplane moves. The tangent to the flight path is the direction of flight velocity (V). The relative wind is in a direction opposite to the flight velocity. If the flight path is confined to the plane of symmetry, then the angle of attack would be the angle between the relative wind direction and the fuselage reference line (FRL) or OXb axis (see Fig.15). However, in a general case the velocity vector (V) will have components both along and perpendicular to the plane of symmetry. The component perpendicular to the plane of symmetry is denoted by ‘v’. The projection of the velocity vector in the plane of symmetry would have components u and w along OXb and OZb axes (Fig.16). With this background, the angle of sideslip and angle of attack are defined below.

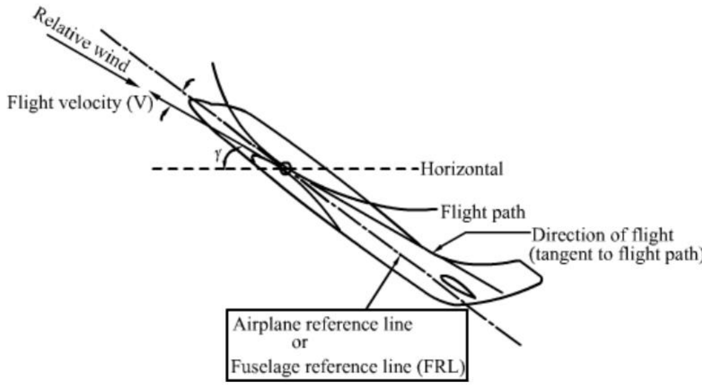


Fig.15 Flight path in the plane of symmetry

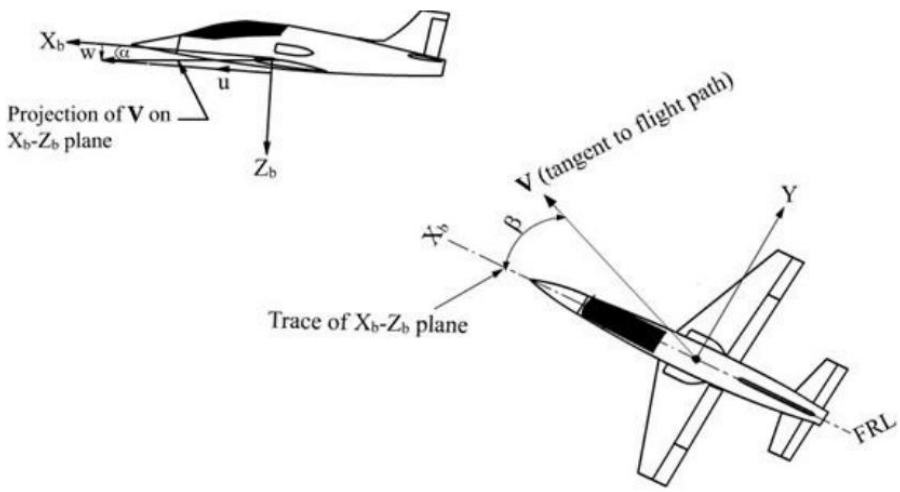


Fig.16: Velocity components in a general case and definition of angle of attack and sideslip

The angle of sideslip (β): is the angle between the velocity vector (V) and the plane of symmetry i.e. $\beta = \sin^{-1}(v/|V|)$; ………. (10)

where |V| is the magnitude of V.

The angle of attack (α): is the angle between the projection of velocity vector (V) in the X_B - Z_B plane and the OXb axis or

$$\alpha = \tan^{-1}\frac{w}{u} = \sin^{-1}\frac{w}{\sqrt{|V|^2 - v^2}} = \sin^{-1}\frac{w}{\sqrt{u^2+w^2}} \quad \text{…..(11)}$$

It is easy to show that, if V denotes magnitude of the velocity (V) ,

then $u = V\cos\alpha\cos\beta$, $v = V\sin\beta$; $w = V\sin\alpha\cos\beta$. …..(12)

Atmospheric flight mechanics is a broad heading that encompasses three major disciplines; namely, performance, flight dynamics, and aero elasticity.

Airplane performance deals with the determination of performance characteristics such as range, endurance, rate of climb, and takeoff and landing distance as well as flight path optimization. To evaluate these performance characteristics, one normally treats the airplane as a point mass acted on by gravity, lift, drag, and thrust. The accuracy of the performance calculations depends on how accurately the lift, drag, and thrust can be determined.

Flight dynamics is concerned with the motion of an airplane due to internally or externally generated disturbances. We particularly are interested in the vehicle's stability and control capabilities. To describe adequately the rigid-body motion of an airplane one needs to consider the complete equations of motion with six degrees of freedom. Again, this will require accurate estimates of the aerodynamic forces and moments acting on the airplane.

Aeroelasticity deals with both static and dynamic aeroelastic phenomena. Basically, Aeroelasticity is concerned with phenomena associated with interactions between inertial, elastic, and aerodynamic forces.