Tishk International University
Architectural Engineering Department
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Calculus

Lecture -4- Polynomial Functions

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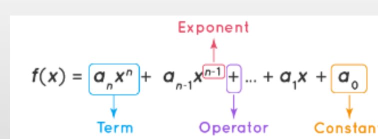
1. Polynomial Functions

- ✓ **Polynomial functions** are expressions that may contain variables of varying degrees, coefficients, positive exponents, and constants.
- ✓ A polynomial function is a function such as a quadratic, a cubic, and so on, involving only non-negative integer powers of x. We can give a general definition of a polynomial and define its degree.
- ✓ Here are some examples of polynomial functions.
  - •$f(x) = 3x^2 - 5$
  - •$g(x) = -7x^3 + (1/2)\, x - 7$
  - •$h(x) = 3x^4 + 7x^3 - 12x^2$



- ✓ We generally represent polynomial functions in decreasing order of the power of the variables i.e. from left to right.
- ✓ A polynomial of degree <u>*n*</u> is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

- ✓ where the *a*'s are real numbers (sometimes called the coefficients of the polynomial). Although this general formula might look quite complicated, examples are much simpler

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❖ For example,

$$f(x) = 4x^3 - 3x^2 + 2$$

is a polynomial of degree 3, as 3 is the highest power of *x* in the formula. This is called a cubic polynomial, or just a cubic.

❖ And another example

$$f(x) = x^7 - 4x^5 + 1$$

is a polynomial of degree 7, as 7 is the highest power of *x*. Notice here that we don't need every power of *x* up to 7: we need to know only the highest power of *x* to find out the degree.

❖ An example you may be familiar with is

$$f(x) = 4x^2 - 2x - 4$$

which is a polynomial of degree 2, as 2 is the highest power of *x*. This is called a quadratic. Functions containing other operations, such as square roots, are not polynomials. For example

$$f(x) = 4x^3 + \sqrt{x} - 1$$

is not a polynomial as it contains a square root. And

$$f(x) = 5x^4 - 2x^2 + 3/x$$

is not a polynomial as it contains a 'divide by *x*'.

2. Types of Polynomial Function

Some of the different types of polynomial functions on the basis of its degrees are given below :

•**Constant Polynomial Function** - A constant polynomial function is a function whose value does not change. It remains the same and it does not include any variables.

•**Zero Polynomial Function** - Polynomial functions with a degree of 1 are known as Linear Polynomial functions.

•**Linear Polynomial Function** - Polynomial functions with a degree of 1 are known as Linear Polynomial functions.

•**Quadratic Polynomial Function** - Polynomial functions with a degree of 2 are known as Quadratic Polynomial functions.

•**Cubic Polynomial Function** - Polynomial functions with a degree of 3 are known as Cubic Polynomial functions.

•**Quartic Polynomial Function** - Polynomial functions with a degree of 4 are known as Quartic Polynomial functions.

General Form of Different Types of Polynomial Function

Degree	Type	General Form
0	Constant	$P(x) = p$
1	Linear	$P(x) = px + q$
2	Quadratic	$P(x) = px^2 + qx + r$
3	Cubic	$P(x) = px^3 + qx^2 + rx + s$
4	Quadratic	$P(x) = px^4 + qx^3 + rx^2 + sx^1 + t$

Standard Form of Different Types of Polynomial Function

Degree	Type	Standard Form
0	Constant	$f(x) = a_0$
1	Linear	$f(x) = a_1x + a_0$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
4	Quadratic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

3. Name of Polynomials

The name of a polynomial is determined by the number of terms in it. The three most common polynomials we usually encounter are monomials, binomials, and trinomials.

- Monomials are polynomials that contain only one term.

  Examples: $15x^2$, $3b$, and $12y^4$

- Binomials are polynomials that contain only two terms.

  Examples: $x^2 + y$, $4x^3 – 7$, and $9x^5 + 2$

- Trinomials are polynomials that contain only three terms.

  Examples: $x^3 – 3 + 5x$, $z^4 + 45 + 3z$, and $x^2 – 12x + 15$

Examples -1-

What is the Degree of the Following Polynomial

i) $5x^4 + 2x^3 + 3x + 4$

Ans: degree is 4

ii) $11x^9 + 10x^5 + 11$

Ans: degree is 9

Examples -2-

Which of the following are polynomial functions?

- $f(x) = 2x^3 \cdot 3x + 4$
- $g(x) = -x(x^2 - 4)$
- $h(x) = 5\sqrt{x} + 2$

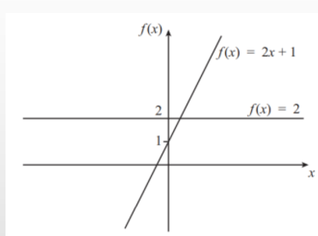
Solution:

- $f(x)$ can be written as $f(x) = 6x^4 + 4$.
- $g(x)$ can be written as $g(x) = -x^3 + 4x$.
- $h(x)$ cannot be written in this form and is therefore not a polynomial function.

4. Graphs of Polynomial Functions

4. Graphs of Polynomial Functions

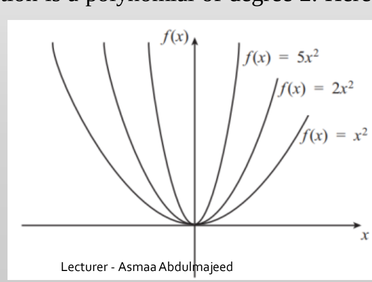
1. We have met some of the basic polynomials already. For example, $f(x) = 2$ is a constant function and $f(x) = 2x + 1$ is a linear function.



It is important to notice that the graphs of constant functions and linear functions are always straight lines

2. We have already said that a quadratic function is a polynomial of degree 2. Here are some examples of quadratic functions:

$$f(x) = x^2, \qquad f(x) = 2x^2, \qquad f(x) = 5x^2$$



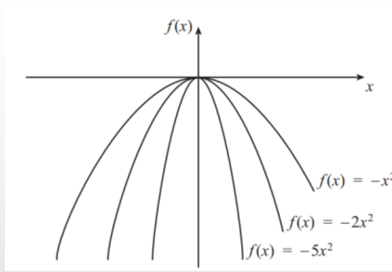
You can see from the graph that, as the coefficient of x^2 is increased, the graph is stretched vertically (that is, in the y direction).

3. What will happen if the coefficient is negative? This will mean that all the positive *f(x)* values will now become negative. So, what will the graphs of the functions look like? The functions are now

$$f(x) = -x^2$$

$$f(x) = -2x^2$$

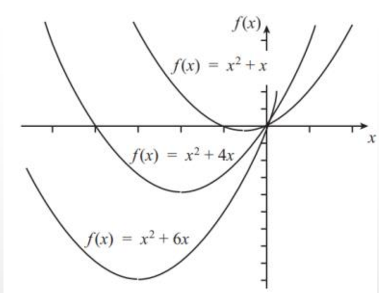
$$f(x) = -5x^2$$



Notice here that all these graphs have been reflected in the *x*-axis. This will always happen for functions of any degree if they are multiplied by −1.

4. Now let us look at some other quadratic functions to see what happens when we vary the coefficient of *x*, rather than the coefficient of x^2. We shall use a table of values in order to plot the graphs, but we shall fill in only those values near the turning points of the functions.

x	-5	-4	-3	-2	-1	0	1	2
x^2+x			6	2	0	0	2	6
x^2+4x		0	-3	-4	-3	0		
x^2+6x	-5	-8	-9	-8	-5			



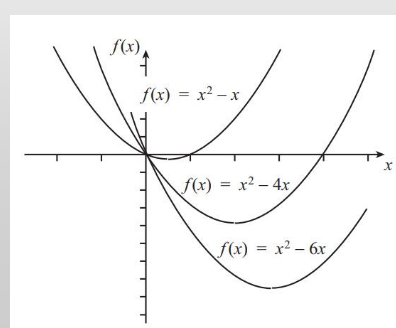
You can see the symmetry in each row of the table, demonstrating that we have concentrated on the region around the turning point of each function. We can now use these values to plot the graphs.

As you can see, increasing the positive coefficient of x in this polynomial moves the graph down and to the left.

5. What happens if the coefficient of *x* is negative?

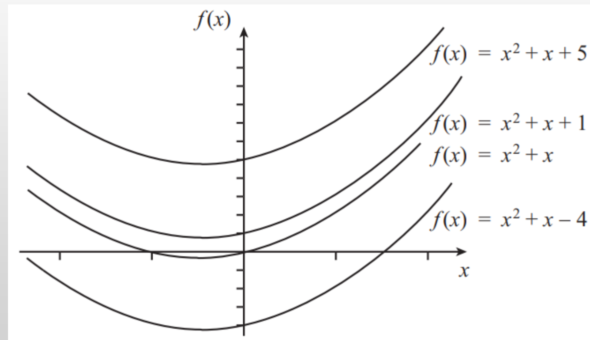
Again, we can use these tables of values to plot the graphs of the functions. As you can see, increasing the negative coefficient of *x* (in absolute terms) moves the graph down and to the right.

x	-2	-1	0	1	2	3	4	5
x^2-x	6	2	0	0	2	6		
x^2-4x			0	-3	-4	-3	0	
x^2-6x				-5	-8	-9	-8	-5



6. So now we know what happens when we vary the x^2 coefficient, and what happens when we vary the x coefficient. But what happens when we vary the constant term at the end of our polynomial? We already know what the graph of the function $f(x) = x^2 + x$ looks like, so how does this differ from the graph of the functions $f(x) = x^2 + x + 1$, or $f(x) = x^2 + x + 5$, or $f(x) = x^2 + x - 4$? As usual, a table of values is a good place to start.

x	-2	-1	0	1	2
x^2+x	2	0	0	2	6
x^2+x+1	3	1	1	3	7
x^2+x+5	7	5	5	7	11
x^2+x-4	-2	-4	-4	-2	2

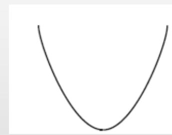


5. Turning Points of Polynomial Functions

5. Turning Points of Polynomial Functions

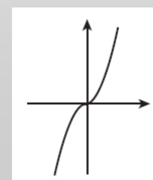
- A turning point of a function is a point where the graph of the function changes from sloping downwards to sloping upwards, or vice versa. So, the gradient changes from negative to positive, or from positive to negative. Generally speaking, curves of degree ***n*** can have up to ***(n − 1)*** turning points.

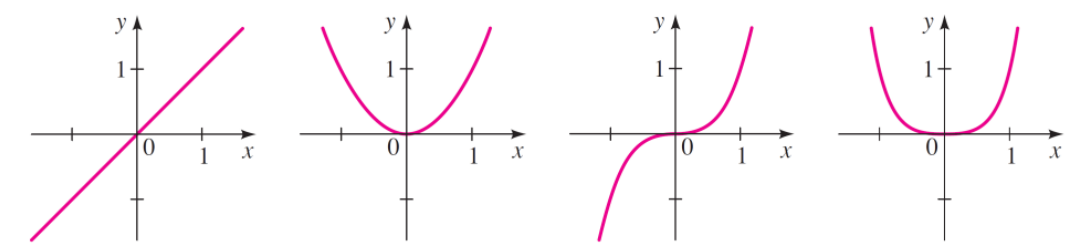
1. For instance, a quadratic has only one turning point.



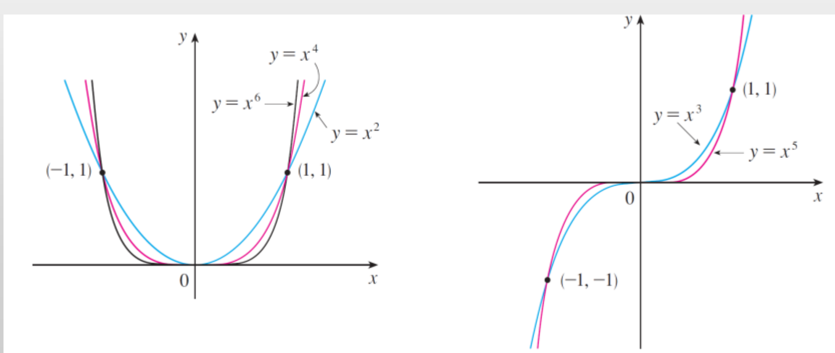
2. A cubic could have up to two turning points, and so would look something like this.

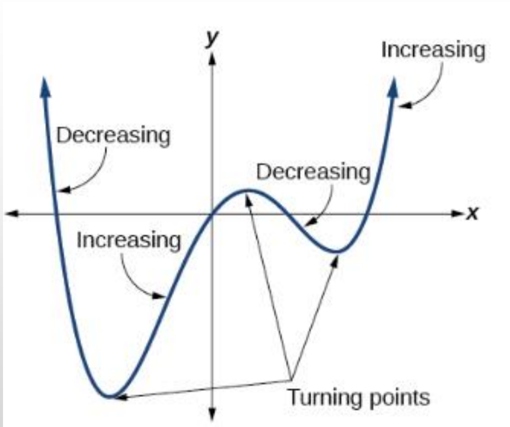
3.However, some cubics have fewer turning points: for example, $f(x) = x^3$. But no cubic has more than two turning points.





(a) $y = x$ (b) $y = x^2$ (c) $y = x^3$ (d) $y = x^4$





6. Roots of Polynomial Function

7. Zeros of Polynomial Functions

8. Description of the Behavior at Each x-intercept

Terminology of Polynomial Functions

We often rearrange polynomials so that the powers are descending.

Leading coefficient — Degree

$$f(x) = \underbrace{a_n x^n}_{\text{Leading term}} + \ldots.. + a_2 x^2 + a_1 x + a_0$$

When a polynomial is written in this way, we say that it is in ***general form***.

Given a polynomial function, identify the degree and leading coefficient

1. Find the highest power of x to determine the degree function.
2. Identify the term containing the highest power of x to find the leading term.
3. Identify the coefficient of the leading term.

Examples -3-

Identify the degree, leading term, and leading coefficient of the following polynomial functions.

$$f(x) = 3 + 2x^2 - 4x^3$$
$$g(t) = 5t^5 - 2t^3 + 7t$$
$$h(p) = 6p - p^3 - 2$$

Solution :

For the function $f(x)$, the highest power of x is 3, so the degree is 3. The leading term is the term containing that degree, $-4x^3$. The leading coefficient is the coefficient of that term, −4.

For the function $g(t)$, the highest power of t is 5, so the degree is 5. The leading term is the term containing that degree, $5t^5$. The leading coefficient is the coefficient of that term, 5.

For the function $h(p)$, the highest power of p is 3, so the degree is 3. The leading term is the term containing that degree, $-p^3$; the leading coefficient is the coefficient of that term, −1.

Examples -4-

Identify the degree, leading term, and leading coefficient of the following polynomial functions.

$$f(x) = 4x^2 - x^6 + 2x - 6.$$

Solution :

Degree:

Leading term:

Leading coefficient:

6. Roots of Polynomial Functions

- You may recall that when $(x - a)(x - b) = 0$, we know that a and b are roots of the function $f(x) = (x - a)(x - b)$.
- Now we can use the converse of this, and say that if a and b are roots, then the polynomial function with these roots must be $f(x) = (x - a)(x - b)$, or a multiple of this.
- For example, if a quadratic has roots $x = 3$ and $x = -2$, then the function must be $f(x) = (x-3)(x+2)$, or a constant multiple of this.
- This can be extended to polynomials of any degree. For example, if the roots of a polynomial are x = 1, x = 2, x = 3, x = 4, then the function must be

$$f(x) = (x-1)(x-2)(x-3)(x-4),$$

7. Zeros of Polynomial Functions

❖ You will need to set the function equal to zero and then use the Zero Product Property to find the x-intercept(s). That means if ab = 0, then either a = 0 or b = 0 .

❖ To find the y-intercept of a function, you will find f (0).

❖ In some problems, one or more of the factors will appear more than once when the function is factored.

❖ The power of a factor is called its *multiplicity*. So given $P(x) = x^3(x-3)^2(x+2)^1$, then the multiplicity of the first factor is 3, the multiplicity of the second factor is 2 and the multiplicity of the third factor is 1.

8. Description of the Behavior at Each x-intercept

1. ***Even Multiplicity***: The graph touches the x-axis, but does not cross it. It looks like a parabola there.
2. Multiplicity of 1: The graph crosses the x-axis. It looks like a line there.
3. ***Odd Multiplicity greater than or equal 3***: The graph crosses the x-axis. It looks like a cubic there.

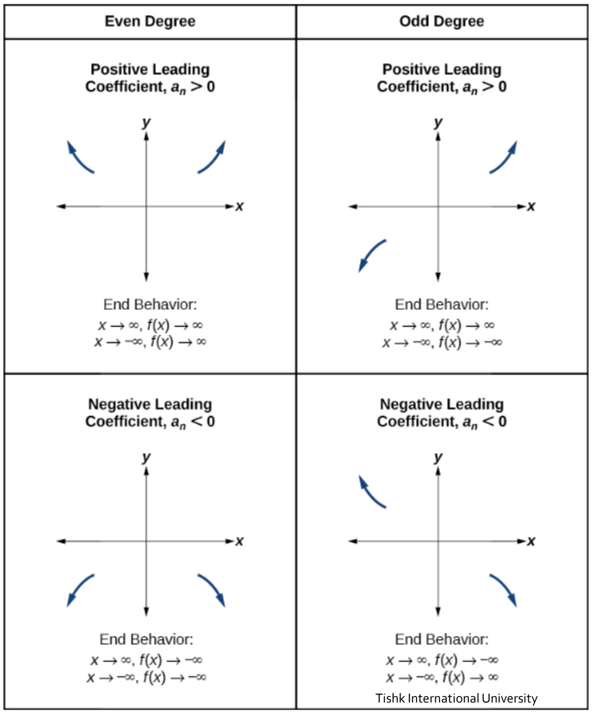
You can use all of this information to generate the graph of a polynomial function.

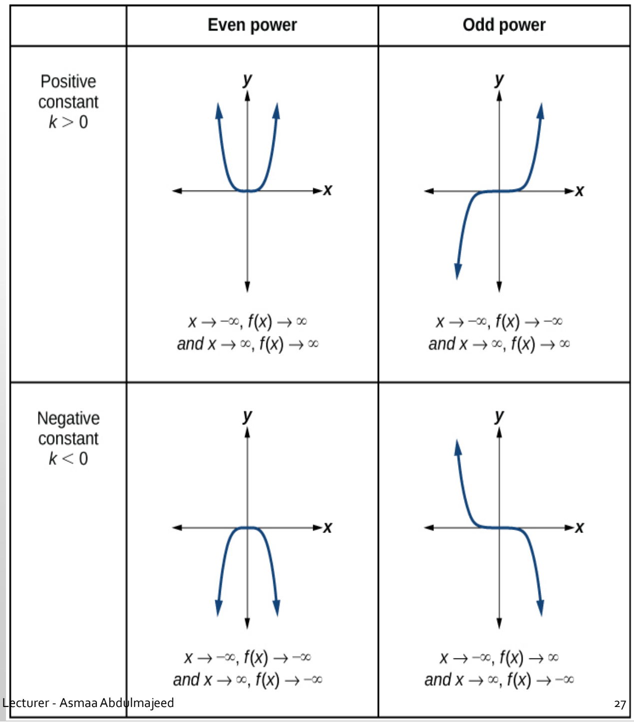
· degree of the function

· end behavior of the function

· x and y intercepts (and multiplicities)

· behavior of the function through each of the x intercepts (zeros) of the function





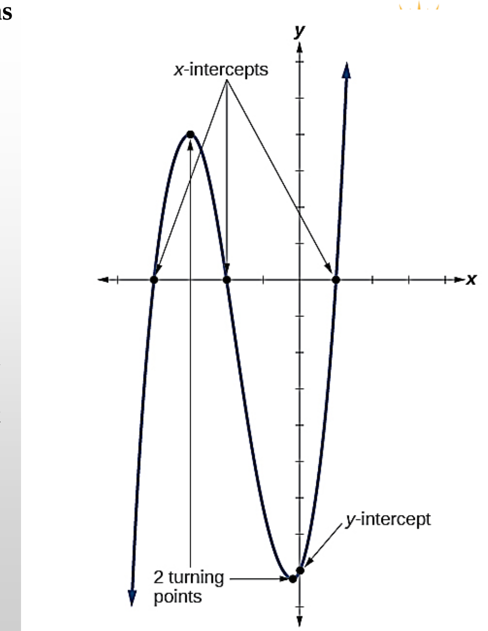
Identifying Local Behavior of Polynomial Functions

In addition to the end behavior of polynomial functions, we are also interested in what happens in the "middle" of the function. In particular, we are interested in locations where graph behavior changes.

1. A ***turning point*** is a point at which the function values change from increasing to decreasing or decreasing to increasing.

2. The ***y-intercept*** is the point at which the graph intersects the *vertical axis*. The point corresponds to the coordinate pair in which the input value is zero. Because a polynomial is a function, *only one Output* value corresponds to each Input value so there can be *only one y- intercept* $(0,a_0)$.

3. The ***x-intercepts*** occur at the input values that correspond to an output value of zero. It is possible to have *more than one* x-intercept .



Steps to Graphing other Polynomials:

1. Determine the ***leading term***. Is the degree even or odd? Is the sign of the leading coefficient positive or negative?
2. Determine the ***end behavior***. Which one of the 4 cases will it look like on the ends?
3. Factor the polynomial.
4. Make a table listing the factors, ***x-intercepts***, ***multiplicity***, and describe the behavior at each x intercept.
5. Find the ***y- intercept***.
6. Draw the graph, being careful to make a nice smooth curve with no sharp corners.

SUMMARY OF TERMINOLOGIES

A **polynomial** is function that can be written as $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

Each of the a_i constants are called **coefficients** and can be positive, negative, or zero, and be whole numbers, decimals, or fractions.

A **term** of the polynomial is any one piece of the sum, that is any a_ix^i. Each individual term is a transformed power function.

The **degree** of the polynomial is the highest power of the variable that occurs in the polynomial.

The **leading term** is the term containing the highest power of the variable: the term with the highest degree.

The **leading coefficient** is the coefficient of the leading term.

Because of the definition of the "leading" term we often rearrange polynomials so that the powers are descending.
$f(x) = a_nx^n + \cdots + a_2x^2 + a_1x + a_0$

- Let us also think about the function $f(x) = (x - 2)^2$. We can see straight away that $x - 2 = 0$, so that $x = 2$. For this function we have only one root. This is what we call a *repeated root*, and a root can be repeated any number of times. For example, $f(x) = (x - 2)^3(x + 4)^4$ has a repeated root $x = 2$, and another repeated root $x = -4$. We say that *the root $x = 2$ has multiplicity 3, and that the root $x = -4$ has multiplicity 4*.

Examples -5-

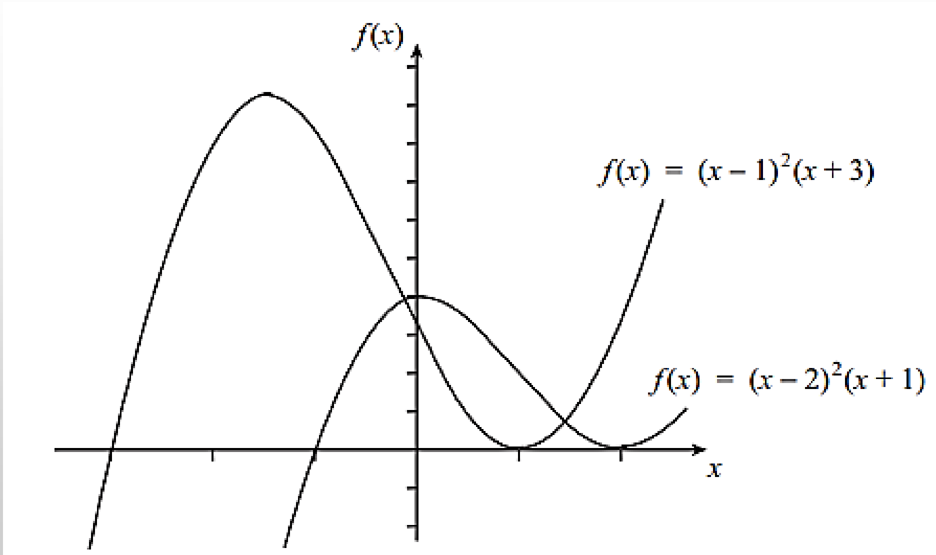
Sketch the following functions:

1. $f(x) = (x - 2)^2(x + 1)$

2. $f(x) = (x - 1)^2(x + 3)$

Solution:

1. To take another example, suppose we have the function $f(x) = (x - 2)^2(x + 1)$.
2. We can see that the largest power of x is 3, and so the function is a cubic.
3. We know the possible general shapes of a cubic, and as the coefficient of x^3 is positive the curve must generally increase to the right and decrease to the left.
4. We can also see that the roots of the function are $x = 2$ and $x = -1$. The root $x = 2$ has <u>*even*</u> multiplicity and so the curve just touches the x-axis here, whilst $x = -1$ has <u>*odd*</u> multiplicity and so here the curve crosses the x-axis.
5. This means we can sketch the graph as follows.



Examples -6-

Given the function $f(x) = -3x^2(x-1)(x+4)$ express the function as a polynomial in general form, and determine the leading term, degree, and end behavior of the function.

Solution:

Obtain the general form by expanding the given expression for $f(x)$.

$$\begin{aligned} f(x) &= -3x^2(x-1)(x+4) \\ &= -3x^2(x^2+3x-4) \\ &= -3x^4-9x^3+12x^2 \end{aligned}$$

The general form is $f(x) = -3x^4 - 9x^3 + 12x^2$. The leading term is $-3x^4$; therefore, the degree of the polynomial is 4. The degree is even (4) and the leading coefficient is negative (–3), so the end behavior is

$$\text{as } x \to -\infty,\ f(x) \to -\infty$$

$$\text{as } x \to \infty,\ f(x) \to -\infty$$

Examples -7-

Given the function $f(x) = 0.2(x-2)(x+1)(x-5)$ express the function as a polynomial in general form, and determine the leading term, degree, and end behavior of the function.

Solution:

The leading term is $0.2x^3$, so it is a degree 3 polynomial. As x approaches positive infinity, $f(x)$ increases without bound; as x approaches negative infinity, $f(x)$ decreases without bound.

Examples -8-

Decide whether the following functions are even, odd or neither.

1. $f(x)=3x^2-4$
2. $g(x) = 1/2x$
3. $f(x) = x^3 - x^2$.

Solution

1.

$$f(-x) = 3(-x)^2 - 4 = 3x^2 - 4 = f(x)$$

The function $f(x) = 3x^2 - 4$ is even.

2.

$$g(-x) = \frac{1}{2(-x)} = \frac{1}{-2x} = -\frac{1}{2x} = -g(x)$$

Therefore, the function g is odd.

3.

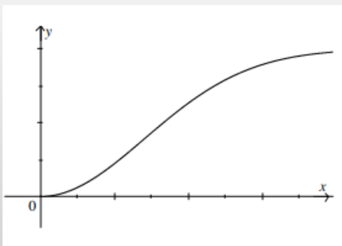
$$f(-x) = (-x)^3 - (-x)^2 = -x^3 - x^2$$

This function is neither even (since $-x^3 - x^2 \neq x^3 - x^2$) nor odd (since $-x^3 - x^2 \neq -(x^3 - x^2)$).

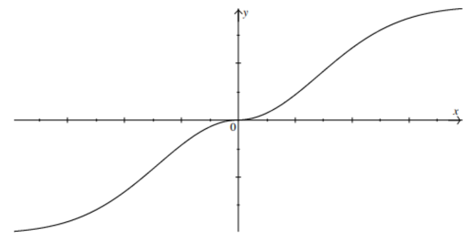
Examples -9-

- Sketched below is part of the graph of y = f(x).
- Complete the graph if y = f(x) is

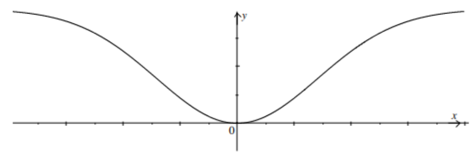
1. Odd
2. Even



Solution



$y = f(x)$ is an odd function.



$y = f(x)$ is an even function.

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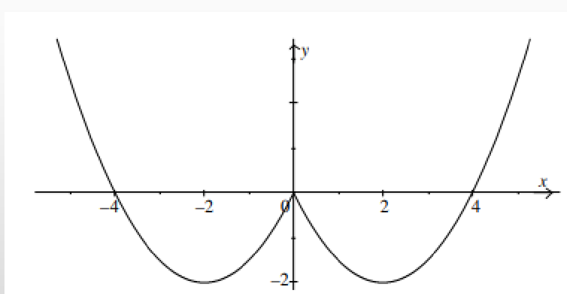
Examples -10-

Is *y* = *f(x)* even, odd or neither?



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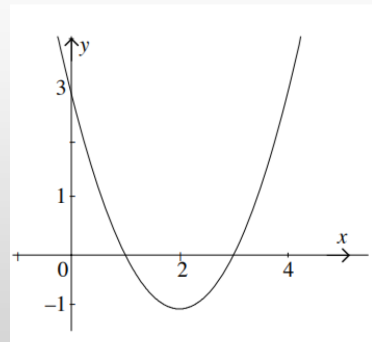
Examples -11-

Let $Q(x) = x^2 - 4x + 3$. We find the zeros of Q(x) by solving the equation Q(x) = 0.

$$\begin{aligned} x^2 - 4x + 3 &= 0 \\ (x-1)(x-3) &= 0 \\ \text{Therefore } x &= 1 \text{ or } 3. \end{aligned}$$

The roots are rational (hence real) and distinct.

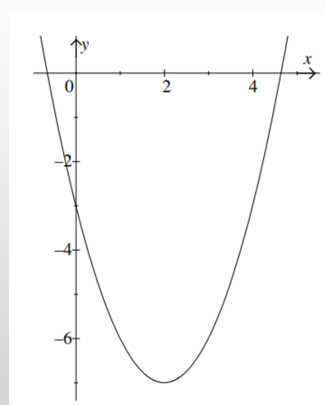


Examples -12-

Let $Q(x) = x^2 - 4x - 3$. Solving the equation Q(x) = 0 we get,

$$\begin{aligned} x^2 - 4x - 3 &= 0 \\ x &= \frac{4 \pm \sqrt{16+12}}{2} \\ \text{Therefore } x &= 2 \pm \sqrt{7}. \end{aligned}$$

The roots are irrational (hence real) and distinct.

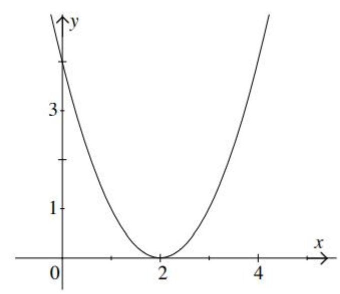


Examples -13-

Let $Q(x) = x^2 - 4x + 4$. Solving the equation Q(x) = 0 we get,

$$\begin{aligned} x^2 - 4x + 4 &= 0 \\ (x-2)^2 &= 0 \\ \text{Therefore } x &= 2. \end{aligned}$$

The roots are rational (hence real) and equal. $Q(x) = 0$ has a repeated or double root at $x = 2$.

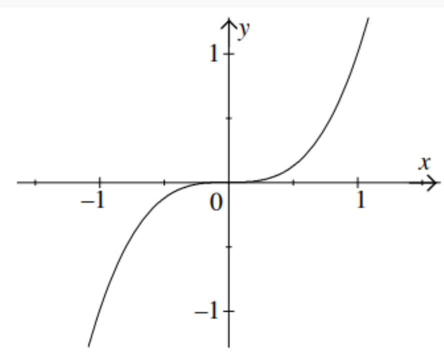


Notice that the graph turns at the double root $x = 2$.

Examples -14-

Let $Q(x) = x^3$. Solving $Q(x) = 0$ we get that $x^3 = 0$.

We can write this as $(x-0)^3 = 0$. So, this equation has three equal real roots at $x = 0$.

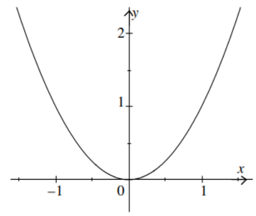


Examples -15-

Sketch the following:

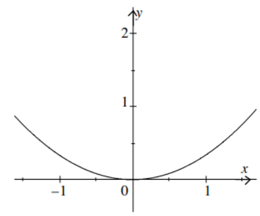
a. $y = x^2$ **b.** $y = \frac{1}{3}x^2$ **c.** $y = -x^2$ **d.** $y = (x+1)^2$

1. a.



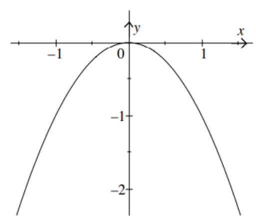
The graph of $y = x^2$.

b.



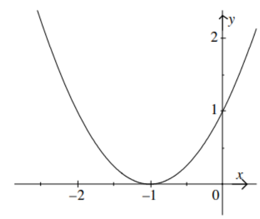
The graph of $y = \frac{x^2}{3}$.

c.



The graph of $y = -x^2$.

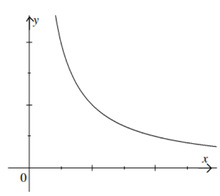
d.



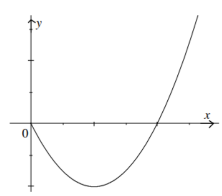
The graph of $y = (x+1)^2$.

Examples -16-

Complete the following functions if they are defined to be (a) even (b) odd.

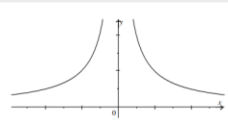


$y = f(x)$



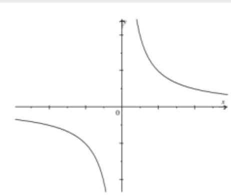
$y = g(x)$

a.



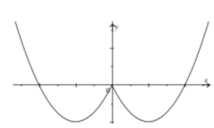
$y = f(x)$ is even.

b.



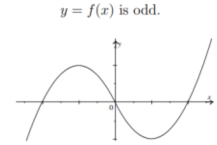
$y = f(x)$ is odd.

a.



$y = g(x)$ is even.

b.



$y = g(x)$ is odd.

Examples -17-

Determine whether the following functions are odd, even or neither.

a. $y = x^4 + 2$ **b.** $y = \sqrt{4 - x^2}$ **c.** $y = 2^x$ **d.** $y = x^3 + 3x$

e. $y = \frac{x}{x^2}$ **f.** $y = \dfrac{1}{x^2 - 4}$ **g.** $y = \dfrac{1}{x^2 + 4}$ **h.** $y = \dfrac{x}{x^3 + 3}$

i. $y = 2^x + 2^{-x}$ **j.** $y = |x - 1| + |x + 1|$

a.	even	**b.**	even	**c.**	neither	**d.**	odd	**e.**	odd
f.	even	**g.**	even	**h.**	neither	**i.**	even	**j.**	even

Examples -18-

Sketch the graphs of the following functions

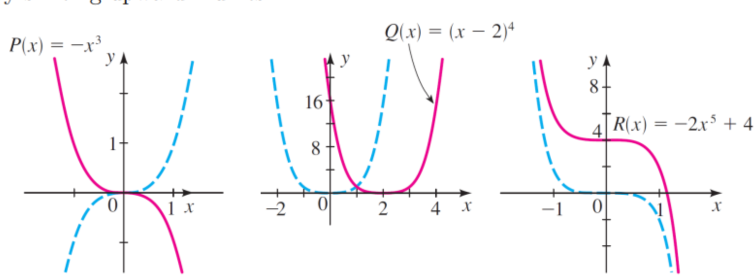
(a) $P(x) = -x^3$ (b) $Q(x) = (x - 2)^4$ (c) $R(x) = -2x^5 + 4$

Solution:

(a) The graph of $P(x) = -x^3$ is the reflection of the graph of $y = x^3$ in the x-axis.

(b) The graph of $Q(x) = (x - 2)^4$ is the graph of $y = x^4$ shifted to the right 2 units.

(c) We begin with the graph of $y = x^5$. The graph of $y = -2x^5$ is obtained by stretching the graph vertically and reflecting it in the x-axis. Finally, the graph of $R(x) = -2x^5 + 4$ is obtained by shifting upward 4 units.



(a) (b) (c)

Examples -19-

Given the polynomial function $f(x) = (x-2)(x+1)(x-4)$ written in factored form for your convenience, determine the y- and x-intercepts.

Solution:

The *y-intercept* occurs when the *Input is zero*, so substitute 0 for *x*.

$$\begin{aligned} f(0) &= (0-2)(0+1)(0-4) \\ &= (-2)(1)(-4) \\ &= 8 \end{aligned}$$

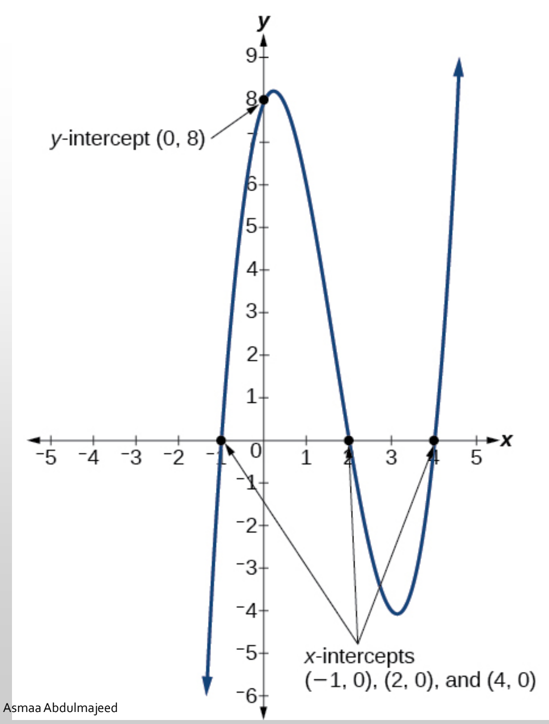
The y-intercept is (0,8).

The *x-intercepts* occur when the *output is zero*.

$$0 = (x-2)(x+1)(x-4)$$

$$\begin{array}{ccccc} x-2=0 & \text{or} & x+1=0 & \text{or} & x-4=0 \\ x=2 & \text{or} & x=-1 & \text{or} & x=4 \end{array}$$

The *x-intercepts* are (2,0),(–1,0), and (4,0).



Examples -20-

Given the polynomial function $f(x) = x^4 - 4x^2 - 45$ written in factored form for your convenience, determine the y- and x-intercepts.

Solution:

The y-intercept occurs when the Input is zero.

$$f(0) = (0)^4 - 4(0)^2 - 45$$
$$= -45$$

The y-intercept is (0,−45).

The x-intercepts occur when the output is zero. To determine when the output is zero, we will need to factor the polynomial.

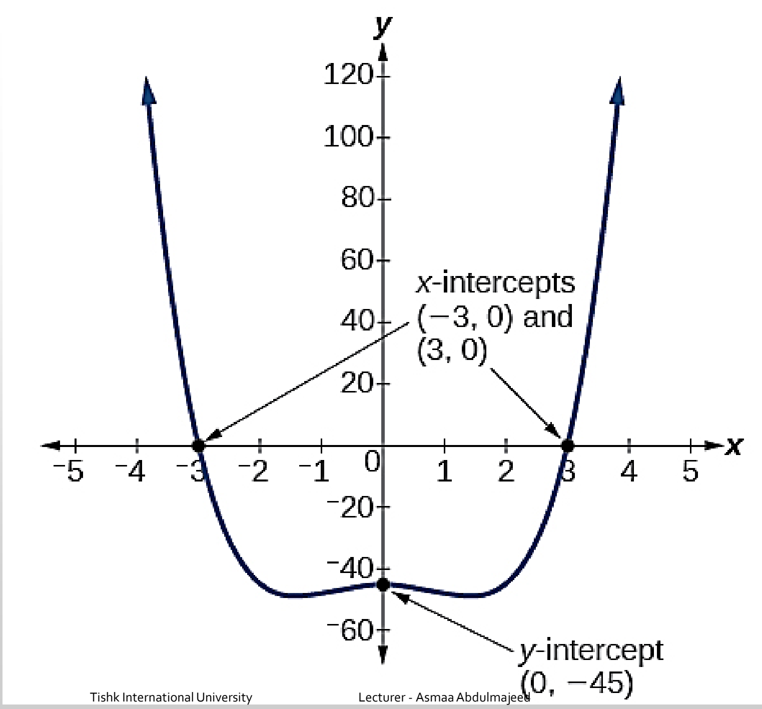
$$f(x) = x^4 - 4x^2 - 45$$
$$= (x^2 - 9)(x^2 + 5)$$
$$= (x - 3)(x + 3)(x^2 + 5)$$

$$0 = (x - 3)(x + 3)(x^2 + 5)$$

$x - 3 = 0$ or $x + 3 = 0$ or $x^2 + 5 = 0$

$x = 3$ or $x = -3$ or (no real solution)

The x-intercepts are (3,0) and (–3,0).



Examples -21-

Find the zeros then graph the polynomial. Be sure to label the x-intercepts, y-intercept if possible and have correct end behavior.

$$P(x)= x^4 (x-2)^3 (x+1)^2$$

Solution;

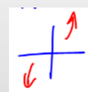
*1. **x** and **y** intercepts*

x- intercept; x= 0, 2, -1

y- intercept; y= 0

2. Leading term = x4.x3.x^2 = x^9

3. End behaviour = odd function x^9, positive.

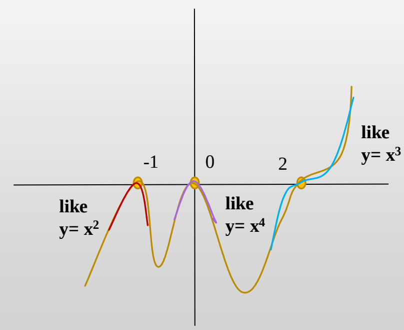


$\longrightarrow$

4. x= -1 $\longrightarrow$ y= x^2

x= 0 $\longrightarrow$ y= x^4

x= 2 y= x^3



Examples -22-

Find the zeros then graph the polynomial. Be sure to label the x intercepts, y intercept if possible and have correct end behavior.

$$P(x)= x^3 (x+2)^1 (x-3)^2$$

Solution;

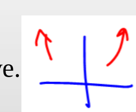
*1. **x** and **y** intercepts*

x- intercept; x= 0, -2, 3

y- intercept; y= 0

2. Leading term = $x^3.x^1.x^2 = x^6$

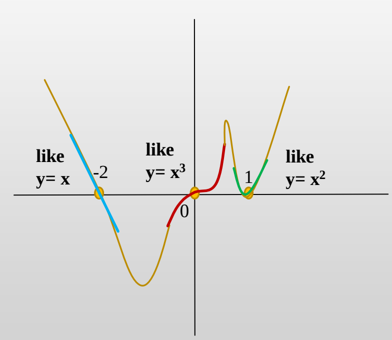
3. End behaviour = even function x^6, positive.



4. x= -2 $\longrightarrow$ y= x^2

x= 0 $\longrightarrow$ y= x^4

x= 3 $\longrightarrow$ y= x^3



Examples -23-

Find the zeros then graph the polynomial. Be sure to label the x-intercepts, y-intercept if possible and have correct end behavior.

$$P(x) = -2(x+1)^2\,(x-3)$$

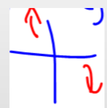
Solution;

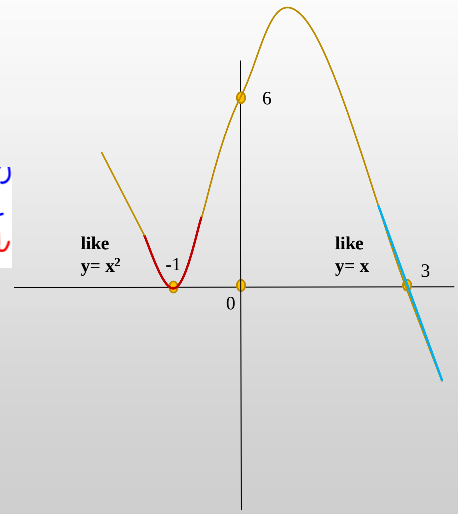
1. ***x*** *and* ***y*** *intercepts*

   x- intercept; x= -1, 3

   y- intercept; $y = -2\,(0+1)^2\,(0-3) = 6$

2. Leading term = $-2.x^2.x^1 = -2x^3$
3. End behaviour = odd function (x^3), negative (-).





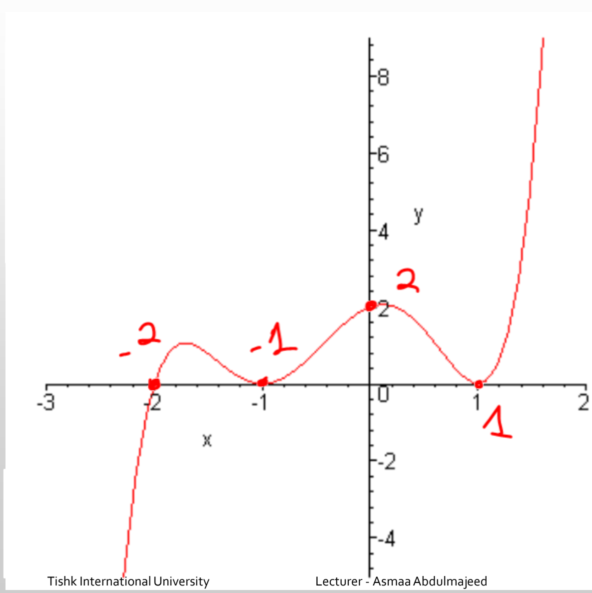
4. $x = -1 \longrightarrow y = x^2$

   $x = 3 \longrightarrow y = x$

   $x = 0 \longrightarrow y = 6$

Examples -24-

Given the graph of a polynomial determine what the equation of that polynomial.



References

- Thomas-Calculus-14th-Edition
- Internet sources