

Lecture 5:

- Powers
- Logarithms



$$\log_{\text{😊}} \text{♥} = \text{🌸}$$

$$\text{😊}^{\text{🌸}} = \text{♥}$$

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What will we learn?

- Power and n th root functions
- Fractional powers and n th roots
- Number System Conversion
- Logarithmic Functions



$$1 \text{ GB} = 10^9$$

Powers / Exponents



Exponents and Viral Marketing

If One Person , tells another 10 people, and then each of these 10 people tell another 10 people, and so on, we get rapid spreading of a message, video, photo, news item, or product across the Internet.

Level	0	1	2	3	4	etc
Spread	1	+ 10	+ 100	+ 1000	+ 10 000	
Powers	10^0	10^1	10^2	10^3	10^4	

Spread = 10^{Level}



Image Source: <http://m5.paperblog.com>

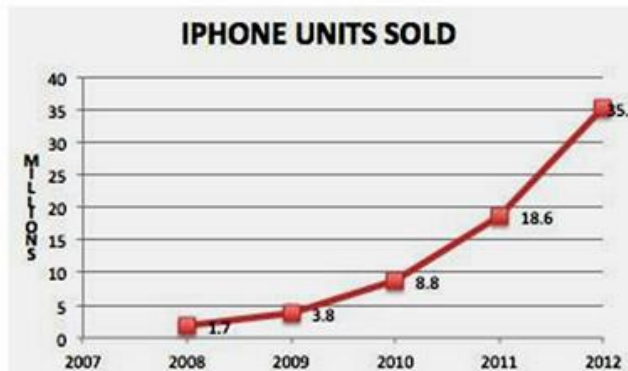
EARTHQUAKES



Exponents are used to measure the strength of earthquakes. A level 1 earthquake is 1×10^1 , a level 2 earthquake is 1×10^2 , a level 3 is 1×10^3 , etc.

Smart Phone Uptake and Sales

At first only a few people had smart phones, then within only a few years, it seems that everybody has an iPhone or similar. Eg. The Growth in Smart Phone usage has been Exponential.



Exponents in Computer Games

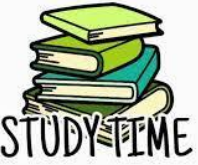
Computer Games use "Game Physics Engines" which are low level programs inside the game to calculate the movement, interactions, and the geometry involved with the game.

These programs use lots of Algebra formulas in their Algorithms, and many of these formulas involve multiplying powers terms containing exponents.

If the mathematics isn't correct in the game engine, then the game is not going to play at all like we would expect it to.



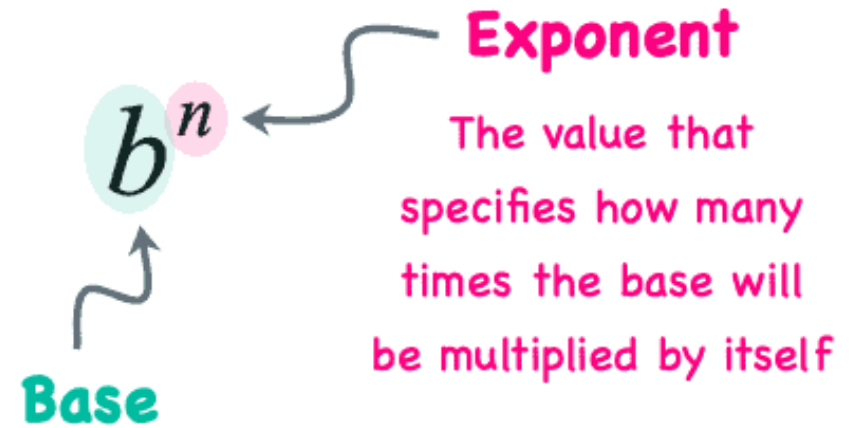
Exponents / Powers



$$\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}_{8 \text{ times}} = 5^8$$

$$\begin{array}{ccccccccc} 2 & \times & 2 & \times & 2 & \times & 2 & \times & 2 & = & 32 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\ \boxed{1} & & \boxed{2} & & \boxed{3} & & \boxed{4} & & \boxed{5} & & \\ \underbrace{\hspace{10em}} & & & & & & & & & & \\ & & & & & & \boxed{2^5} & & & & \end{array}$$

$$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^9$$



The number or variable that is being multiplied repeatedly in the expanded form

Exponents/Powers Properties

RULES

Law	Example
$a^m a^n = a^{m+n}$	$2^3 2^4 = 2^{3+4} = 2^7 = 128$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12} = 4096$
$(ab)^n = a^n b^n$	$(20)^3 = (2 \cdot 10)^3 = 2^3 \cdot 10^3 = 8 \cdot 1000 = 8000$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$
$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2} = \frac{1}{4}$

1. $3 \cdot 4^3$

2. $4x^3 \cdot 2x^3$

3. $x^5 \cdot x^3$

4. $2x^3 \cdot 2x^2$

5. $\frac{6^5}{6^3}$

6. $\frac{x^4}{x^7}$

7. 8^0

8. $-(9x)^0$

9. $(y^4)^3$

10. $(x^2y)^4$

11. $\frac{6x^7}{2x^4}$

12. $\frac{8x^5}{4x^2}$

13. $(2cd^4)^2(cd)^5$

14. $(2fg^4)^4(fg)^6$

15. $\frac{x^5y^6}{xy^2}$

16. $\frac{x^2y^5}{xy^4}$

17. $\left(\frac{4x^5y}{16xy^4}\right)^3$

18. $\left(\frac{5x^3y}{20xy^5}\right)^4$

19. y^{-7}

20. 7^{-2}

21. $\frac{1}{x^{-5}}$

22. $\frac{1}{2^{-4}}$

23. $x^5 \cdot x^{-1}$

24. x^{-6}

25. $x^9 \cdot x^{-7}$

26. $(j^{-13})(j^4)(j^6)$

27. $\frac{x^{-1}}{x^{-8}}$

28. $\frac{52x^6}{13x^{-7}}$

29. $f^{-3}(f^2)(f^{-3})$

30. $\frac{x^{-4}}{x^{-9}}$

31. $\frac{24x^6}{12x^{-8}}$

32. $\frac{3x^2y^{-3}}{12x^6y^3}$

33. $(2x^3y^{-3})^{-2}$

34. $\frac{2x^4y^{-4}}{8x^7y^3}$

35. $(4x^4y^{-4})^3$

36. $5x^2y(2x^4y^{-3})$

37. $\left(\frac{-7a^2b^3c^0}{3a^3b^4c^3}\right)^{-4}$

38. $\left(\frac{-2a^3b^2c^0}{3a^2b^3c^7}\right)^{-2}$

ANSWERS

1. 192

2. $8x^6$

3. x^8

4. $4x^5$

5. 36

6. $\frac{1}{x^3}$

7. 1

8. -1

9. y^{12}

10. x^8y^4

11. $3x^3$

12. $2x^3$

13. $4c^7d^{13}$

14. $16f^{10}g^{22}$

15. x^4y^4

16. xy

17. $\frac{x^{12}}{64y^9}$

18. $\frac{x^8}{256y^{16}}$

19. $\frac{1}{y^7}$

20. $\frac{1}{49}$

21. x^5

22. 16

23. x^4

24. $\frac{1}{x^6}$

25. x^2

26. $\frac{1}{j^3}$

27. x^7

28. $4x^{13}$

29. $\frac{1}{f^4}$

30. x^5

31. $2x^{14}$

32. $\frac{1}{4x^4y^6}$

33. $\frac{y^6}{4x^6}$

34. $\frac{1}{4x^3y^7}$

35. $\frac{64x^{12}}{y^{12}}$

36. $\frac{10x^6}{y^2}$

37. $\frac{81a^4b^4c^{12}}{2401}$

38. $\frac{9b^2c^{14}}{4a^2}$

Exponents/Powers with Roots

RULES

Index Number

Radical

Radicaud

$$\sqrt[n]{a} = a^{1/n}$$

$n > 1$

The index number becomes the denominator of the exponent.

*n*th root function or fractional power:

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$\sqrt[3]{7} = 7^{\frac{1}{3}}$$

$$\sqrt[4]{5^3} = 5^{\frac{3}{4}}$$

...

Negative Exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{For } a \neq 0$$

a^{-n} is a reciprocal of a^n

Example:

$$3^{-2} = \frac{1}{3^2}$$

$$\left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^6$$

$$\star 3^{1/4} \cdot 3^{3/4}$$

$$\star (3^3 \cdot 6^3)^{-1/3}$$

$$\star \left(\frac{54^{1/4}}{27^{1/4}} \right)^2$$

$$\star (64^{1/3} \cdot 8^{1/3})^2$$

$$\star \frac{4}{4^{1/2}}$$

$$\star \sqrt[3]{25} \cdot \sqrt[3]{5}$$

$$\star \frac{\sqrt[3]{32}}{\sqrt[3]{4}}$$

$$\star \sqrt[4]{27} \cdot \sqrt[4]{3}$$

$$\star \frac{\sqrt[3]{625}}{\sqrt[3]{5}}$$

$$\star 6^{1/2} \cdot 6^{1/3}$$

$$\star (4^3 \cdot 2^3)^{-1/3}$$

$$\star (27^{1/3} \cdot 6^{1/4})^2$$

$$\star \frac{6}{6^{3/4}}$$



$$(a) x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$$

$$(b) a^{\frac{3}{5}} \cdot a^{\frac{12}{5}}$$

$$(c) (9x)^{\frac{1}{2}} \cdot (4x^{\frac{1}{4}})$$

$$(d) ((2b)^{\frac{2}{9}})^3 \cdot (2b)^{\frac{1}{3}}$$

$$(e) \frac{x^{\frac{3}{2}} x^{\frac{1}{2}}}{x^{\frac{5}{2}}}$$

$$(f) (27z^3)^{-\frac{2}{3}}$$

$$(g) (x^5 y^4)^{-\frac{1}{2}}$$

$$(h) (-8x^6 y^{-18})^{-\frac{1}{3}}$$

$$y^{-1} (yx^{\frac{1}{2}})^{\frac{2}{3}}$$

$$(i) \left(\frac{a^{\frac{3}{2}}}{b^{-\frac{1}{2}}} \right)^4 \left(\frac{a^{-2}}{b^3} \right)$$

$$(j) \left(\frac{x^6 y^{-3}}{27y^{\frac{3}{5}}} \right)^{-\frac{1}{3}}$$

$$(a) x^2 \quad (b) a^3 \quad (c) 12x^{\frac{3}{4}} \quad (d) 2b \quad (e) \frac{1}{x^{\frac{1}{2}}} \quad (f) \frac{1}{9z^2} \quad (g) \frac{1}{x^{\frac{5}{2}} y^2} \quad (h) \frac{-y^6}{2x^2}$$

$$(i) \frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}} \quad (j) \frac{a^4}{b} \quad (k) \frac{3y^{\frac{6}{5}}}{x^2}$$

Express the following in the form x^r .

(a) $(\sqrt[5]{x})^6$ (b) $\sqrt[8]{x^3}$ (c) $\frac{1}{(\sqrt{x})^5}$ (d) $\frac{1}{\sqrt[3]{x^4}}$ (e) $\sqrt[4]{\sqrt[3]{x}}$ (f) $\sqrt{\frac{1}{\sqrt[5]{x}}}$

Express the following in the form x^r .

(a) $x^{\frac{5}{2}}x^3$ (b) $\frac{x^{\frac{6}{7}}}{x^4}$ (c) $(x^3)^{-\frac{4}{5}}$ (d) $x^{\frac{7}{5}}x^{-\frac{8}{3}}$ (e) $(x^{\frac{2}{3}})^{\frac{4}{9}}$ (f) $\frac{1}{x^{\frac{5}{2}}}$
(g) $\left(\frac{1}{x^3}\right)^{-\frac{2}{3}}$ (h) $\frac{1}{x\sqrt{x}}$ (i) $x^2(\sqrt[3]{x})$ (j) $\frac{x}{x^{\frac{2}{5}}}$ (k) $\frac{x^{\frac{1}{3}}}{x}$ (l) $\frac{1}{x^{-\frac{5}{4}}}$

(a) $x^{\frac{6}{5}}$ (b) $x^{\frac{3}{8}}$ (c) $x^{-\frac{5}{2}}$ (d) $x^{-\frac{4}{3}}$ (e) $x^{\frac{1}{12}}$ (f) $x^{-\frac{1}{10}}$

(a) $x^{\frac{11}{2}}$ (b) $x^{-\frac{22}{7}}$ (c) $x^{-\frac{12}{5}}$ (d) $x^{-\frac{19}{15}}$ (e) $x^{\frac{8}{27}}$ (f) $x^{-\frac{5}{2}}$ (g) x^2 (h) $x^{-\frac{3}{2}}$

(i) $x^{\frac{7}{3}}$ (j) $x^{\frac{3}{5}}$ (k) $x^{-\frac{2}{3}}$ (l) $x^{\frac{5}{4}}$



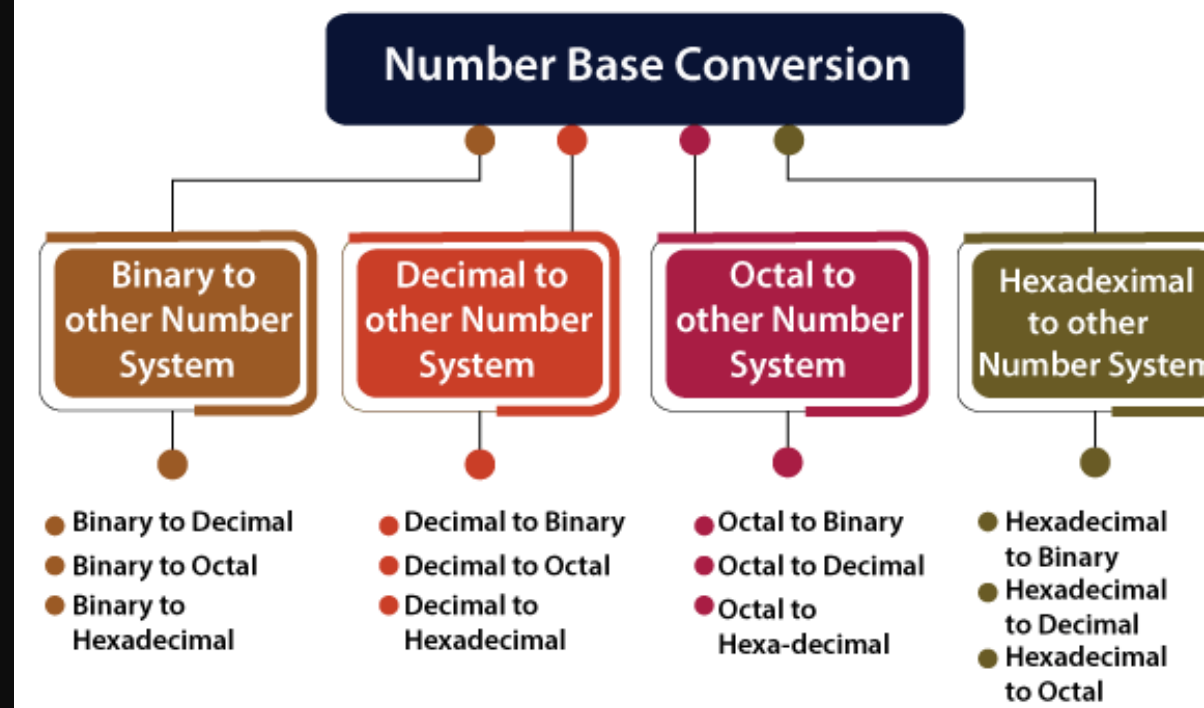
Number System Conversion

Numeral Systems in Computer Science refer to the numeric base systems used for **performing computation, storing and representing data**. They include the binary (base-2), decimal (base-10), octal (base-8), and hexadecimal (base-16) systems.

- Database systems use **decimal numbers** for all financial data, dates, and time. In fact, for any numeral data that is input or output in interaction with a user, the decimal system is ordinarily used.
- In scientific computing, **decimal numbers** are essential for complex calculations and computations involving real numbers, representing them more naturally.
- For error-correcting codes, the decimal system is used to represent parity bits. In cryptography, **decimal numbers** are used in key generation operations.
- **Binary code** governs the realm of machine language, the lowest level of programming languages. Each assembly instruction corresponds to a unique binary code, instruction set architecture being processor-specific.
- Bit manipulation tasks often utilise **binary operations**, which can prove more efficient and swift.

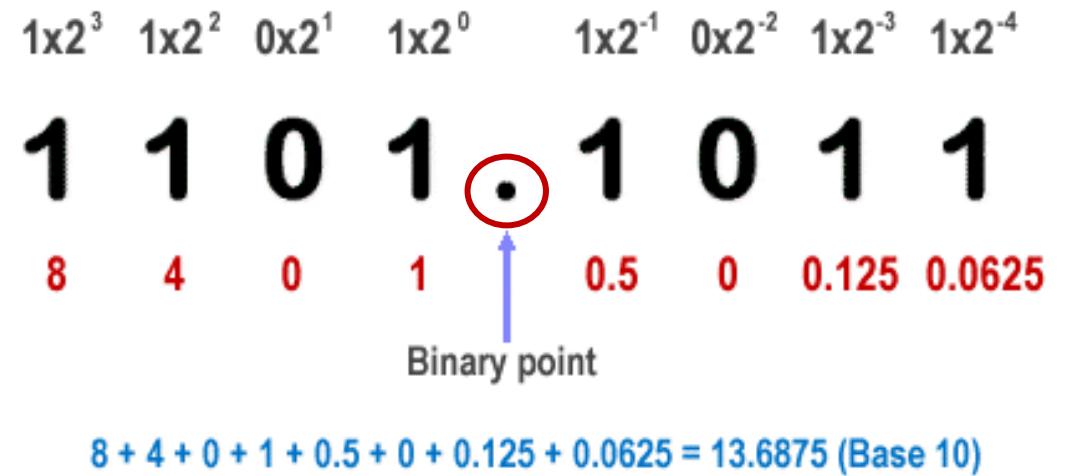
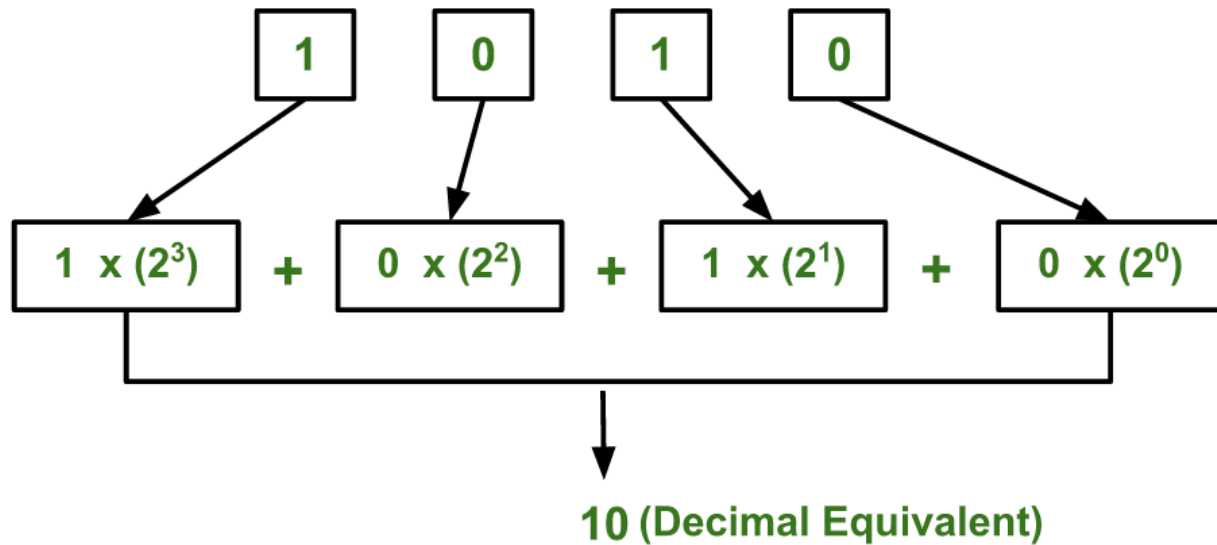


• Although machines operate using binary, human-interface devices primarily use the decimal system. It forms the backbone of numeric data in user-centric applications. When processing numeric data, computers convert decimal values into binary and perform computations. The resulting binary data is then transformed back into decimal form for user-friendly output.



Binary to Decimal

Binary number - 1010



Convert the following binary numbers to equivalent decimal numbers.

- (a) $(1101)_2$
- (b) $(11101)_2$
- (c) $(0101\ 1101)_2$
- (d) $(1101\ 1101)_2$
- (e) $(1111\ 1111)_2$
- (f) $(0101\ 1001)_2$
- (g) $(1101\ 1101\ 0101)_2$
- (h) $(11100.101)_2$

Successive Division Method

Decimal Number:

75

$$75 \div 2 = 37 (R = 1)$$

$$37 \div 2 = 18 (R = 1)$$

$$18 \div 2 = 9 (R = 0)$$

$$9 \div 2 = 4 (R = 1)$$

$$4 \div 2 = 2 (R = 0)$$

$$2 \div 2 = 1 (R = 0)$$

$$1 \div 2 = 0 (R = 1)$$



← LSB (Least significant bit)

1001011

← MSB (Most significant bit)

Decimal Number:

$$142 = 10001110$$

$$339 = 101010011$$



Decimal Number:

$$75 = 64 + 8 + 2 + 1$$

Decimal Number:

$$142 = 10001110$$

$$339 = 101010011$$

MSB	Binary Digit							LSB
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1

Highest base 2 number, that is less than 75 is 64:

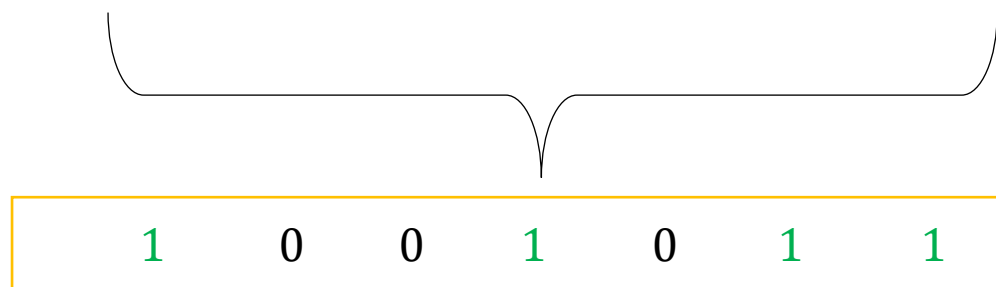
Highest base 2 number, that is less than 11 is 8:

1

$$75 - 64 = 11$$

$$11 - 8 = 3$$

$$3 - 2 = 1$$



Convert the following decimal numbers to equivalent binary numbers.

- $(a) (57)_{10}$

- $(b) (45)_{10}$

- $(c) (255)_{10}$

- $(d) (256)_{10}$

- $(e) (2416)_{10}$

- $(f) (4195)_{10}$

PRACTICE

DECIMAL TO BINARY

41	
30	
5	
10	
99	
123	
244	
13	
78	
143	
94	
58	
190	
202	
6	

BINARY TO DECIMAL

1111	
1101	
100101	
10	
00111100	
100	
110	
11111101	
1000100	
100001	
11010	
10101011	
10011001	
1110111	
11111	

Logarithms

Logarithms play a crucial role in various aspects of computer science, from algorithms and data structures to cryptography and information theory.





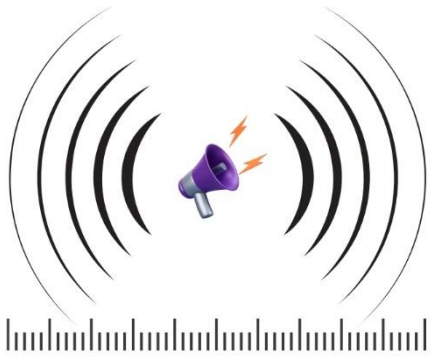
Logarithms

- In computer science, we almost always work with logarithms base 2, because we work with bits
- $\log_2 n$ (or we can just write $\log n$) tells us how many bits we need to represent n possibilities
 - Example: To represent 10 digits, we need $\log 10 = 3.322$ bits
 - Since we can't have fractional bits, we need 4 bits, with some bit patterns not used: 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, and not 1010, 1011, 1100, 1101, 1110, 1111
- Logarithms also tell us how many times we can cut a positive integer in half before reaching 1
 - Example: $16/2=8$, $8/2=4$, $4/2=2$, $2/2=1$, and $\log 16 = 4$
 - Example: $10/2=5$, $5/2=2.5$, $2.5/2=1.25$, and $\log 10 = 3.322$

Data compression

- Ever wondered how such large data obtained from various organizations and governments is stored and used? Storing such large data requires various storage devices, which will still be insufficient. In that case, the data compression technique is used, which uses logarithms to simplify and compress the data.
 - The data is compressed using different coding processes, such as arithmetic, transform, Huffman, Delta, entropy, Shannon-Fano, run-length encoding, etc., based on logarithm and its application.
-





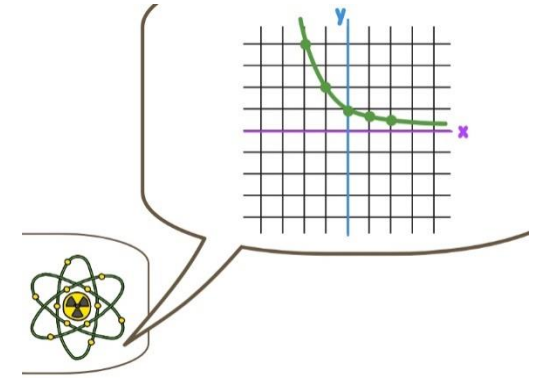
Measuring the sound intensity



Analyzing drug concentrations in medicines



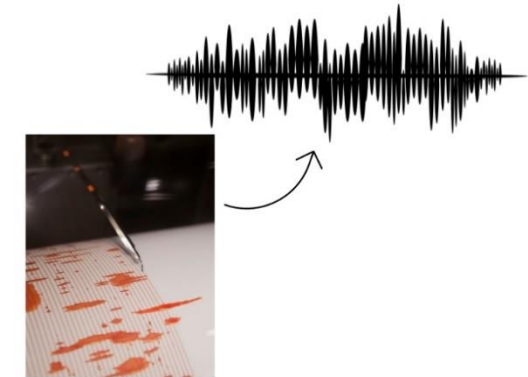
Stock market analysis



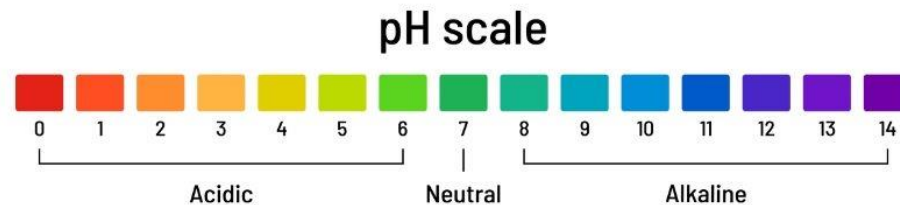
Studying the process of decay of radioactive elements



Calculating the growth of the human species or other living species



Assessing the magnitude of earthquakes using the Richter scale



Measuring pH levels of chemicals

Logarithmic Function

$$\log_a x = y \text{ means } a^y = x$$

exponent
base

$$a > 0, a \neq 1, y \neq 0$$

Example:

$$\log_2 8 = 3 \text{ means } 2^3 = 8$$

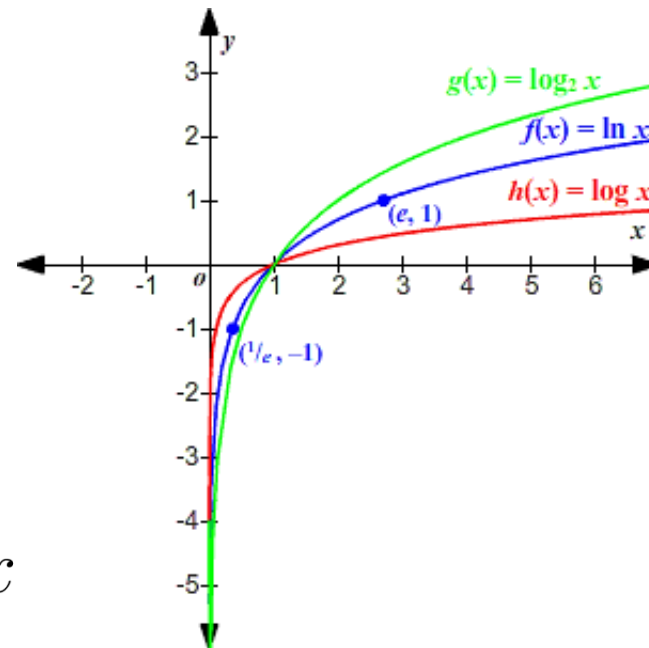
Natural Logarithm

$$\log_e x = \ln x$$

$$e \approx 2.718281828459045$$

A special form of logarithms in which
the base is mathematical constant e

Graph of logarithmic functions



List of logarithm rules

$$\log_a(bc) = \log_a(b) + \log_a(c)$$

$$\log_a(b^c) = c \log_a(b)$$

$$\log_a(1/b) = -\log_a(b)$$

$$\log_a(1) = 0$$

$$\log_a(a) = 1$$

$$\log_a(a^r) = r$$

$$\log_{1/a}(b) = -\log_a(b)$$

$$\log_a(b) \log_b(c) = \log_a(c)$$

$$\log_b(a) = \frac{1}{\log_a(b)}$$

$$\log_{a^m}(a^n) = \frac{n}{m}, \quad m \neq 0$$

Write the following equalities in exponential form.

$$\begin{array}{llll} (1) \log_3 81 = 4 & (2) \log_7 7 = 1 & (3) \log_{\frac{1}{2}} \frac{1}{8} = 3 & (4) \log_3 1 = 0 \\ (5) \log_4 \frac{1}{64} = -3 & (6) \log_6 \frac{1}{36} = -2 & (7) \log_x y = z & (8) \log_m n = \frac{1}{2} \end{array}$$

$$\begin{array}{l} (1) 3^4 = 81 \\ (2) 7^1 = 7 \\ (3) \left(\frac{1}{2}\right)^3 = \frac{1}{8} \\ (4) 3^0 = 1 \\ (5) 4^{-3} = \frac{1}{64} \\ (6) 6^{-2} = \frac{1}{36} \\ (7) x^z = y \\ (8) m^{\frac{1}{2}} = n \end{array}$$

Write the following equalities in logarithmic form.

$$\begin{array}{llll} (1) 8^2 = 64 & (2) 10^3 = 10000 & (3) 4^{-2} = \frac{1}{16} & (4) 3^{-4} = \frac{1}{81} \\ (5) \left(\frac{1}{2}\right)^{-5} = 32 & (6) \left(\frac{1}{3}\right)^{-3} = 27 & (7) x^{2z} = y & (8) \sqrt{x} = y \end{array}$$

$$\begin{array}{l} (1) \log_8 64 = 2 \\ (2) \log_{10} 10000 = 3 \\ (3) \log_4 \frac{1}{16} = -2 \\ (4) \log_3 \frac{1}{81} = -4 \\ (5) \log_{\frac{1}{2}} 32 = -5 \\ (6) \log_{\frac{1}{3}} 27 = -3 \\ (7) \log_x y = 2z \\ (8) \log_x y = \frac{1}{2} \end{array}$$

1. Find the value of y .

(1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \frac{1}{8} = y$
(5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \frac{1}{7} = y$ (8) $\log_3 \frac{1}{9} = y$
(9) $\log_y 32 = 5$ (10) $\log_9 y = -\frac{1}{2}$ (11) $\log_4 \frac{1}{8} = y$ (12) $\log_9 \frac{1}{81} = y$

2. Evaluate.

(1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (5) $\log_{25} 5^3$ (6) $16^{\log_4 8}$

1. (1) 2
- (2) 0
- (3) $\frac{1}{2}$
- (4) -3
- (5) 0
- (6) 3
- (7) -1
- (8) -2
- (9) 2
- (10) $\frac{1}{3}$
- (11) $-\frac{3}{2}$
- (12) -2

2. (1) 0
- (2) 1
- (3) 3
- (4) 3
- (5) $\frac{3}{2}$
- (6) 64

True or False?

$$(1) \log\left(\frac{x}{y^3}\right) = \log x - 3 \log y \quad (2) \log(a - b) = \log a - \log b \quad (3) \log x^k = k \cdot \log x$$

$$(4) (\log a)(\log b) = \log(a + b) \quad (5) \frac{\log a}{\log b} = \log(a - b) \quad (6) (\ln a)^k = k \cdot \ln a$$

$$(7) \log_a a^a = a \quad (8) -\ln\left(\frac{1}{x}\right) = \ln x \quad (9) \ln_{\sqrt{x}} x^k = 2k$$

(1) True

(2) False

(3) True

(4) False

(5) False

(6) False

(7) True

(8) True

Solve the following logarithmic equations.

$$(1) \ln x = -3 \quad (2) \log(3x - 2) = 2$$
$$(3) 2 \log x = \log 2 + \log(3x - 4) \quad (4) \log x + \log(x - 1) = \log(4x)$$
$$(5) \log_3(x + 25) - \log_3(x - 1) = 3 \quad (6) \log_9(x - 5) + \log_9(x + 3) = 1$$
$$(7) \log x + \log(x - 3) = 1 \quad (8) \log_2(x - 2) + \log_2(x + 1) = 2$$

$$(1) S = \{e^{-3}\}$$

$$(2) S = \{34\}$$

$$(3) S = \{2, 4\}$$

$$(4) S = \{5\}$$

$$(5) S = \{2\}$$

$$(6) S = \{6\}$$

$$(7) S = \{5\}$$

$$(8) S = \{3\}$$

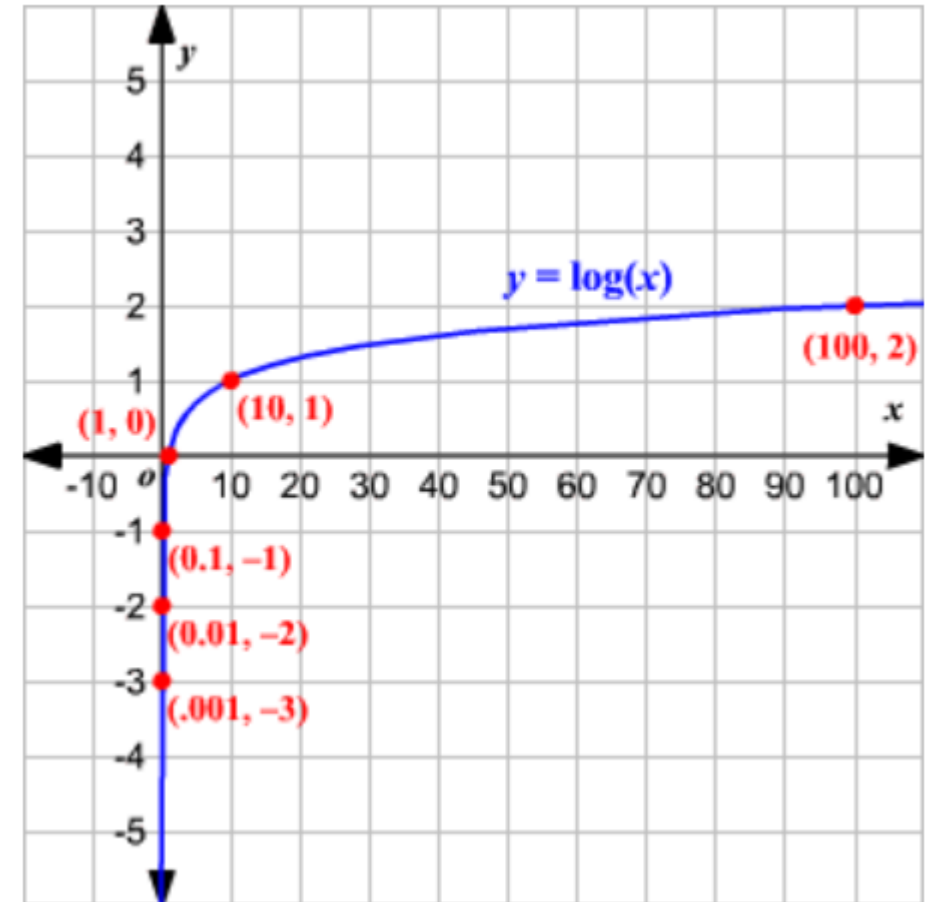
Graphing Logarithmic Functions

- When no base is written, assume that the log is base 10.

$$y = \log x$$

x	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
$y = \log x$	-3	-2	-1	0	1	2	3

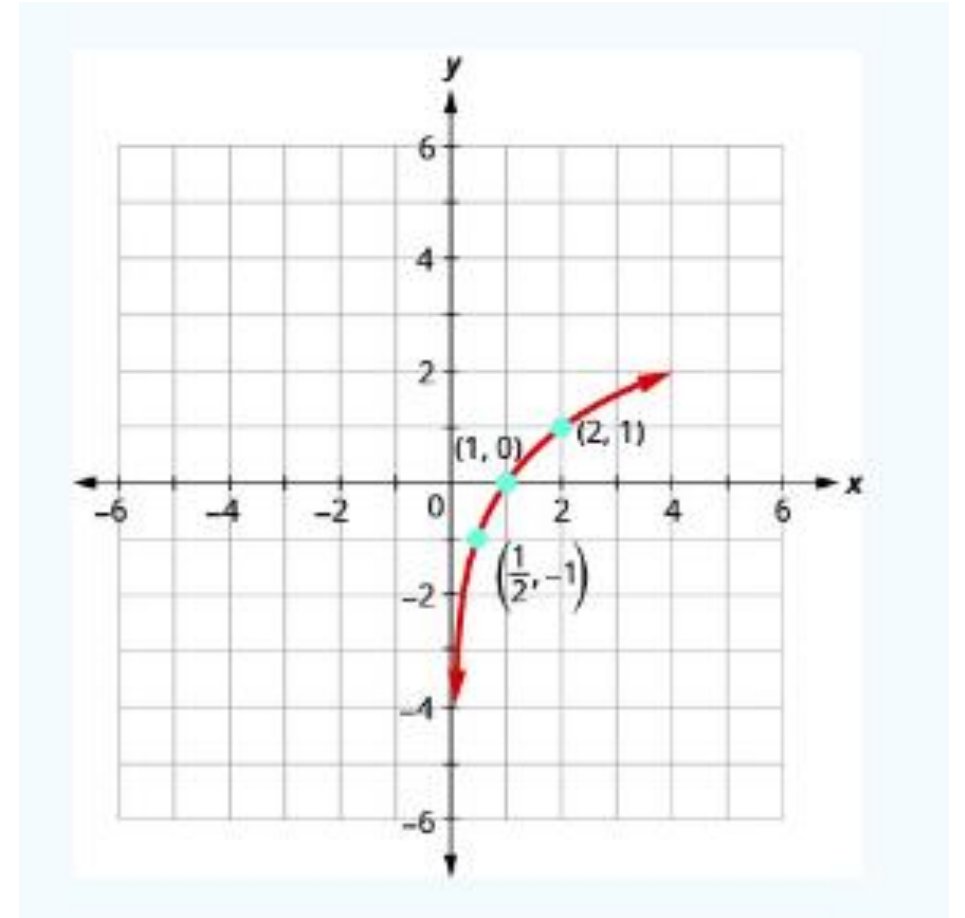
- The **domain** of log function $y = \log x$ is $x > 0$ (or) $(0, \infty)$.
- The **range** of any log function is the set of all real numbers (R)



The graph of the function $y = \log_2 x$.

$$\downarrow$$
$$x = 2^y$$

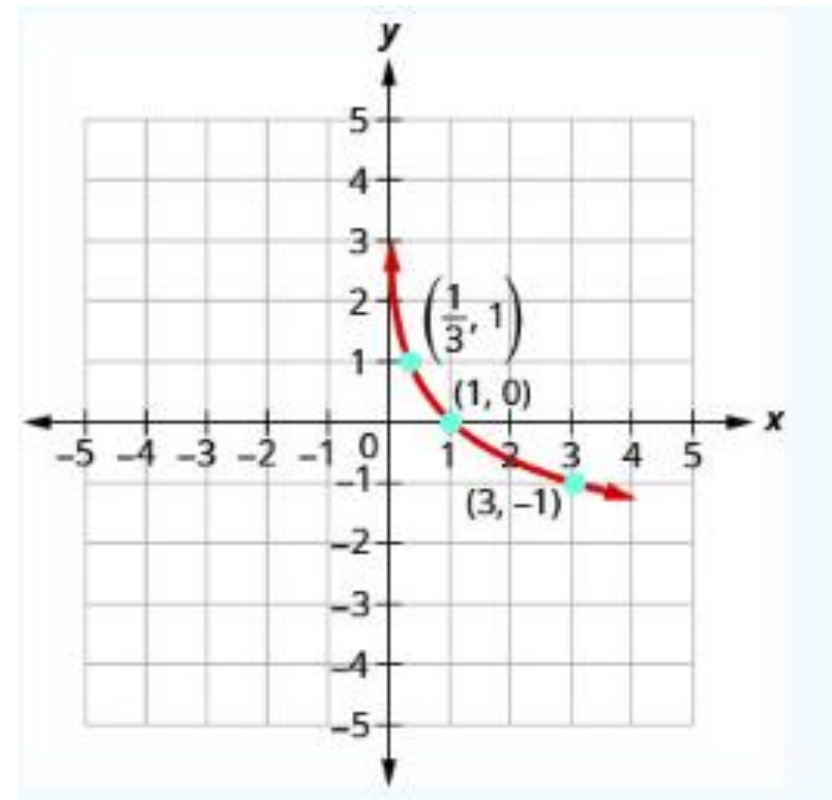
y	$2^y = x$	(x, y)
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(\frac{1}{4}, -2)$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	$(\frac{1}{2}, -1)$
0	$2^0 = 1$	$(1, 0)$
1	$2^1 = 2$	$(2, 1)$
2	$2^2 = 4$	$(4, 2)$
3	$2^3 = 8$	$(8, 3)$



The graph of the function $y = \log_{\frac{1}{3}} x$.

$$\begin{array}{c} \downarrow \\ x = \frac{1}{3}^y \end{array}$$

y	$(\frac{1}{3})^y = x$	(x, y)
-2	$(\frac{1}{3})^{-2} = 3^2 = 9$	$(9, -2)$
-1	$(\frac{1}{3})^{-1} = 3^1 = 3$	$(3, -1)$
0	$(\frac{1}{3})^0 = 1$	$(1, 0)$
1	$(\frac{1}{3})^1 = \frac{1}{3}$	$(\frac{1}{3}, 1)$
2	$(\frac{1}{3})^2 = \frac{1}{9}$	$(\frac{1}{9}, 2)$
3	$(\frac{1}{3})^3 = \frac{1}{27}$	$(\frac{1}{27}, 3)$



Graph the logarithmic function $f(x) = 2 \log_3 (x + 1)$.

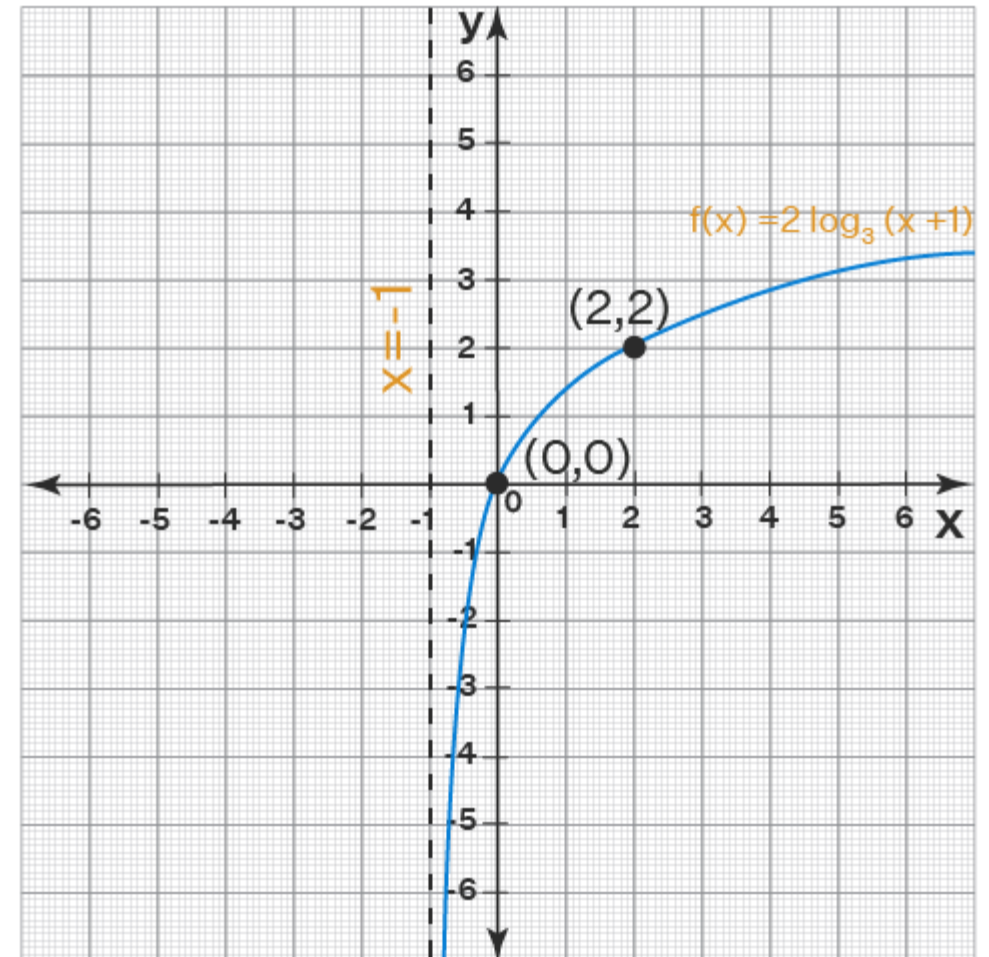
For domain: $x + 1 > 0 \Rightarrow x > -1$. So domain = $(-1, \infty)$.

Range = R .

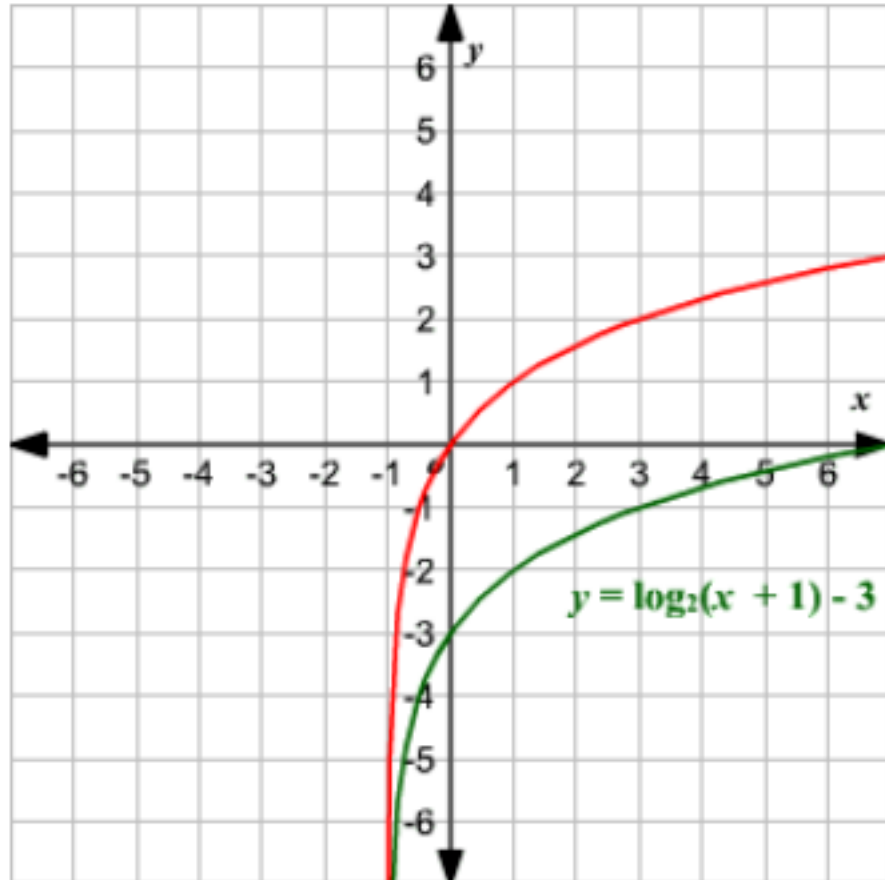
Vertical asymptote is $x = -1$.

• At $x = 0, y = 2 \log_3 (0 + 1) = 2 \log_3 1 = 2(0) = 0$

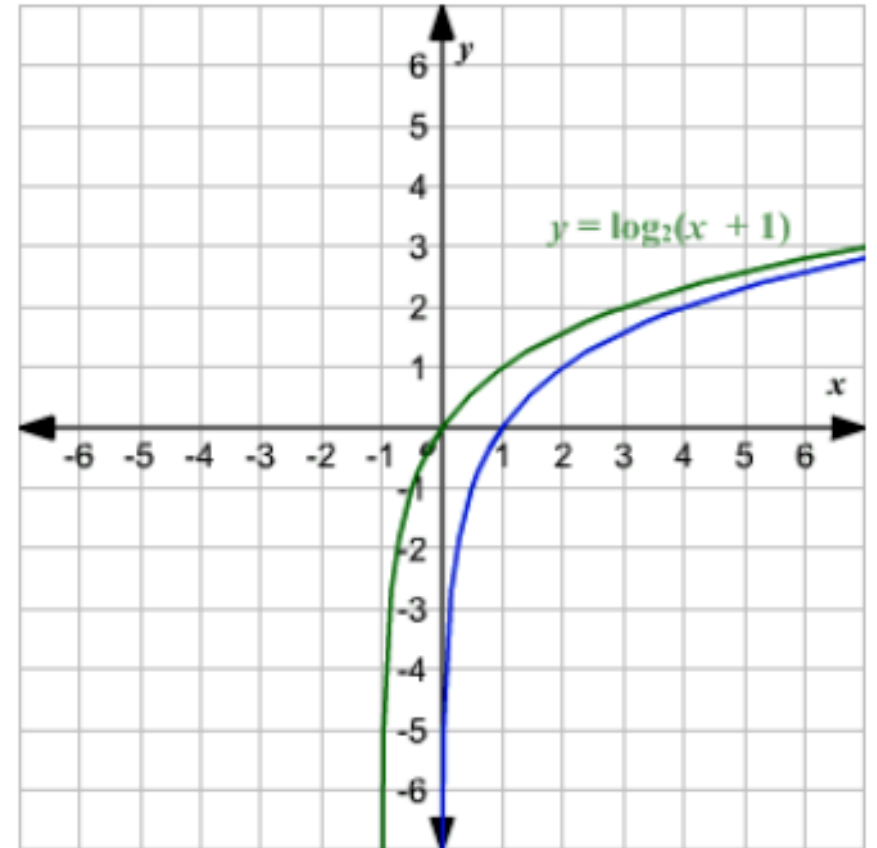
• At $x = 2, y = 2 \log_3 (2 + 1) = 2 \log_3 3 = 2(1) = 2$



The graph of the function
 $y = \log_2(x + 1) - 3$.

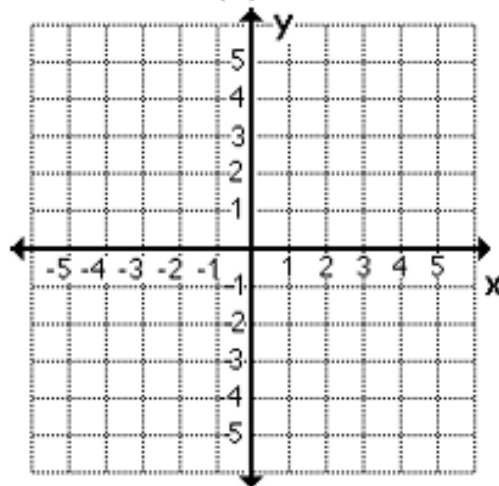


The graph of the function
 $y = \log_2(x + 1)$.

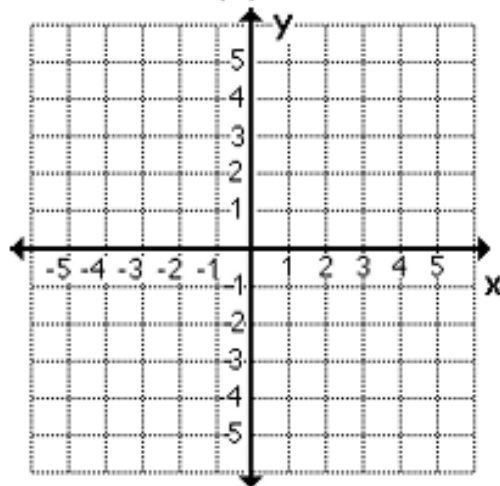


Graph each of the following logarithmic functions. Label the key point for each.

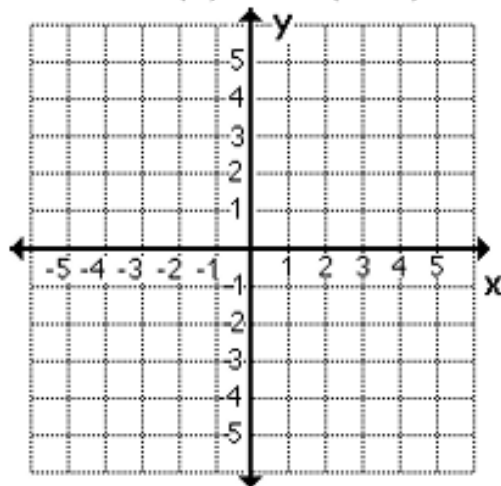
13 $f(x) = \log_2 x$



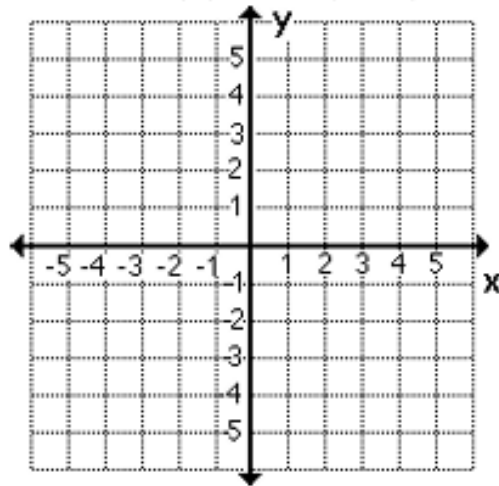
14 $f(x) = \log_4 x$



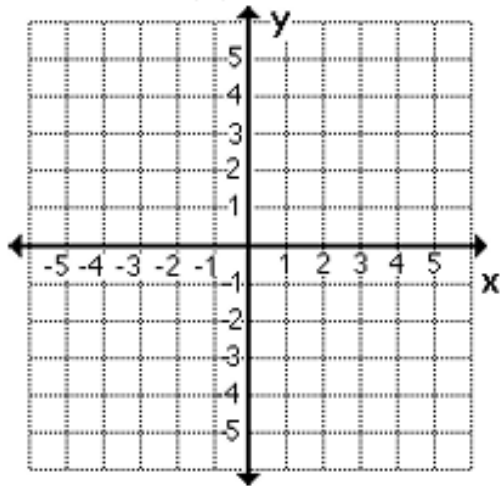
15 $f(x) = \log_4(x-3)$



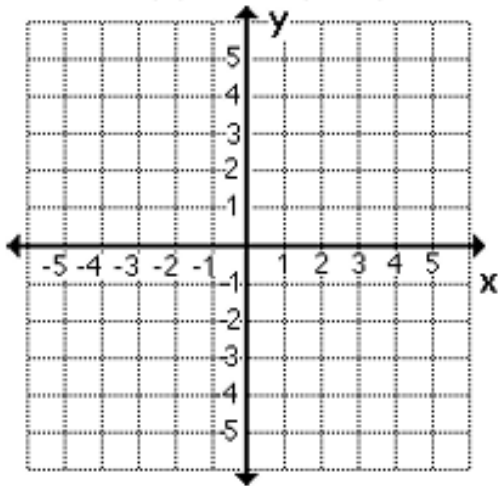
16 $f(x) = \log_2(x+2)$



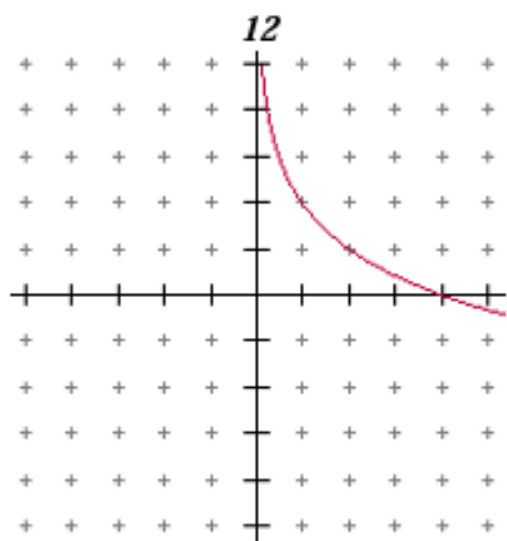
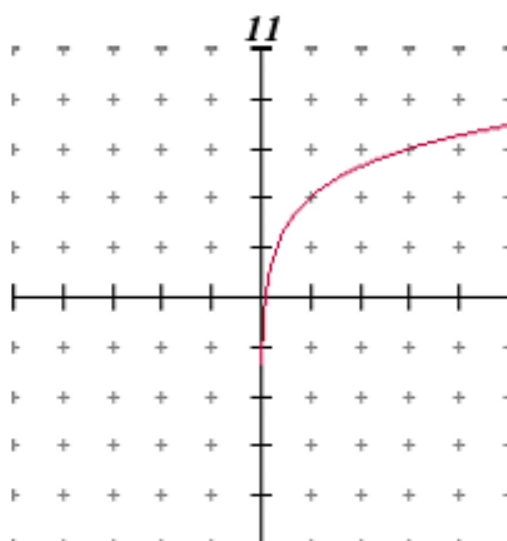
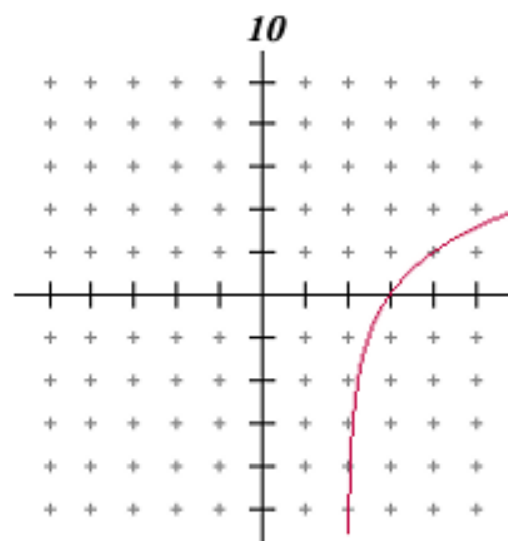
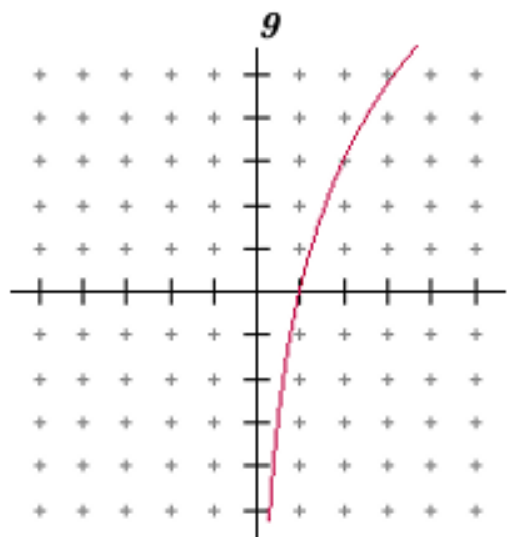
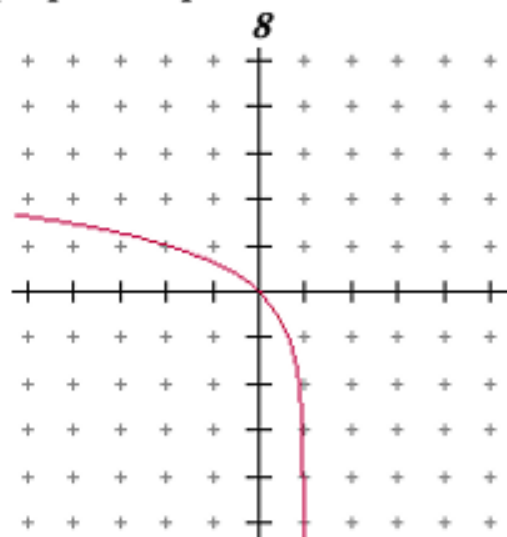
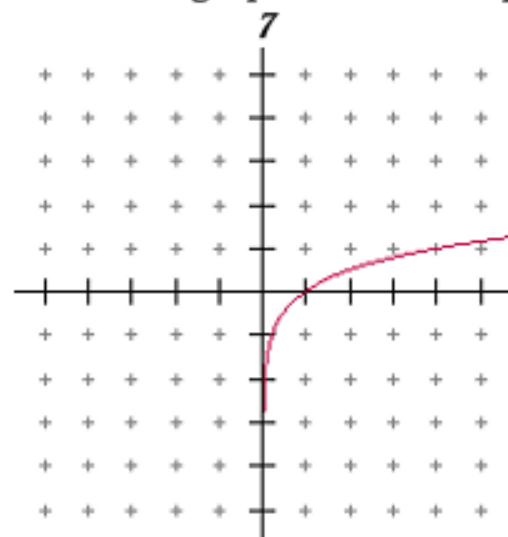
17 $f(x) = \log_3 x + 2$



18 $f(x) = \log_5(x-3) + 2$



Match the graphs with their appropriate equation below.



- A) $f(x) = \log_2(x-2)$
- B) $f(x) = \log_3(1-x)$
- C) $f(x) = -\log_2 x + 2$
- D) $f(x) = \log_3 x + 2$
- E) $f(x) = \frac{1}{2} \log_2 x$
- F) $f(x) = 3 \log_2 x$

1. $f(x) = \log_2(x+5) - 3$

2. $f(x) = \log_5(x-3) + 1$

3. $f(x) = \log_3(x-4) + 2$

4. $f(x) = 3\log_2(x-1) + 2$

5. $f(x) = \frac{1}{2}\log_4(x-6) - 5$

6. $f(x) = -4\log_2(x-2)$