Tishk International University
Architecture Department
First Grade
Fall semester 2023-2024

Calculus

Lecture -3-
Non-Linear Functions

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Non-Linear Function

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Non-Linear Functions

- A function or equation having a degree greater than 1 with dependent and independent variable(s) will be called a nonlinear function.
- Such functions, when plotted, do not form a straight line. Alternatively, if any function is not linear, then it will certainly be a nonlinear function. Nonlinear equations are generally written as:

$$f(x) = y = ax^2 + bx + c$$

- A **nonlinear function** is a function whose graph is NOT a straight line. Its graph can be any curve other than a straight line. For example, if there are 100 fishes in a pond initially and they become double every week, then this situation can be modeled by the function $f(x) = 100\ (2)^x$, where x is the number of weeks and f(x) is the number of fishes. Let us make a table and graph this function making use of the table.

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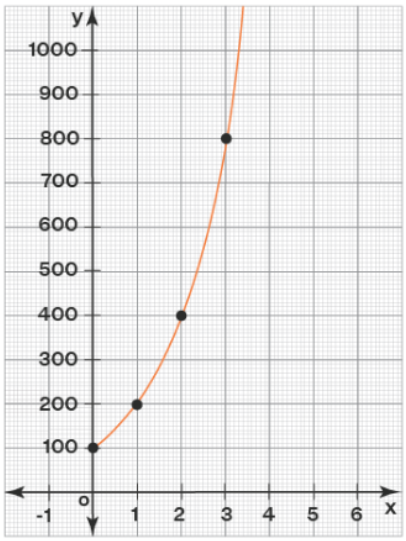
$f(x) = 100\ (2)^x$

x	y
0	100
1	200
2	400
3	800

Let's graph the table now.



- The shown graph is NOT a line and hence it represents a nonlinear function. From the graph, we can say that the slope is not uniform on a nonlinear function. A nonlinear function can be described using a table of values, an equation, or a graph. Let us see each of them now. Some of the examples of nonlinear functions include quadratic functions, cubic functions, polynomial functions.

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The steps to determine whether a table of values determine a linear function are:

1.Find the differences between every two consecutive x values.

2.Find the differences between every two consecutive y values.

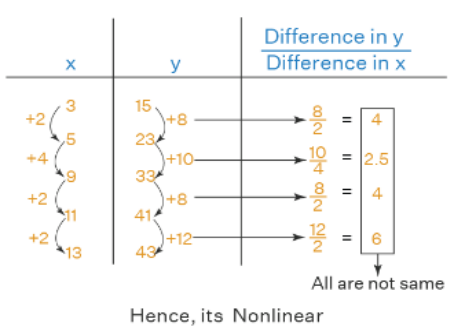
3.Find the corresponding ratios of differences of y and differences of x.

4.If all the ratios are NOT same, then only the function is linear.

Consider the following table of values.

x	y
3	15
5	23
9	33
11	41
13	43

- Let us determine whether this table denotes a nonlinear function by using the steps mentioned above.
- Since all the ratios of differences of y to the differences of x are NOT same, the function is a nonlinear function.



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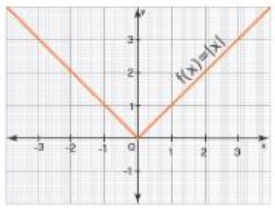
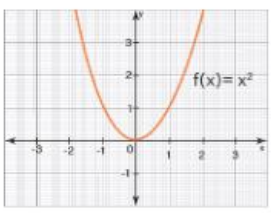
- Some examples of nonlinear functions are:

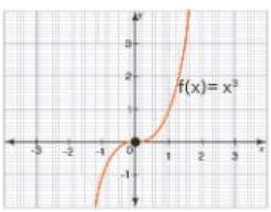
•$f(x) = x^2$ is nonlinear as it is a quadratic function.

•$f(x) = 2^x$ is nonlinear as it is an exponential function.

•$f(x) = x^3 - 3x$ is nonlinear as it is a cubic function.







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Here are the differences between linear and nonlinear functions.

Linear Functions	Nonlinear Functions
A linear function is a function whose graph is a line.	A nonlinear function is a function whose graph is NOT a line.
Its equation is of the form f(x) = ax + b.	Its equation can be in any form except of the form f(x) = ax + b.
Its slope is constant for any two points on the curve.	The slope of every two points on the graph is NOT the same.
In the table of a linear function, the ratio of difference of y and difference of x is a constant.	In the table of a nonlinear function, the ratio of difference of y and difference of x is NOT a constant.

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Example -1-

Plot the graph for the nonlinear function $f(x) = x^2 - 6x + 12$

Solution:

We will solve the nonlinear function at x=1,2,3,4 and 5

$y = x^2 - 6x + 12$

When $x = 1$

$y = 1^2 - 6(1) + 12 = 7$

When $x = 2$

$y = 2^2 - 6(2) + 12 = 4$

When $x = 4$

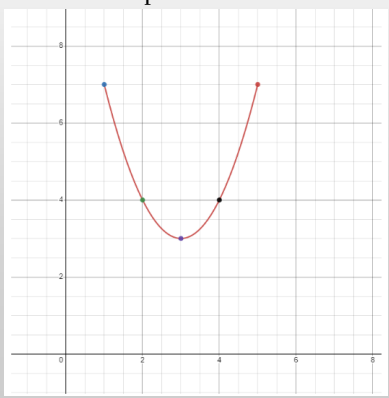
$y = 4^2 - 6(4) + 12 = 4$

When $x = 5$

$y = 5^2 - 6(5) + 12 = 7$

Let us form the table so we can easily plot our ordered pairs.

x	y
1	7
2	4
3	3
4	4
5	7



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Example -2-

Plot the graph for the nonlinear function $y = |x|$

Solution:

As "y" is equal to the absolute of "x," "y" cannot be negative. Hence, we will have a bell-shaped graph. The value of "y" will be the same for every value of x.

When $x = 1$

$y = |1| = 1$

When $x = -1$

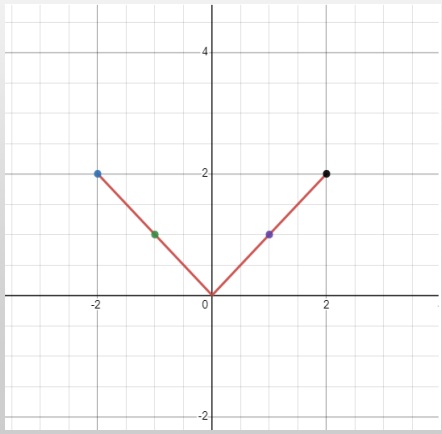
$y = |-1| = 1$

When $x = 2$

$y = |2| = 2$

When $x = -2$

$y = |-2| = 2$



We will have a "V" shaped graph, but as it is not a straight line, it is a nonlinear graph.

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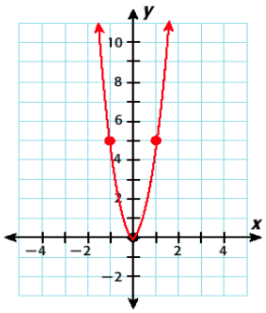
Example -3-

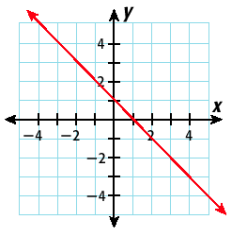
Tell whether the graph is linear or nonlinear.

A.



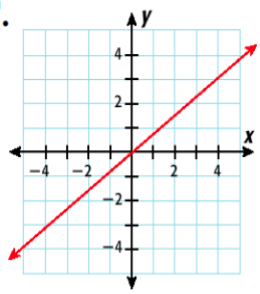
The graph is not a straight line, so it is nonlinear.

B.



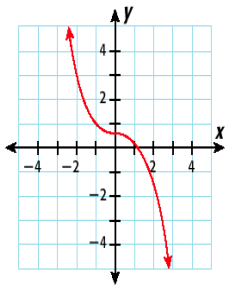
The graph is a straight line, so it is linear.

C.



The graph is a straight line, so it is linear.

D.



The graph is not a straight line so, it is nonlinear.

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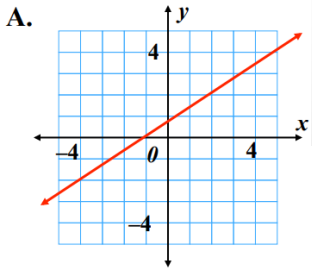
Example -4-

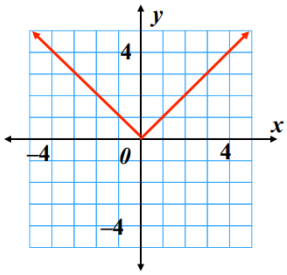
Tell whether the graph is linear or nonlinear.

A.



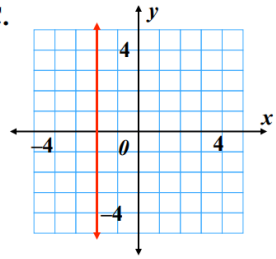
The graph is a straight line, so the graph is linear.

B.



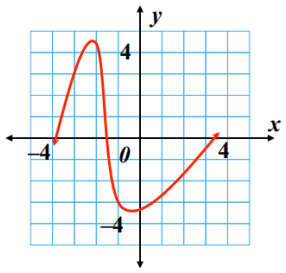
The graph is not a straight line, so it is nonlinear.

C.



The graph is a straight line, so the graph is linear.

D.



The graph is not a straight line, so it is nonlinear.

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Example -5-

Tell whether the function in the table has a linear or nonlinear relationship.

A.

Input	Output
1	2
2	5
3	11

Solution:

A.

Input	Output
1	2
2	5
3	11

difference = 1 (1 → 2)

difference = 1 (2 → 3)

difference = 3 (2 → 5)

difference = 6 (5 → 11)

The difference between consecutive input values is constant.

The difference between consecutive output values is not constant.

The function represented in the table is nonlinear.

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Tell whether the function in the table has a linear or nonlinear relationship.

B.

Input	Output
1	3
2	6
3	9

Solution:

Input	Output
1	3
2	6
3	9

difference = 1
difference = 1

difference = 3
difference = 3

The difference between consecutive input values is constant.

The difference between consecutive output values is constant.

The function represented in the table is linear.

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Tell whether the function in the table has a linear or nonlinear relationship.

C.

Input	Output
1	1
2	4
3	9

Solution:

Input	Output
1	1
2	4
3	9

difference = 1
difference = 1

difference = 3
difference = 5

The difference between consecutive input values is constant.

The difference between consecutive output values is not constant.

The function represented in the table is nonlinear.

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Example -6-

Determine whether each table represents a linear or nonlinear function.

	x	y	
+2	2	−1	+3
+2	4	2	+3
+2	6	5	+3
	8	8	

Linear. As x increases by 2, y increases by 3. The rate of change is constant.

	x	y	
+2	1	1	+8
+2	3	9	+16
+2	5	25	+24
	7	49	

Nonlinear. As x increases by 2, y increases by a greater amount each time. The rate of change is NOT constant.

	x	y	
+2	0	0	+2
+2	2	2	+4
+2	4	6	+6
	6	12	

Nonlinear. As x increases by 2, y increases by a greater amount each time. The rate of change is NOT constant.

	x	y	
+4	4	2	−2
+4	8	0	−2
+4	12	−2	−2
	16	−4	

Linear. As x increases by 4, y decreases by 2 each time. The rate of change is constant.

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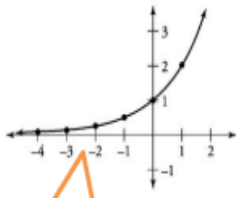
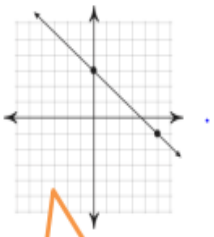
Example -7-

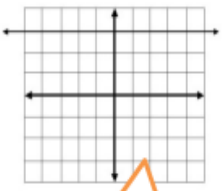
Determine whether each equation represents a linear or nonlinear function.



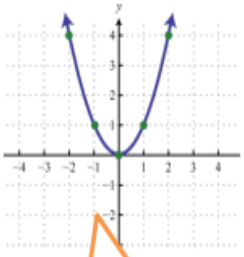
Nonlinear. Graph is not a straight line



Linear. Graph is a straight line



Linear. Graph is a straight line



Nonlinear. Graph is not a straight line

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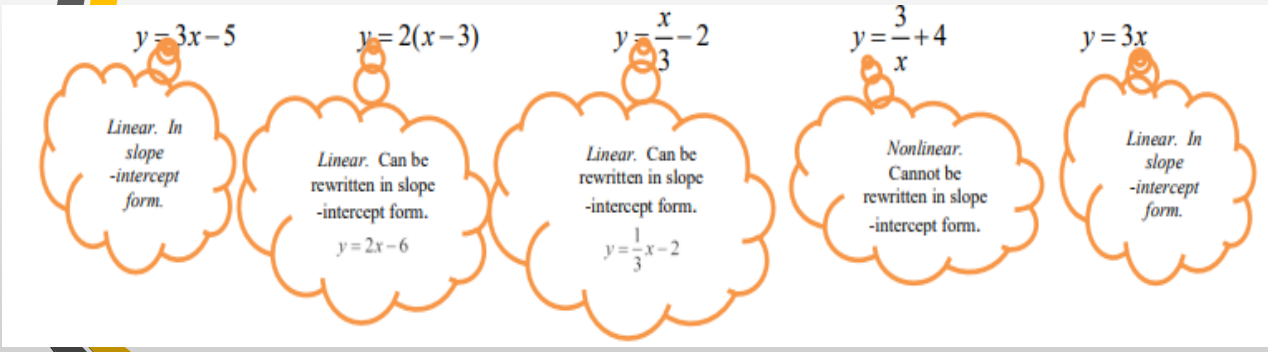
Example -8-

Determine whether each equation represents a linear or nonlinear function. Remember that all linear functions can be written in the slope-intercept form.



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Example -9-

The following table shows the bank balances of Joe and Mitchell for the last 55 years. Graph the data and check if there has been any constant growth for both.

Year (x)	Joe	Mitchell
1	$110	$110
2	$210	$250
3	$310	$160
4	$410	$280
5	$510	$400

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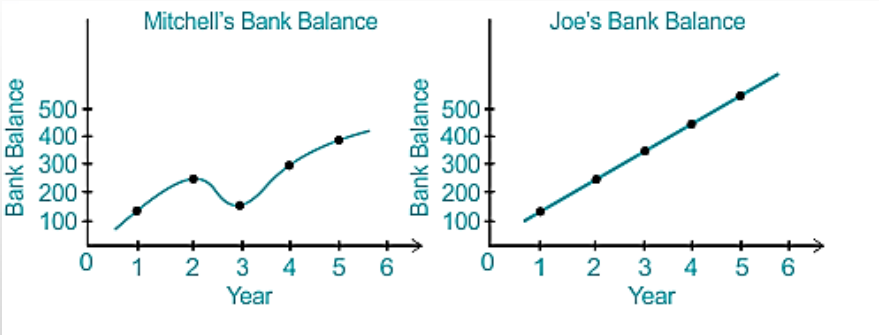
Solution:

We will plot the points for both Joe and Mitchell.



- Joe's points are displayed on a straight line, while Mitchell's points are on a curved line, both with positive slopes.
- It is evident that Joe's graph maintains a constant growth rate with a consistent rate of change of $100, while Mitchell's graph portrays an inconsistent growth pattern with a curve.
- These observations indicate that Joe's growth rate has been constant over the past five years.

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References

- Thomas-Calculus-14th-Edition
- Internet sources

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